Form factors from dimensional reduction

Manuel Accettulli Huber

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SAGEX
Scattering Amplitudes: from Geometry to Experiment

Example: 4 gluon, 1 loop, all plus configuration

- four gluons

[Cheung,O'Connell '09; Davies '11]

$$A_4(1_{\alpha\dot{\alpha}}, 2_{\beta\dot{\beta}}, 3_{\gamma\dot{\gamma}}, 4_{\delta\dot{\delta}}) = \frac{-\imath}{s_{12}s_{23}} \langle 1_\alpha 2_\beta 3_\gamma 4_\delta \rangle [1_{\dot{\alpha}} 2_{\dot{\beta}} 3_{\dot{\gamma}} 4_{\dot{\delta}}]$$

- two gluons and two scalars

$$A_4(1_{\alpha\dot{\alpha}}, 2_{\beta\dot{\beta}}, 3, 4) = \frac{-\imath}{4s_{12}s_{23}} \langle 1_\alpha 2_\beta 3_\gamma 3^\gamma \rangle [1_{\dot{\alpha}} 2_{\dot{\beta}} 3_{\dot{\gamma}} 3^{\dot{\gamma}}]$$

$$= -2\frac{[12][34]}{\langle 12\rangle\langle 34\rangle} \underbrace{\frac{\mu^4}{(l_2+p_2)^2(l_1-p_4)^2}}_{\text{uplifts to box integral}}\Bigg|_{6D}$$

$$I_n^{D=4-2\epsilon}[\mu^4] = -\epsilon(1-\epsilon) I_n^{D=8-2\epsilon} \longrightarrow I_4^{4-2\epsilon}[\mu^4] \xrightarrow{\epsilon\to 0} -\frac{1}{6}$$

[Bern, Morgan '96]

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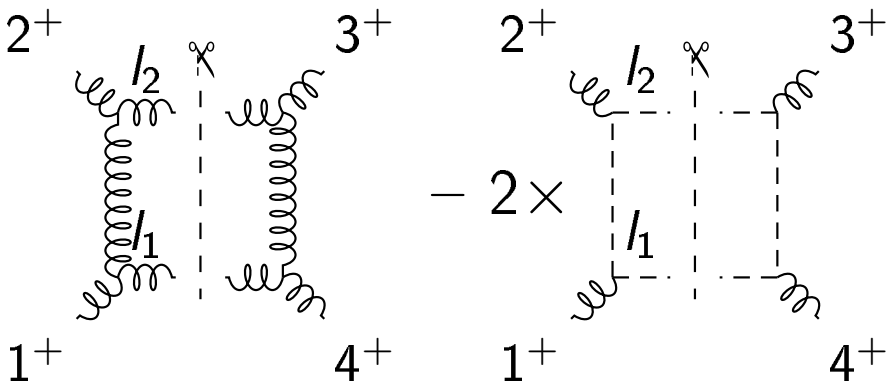

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Form Factors

[Brandhuber,Travaglini et al; Dawson,Lewis,Zeng; Buchmuller,Wyler]

Definition:

$$F_{\mathcal{O}}(1,\ldots,n;q) := \int d^4x\, e^{-\imath q\cdot x}\langle 1\ldots n|\mathcal{O}(x)|0\rangle$$
$$= (2\pi)^4\,\delta^{(4)}(q-\sum_i p_i)\langle 1\ldots n|\mathcal{O}(0)|0\rangle$$

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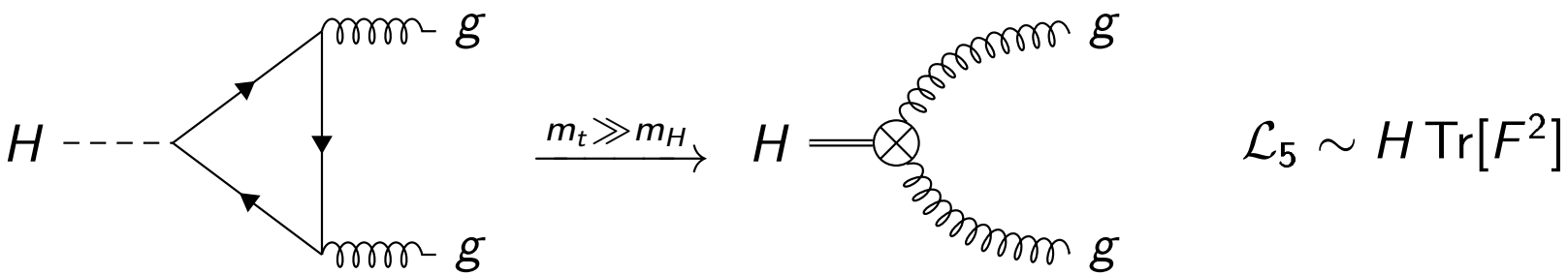
Where do form factors appear?

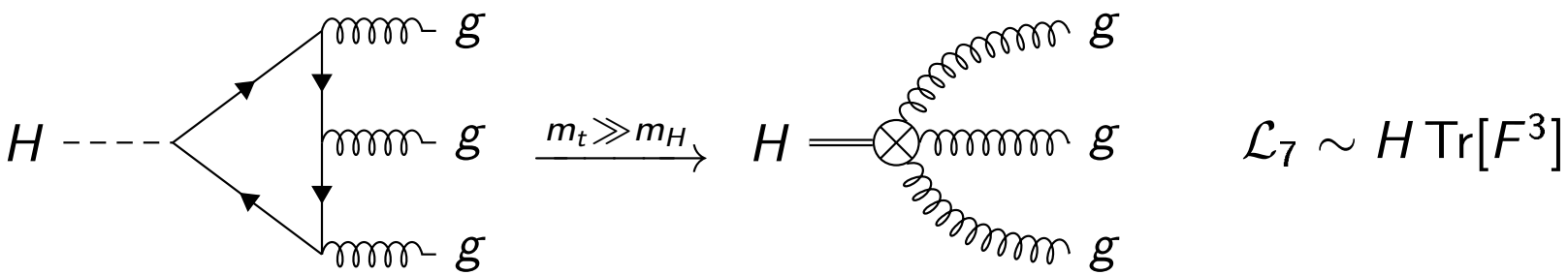
- Form factor of the electromagnetic current, which computes $g-2$
- FF of the hadronic electromagnetic current, which are building blocks of the $e^+e^- \to$ hadrons
- FF of the effective Higgs+gluons interaction in the $m_t \gg m_H$ limit

Effective Higgs + gluons interaction

1-loop: Higgs couples to gluons through a quark loop.
The main contribution given by the top quark.

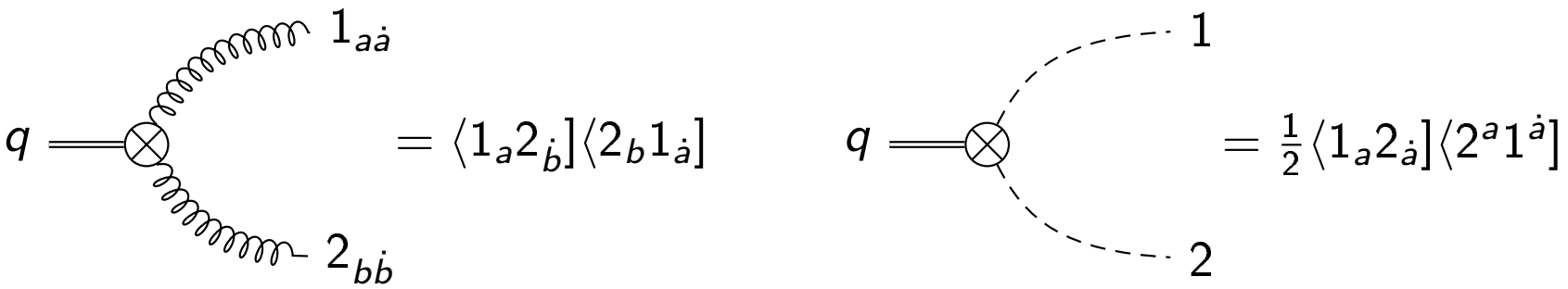
Effective field theory



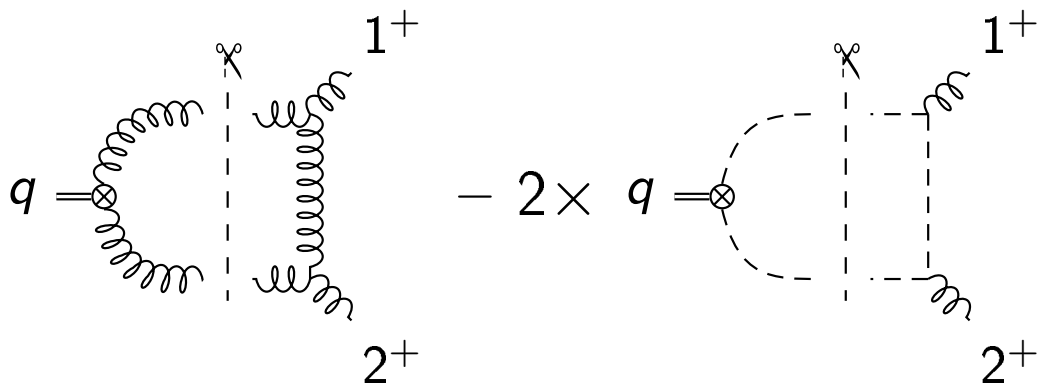


Tr[F^2] through SpinorHelicity6D package

Tree level in 6 dimensions:



One loop:



Tr[F^2] 1-loop 6D, gluons

In[20]:= `KillMasses[{1, 2}]`

In[21]:= `TrF2tree = ⟨1₁, 2₁] ⟨2₁, 1₁]`

Out[21]= $-[1\ \ 2]^2$

In[22]:=
```
TrF2loop =
  Sum[⟨l1ₐ, l2_bdot] * ⟨l2_b, l1_adot]/TrF2tree * -I/(⟨1 2⟩ [2 1] S[2, l2]) * ⟨1₁, 2₁, l2_cdot, l1_ddot⟩ *
    [1₁, 2₁, l2_cdot, l1_ddot] * ϵ^(a d) * ϵ^(b c) * ϵ^(adot ddot) * ϵ^(bdot cdot), {a, 2}, {b, 2},
   {c, 2}, {d, 2}, {adot, 2}, {bdot, 2}, {cdot, 2}, {ddot, 2}] // Expand
```

Out[22]=
$$\frac{2\,i\,M[l1]\,M[l2]\,\tilde{M}[l1]\,\tilde{M}[l2]\,\left\langle l1\,\overline{l2}\right\rangle^2\left\langle l2\,\overline{l1}\right\rangle^2}{S[2,l2]\,\langle 1\ \ 2\rangle\left\langle l1\,\overline{l1}\right\rangle^2\left\langle l2\,\overline{l2}\right\rangle^2[1\ \ 2]}+$$

$$\frac{4\,i\,M[l1]\,M[l2]\,\tilde{M}[l1]\,\tilde{M}[l2]\,\langle l1\,l2\rangle\left\langle l1\,\overline{l2}\right\rangle\left\langle l2\,\overline{l1}\right\rangle\left\langle \overline{l1}\,\overline{l2}\right\rangle}{S[2,l2]\,\langle 1\ \ 2\rangle\left\langle l1\,\overline{l1}\right\rangle^2\left\langle l2\,\overline{l2}\right\rangle^2[1\ \ 2]}+$$

$$\frac{2\,i\,M[l1]\,M[l2]\,\tilde{M}[l1]\,\tilde{M}[l2]\,\langle l1\,l2\rangle^2\left\langle \overline{l1}\,\overline{l2}\right\rangle^2}{S[2,l2]\,\langle 1\ \ 2\rangle\left\langle l1\,\overline{l1}\right\rangle^2\left\langle l2\,\overline{l2}\right\rangle^2[1\ \ 2]}+$$

$$\frac{i\,M[l2]\,\tilde{M}[l1]\,\langle l1\,l2\rangle\left\langle l1\,\overline{l2}\right\rangle\left\langle l2\,\overline{l1}\right\rangle[l1\,l2]}{S[2,l2]\,\langle 1\ \ 2\rangle\left\langle l1\,\overline{l1}\right\rangle\left\langle l2\,\overline{l2}\right\rangle[1\ \ 2]}+$$

$$\frac{i\,M[l1]\,M[l2]\,\tilde{M}[l1]^2\,\langle l1\,l2\rangle\,\langle 2\,l2\,\overline{l1}\rangle\,\langle \overline{l1}\,\overline{l2}\rangle\,[2\,\overline{l1}]}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l1\,\overline{l1}\rangle^2\,\langle l2\,\overline{l2}\rangle\,[1\;\;2]\,[l1\,\overline{l1}]} -$$

$$\frac{i\,M[l1]^2\,\tilde{M}[l1]\,\tilde{M}[l2]\,\langle l1\,l2\rangle\,\langle 2\,l2\,\overline{l1}\rangle\,\langle \overline{l1}\,\overline{l2}\rangle\,[2\,\overline{l1}]}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l1\,\overline{l1}\rangle^2\,\langle l2\,\overline{l2}\rangle\,[1\;\;2]\,[l1\,\overline{l1}]} -$$

$$\frac{2\,i\,M[l1]\,\tilde{M}[l1]\,\langle l1\,l2\rangle\,\langle 2\,l2\,\overline{l1}\rangle\,[l1\,l2]\,[2\,\overline{l1}]}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l1\,\overline{l1}\rangle\,[1\;\;2]\,[l1\,\overline{l1}]} + \frac{i\,M[l1]^2\,\tilde{M}[l1]^2\,\langle 2\,l2\,\overline{l1}\rangle^2\,[2\,\overline{l1}]^2}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l1\,\overline{l1}\rangle^2\,[1\;\;2]\,[l1\,\overline{l1}]^2} +$$

$$\frac{i\,M[l2]^2\,\tilde{M}[l2]^2\,\langle l1\,\overline{l2}\rangle^2\,[l1\,\overline{l2}]^2}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l2\,\overline{l2}\rangle^2\,[1\;\;2]\,[l2\,\overline{l2}]^2} - \frac{i\,M[l2]^2\,\tilde{M}[l1]\,\tilde{M}[l2]\,\langle l1\,\overline{l2}\rangle^2\,\langle 2\,l2\,\overline{l1}\rangle\,[l1\,\overline{l2}]}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l1\,\overline{l1}\rangle\,\langle l2\,\overline{l2}\rangle^2\,[1\;\;2]\,[l2\,\overline{l2}]} -$$

$$\frac{i\,M[l1]\,M[l2]\,\tilde{M}[l2]^2\,\langle l1\,\overline{l2}\rangle^2\,\langle 2\,l2\,\overline{l1}\rangle\,[l1\,\overline{l2}]}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l1\,\overline{l1}\rangle\,\langle l2\,\overline{l2}\rangle^2\,[1\;\;2]\,[l2\,\overline{l2}]} -$$

$$\frac{i\,M[l2]^2\,\tilde{M}[l1]\,\tilde{M}[l2]\,\langle l1\,l2\rangle\,\langle l1\,\overline{l2}\rangle\,\langle \overline{l1}\,\overline{l2}\rangle\,[l1\,\overline{l2}]}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l1\,\overline{l1}\rangle\,\langle l2\,\overline{l2}\rangle^2\,[1\;\;2]\,[l2\,\overline{l2}]} -$$

$$\frac{i\,M[l1]\,M[l2]\,\tilde{M}[l2]^2\,\langle l1\,l2\rangle\,\langle l1\,\overline{l2}\rangle\,\langle \overline{l1}\,\overline{l2}\rangle\,[l1\,\overline{l2}]}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l1\,\overline{l1}\rangle\,\langle l2\,\overline{l2}\rangle^2\,[1\;\;2]\,[l2\,\overline{l2}]} -$$

$$2\,i\,M[l2]\,\tilde{M}[l2]\,\langle l1\,l2\rangle\,\langle l1\,\overline{l2}\rangle\,[l1\,l2]\,[l1\,\overline{l2}]$$

$$
\begin{aligned}
&S[2, l2]\,\langle 1\ \ 2\rangle\,\langle l2\,l2\rangle\,[1\ \ 2]\,[l2\,l2] \\
&\frac{2\,i\,M[l1]\,M[l2]\,\tilde{M}[l1]\,\tilde{M}[l2]\,\langle l1\,l2\rangle\,\langle \bar{l1}\,\bar{l2}\rangle\,[l1\,\bar{l2}]\,[l2\,\bar{l1}]}{S[2, l2]\,\langle 1\ \ 2\rangle\,\langle l1\,\bar{l1}\rangle\,\langle l2\,\bar{l2}\rangle\,[1\ \ 2]\,[l1\,\bar{l1}]\,[l2\,\bar{l2}]} - \\
&\frac{2\,i\,M[l1]\,M[l2]^2\,\tilde{M}[l1]\,\tilde{M}[l2]^2\,\langle l1\,\bar{l2}\rangle\,\langle \bar{l1}\,\bar{l2}\rangle\,[l1\,\bar{l2}]\,[\bar{l1}\,\bar{l2}]}{S[2, l2]\,\langle 1\ \ 2\rangle\,\langle l1\,\bar{l1}\rangle\,\langle l2\,\bar{l2}\rangle^2\,[1\ \ 2]\,[l1\,\bar{l1}]\,[l2\,\bar{l2}]^2} + \\
&\frac{i\,M[l1]\,M[l2]^2\,\tilde{M}[l1]^2\,\tilde{M}[l2]\,\langle l1\,\bar{l2}\rangle\,\langle l2\,\bar{l1}\rangle\,\langle \bar{l1}\,\bar{l2}\rangle\,[\bar{l1}\,\bar{l2}]}{S[2, l2]\,\langle 1\ \ 2\rangle\,\langle l1\,\bar{l1}\rangle^2\,\langle l2\,\bar{l2}\rangle^2\,[1\ \ 2]\,[l1\,\bar{l1}]\,[l2\,\bar{l2}]} + \\
&\frac{i\,M[l1]^2\,M[l2]\,\tilde{M}[l1]\,\tilde{M}[l2]^2\,\langle l1\,\bar{l2}\rangle\,\langle l2\,\bar{l1}\rangle\,\langle \bar{l1}\,\bar{l2}\rangle\,[\bar{l1}\,\bar{l2}]}{S[2, l2]\,\langle 1\ \ 2\rangle\,\langle l1\,\bar{l1}\rangle^2\,\langle l2\,\bar{l2}\rangle^2\,[1\ \ 2]\,[l1\,\bar{l1}]\,[l2\,\bar{l2}]} + \\
&\frac{i\,M[l1]\,M[l2]^2\,\tilde{M}[l1]^2\,\tilde{M}[l2]\,\langle l1\,l2\rangle\,\langle \bar{l1}\,\bar{l2}\rangle^2\,[\bar{l1}\,\bar{l2}]}{S[2, l2]\,\langle 1\ \ 2\rangle\,\langle l1\,\bar{l1}\rangle^2\,\langle l2\,\bar{l2}\rangle^2\,[1\ \ 2]\,[l1\,\bar{l1}]\,[l2\,\bar{l2}]} + \\
&\frac{i\,M[l1]^2\,M[l2]\,\tilde{M}[l1]\,\tilde{M}[l2]^2\,\langle l1\,l2\rangle\,\langle \bar{l1}\,\bar{l2}\rangle^2\,[\bar{l1}\,\bar{l2}]}{S[2, l2]\,\langle 1\ \ 2\rangle\,\langle l1\,\bar{l1}\rangle^2\,\langle l2\,\bar{l2}\rangle^2\,[1\ \ 2]\,[l1\,\bar{l1}]\,[l2\,\bar{l2}]} - \\
&\frac{2\,i\,M[l1]\,M[l2]\,\tilde{M}[l1]\,\tilde{M}[l2]\,\langle l1\,\bar{l2}\rangle\,\langle l2\,\bar{l1}\rangle\,[l1\,l2]\,[\bar{l1}\,\bar{l2}]}{S[2, l2]\,\langle 1\ \ 2\rangle\,\langle l1\,\bar{l1}\rangle\,\langle l2\,\bar{l2}\rangle\,[1\ \ 2]\,[l1\,\bar{l1}]\,[l2\,\bar{l2}]} -
\end{aligned}
$$

$$\frac{2\,i\,M[l1]^2\,M[l2]\,\tilde{M}[l1]^2\,\tilde{M}[l2]\,\langle l2\,\overline{l1}\rangle\,\langle \overline{l1}\,\overline{l2}\rangle\,[l2\,\overline{l1}]\,[\overline{l1}\,\overline{l2}]}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l1\,\overline{l1}\rangle^2\,\langle l2\,\overline{l2}\rangle\,[1\;\;2]\,[l1\,\overline{l1}]^2\,[l2\,\overline{l2}]} +$$

$$\frac{i\,M[l1]^2\,M[l2]^2\,\tilde{M}[l1]^2\,\tilde{M}[l2]^2\,\langle \overline{l1}\,\overline{l2}\rangle^2\,[\overline{l1}\,\overline{l2}]^2}{S[2,l2]\,\langle 1\;\;2\rangle\,\langle l1\,\overline{l1}\rangle^2\,\langle l2\,\overline{l2}\rangle^2\,[1\;\;2]\,[l1\,\overline{l1}]^2\,[l2\,\overline{l2}]^2}$$

In[23]:=
```
% /. {extramass[l2] → -extramass[l1], extramasstilde[l2] → -extramasstilde[l1]};
% /. {extramass[l1] → m^2 / extramasstilde[l1]};
SpinorReplaceSequential[%, {μ[l1] → λ[l2], μ[l2] -> λ[l1], μ̃[l2] -> μ̃[l1]}];
TrF2loop = % /. {[l1 l2] → -(⟨1 2⟩ [2 1] - 2 m^2) / ⟨l1 l2⟩} // Expand
```

Out[26]=
$$\frac{2\,i\,m^2}{S[2,l2]} + \frac{i\,\langle 1\;\;2\rangle\,[1\;\;2]}{S[2,l2]}$$

Tr[F^2] scalars

In[27]:=
```
TrF2scal =
  1 / 2 * (Sum[⟨l1_a, l2_bdot] ⟨l2_b, l1_adot] ϵ^(a b) ϵ^(bdot adot) (-I)/(4 * ⟨1 2⟩ [2 1] S[2, l2]) * ⟨1_1, 2_1, l2_c,
      l2_d⟩ * [1_1, 2_1, l2_cdot, l2_ddot] * ϵ^(c d) * ϵ^(cdot ddot), {a, 2}, {adot, 2}, {b, 2},
    {bdot, 2}, {c, 2}, {d, 2}, {cdot, 2}, {ddot, 2}]) / (TrF2tree) // Expand;
% /. {extramass[l2] → -extramass[l1], extramasstilde[l2] → -extramasstilde[l1]};
% /. {extramass[l1] → m^2 / extramasstilde[l1]};
SpinorReplaceSequential[%,
  {μ[l1] → λ[l2], μ[l2] -> λ[l1], μ̃[l1] -> λ̃[l2], μ̃[l2] -> λ̃[l1]}];
TrF2scal = % /. {⟨l1 l2⟩ [l1 l2] → ⟨1 2⟩ [1 2] + 2 m^2 - m^4/(⟨l1 l2⟩ [l1 l2])} // Expand
```

Out[31]=
$$\frac{i\, m^2}{S[2, l2]}$$

In[32]:=
```
TrF2loop - 2 TrF2scal // Expand
```

Out[32]=
$$\frac{i\, \langle 1\ \ 2\rangle\, [1\ \ 2]}{S[2, l2]}$$

Previous work

My master thesis was about:

- Tree level recursion relations, in particular BCFW and Berends-Giele
- Functional reconstruction, i.e. obtain analytic expressions of rational functions from numerical evaluations
- finite fields methods, to allow for efficient numerical evaluations