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Modular Properties of the Low Energy Expansion

Modular forms arise in two rather distinct contexts in type II superstring scattering amplitudes. Firstly, the coefficients of terms in the low energy expansion of the integrand of the genus-one amplitude are integrals of non-holomorphic modular forms over the complex structure, τ, of the world-sheet torus. These are superpositions of modular graph forms (MGFs) which have been the subject of much recent mathematical interest.

The second context in which modular forms arise is in the elucidation of the low energy expansion of type IIB amplitudes, for which $SL(2,\mathbb{Z})$ is the S-duality group and τ is identified with the complex coupling constant. In this case, the coefficient functions, which get contributions from all genera as well as from instantons, are known as S-dual modular forms (SMF). This case has been understood in great detail from the holographic viewpoint where modular covariance is associated with Montonen–Olive duality of $\mathcal{N} = 4$ $SU(N)$ SUSY Yang–Mills.

In both cases the coefficients of the low energy expansion are linear combinations of 'Generalised Eisenstein Series' $\mathcal{E}(s, s_1, s_2)$ that satisfy inhomogeneous Laplace eigenvalue equations of the form
\begin{equation}
\left(\Delta_\tau - s(s-1)\right) \mathcal{E}(s, s_1, s_2) = E(s_1, \tau) E(s_2, \tau),
\end{equation}
where $E(s_i, \tau)$ is a non-holomorphic Eisenstein series. The eigenvalue s is an integer and for MGFs $s_1, s_2 \in \mathbb{N}^+$ while for SMFs $s_1, s_2 \in \mathbb{N}^+ + \frac{1}{2}$. This talk will describe a number of recent results concerning lattice solutions to these equations and their connections to string theory.

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