

BEYOND THE PRODUCTION STRAIGHT

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OVERVIEW

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- Muon Beam Position Determination
- Muon Decay Coordinates Determination
- Muon Beam Velocity Determination
- Corresponding Rotation Operator Calculation
- Boosting to nuSTORM Reference Frame

MUON DECAY COORDINATES

MUON STORAGE RING

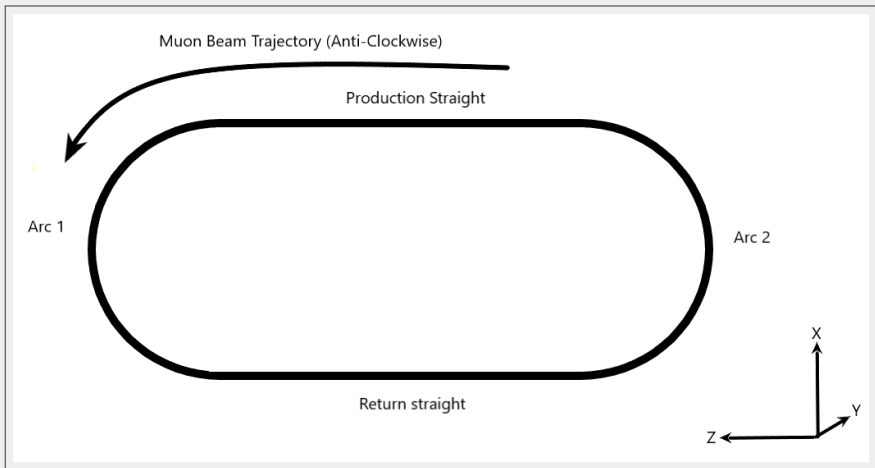


Figure: Muon Storage Ring

POSITION OF THE BEAM

Production Straight:

$X = 0; Y = 0; Z = \text{Longitudinal position along the beam}$

Arc 1:

$X = -(r-r \cos\theta); Y = 0; Z = \text{Production Straight Length} + r \sin\theta$

Return Straight:

$X = -2r; Y = 0; Z = \text{Production Straight Length} - (\text{Longitudinal position along the beam} - \text{Arc Length} - \text{Production Straight Length})$

Arc 2:

$X = -(r-r \cos\theta); Y = 0; Z = r \sin\theta$

POSITION OF THE BEAM

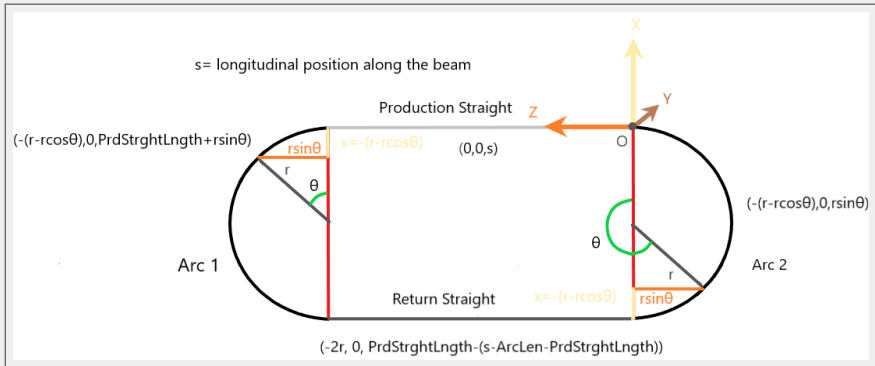


Figure: Position of the beam

MUON DECAY COORDINATES

$$X_m = X_b + X_t \cos\theta$$

$$Y_m = Y_t ; (Y_b = 0)$$

$$Z_m = Z_b + X_t \sin\theta$$

X, Y, Z \rightarrow nuSTORM reference frame coordinates

X_b, Y_b, Z_b \rightarrow Position of the beam

X_t, Y_t \rightarrow Transverse displacements of the muon from the beam

X_m, Y_m, Z_m \rightarrow Muon Decay Coordinates

MUON DECAY COORDINATES

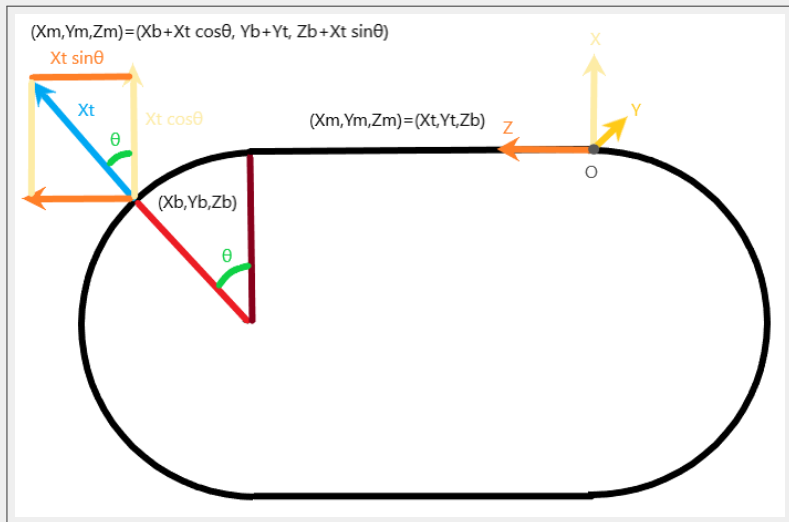


Figure: Muon Decay Coordinates

MUON DECAY COORDINATES

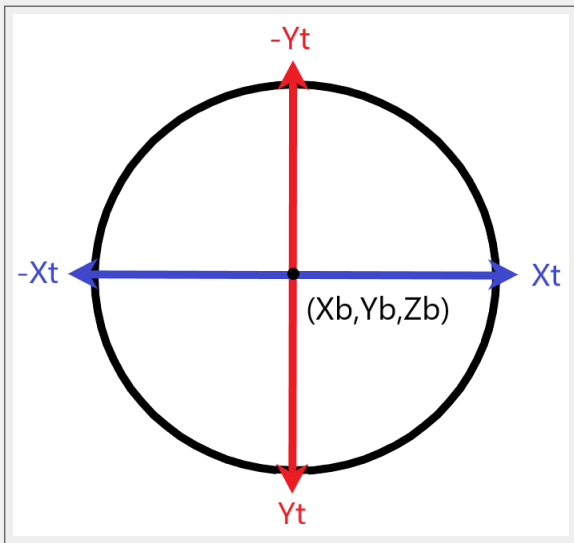


Figure: Transverse Coordinates

BOOSTING TO NUSTORM FRAME

BOOST IN AN ARBITRARY DIRECTION [2]

$$X' = B(\mathbf{v}) X$$

X' = 4-vector observed from muon reference frame

X = 4-vector observed from nuSTORM reference frame

\mathbf{v} = the velocity of the muon observed from the nuSTORM frame

$$B(\mathbf{v}) = \begin{bmatrix} \gamma & -\gamma \mathbf{v}_x / c & -\gamma \mathbf{v}_y / c & -\gamma \mathbf{v}_z / c \\ -\gamma \mathbf{v}_x / c & 1 + (\gamma - 1) \frac{v_x^2}{v^2} & (\gamma - 1) \frac{v_x v_y}{v^2} & (\gamma - 1) \frac{v_x v_z}{v^2} \\ -\gamma \mathbf{v}_y / c & (\gamma - 1) \frac{v_y v_x}{v^2} & 1 + (\gamma - 1) \frac{v_y^2}{v^2} & (\gamma - 1) \frac{v_y v_z}{v^2} \\ -\gamma \mathbf{v}_z / c & (\gamma - 1) \frac{v_z v_x}{v^2} & (\gamma - 1) \frac{v_z v_y}{v^2} & 1 + (\gamma - 1) \frac{v_z^2}{v^2} \end{bmatrix}$$

BEAM VELOCITY IN THE STORAGE RING

$$v_y = 0$$

$$\mathbf{v} = v_z \hat{z} + v_x \hat{x}$$

$$B(\mathbf{v}) = \begin{bmatrix} \gamma & -\gamma v_x/c & 0 & -\gamma v_z/c \\ -\gamma v_x/c & 1 + (\gamma - 1) \frac{v_x^2}{v^2} & 0 & (\gamma - 1) \frac{v_x v_z}{v^2} \\ 0 & 0 & 1 & 0 \\ -\gamma v_z/c & (\gamma - 1) \frac{v_z v_x}{v^2} & 0 & 1 + (\gamma - 1) \frac{v_z^2}{v^2} \end{bmatrix}$$

The motion of the beam is assumed to be confined in the ZX plane of the nuSTORM reference frame.

VELOCITY OF THE BEAM

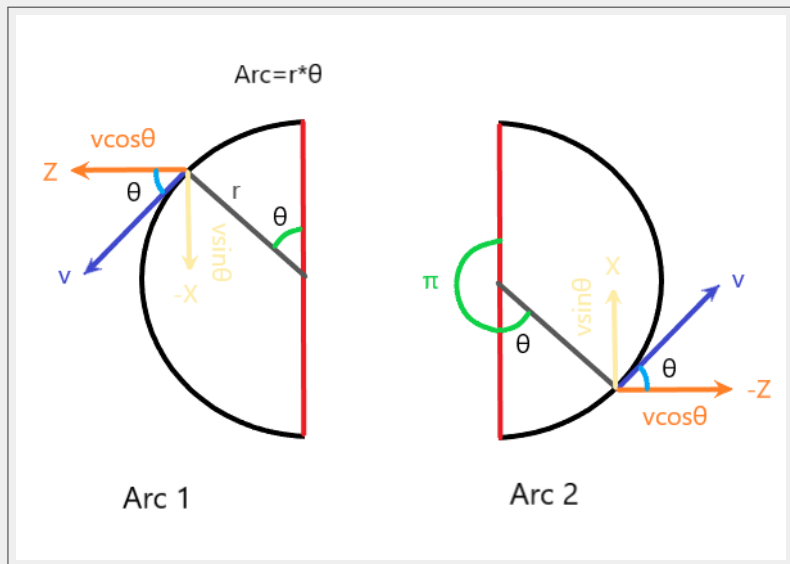


Figure: Velocity of the beam

VELOCITY OF THE BEAM

$$\mathbf{v} = v_z \hat{z} + v_x \hat{x}$$

$$v_z = v \cos\theta$$

$$v_x = -v \sin\theta$$

$$B(\mathbf{v}) = \begin{bmatrix} \gamma & \gamma \frac{v}{c} \sin\theta & 0 & -\gamma \frac{v}{c} \cos\theta \\ \gamma \frac{v}{c} \sin\theta & 1 + (\gamma - 1) \sin^2\theta & 0 & -(\gamma - 1) \sin\theta \cos\theta \\ 0 & 0 & 1 & 0 \\ -\gamma \frac{v}{c} \cos\theta & -(\gamma - 1) \sin\theta \cos\theta & 0 & 1 + (\gamma - 1) \cos^2\theta \end{bmatrix}$$

ROTATION MATRIX

$$\mathbf{B}(\mathbf{v}) = \mathbf{R}^{-1}(\theta) \mathbf{B}_z(\mathbf{v}) \mathbf{R}(\theta)$$

$$\mathbf{R}_y(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\mathbf{B}_z(\mathbf{v}) = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\mathbf{v}/c \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & 0 \\ -\gamma\mathbf{v}/c & 0 & 0 & \gamma \end{bmatrix}$$

BOOSTING FROM MUON REST FRAME TO NUSTORM FRAME

$$X' = B(\mathbf{v}) X$$

$$X' = R^{-1}(\theta) B_z(\mathbf{v}) R(\theta) X$$

$$X = R^{-1}(\theta) B_z^{-1}(\mathbf{v}) R(\theta) X'$$

ALGORITHM

ALGORITHM

- Number of turns calculation (Optional)
- Determination of where the beam is (Production Straight, Arc1, Return straight or Arc2)
- Determination of the angle between the beam velocity and nuSTORM Z axis.
- Calculation of corresponding rotation operator
- Calculation of the beam position coordinates
- Obtaining the transverse coordinates
- Calculation of the muon decay coordinates
- Calculation of beam momentum and muon decay momentum (Yet to be implemented)

PLOTS

LONGITUDINAL POSITION DISTRIBUTION

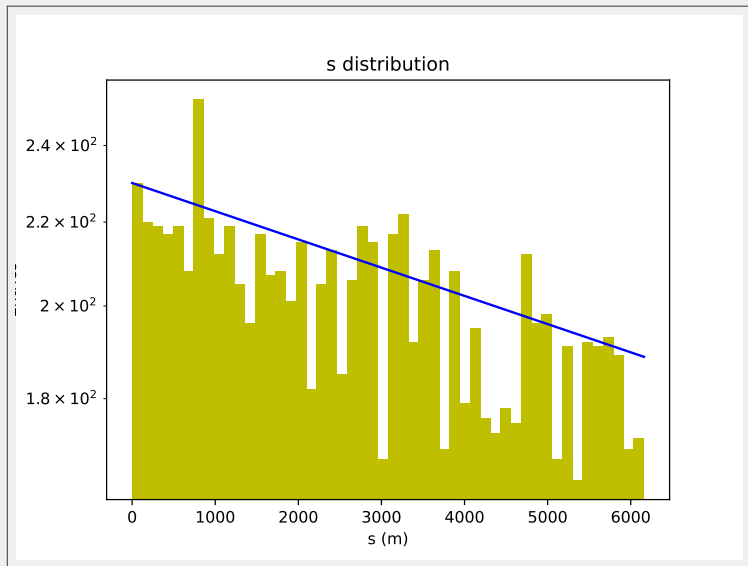


Figure: s distribution for 10 trips around the storage ring

ELECTRON ENERGY DISTRIBUTION

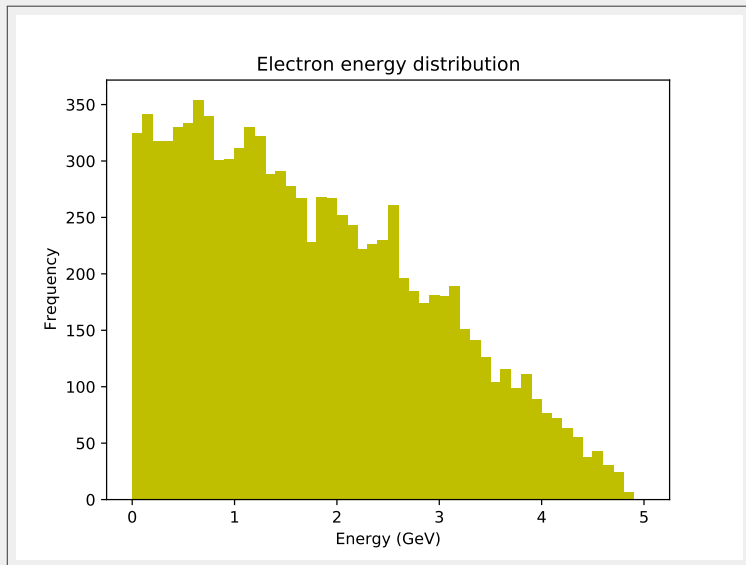


Figure: Electron energy distribution

ELECTRON-NEUTRINO ENERGY DISTRIBUTION

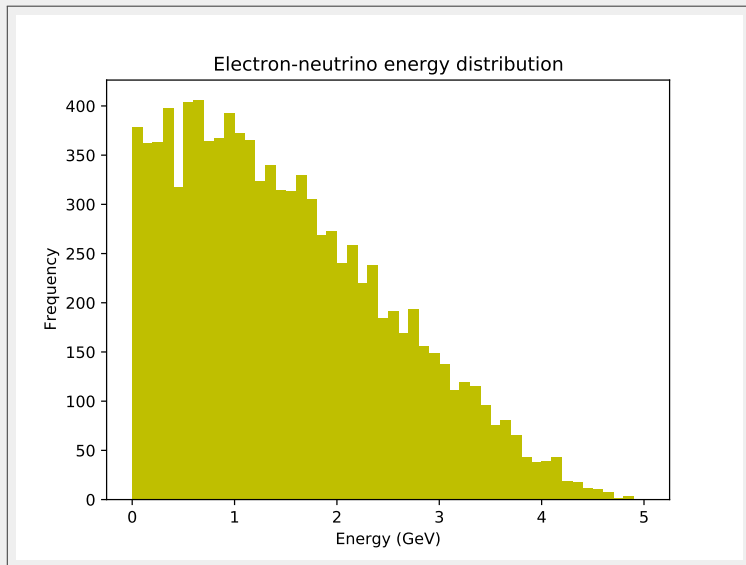


Figure: Electron-Neutrino Energy Distribution

MUON-NEUTRINO ENERGY DISTRIBUTION

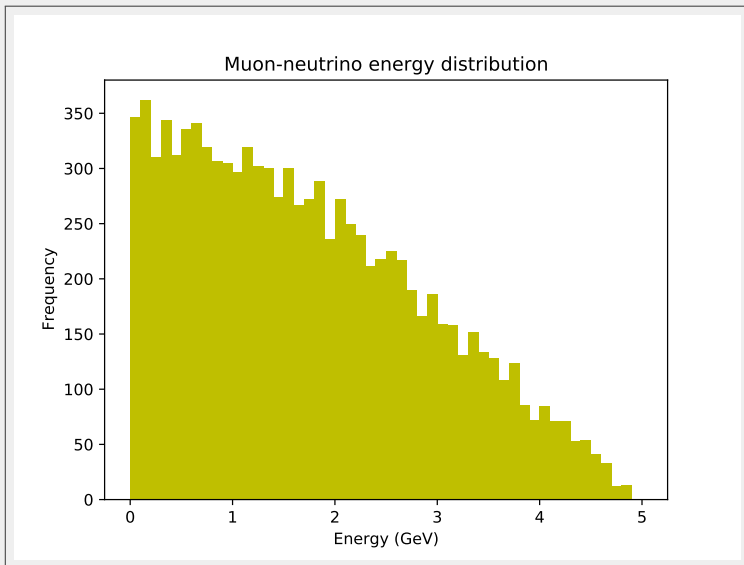
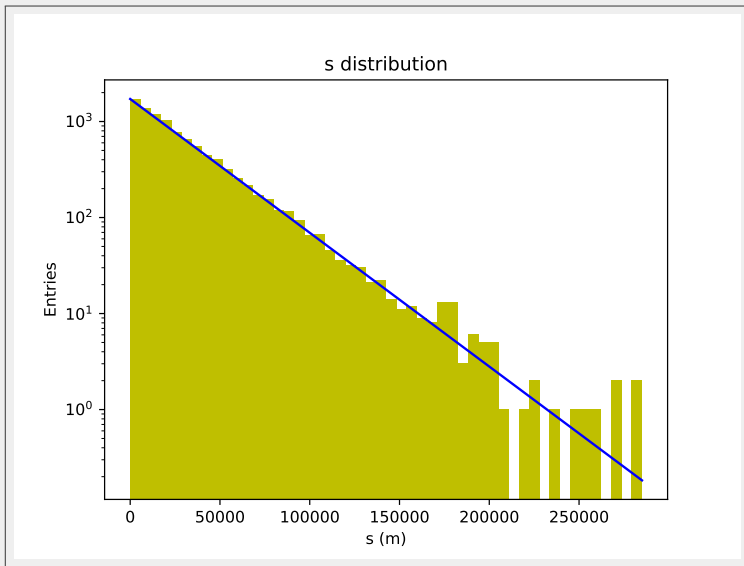


Figure: Muon-Neutrino Energy Distribution

LONGITUDINAL POSITION DISTRIBUTION FOR INFINITE LIFETIME



THANKS!

REFERENCES

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**<https://github.com/ImperialCollegeLondon/nuSTORM>,
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-  **WIKIPEDIA CONTRIBUTORS.**
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