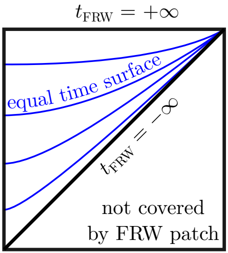
GongShow: Quantum de Sitter Universe

Cambridge University DAMTP

19th April 2023

Cosmological Bootstrap

- Inflation well approximated by de Sitter
- Want to learn about correlators that live on future conformal boundary



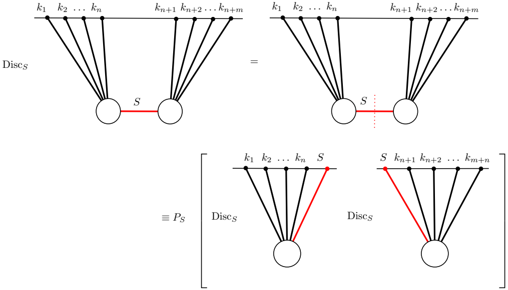
- Impose constraints on the cosmological wavefunction:

$$\Psi[\phi] = \exp\left(- \int \sum_{n=2}^{\infty} \psi_n(k_1, \ldots, k_n) \phi(k_1) \ldots \phi(k_n) \right) .$$

- (Manifest) Locality, (bulk) unitarity, scale invariance
- Can also use full dS isometry (Ward identities)

Unitarity and Locality

- Unitarity (perturbative): Cosmological Optical Theorem



Works for particles of any mass, integer spin.

- Locality: Manifest Locality Test

$$\frac{\partial}{\partial k_c}\psi_n|_{k_c=0} = 0$$

Some results

- Locality + Scale invariance fixes the three point function

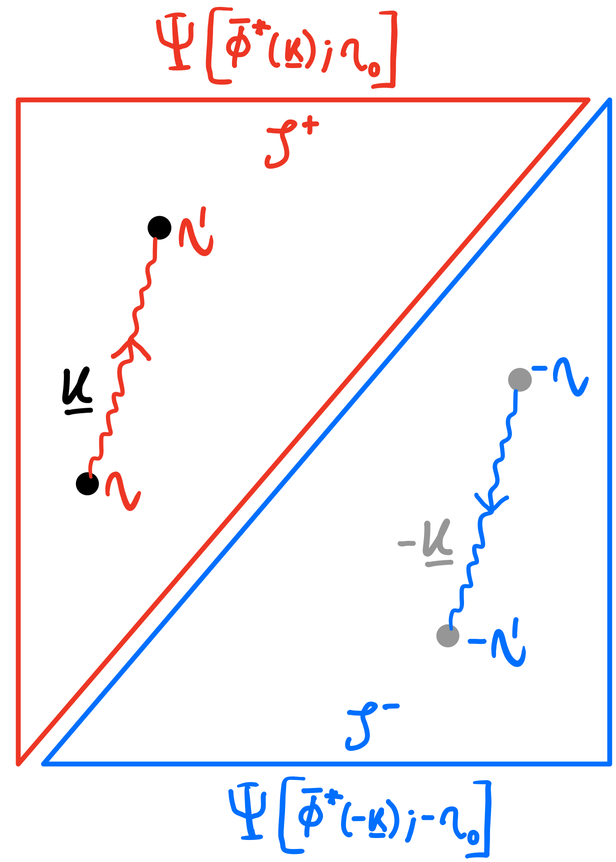
$$\psi_3 = \sum_p C_p \frac{\mathrm{Poly}_{3+p}(k_T, e_2, e_3)}{k_T^p}$$

  The polynomials are NOT arbitrary!
- Constraints on parity odd correlators
  - Example: No parity odd trispectrum from contact diagrams (due to unitarity+scale invariance)
  - No tree level exchange trispectrum for massless scalars, the simplest signal starts at loop level!
- Future: positivity bounds?

The Cosmological CPT Theorem

AYNGARAN THAVANESAN (HE/HIM)

(w/ C. Duaso Pueyo, H. Goodhew, A. Wall)



UNIVERSITY OF CAMBRIDGE

IOP Institute of Physics

Quantum de Sitter Universe

Wavefunction of the Universe (WFU)

QFT in dS path integral

CPT invariant vacuum

any field

$$\Psi[\bar\phi(\mathbf{k});\eta_0] \doteq \int_{\phi(-\infty)=\Omega}^{\phi(\eta_0)=\bar\phi(\mathbf{k})} \mathcal{D}\phi\, e^{iS[\phi]}$$

$$\propto \exp\left\{-\sum_{n=2}^{\infty}\frac{1}{n!}\int_{\mathbf{k}_1,\ldots,\mathbf{k}_n} \psi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)\bar\phi(\mathbf{k}_1)\ldots\bar\phi(\mathbf{k}_n)\right\}$$

WFU coefficients
(boundary correlators)

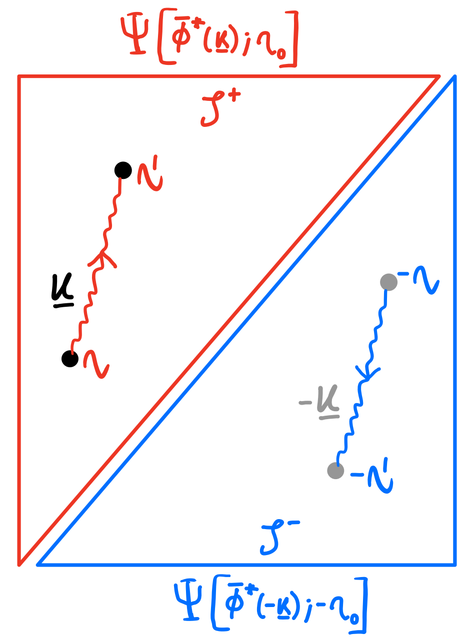
CPT Transformation

$$\Psi^*[\phi^*(\mathbf{k}_a);-\eta_0] = \Psi[\phi(\mathbf{k}_a);\eta_0]\,, \quad \text{for even } D,$$

$$\Psi^*[\phi^*(\mathbf{k}_a);-\eta_0] = -\Psi[\phi(\mathbf{k}_a);\eta_0]\,, \quad \text{for odd } D.$$

$$\psi_n(\omega_a,\mathbf{k}_a;\eta_0) = \psi_n^*(\omega_a,\mathbf{k}_a;-\eta_0)\,, \quad \text{for even } D,$$

$$\psi_n(\omega_a,\mathbf{k}_a;\eta_0) = -\psi_n^*(\omega_a,\mathbf{k}_a;-\eta_0)\,, \quad \text{for odd } D.$$



Cosmological CPT Theorem

Unitarity + ISO(d) w/ Dilations + CPT inv. vacuum $\overset{?}{\Longrightarrow}$ CPT

Unitarity + ISO(d) + BD vacuum $\Longrightarrow$ COT $\psi_n(\omega_a, \mathbf{k}_a; \eta_0) = -\psi_n^*(-\omega_a, -\mathbf{k}_a; \eta_0)$

Scale Invariance $\Longrightarrow$ $\psi_n(\omega_a, \eta_0) = \omega^d f(-\omega_a \eta_0)$

$\Longrightarrow$ $\psi_n(-\omega_a, \eta_0) = (-1)^d \psi_n(\omega_a, -\eta_0)$

dS Horizon Scattering and Sphere Partition Functions

Y.T. Albert Law

Center for the Fundamental Laws of Nature, Harvard University

Black Hole Initiative, Harvard University

Based on 2207.07024 and 2211.16644 with M. Grewal and K. Parmentier

2009.12464 with D. Anninos, F. Denef and Z. Sun

Croucher Foundation 裘槎基金會

BLACK HOLE INITIATIVE HARVARD UNIVERSITY

JOHN TEMPLETON FOUNDATION

GORDON AND BETTY MOORE FOUNDATION

What is the Lorentzian meaning of the S^{d+1} path integral?

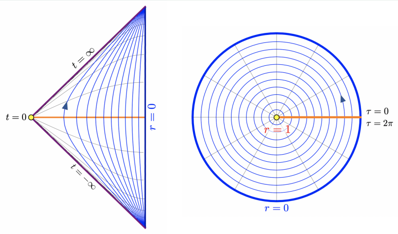
- Gibbons-Hawking: S^{d+1} path integrals compute corrections to dS entropy

$$\mathcal{S}_{\rm dS} = \cdots + \underbrace{b \log \mathcal{S}_0 + c}_{\text{1-loop}} + \cdots \, .$$

- E.g. scalar at 1 loop: $Z_{\rm PI} = \det\left(-\nabla^2 + m^2\right)^{-1/2}$

Q: In what sense the S^{d+1} path integral is computing a trace?

$$Z_{\rm PI} \overset{?}{=} Z_{\rm bulk} \overset{?}{\equiv} \text{“Tr''}\; e^{-\beta_{\rm dS} \hat{H}}$$



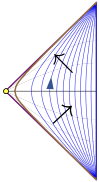
“Tr” is ill-defined: density of normal modes $\rho(\omega) = \infty$ ($\sim$ Type-III vN algebra)

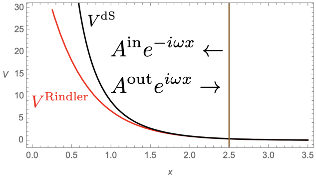
Making sense of "Tr"

- (Renormalized) density of single-particle states/normal modes:

$$\boxed{\Delta\rho_l(\omega) \equiv \rho_l(\omega) - \rho^{\text{Rindler}}(\omega) = \frac{1}{2\pi i}\partial_\omega \log \frac{\mathcal{S}_l(\omega)}{\mathcal{S}^{\text{Rindler}}(\omega)}}\,.$$

- $\mathcal{S}_l(\omega) = \mathcal{S}_l^{\text{dS}}(\omega)\mathcal{S}^{\text{Rindler}}(\omega)$ and $\mathcal{S}^{\text{Rindler}}(\omega)$ are **scattering phases**





- Renormalized thermal canonical partition function

$$\boxed{\widetilde{Z}_{\text{bulk}} \equiv \widetilde{\text{Tr}}\, e^{-\beta_{\text{dS}}\hat{H}} \equiv \frac{\text{Tr}\, e^{-\beta_{\text{dS}}\hat{H}}}{\text{Tr}\, e^{-\beta_{\text{dS}}\hat{H}_{\text{Rindler}}}}}\,.$$

$$\log \widetilde{Z}_{\text{bulk}} = \int_0^\infty \frac{dt}{2t}\frac{1+e^{-2\pi t/\beta_{\text{dS}}}}{1-e^{-2\pi t/\beta_{\text{dS}}}}\chi_{\text{QNM}}(t)\,, \qquad \chi_{\text{QNM}}(t) \equiv \sum_z N_z\, e^{-izt}$$

Result: $Z_{\text{PI}} = \widetilde{Z}_{\text{bulk}}/Z_{\text{edge}}$

- Z_{edge} only exists for **spin** $s \geq 1$
- Related to QNMs (as a function of m^2) that **fail** to Wick-rotate ($t \to -i\tau$) to a subset of Euclidean eigenfunctions (**regular** at the origin)
- Example: for a massive vector, QNMs with $iz(m^2) = \Delta + l - 1$ fail to Wick-rotate to the $k = 0$ eigenfunctions of the massive vector Laplacian $-\nabla^2_{(1)} + m^2$.
- **Upcoming:** Z_{edge} from $SO(d+2) \to U(1) \times SO(d)$ analysis
- Remarks/work in progress:
  1. Scattering formulation of interacting QFTs in static patch?
  2. Why does the (free particle) horizon S-matrix $\mathcal{S}_l(\omega)$ **factorize**?
  3. Canonical/Lorentzian analysis for Z_{edge}?

Thank you!

Cosmological quantum states of de Sitter-Schwarzschild are static patch partition functions

Matthew J. Blacker and Sean A. Hartnoll

Based on arXiv:[2304.06865]

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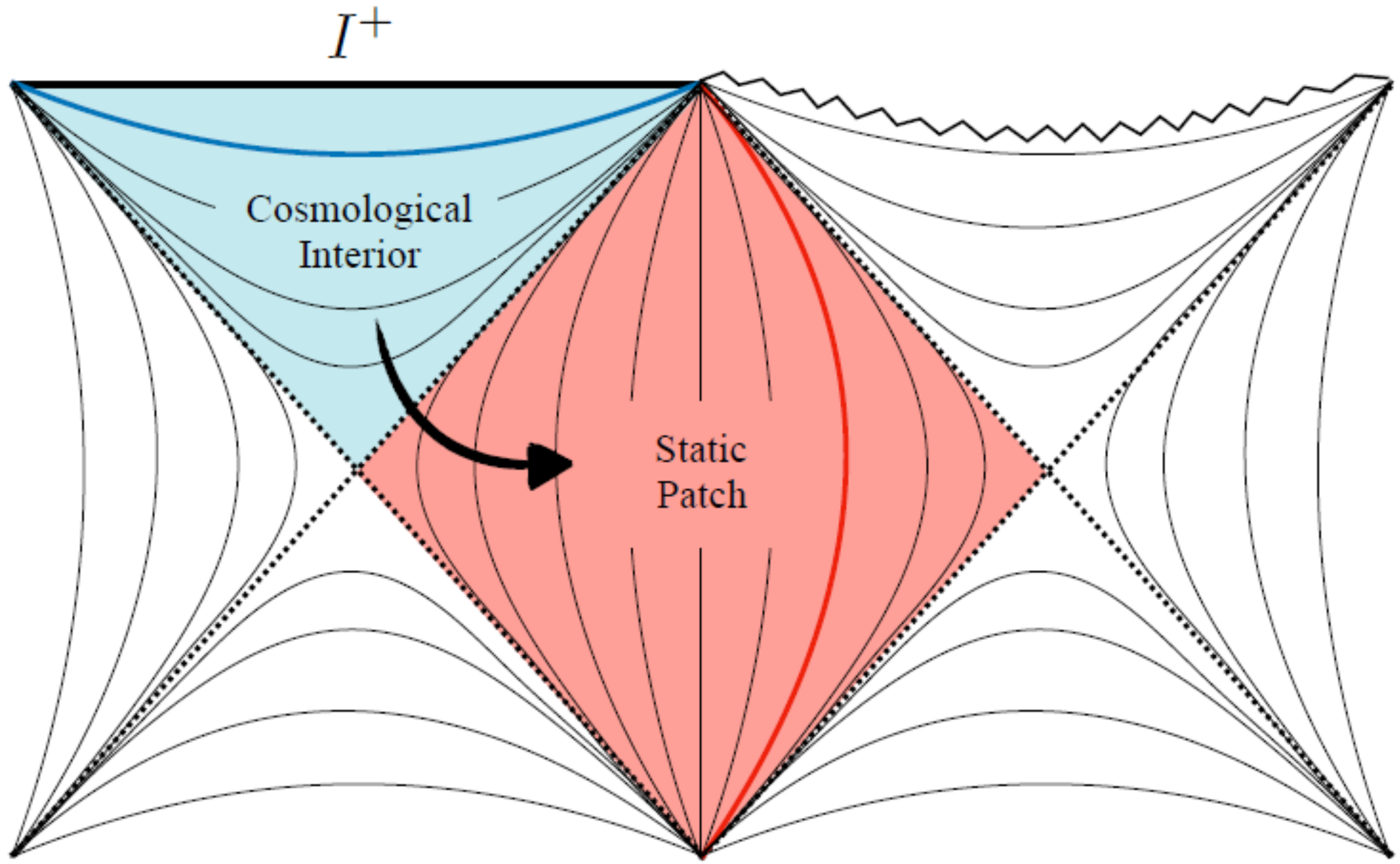
Our Aim



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Solving the WdW Equation

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Solving the WdW Equation

$$ds^2 = -N(r)^2 dr^2 + g_{tt}(r)dt^2 + R(r)^2 d\Omega^2_{S^2}$$ → Mini superspace

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Solving the WdW Equation

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$$\Psi\left(g_{tt}, R; c\right) = e^{iS(g_{tt}, R; c)}$$ → Basis of Semi-classical solutions

where $$S\left(g_{tt}, R; c\right) = \frac{1}{2G_N}\left(\frac{g_{tt}R}{c} + cR\left(\frac{R^2}{L^2} - 1\right)\right)$$

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$$\Psi\left(g_{tt}, R\right) = \int \frac{dc}{2\pi} \beta(c) e^{iS(g_{tt}, R; c)}$$ → Build wavepackets

$$\partial_c S = -M$$ → Stationary phase: dS Schwarzschild solution

Based on arXiv:[2304.06865]

The Proposed Duality

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$$\exp\left(i\left(S - 2g_{tt}\langle\pi_{tt}\rangle\right)\right) = \exp\left(icM\right) \quad \text{where} \quad \pi_{tt} = \frac{\partial S}{\partial g_{tt}}$$

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Motivates our proposal

$$\Psi\left(g_{tt}, R; c\right) e^{-i2g_{tt}\langle\pi_{tt}\rangle_{\Psi}} = \mathrm{Tr}\left[e^{-icH(g_{tt},R)}\right]$$

where $\langle H\left(g_{tt}, R\right)\rangle = M$

Based on arXiv:[2304.06865]

What can we say about the theory?

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Entropy: Fourier transform onto VEV

$$\widetilde{Z}\left(R;c\right) = \int dg_{tt} e^{-iv\pi_v} \mathrm{Tr}\left[e^{icH(g_{tt},R)}\right]$$

$$\widetilde{Z}\left(R_{\mathcal{H}}, -i\beta_{\mathcal{H}}\right) = e^{S_{BH}}$$

What can we say about the theory?

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The operators $H,\ \pi_{tt},\ \pi_R$ can be mapped to

$$i[X,D] = X\,, \qquad i[P,D] = -P\,, \qquad i[X,P] = h(R)\,,$$

The massless spin-3/2 field on dS_D cannot be unitary unless $D = 4$

Vasileios A. Letsios
Department of Mathematics, University of York
Based on https://arxiv.org/abs/2303.00420

April 2023

- **Elementary particles in dS_D $\rightarrow$ UIR's** of the **dS algebra spin$(D,1)$**.
- Interested in "one-particle Hilbert space $\Longleftrightarrow$ UIR's" dictionary for fields on dS_D
- Known for the case of bosons with arbitrary spin [Higuchi; Boulanger, Bekaert and Basile]
- **Not** studied for higher-spin fermions
- Rep. theory of gauge-invariant higher-spin fermions turns out to be very interesting

The massless spin-3/2 field on dS_D

Massless spin-3/2 field $\rightarrow$ gauge-invariant vector-spinor

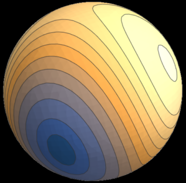
- Onshell conditions

$$\gamma^a \nabla_a \Psi_\mu = -M \Psi_\mu$$
$$\gamma^a \Psi_a = \nabla^a \Psi_a = 0,$$

- **Imaginary(!) mass parameter $M = i(D-2)/2$**
- Eigenmodes form a rep. of spin$(D, 1)$.
- Unitary if:positivity and dS invariance of norm.

The gravitino field on dS_D

- **Aim:** Investigate when this rep. is unitary.
- **Method:** Study group-theoretical properties of the eigenmodes.
- **Main result:** Unitarity only for $D = 4$.



A large-j spin model for the dS_2 principal series

Klaas Parmentier

in collaboration with

Frederik Denef

Columbia University

exclude

take specific microscopic proposal, see if subleading entropy corrections match the sphere calculation

construct

reproduce character $\chi(t) = \mathrm{tr}\, \mathrm{e}^{-\mathrm{i}tH}$ from large N QM

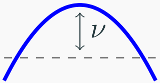
minimal features

- approximate upside-down harmonic oscillator
- map $H \to -H$

spin model

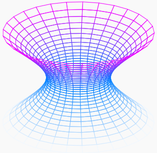
$$H = \frac{\mathrm{i}}{8j}(\mathcal{J}_+^2 - \mathcal{J}_-^2) - \frac{\nu}{j}\,\mathcal{J}_3$$

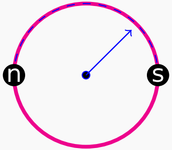
acts on spin-j rep of $\mathfrak{su}(2)$

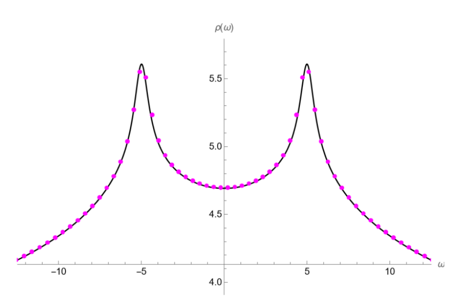


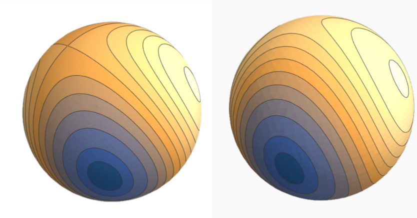
$$\Delta = \frac{1}{2} + \mathrm{i}\nu$$

$$\chi(t) = \frac{\mathrm{e}^{-\Delta t} + \mathrm{e}^{-\bar{\Delta} t}}{|1 - \mathrm{e}^{-t}|}$$









large-j limit

recover $\chi(t)$ and $\rho(\omega)$ (numerically),
captures $\sim j$ lowest overtones,

semiclassical analysis

spherical phase space (Berezin),
log tails of $\rho(\omega)$ from periods

Reissner-Nordström-de Sitter: Geometry

arxiv.2212.12713

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2_{D-2}$$

$$f(r) = 1 - \frac{r^2}{l^2} - \frac{m}{r^{D-3}} + \frac{q^2}{r^{2(D-3)}}$$

$$M = \frac{(D-2)\,\Omega_{D-2}}{16\pi G}\, m\,, \qquad Q = \sqrt{\frac{(D-3)(D-2)}{8\pi G}}\, q$$

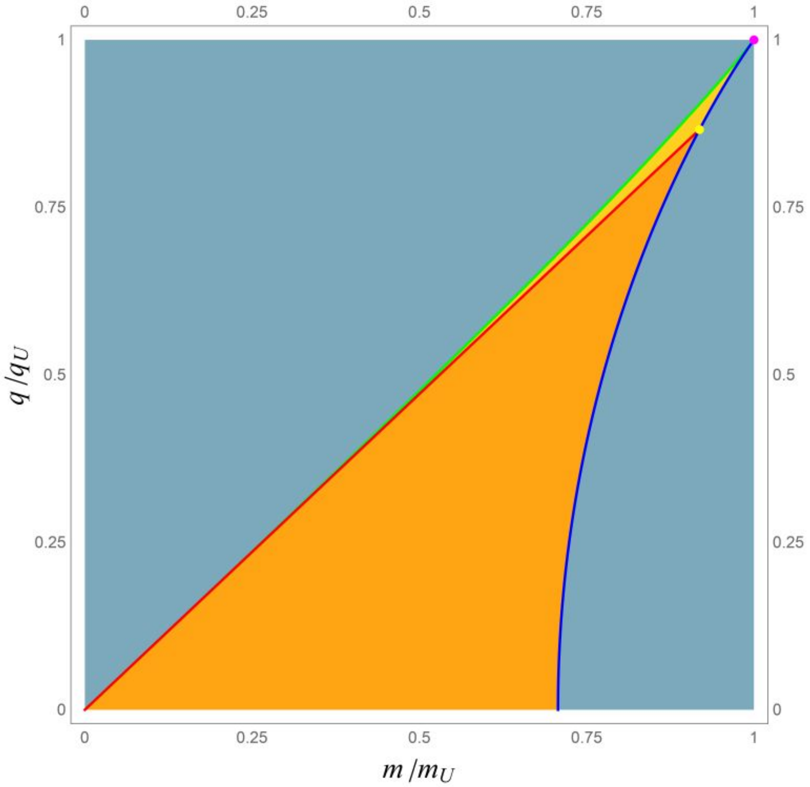
3 Horizons

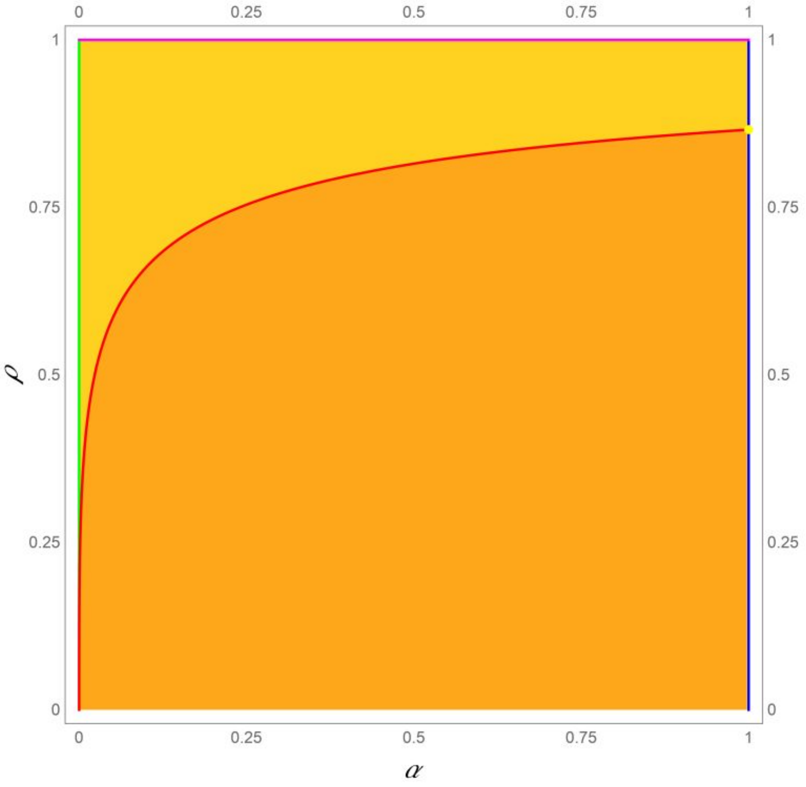
r_- $\quad r_+$ $\quad r_{++}$

2 Temperatures

T_+ $\quad T_{++}$

Reissner-Nordström-de Sitter: Geometry





Reissner-Nordström-de Sitter: Tunneling

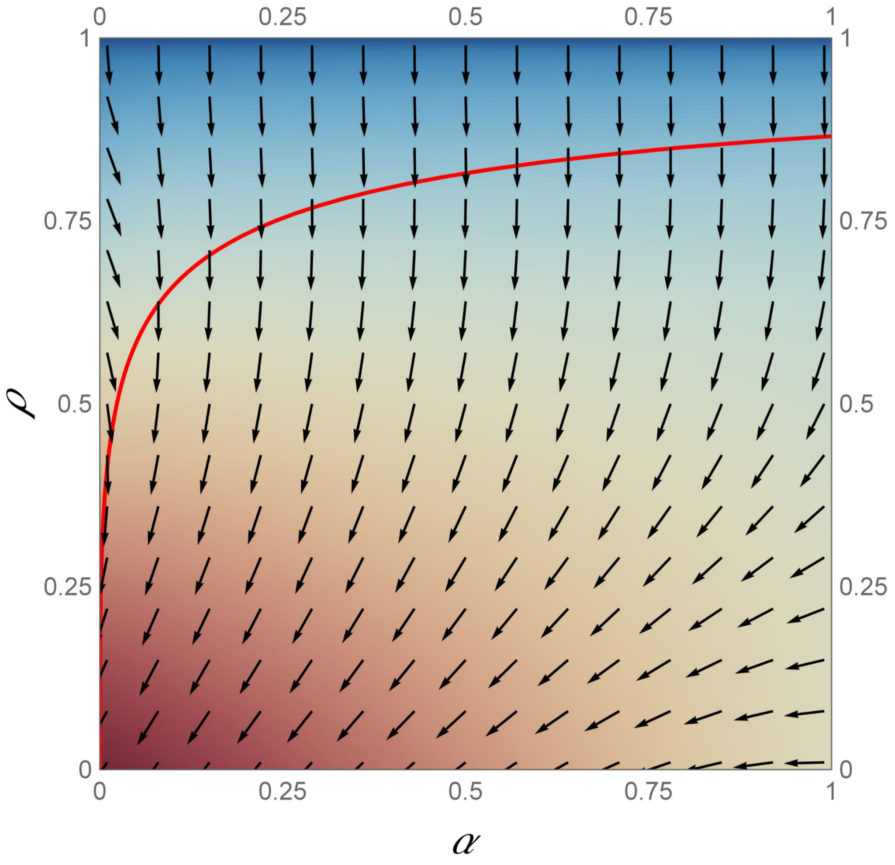
$$I_{RNdS} = -\frac{\mathcal{A}_+ + \mathcal{A}_{++}}{4G} = -(S_+ + S_{++})$$

$$\int [dg][dA]\, e^{-I[g,A]} = \int d\alpha \int d\rho \int d\lambda_m \int d\lambda_q \int [dg][dA]\, e^{-I[g,A]+\lambda_m(\mathcal{C}[g,A]-\alpha)+\lambda_q(\mathcal{D}[g,A]-\rho))}$$

$$\Gamma = \int d\alpha\, d\rho\, e^{-(I_{RNdS}-I_{dS})}$$

$$\boxed{\Gamma \approx \frac{e^{-(S_{dS}-S_U)}}{S_{dS}-S_N^0}\Big(\gamma(\sigma_N) - \gamma(\sigma_{dS}) + \log\left(\sigma_N/\sigma_{dS}\right)\Big)} \qquad \gamma(\sigma) \equiv \int_\sigma^\infty t^{-1} e^{-t}\, dt$$

Reissner-Nordström-de Sitter: Action



Goal: Micro dS_2

SYK $\longleftrightarrow$ JT grav AdS_2

$S = S_0 + T$

$U(\phi)$
$\wr$
$T(S - S_0)$

$H = H_q + s H_{\tilde{q}}$

$$R = -V'(\phi)$$

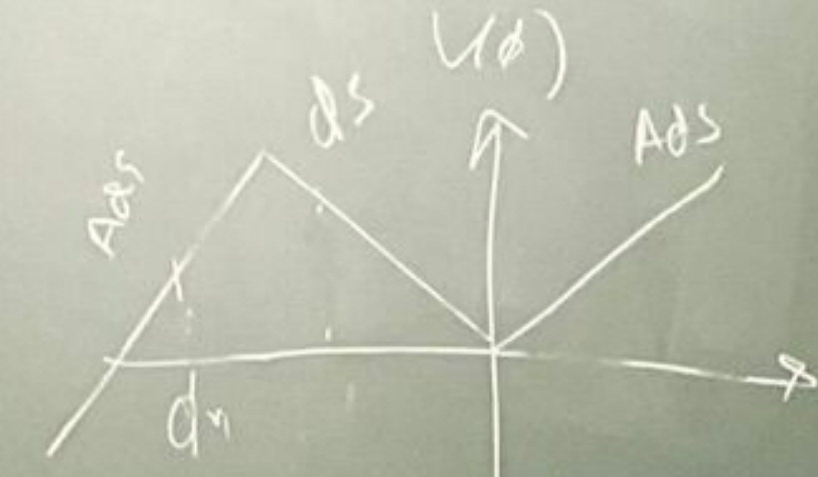
$$C \sim \frac{V(\phi_h)}{V'(\phi_h)}$$

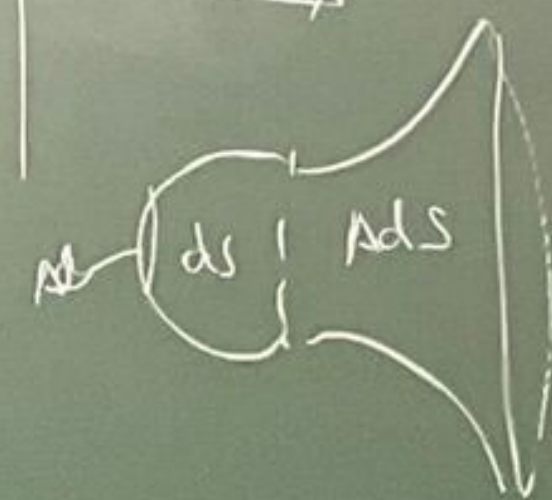
JT : $V(\phi) = 2\phi$

dS_2 : $V(\phi) = -2\phi$

1) Dirichlet : ϕ_b finite $\quad C > 0$

2) Double Interpolating





$$C|\Delta, n\rangle = \Delta(\Delta-1)|\Delta, n\rangle$$

$$L_0|\Delta, n\rangle = -n|\Delta, n\rangle$$

$P_\nu \quad \Delta = \frac{1}{2} + i\nu$

$C_\Delta \quad \Delta \in (0, 1)$

$D_\Delta^\pm \quad \Delta = \mathbb{Z}_+$

Models

$$\Box_{dS}\phi = \Delta(1-\Delta)\phi$$

JT $\quad R = 2$

$$\Box_{dS}\,\omega = -2\omega$$

$$P_\nu \otimes P_\nu = \int P_\nu + \sum D_\Delta$$

$$S = \int d^2x \sqrt{g}\,(\phi R + V(\phi))$$

$$R = -V'(\phi)$$

$$c \sim \frac{V(\phi_h)}{V'(\phi_h)}$$

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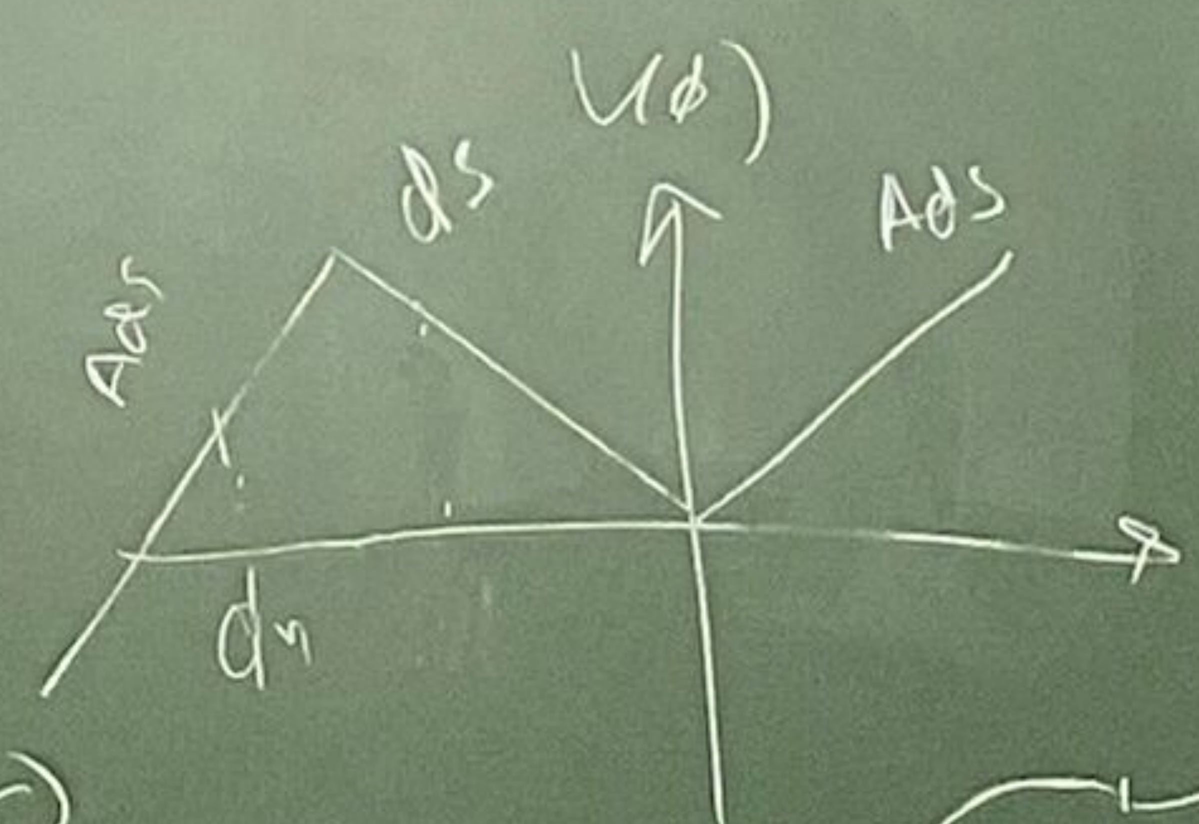
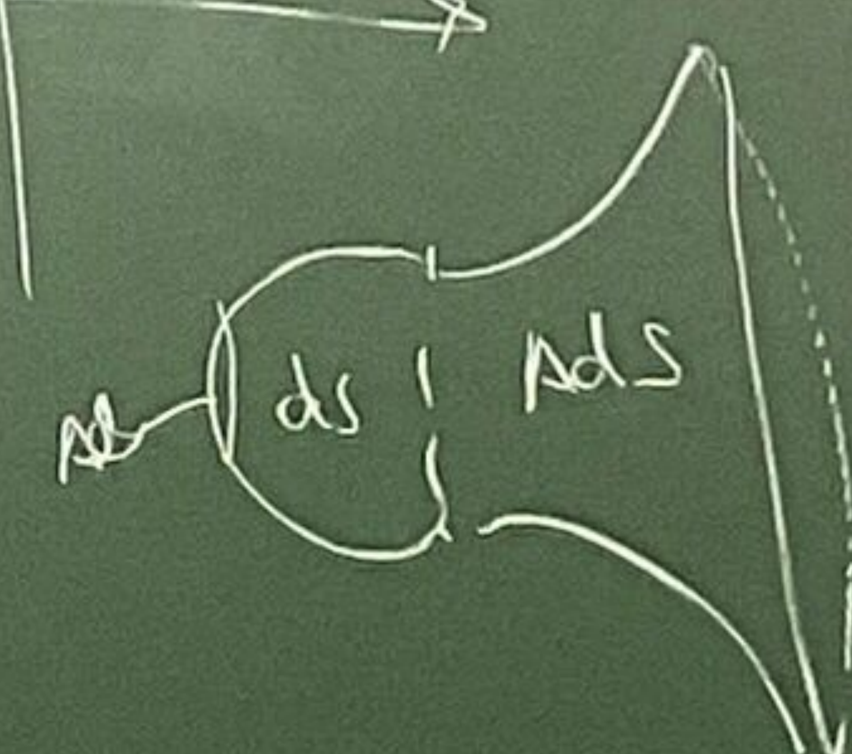
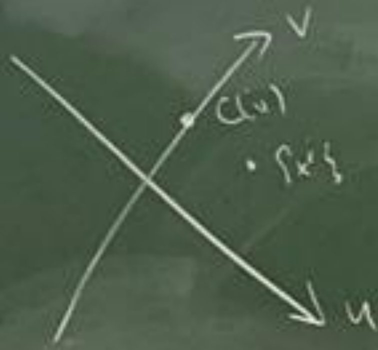




$$L = f(g, Ric, \nabla^{(q)} Ric, \phi, \nabla^{(p)}\phi, V, \nabla^{(r)} V)$$

$$E_{vv} \supset F(\partial_v V_i) = D_i(\ldots)$$



$$\nabla \sim \partial_u, \partial_v, D_i$$

$$E_{vv} \to \partial_v | V_i$$

$$\partial_v, \partial_v \quad g, \phi, A$$

$$\partial_v^2 S = -2\pi \int_{c(v)} E_{vv}$$

$$f(Ric) + V, \nabla V$$

$$S = -2\pi \int \left(4 \frac{\partial L}{\partial R_{uvuv}} + \left(4 K_{ij}^{(u)} \frac{\partial}{\partial R_{uiuj}} - V_u \frac{\partial}{\partial(\nabla_u V_u)} \right) \left(4 K_{kl}^{(v)} \frac{\partial}{\partial R_{vkvl}} - V_v \frac{\partial}{\partial(\nabla_v V_v)} \right) L \right)$$