Cauchy Slice Holography in a Closed Universe

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Based on:
2204.00591 (w/ Rifath Khan and Aron Wall)
2212.03219

Outline

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Diffeomorphism invariance $\Longrightarrow$ constraints:

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Ψ lives on a closed spatial slice Σ.

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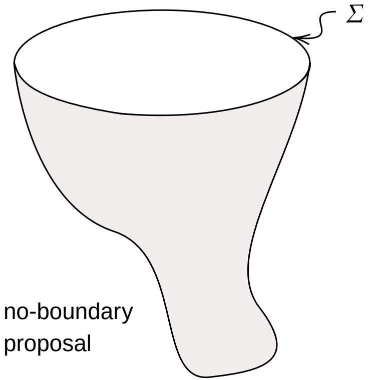
$\Psi[g] = Z[g]$



Questions

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What are some of the implications of such a definition of quantum gravity?

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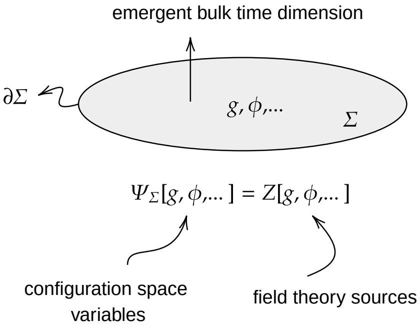
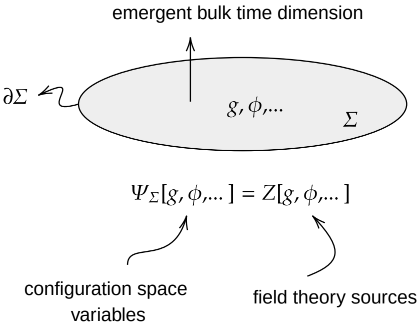
Basic idea


emergent bulk time dimension
$\partial\Sigma$
$g, \phi, \ldots$
Σ
$\Psi_\Sigma[g, \phi, \ldots] = Z[g, \phi, \ldots]$
configuration space variables
field theory sources

Basic idea



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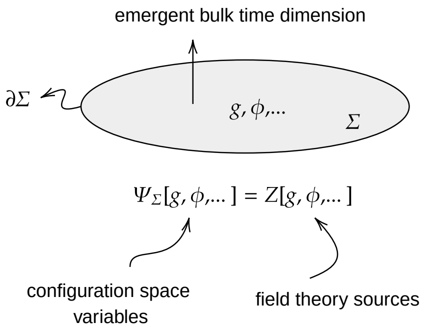

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From now on Σ will be closed.

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They both live on the SAME spatial slice! The one on which the state Ψ is defined.

Quantum Gravity = RG Flow

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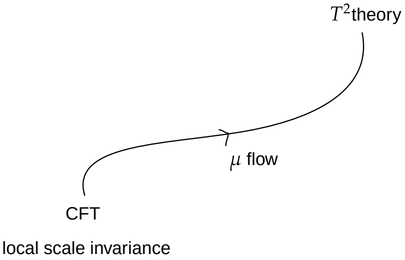
But instead we should think of it as:

Key message

QG is dual to an RG flow line!

$$[H(x), H(y)] = iD^a(x)\partial_a^x \delta(x-y) - (x \leftrightarrow y)$$

$$H(x) = 0$$



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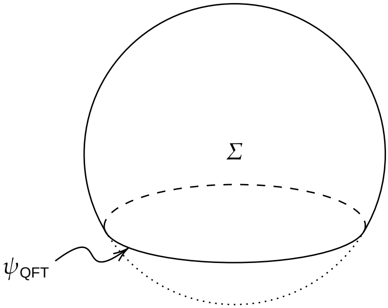
So recover CFT behaviour in the IR limit $\iff$ large volume limit of quantum gravity.

Outline

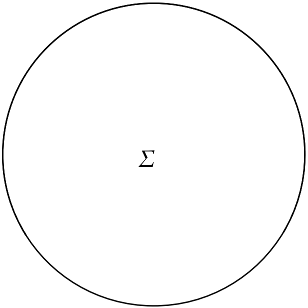
Boundary Information

Open Cosmology



$$\Psi[g] = Z[g; \psi_{\text{QFT}}]$$

Closed Cosmology



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But there is still only one state satisfying the full Hamiltonian constraint.

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Should it not be the case that there is a single holographic dual to a given quantum gravity theory?

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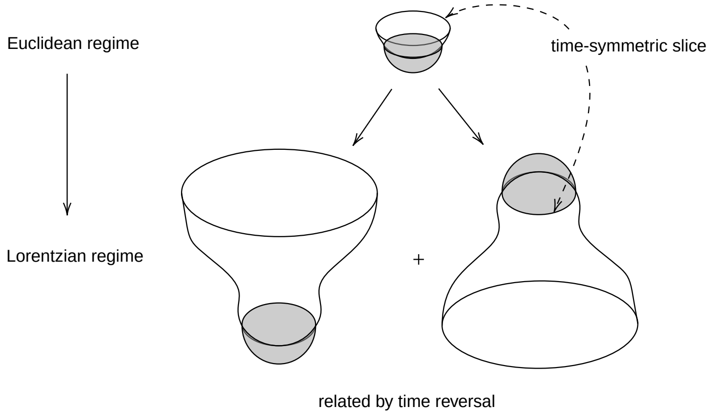
Phase Transition

$$Z_{\mathrm{UV}}[g] \longrightarrow e^{i\varphi} Z_+[g] + e^{-i\varphi} Z_-[g] \quad \text{in the IR limit}$$

Spontaneous CPT Breaking

Bulk picture suggests this is exactly what happens:



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How is this encoded in $Z[g]$?

No-boundary State

The no-boundary state is of the form:

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Thus it should use the same class $\mathcal{C}$ we used to define the QG theory we are working with.

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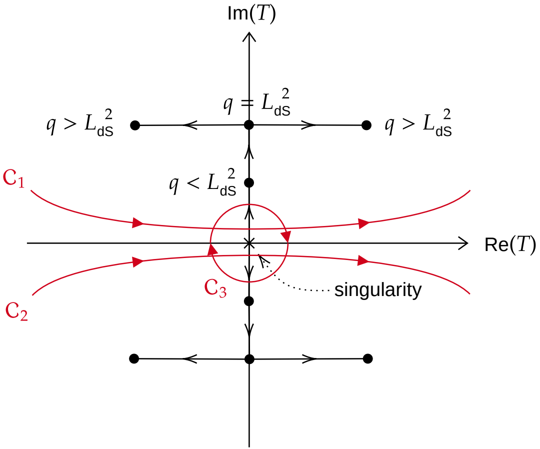

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This is in agreement with a given dual partition function, $Z[g]$, providing a definition of a given QG theory.

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- Holography suggests unique state of the Universe.

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Thank you for listening!

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$$O_\pm(\lambda) = \frac{1}{2}\int_\Sigma \left\{ -\frac{c_\pm}{24\pi}\sqrt{g}R + \frac{24\pi}{c_\pm}\lambda\frac{1}{\sqrt{g}} : \left(\Pi_{ab}\Pi^{ab} - \Pi^2\right) : \right\}$$

$$\mathrm{CT}_\pm[g] = \frac{c_\pm}{12\pi\mu}\int_\Sigma \sqrt{g}$$

where $\Pi^{ab} = -i\frac{\delta}{\delta g_{ab}}$.

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Take two particular superpositions:

$$\Psi_{\text{HH}}(q) = \frac{Z_+(q) + Z_-(q)}{2}$$
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Familiar Solutions