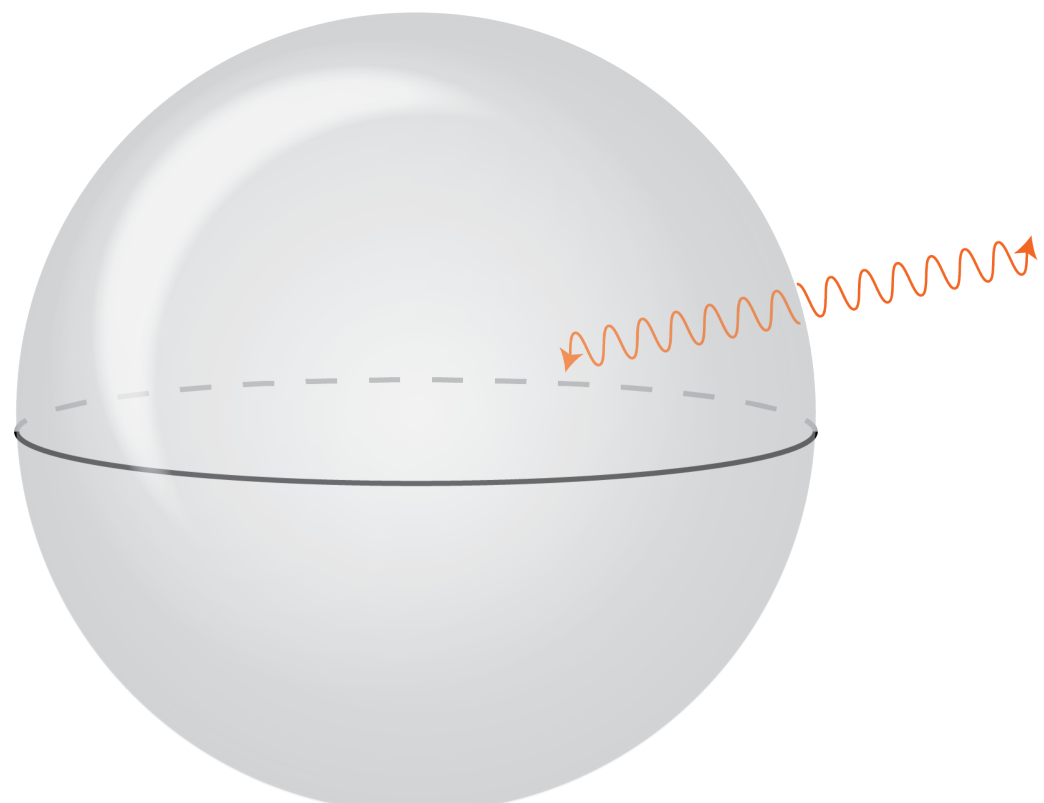
Entanglement and Islands in de Sitter JT Gravity



Lars Aalsma

Arizona State University

Quantum de Sitter Universe Workshop
April 21, 2023

Work with:

W. Sybesma
W. Sybesma + S. Aguilar-Gutierrez

Layout

- **Motivation**
- Symmetries of the Cosmological Horizon
- JT Gravity + Matter
- Entanglement and Islands
- Outlook

Motivation

At this workshop, we've heard a lot about the different approaches to de Sitter quantum gravity.

Most approaches have in common that they look for signatures/interpretations/implications of the finite de Sitter entropy.

This makes sense, because $S_{\rm dS}$ is a truly quantum gravity effect:

$$G_N \to 0 \quad \Rightarrow \quad S_{\rm dS} \to \infty$$

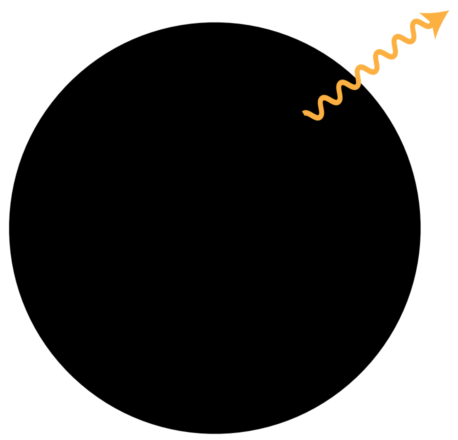
Allowing for (small) fluctuations of the background should introduce finite entropy corrections.

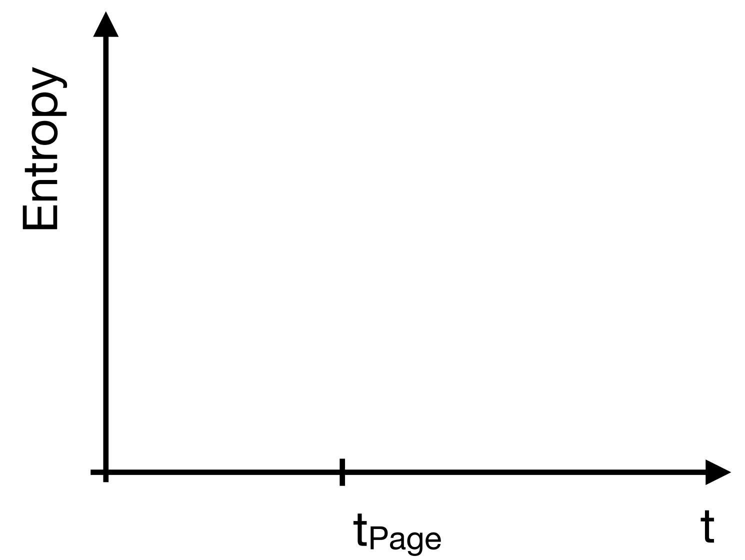
Probing $S_{\rm dS}$ is a window into de Sitter quantum gravity.

Motivation

One way of probing S_{dS} is through entanglement entropy.

This approach as been fruitful for black holes.





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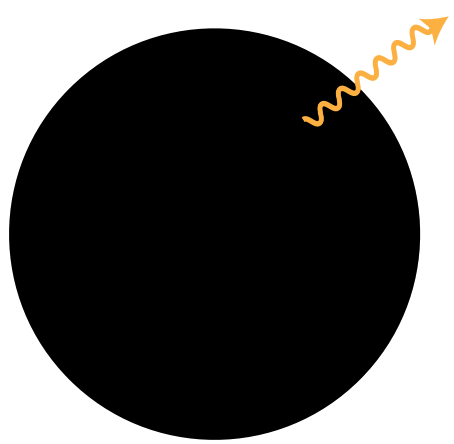

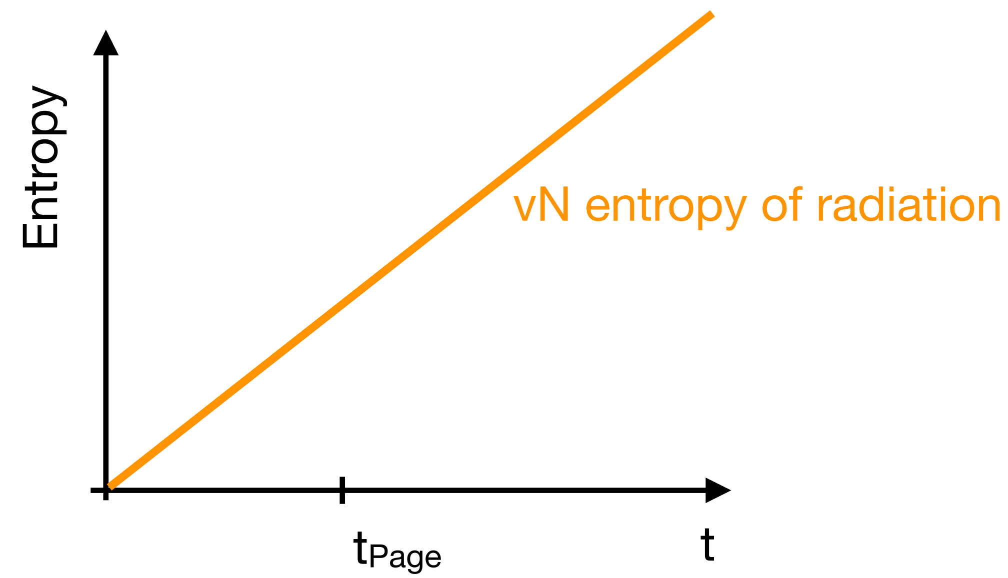

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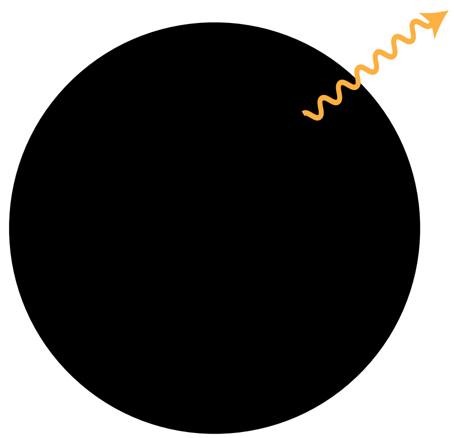

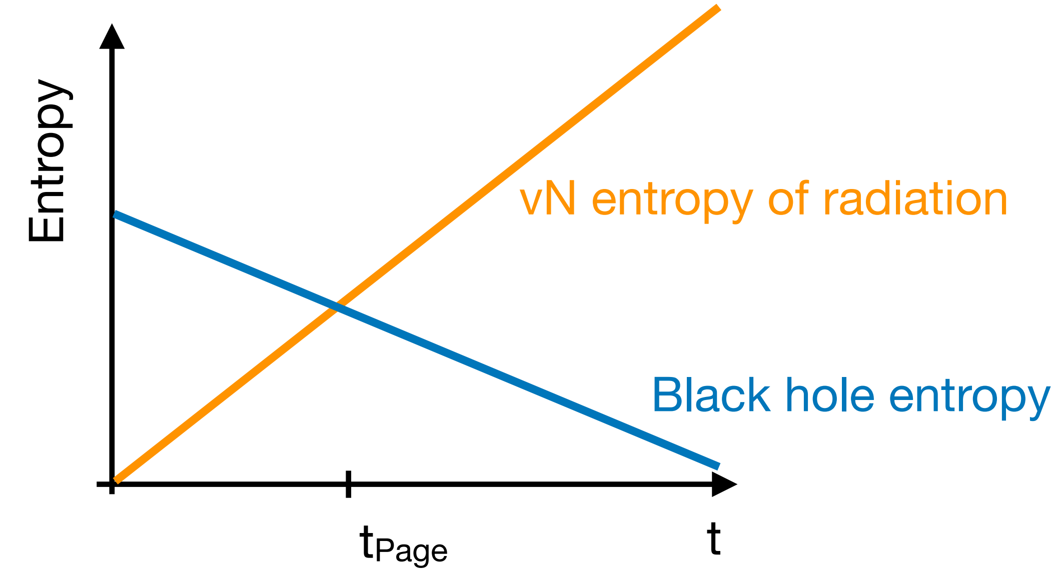

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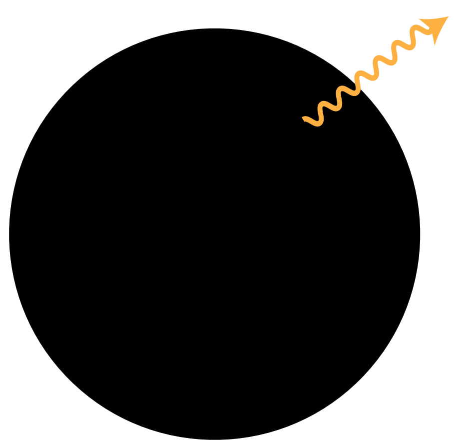
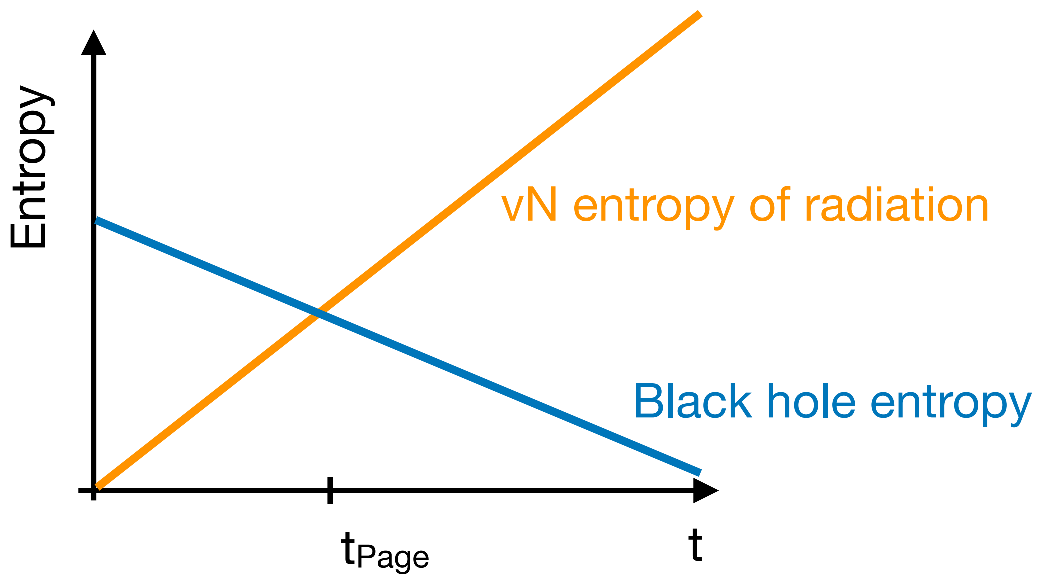


S_{Hawking} naively exceeds S_{BH} !



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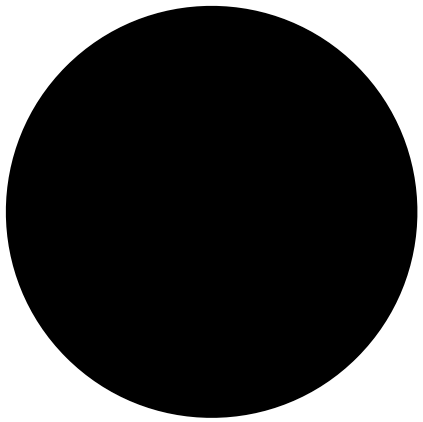
Central Dogma's

First, we need to understand what the entropy is supposed to be counting.

For black holes this is formulated in the "central dogma": [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini '20]

> As seen from the outside, a black hole can be described in terms of a quantum system with Area/$(4G_N)$ degrees of freedom, which evolves unitarily under time evolution.



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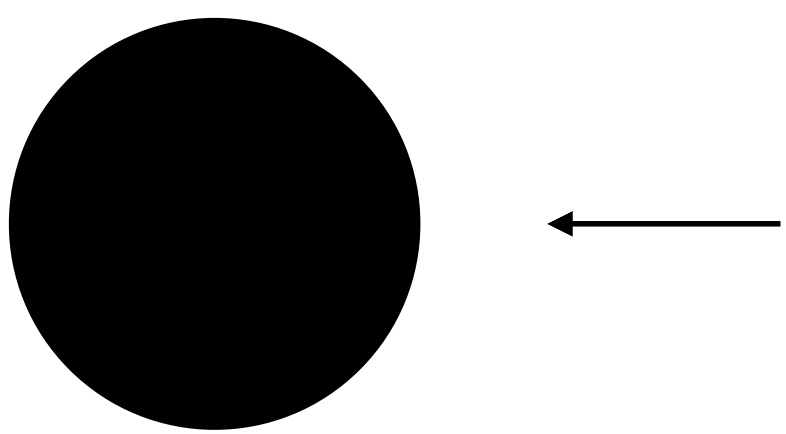

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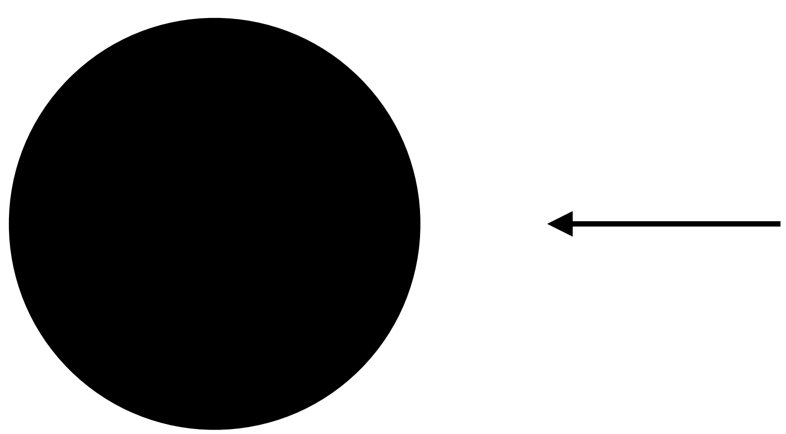

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Significant evidence!

Central Dogma's

This has been extended to de Sitter space: [Shaghoulian '21]

> We will assume that the de Sitter horizon, as viewed from the observer's vantage point, can be thought of as a quantum system with $e^{A/(4G)}$ degrees of freedom.

However, empty de Sitter space has maximum entropy.

Implies the Hilbert space contains "everything".

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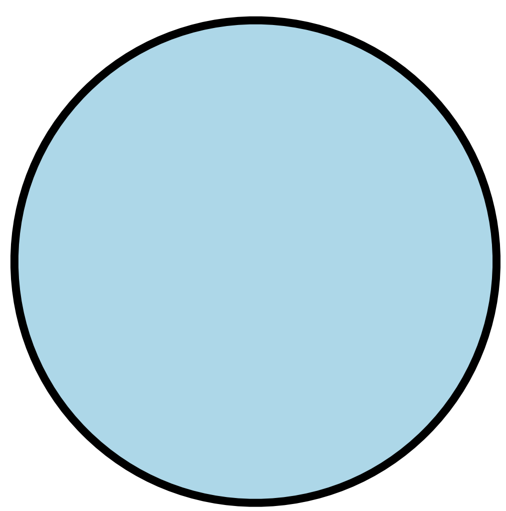

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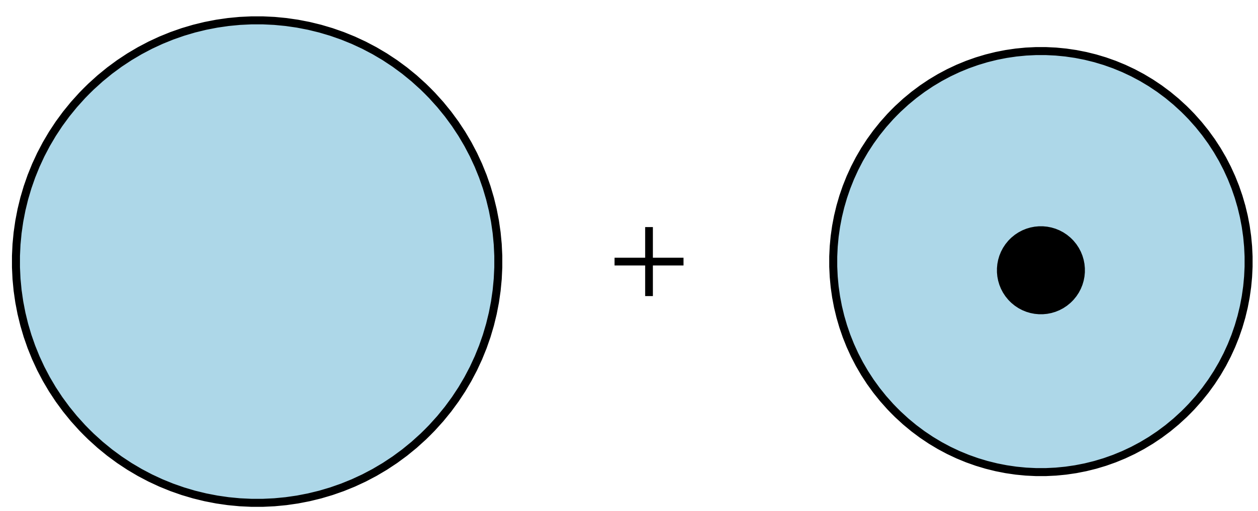

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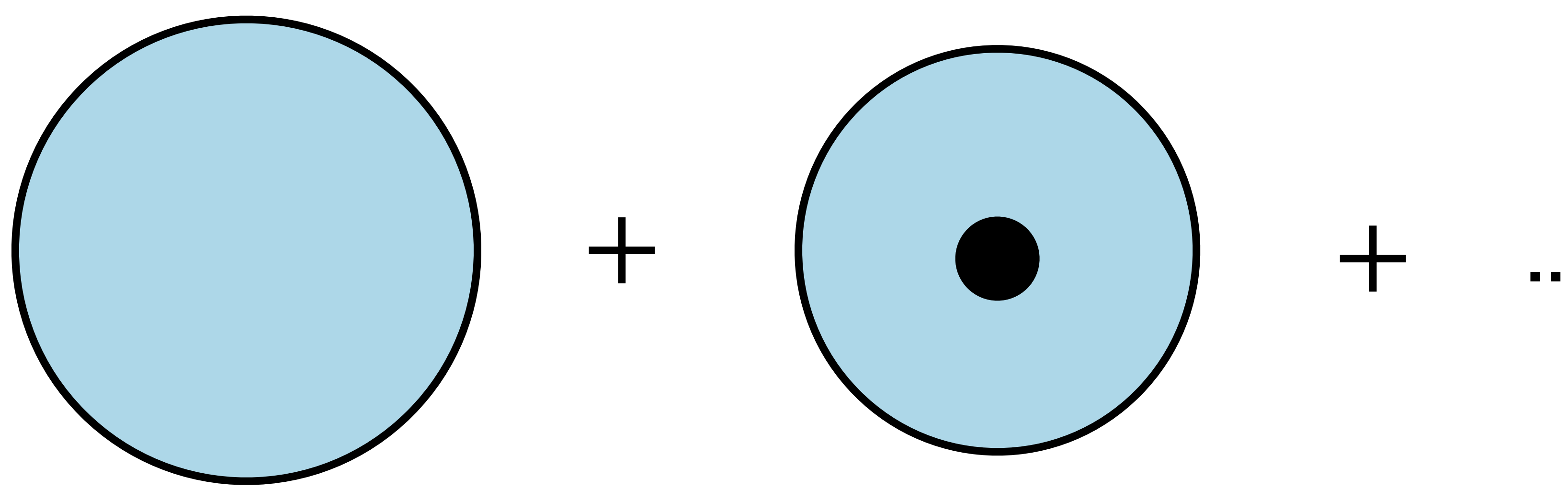

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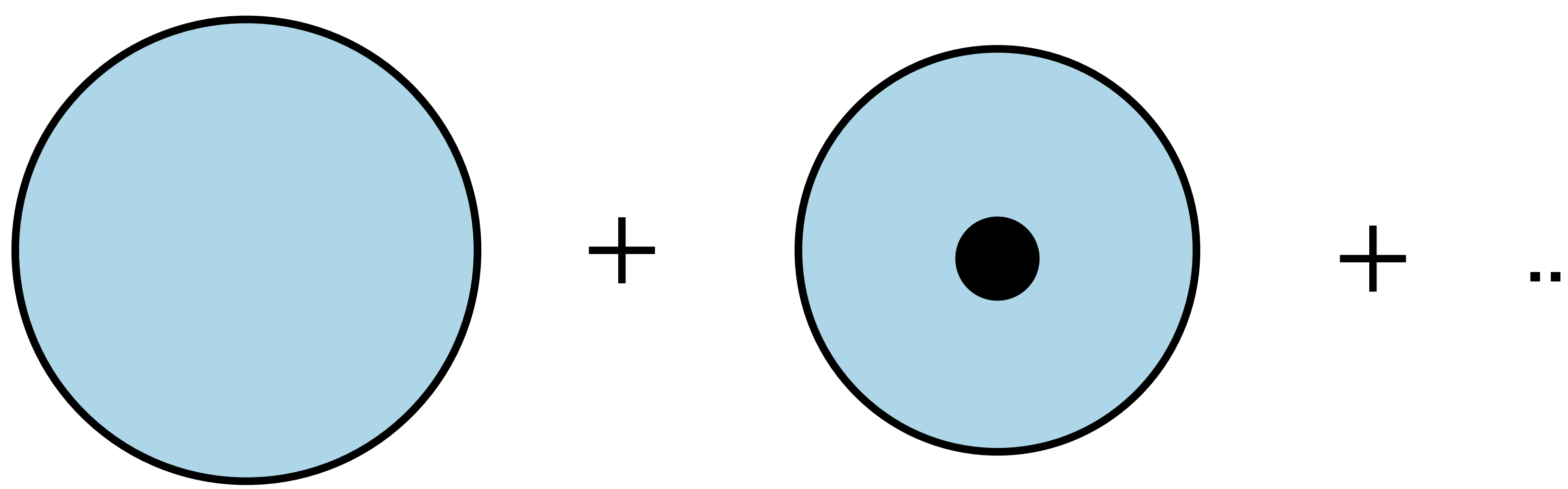

Described by type II_1 von Neumann algebra

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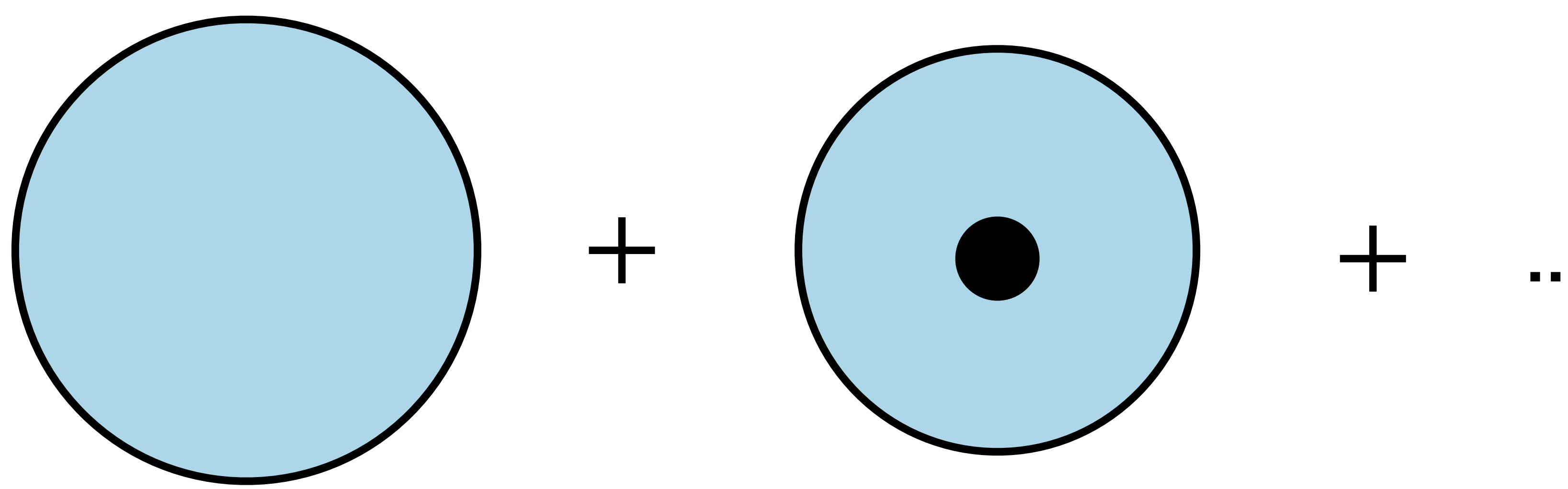

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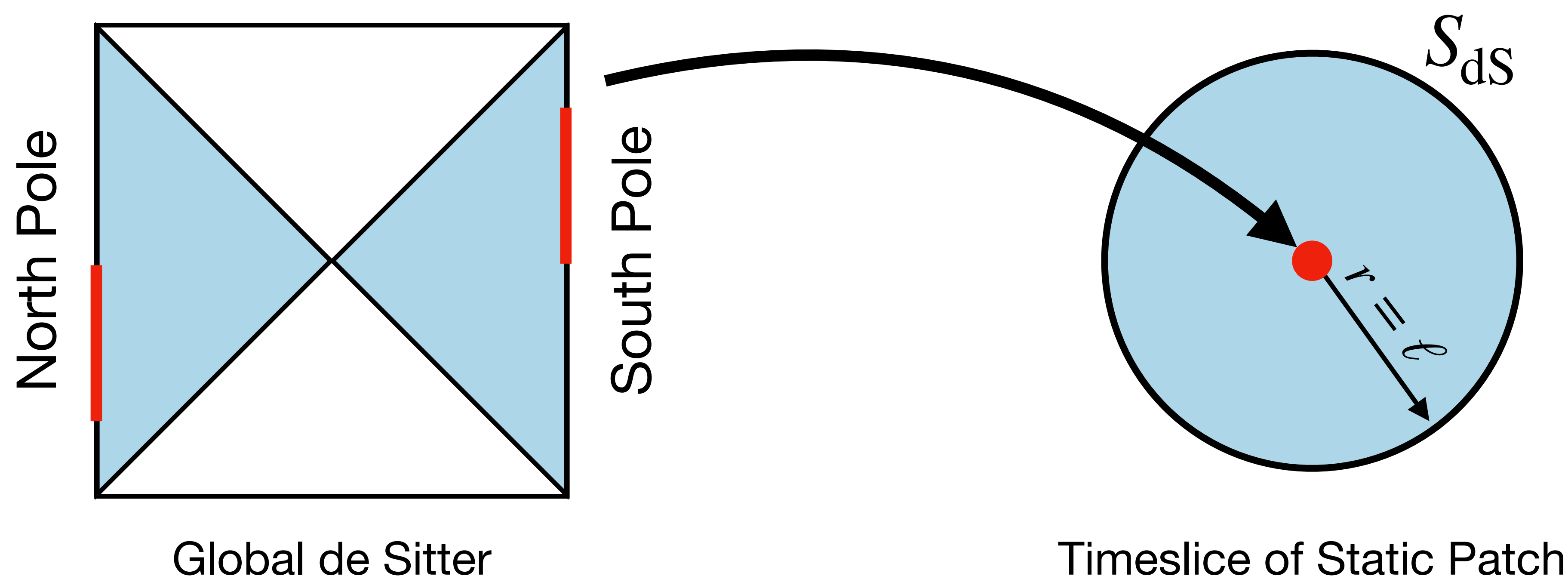
Compelling evidence

Role of the Observer

To define the quantum system the entropy is supposed to be counting, we need to introduce a (pair of) observers.



The observer spontaneously breaks the isometry group of de Sitter:

$$SO(d,1) \rightarrow SO(d-1) \times \mathrm{R}$$

Other isometries "mix" static patches.

Symmetries of dS QG

This symmetry breaking actually seems to be required!

- Finite entropy is incompatible with symmetry generators that mix different static patches. [Goheer, Kleban, Susskind '03]

- Said differently, $SO(1,d)$ has no finite dimensional representations, so it cannot act on $\mathscr{H}$. [Witten '04] [Parikh, E. Verlinde '04]

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"clock" that breaks time reversal [César's + Leonard's talk]

Implications

In de Sitter, spatial slices are compact so gauge charges associated to isometries are constraints.

- For QFT on fixed dS background, $G_N = 0$ and $S_{\rm dS}$ infinite. No issue with full isometry group.

- In dS quantum gravity, $G_N \neq 0$, $S_{\rm dS}$ is finite and we only require invariance under static patch isometries.

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Let's see how this appears in entropy computations

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JT Gravity on de Sitter Space

We'll now see how to apply these ideas to dS JT gravity.

$$I = I_0(\Phi_0) + \frac{1}{2\kappa^2}\int d^2x\sqrt{-g}\,\Phi(R - 2/\ell^2) + \textbf{(matter)}$$

Large entropy   Dynamics

Leads to EOM:

$$-\nabla_a\nabla_b\Phi + g_{ab}\Box\Phi + \frac{2\Phi}{\ell^2}g_{ab} = \kappa^2\langle T_{ab}\rangle$$

$$R = 2/\ell^2$$

Different solutions for the dilaton spontaneously break some of the de Sitter isometries → "Nearly dS_2 gravity".

Can classify solutions based on the different Killing vectors. [Maldacena, Turiaci, Yang '19]

Static Solutions

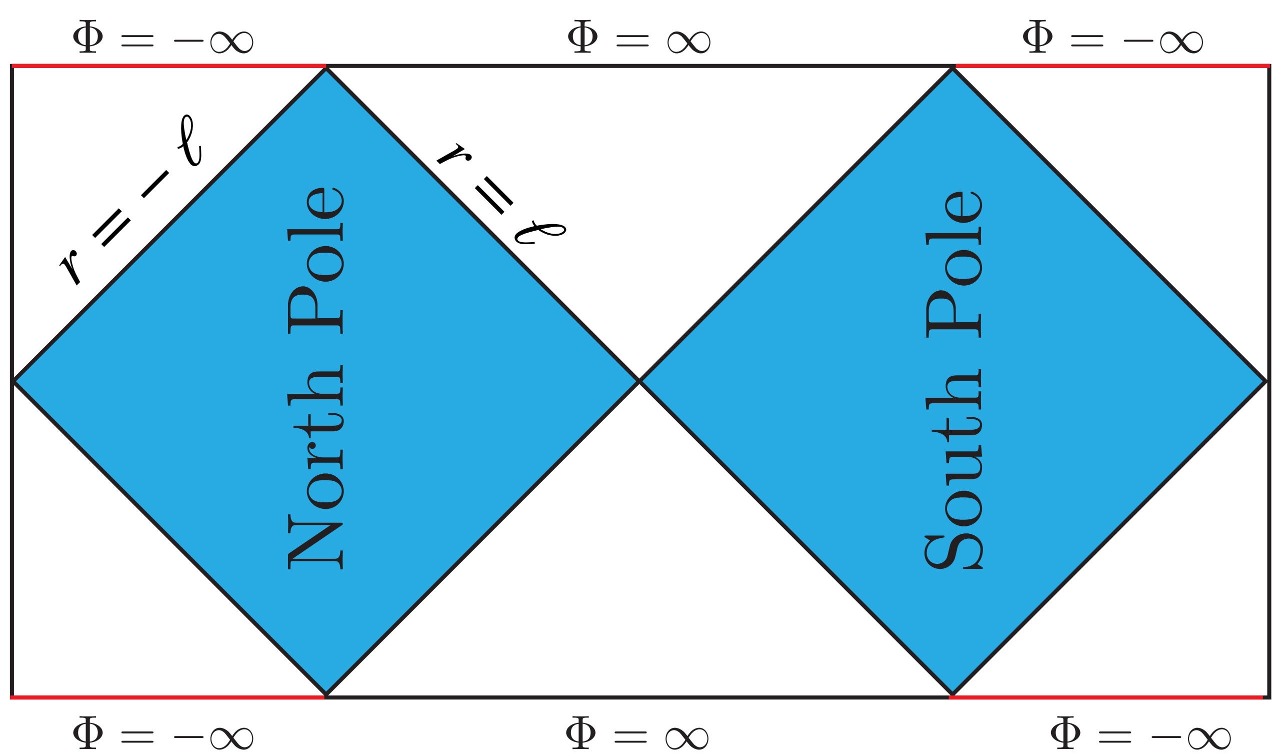
We'll use coordinates that cover a single static patch. First set $T_{ab} = 0$:

$$ds^2 = -\left(1 - r^2/\ell^2\right) dt^2 + \left(1 - r^2/\ell^2\right)^{-1} dr^2$$

$$\Phi = \phi_0 \frac{r}{\ell}$$

Entropy of BH + dS horizon:

$$S = \frac{4\pi}{\kappa^2} \Phi_0$$



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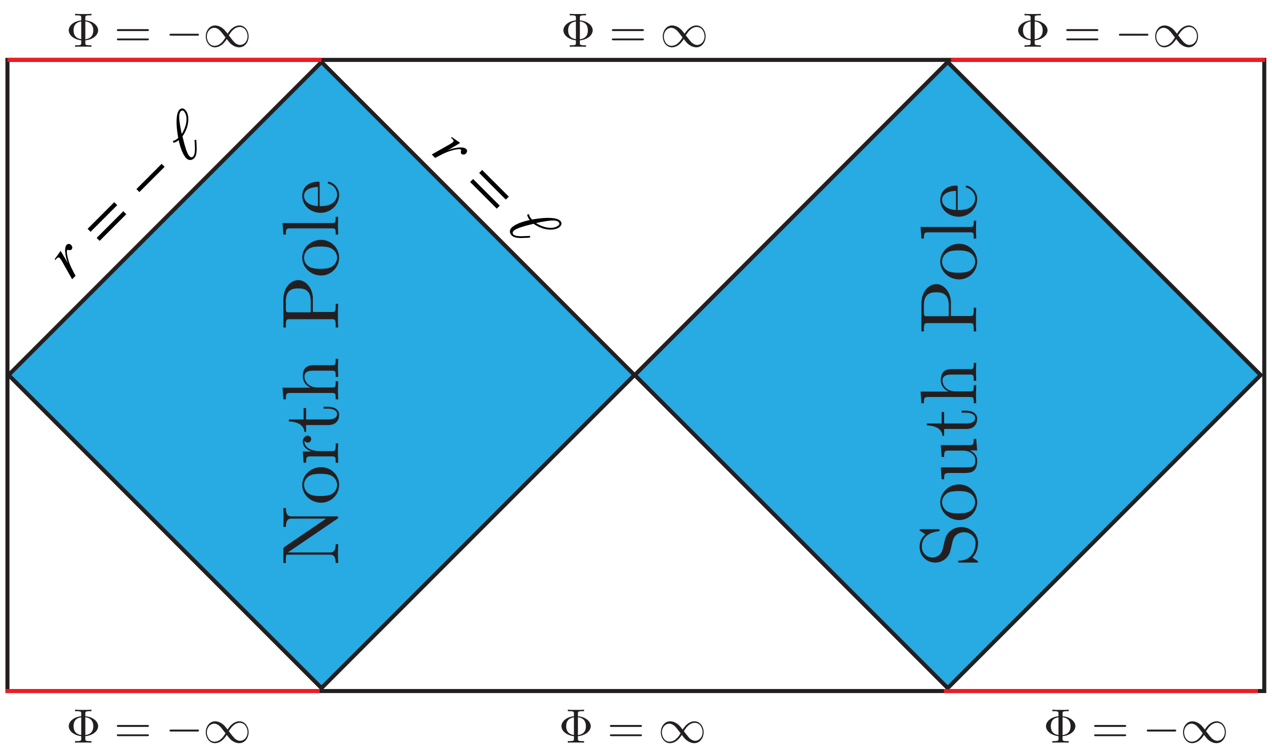

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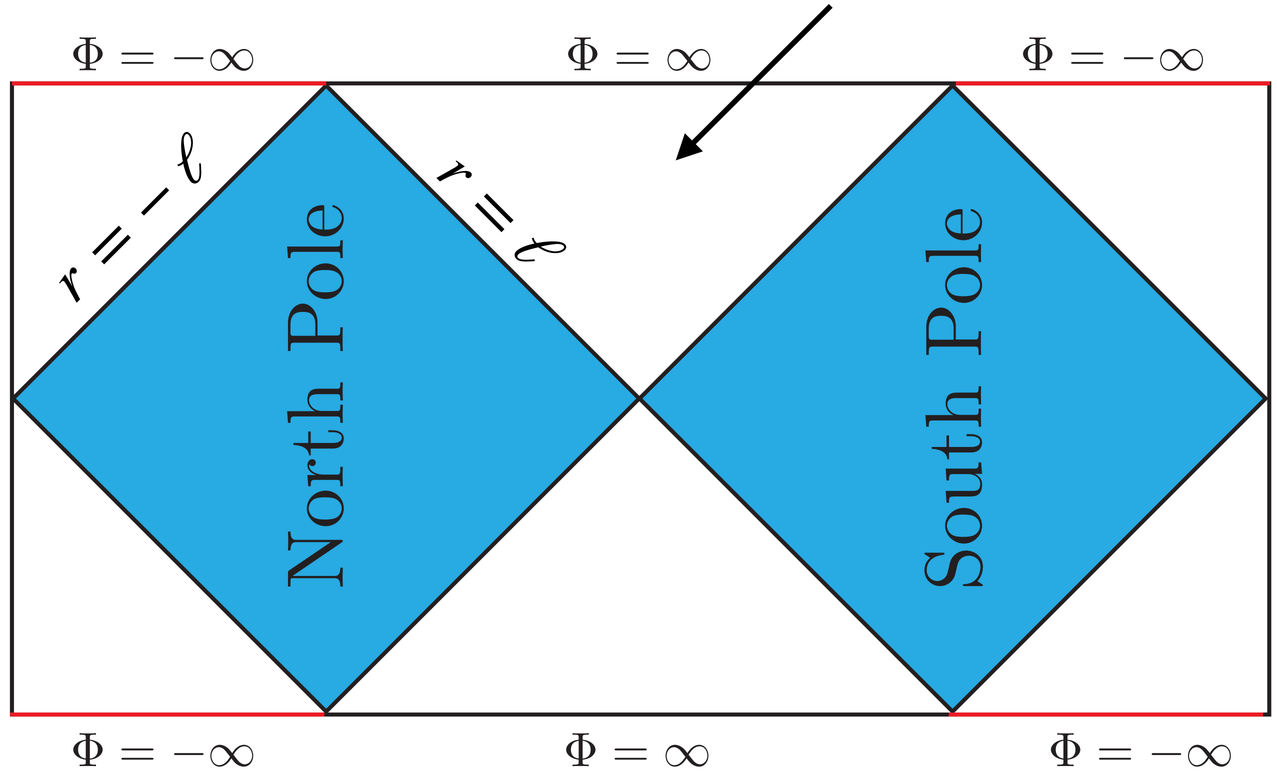

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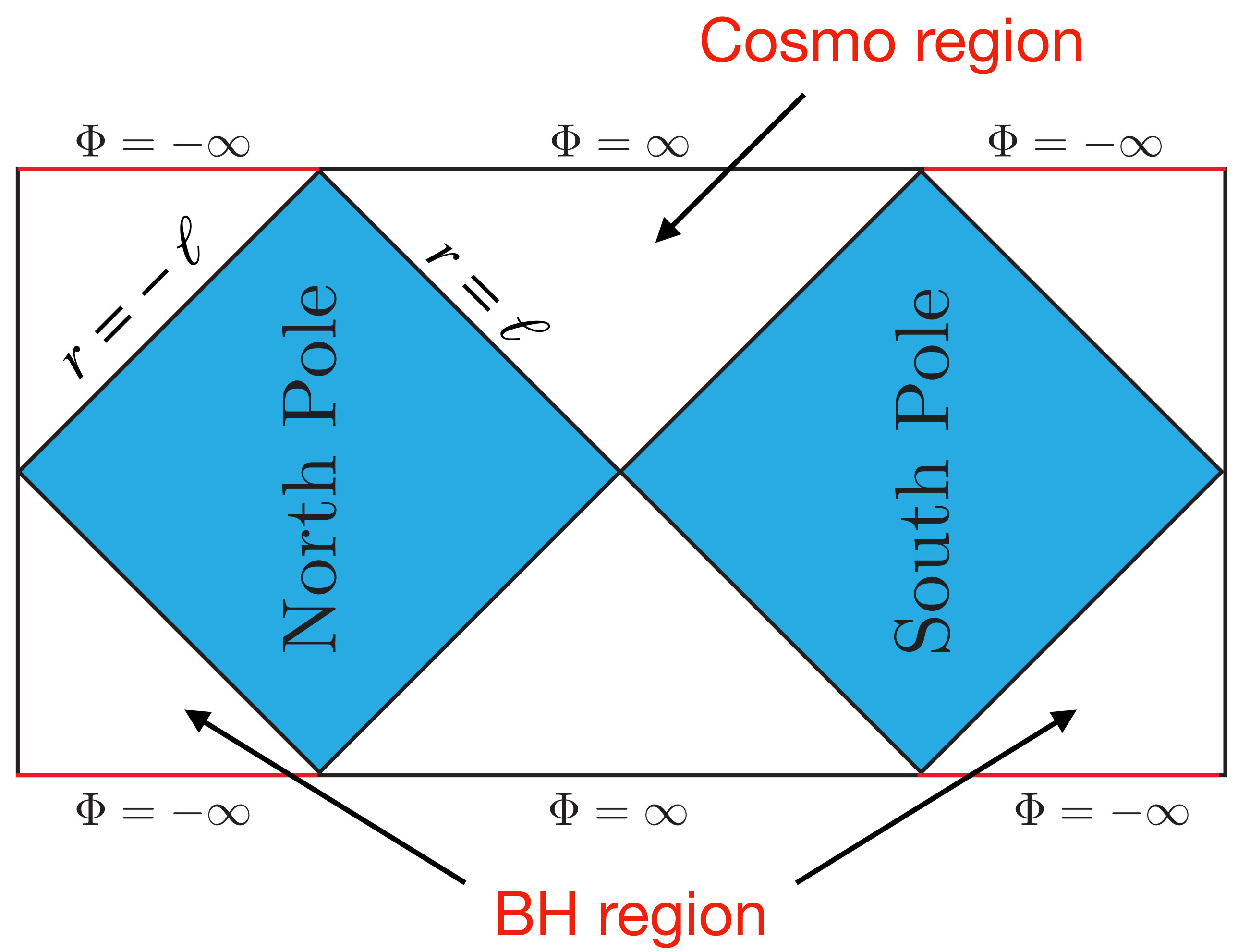

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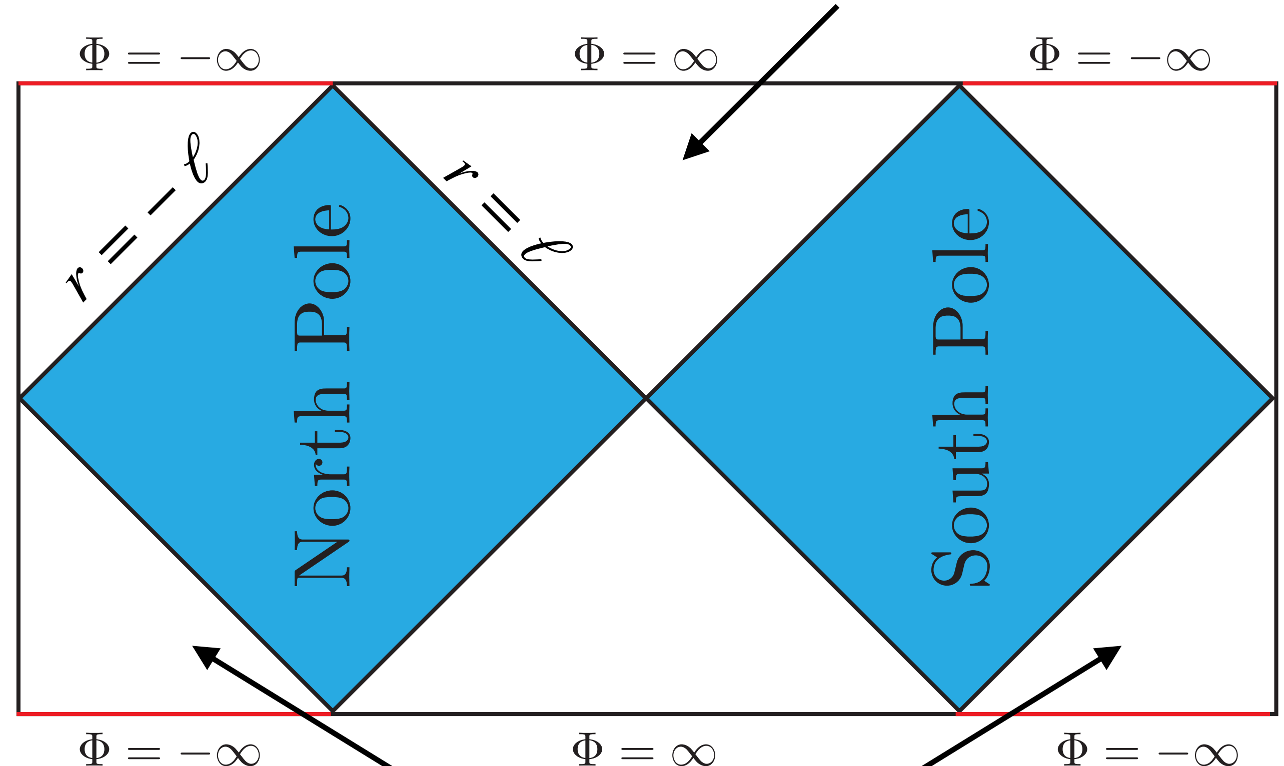
Since $\Phi \sim 1/G_{\text{eff}}$ only non-gravitational regions are at $I^{\pm}$

Cosmo region

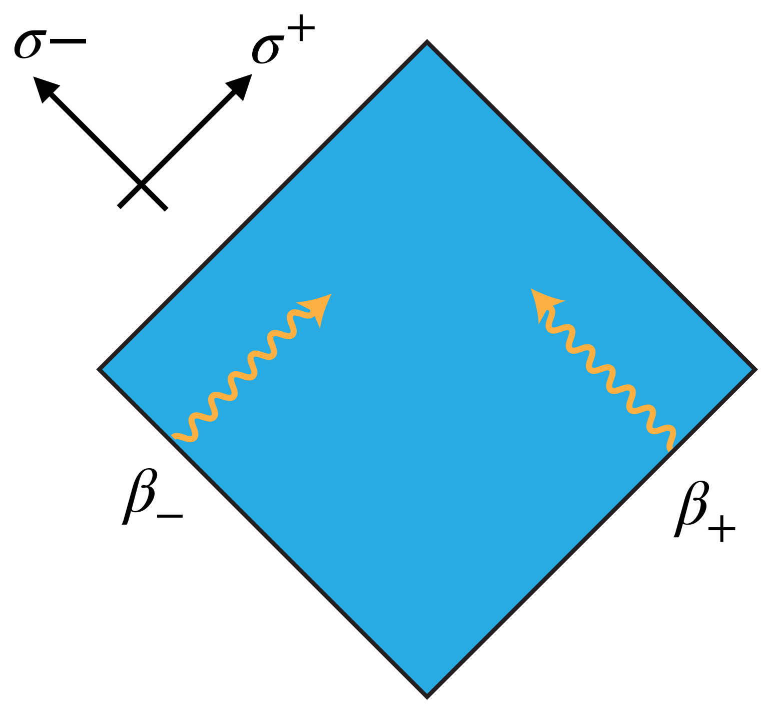


BH region

Bunch-Davies Vacuum

Now we turn on (conformal) matter. The standard vacuum state is Bunch-Davies.

We can exactly solve backreaction. Using null coordinates $\sigma^{\pm} = t \pm r_*$, we find:



$$\langle T_{\pm\pm}(\sigma^{\pm})\rangle = \frac{\pi c}{12\beta_{\pm}^2}$$

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This is a thermal equilibrium, no time dependence!

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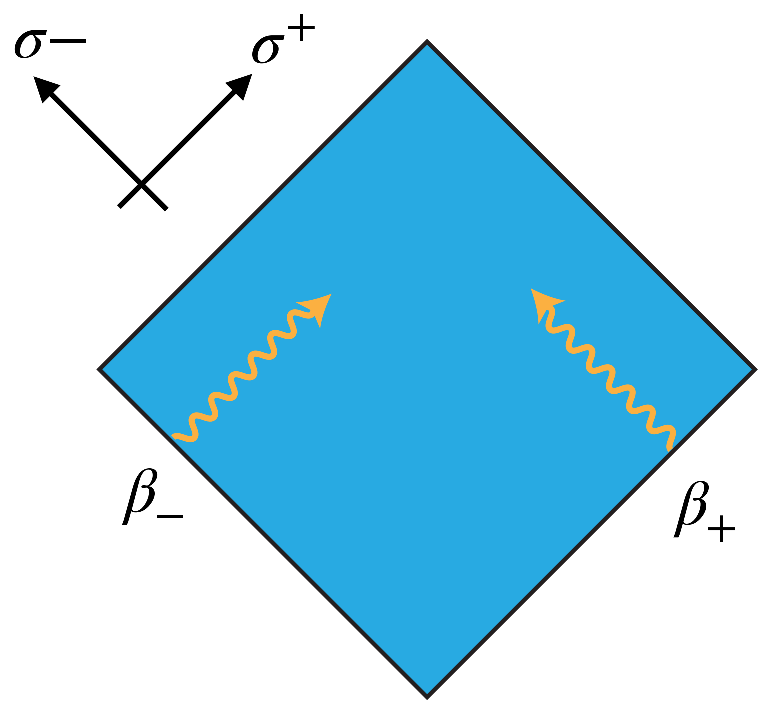

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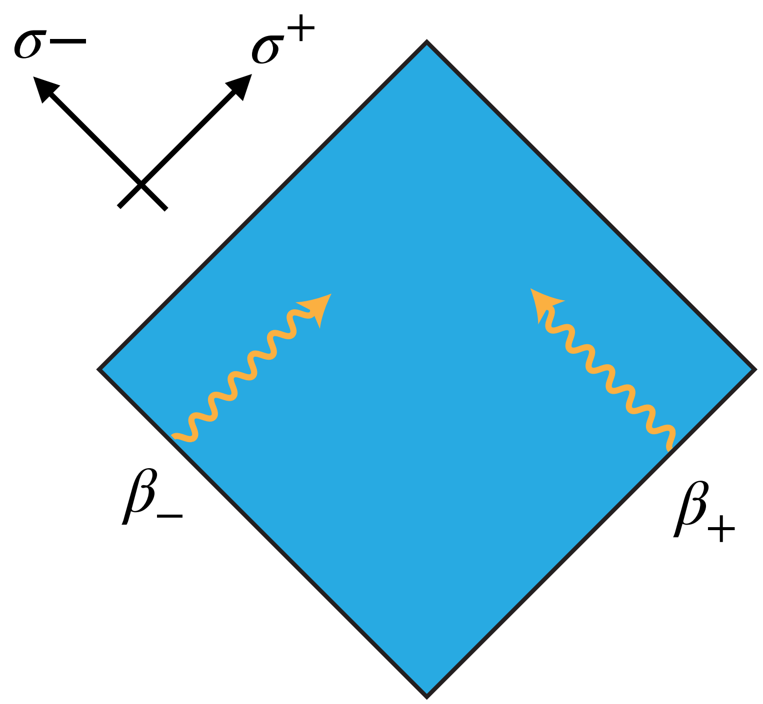

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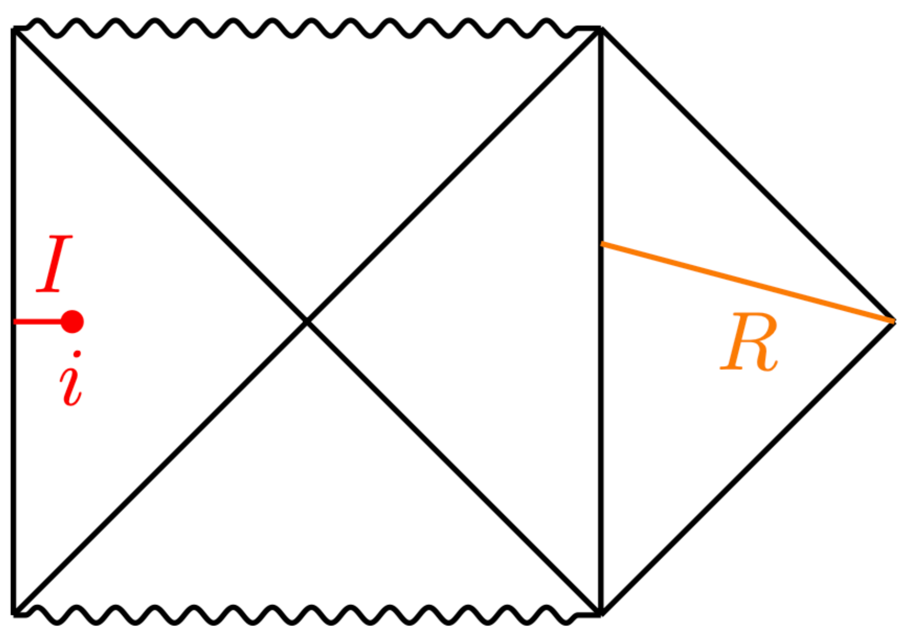
Entropy of Radiation

We'll now compute the entropy to see if there's an information paradox.

To compute the entropy of the radiation, we use the island formula.
[Engelhardt, Wall '14] [Review: Maldacena, Turiaci, Yang '19][Svesko, Verheijden, E. Verlinde, Visser '22] + [...]



$$S(R) = \textbf{min, ext}_i \left[\frac{A(i)}{4G_N} + S_{\rm vN}(R \cup I) \right]$$

Two options:

1. $I = \varnothing$: (Hawking saddle)

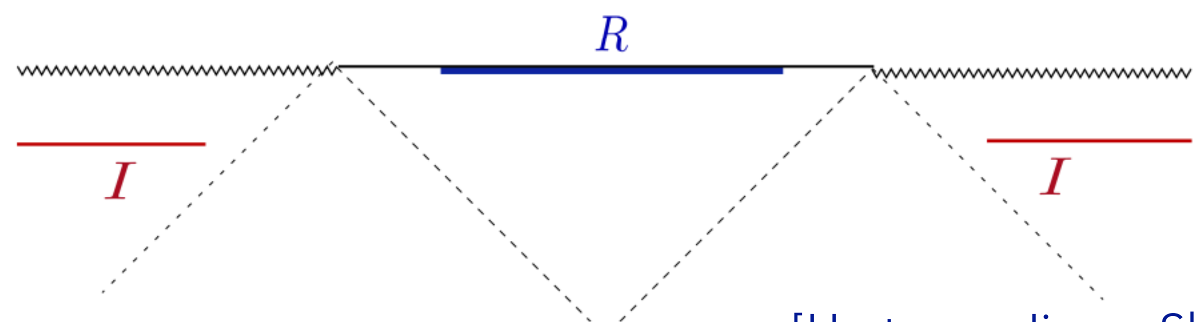
2. $I =$ non-trivial (Page saddle)

Best understood when region R is non-gravitational.

Islands in de Sitter

Where can we put a “reservoir” in 2d de Sitter? Different approaches have been taken:

Black Hole in dS island:

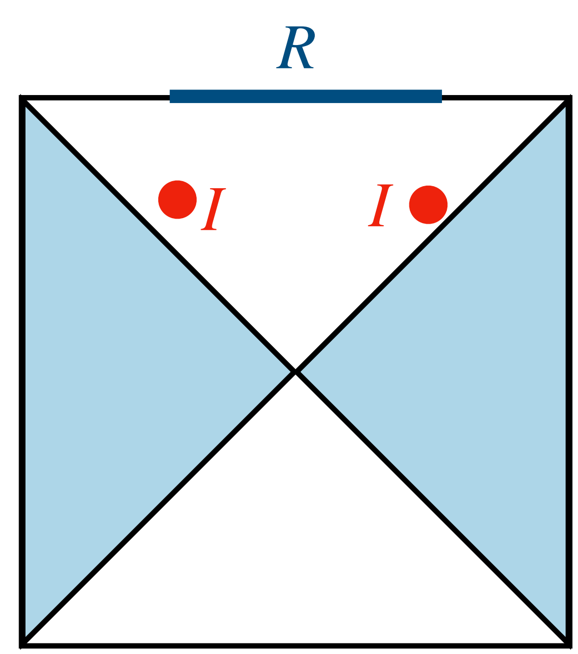


+ : Conventional islands, finite entropy

- : Island associated to the BH

[Hartman, Jiang, Shaghoulian ’20]
[Balasubramanian, Kar, Ugajin ’20]
[Levine, Shaghoulian ‘22]
+ […]

Inflationary island:



+ : Pure dS space, finite entropy

- : Islands are timelike separated, contour ambiguity

[Chen, Gorbenko, Maldacena ’20]
+ […]

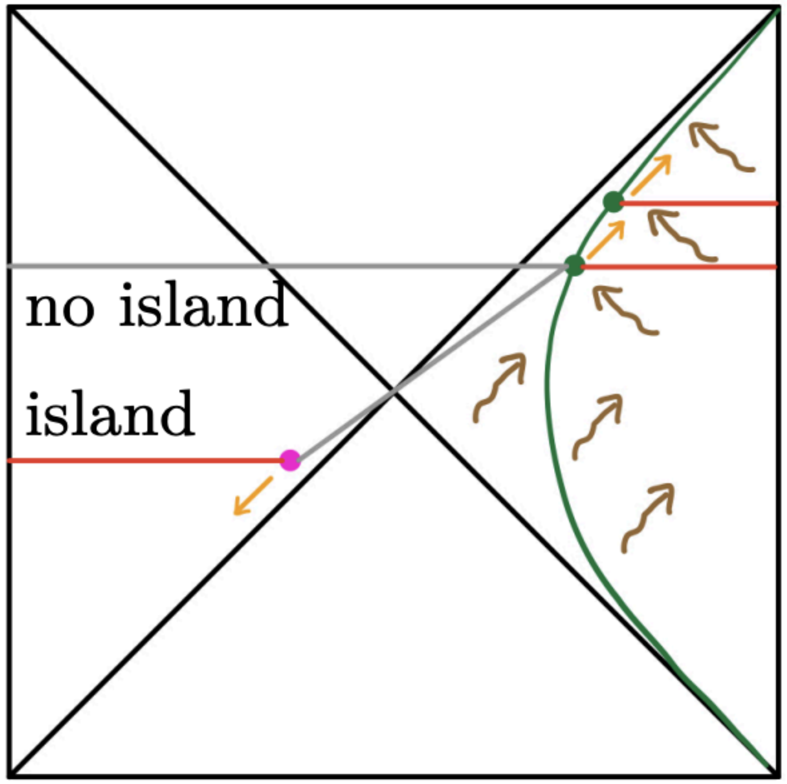
Islands in de Sitter

Let's take the perspective from a static observer.

Static reservoir island:

+ : Pure dS, static patch perspective

- : Freeze gravity by hand,
  no dynamics, island backwards
  in time



[Sybesma '20]

To have a sensible island for a static patch observer we want:

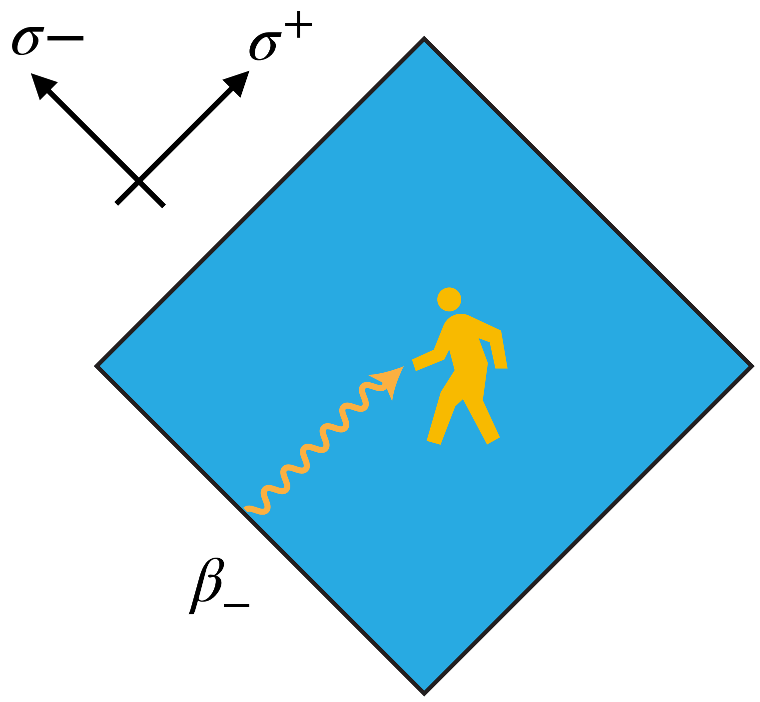
1. Region of weak gravity.

2. Dynamics.

Breaking Thermal Equilibrium

We are interested in computing entropy collected by an observer.

The appropriate state is non-equilibrium. [LA, Parikh, van der Schaar '19][LA, Sybesma '21]



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Leads to non-trivial backreaction:

$$\Phi = \phi_0 \frac{r}{\ell} - \frac{c\kappa^2}{96\pi}\left(1 + 2\frac{r}{\ell}\log(x^+/\ell)\right)$$

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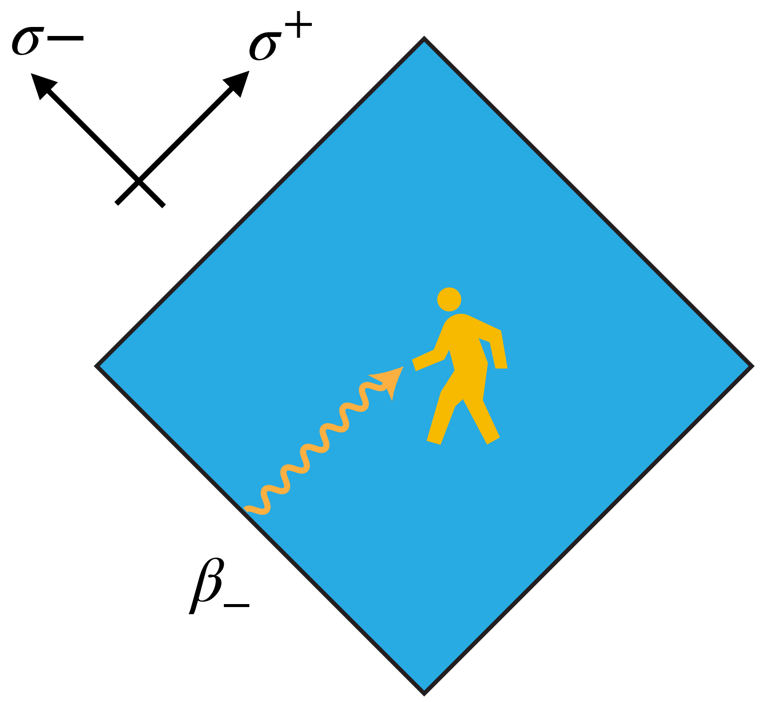

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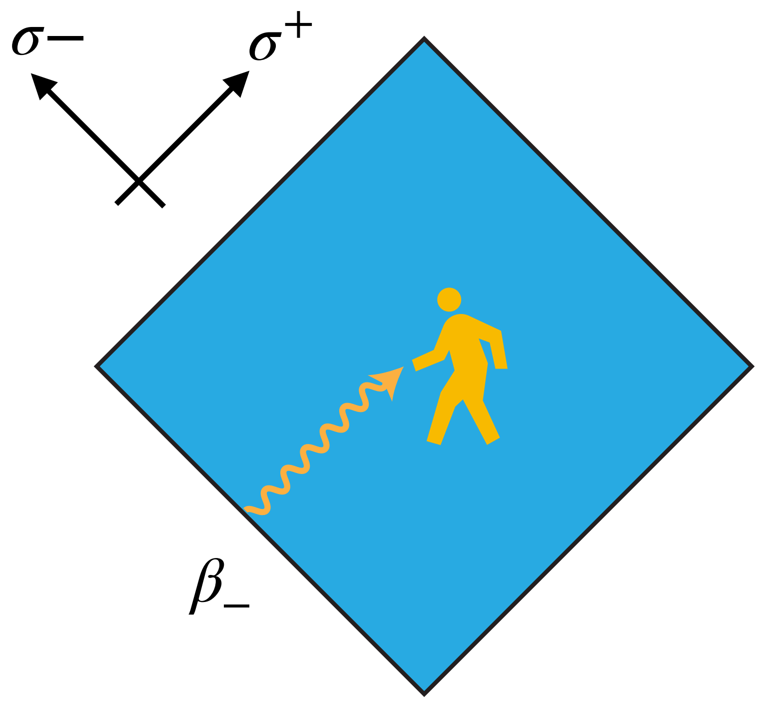

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Time dependence!

Modified Penrose Diagram

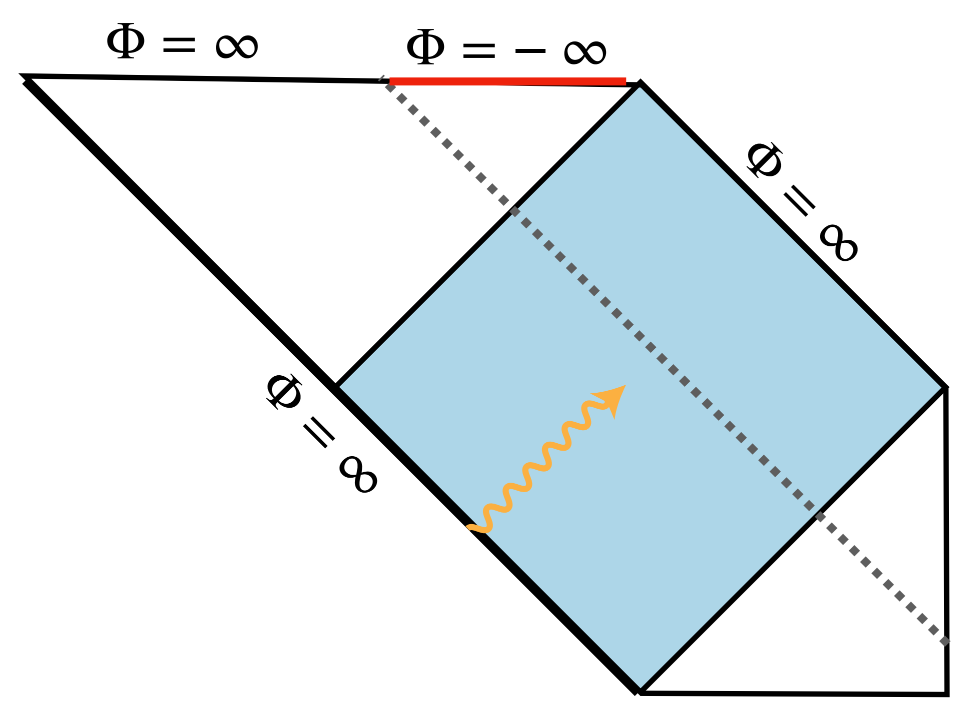
The Penrose diagram now contains additional regions of weak gravity + singularities.

In $x^{\pm}$ coordinates, the stress tensor is:

$$x^{\pm} = \pm \ell e^{\pm\sigma^{\pm}/\ell}$$

$$T_{++}(x^{+}) = -\frac{c}{48\pi(x^{+})^2} \longleftarrow \text{signals flux}$$

$$T_{--}(x^{-}) = 0$$



This kills two birds with one stone!

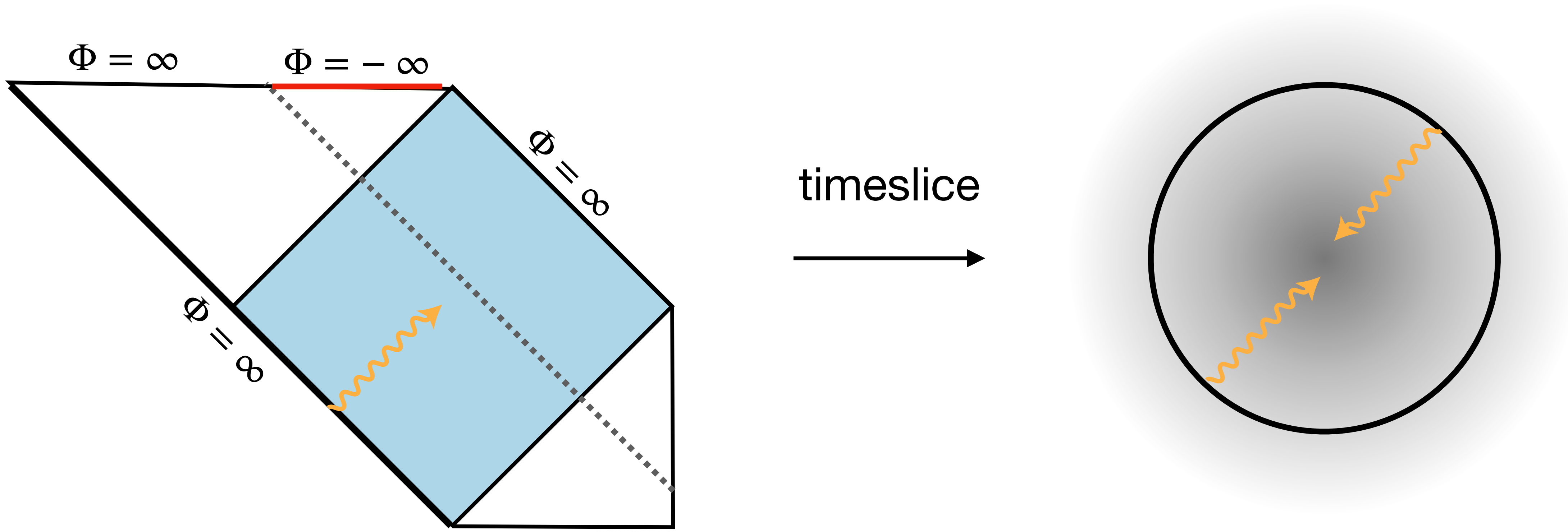
1. Generates a flux of radiation that we can compute the entropy of.

2. Introduces a region where gravity decouples.

Evolution of the Solution

The chosen vacuum state has an "eternal" net flux.

This leads to backreaction:

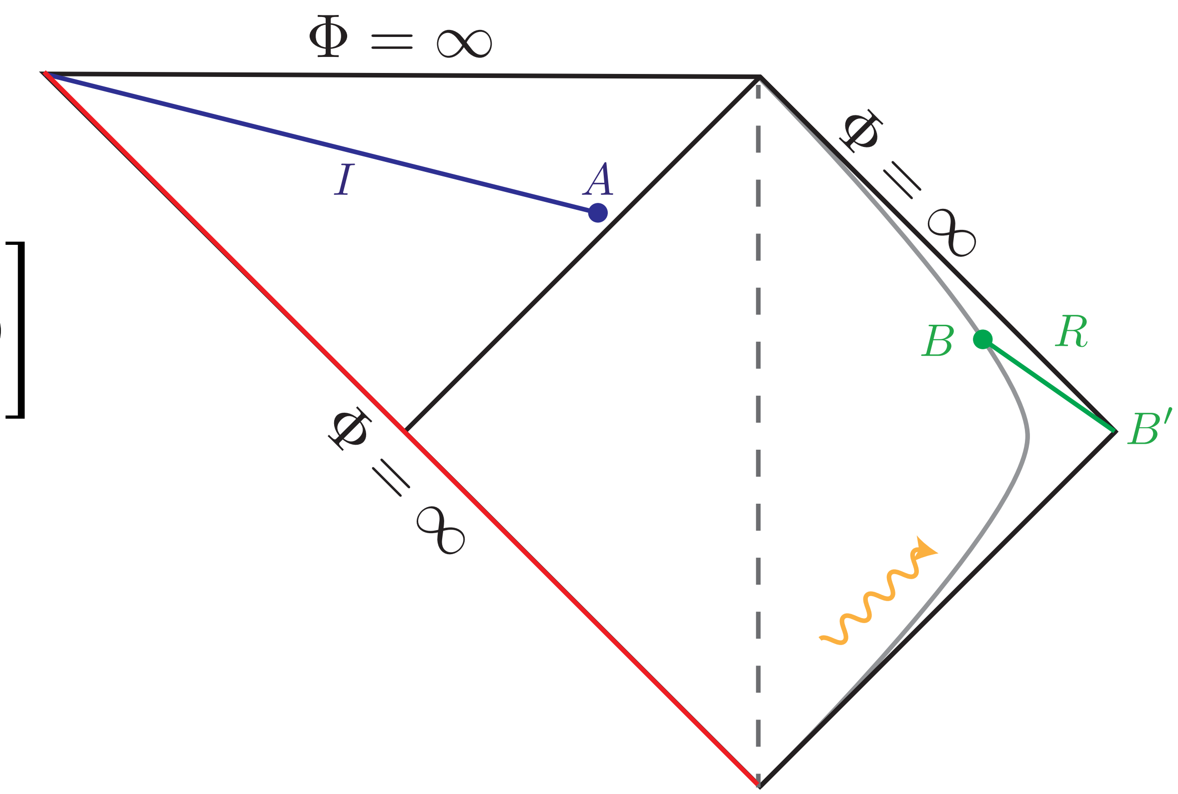


The cosmological horizon shrinks. When $t \simeq \left(\phi_0/c\right)\ell$, a singularity forms.

How does this compare with a putative Page time?

Entropy in de Sitter Space

We now compute the entropy of radiation in our non-equilibrium state.



$$S(R) = \textbf{min, ext}_i \left[\frac{2\pi}{\kappa^2} \left(\Phi_0 + \Phi(i) \right) + S_{\mathrm{vN}}(R \cup I) \right]$$

Extremizing, we find:

1. Trivial island: $S(R) = S_{\mathrm{vN}}(R) = \dfrac{c}{12\ell} t$

2. Non-trivial: $S(R) \simeq \dfrac{2\pi}{\kappa^2} \left(\Phi_0 + \Phi(A) \right)$

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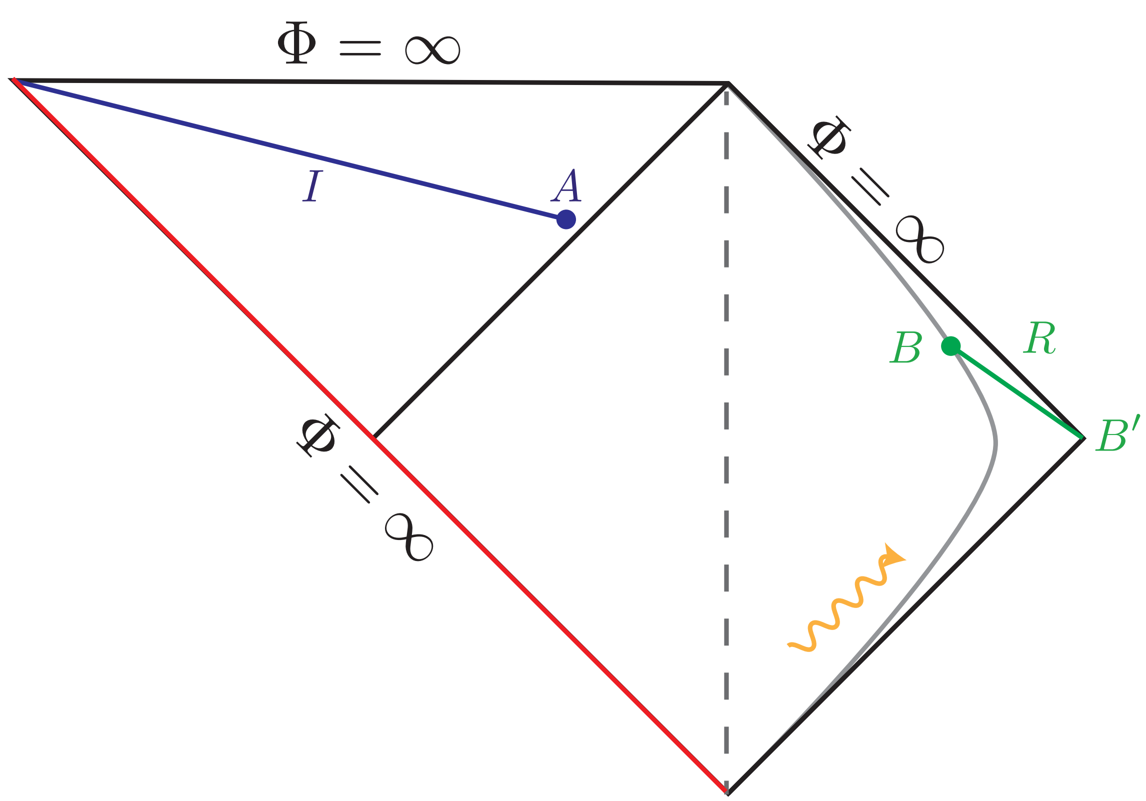

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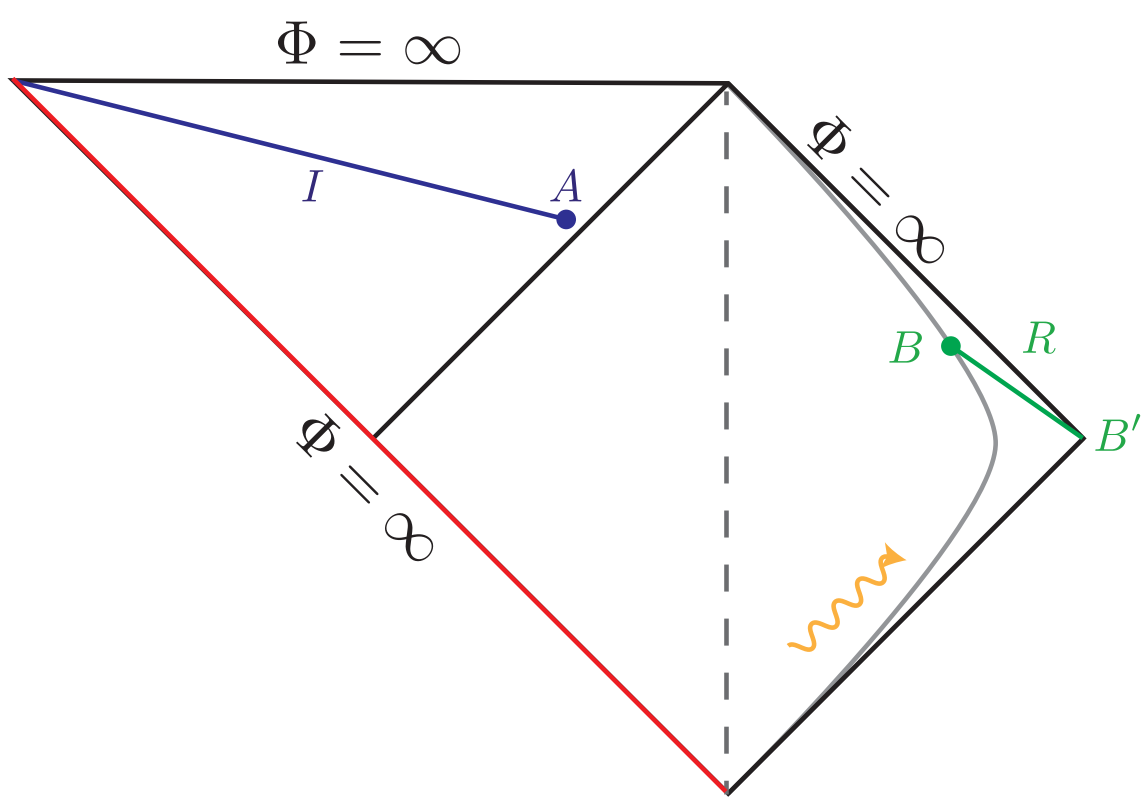

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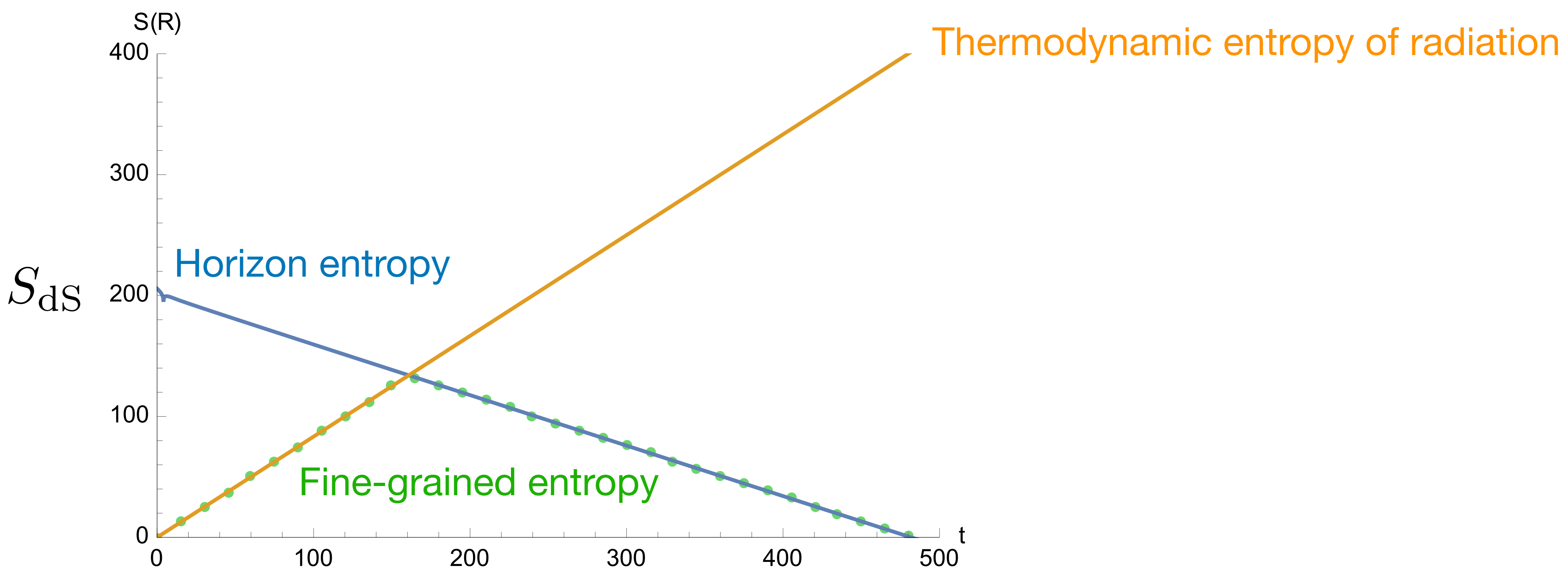
2. Non-trivial: $S(R) \simeq \dfrac{2\pi}{\kappa^2} \big(\Phi_0 + \Phi(A)\big)$ ⟵ Decreasing horizon entropy

Page Curve for de Sitter

Taking the minimum of the two saddles, we get a Page curve.



An estimate is given by the value of the dilaton at the horizon. This becomes zero when:

$$t_{\rm end} \simeq \frac{\ell}{c}\left(\Phi_0 + \Phi(t=0)\right) \quad \Rightarrow \quad t_{\rm Page} = t_{\rm end}/3$$

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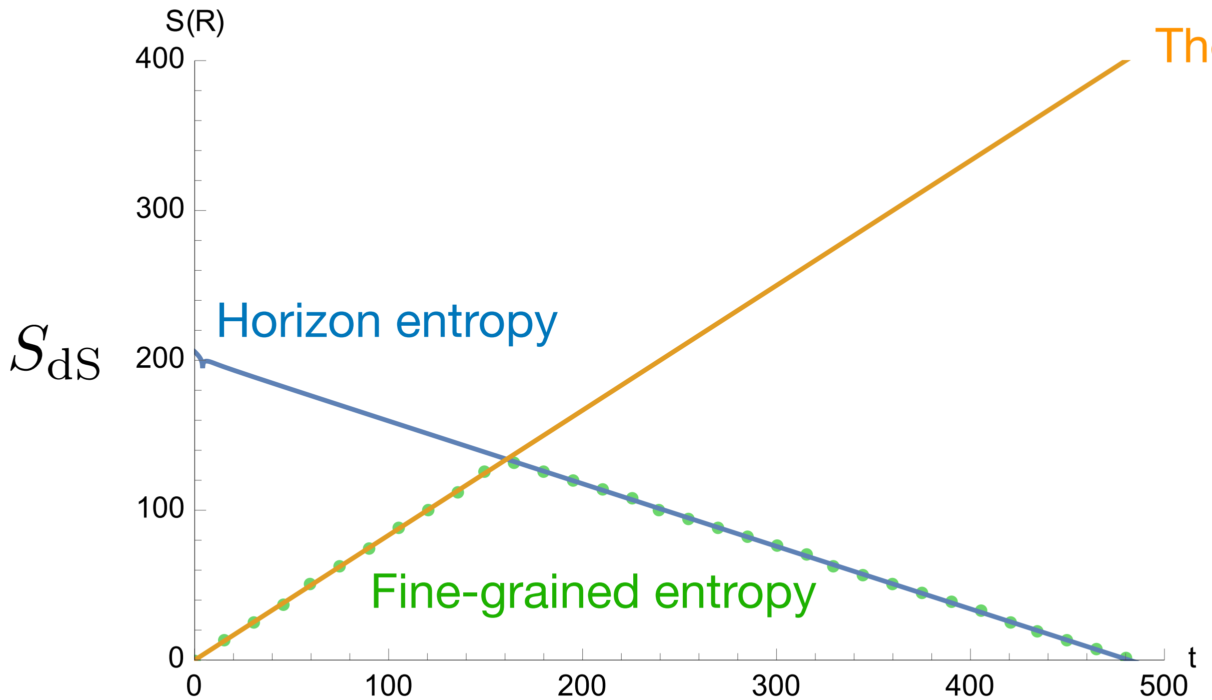


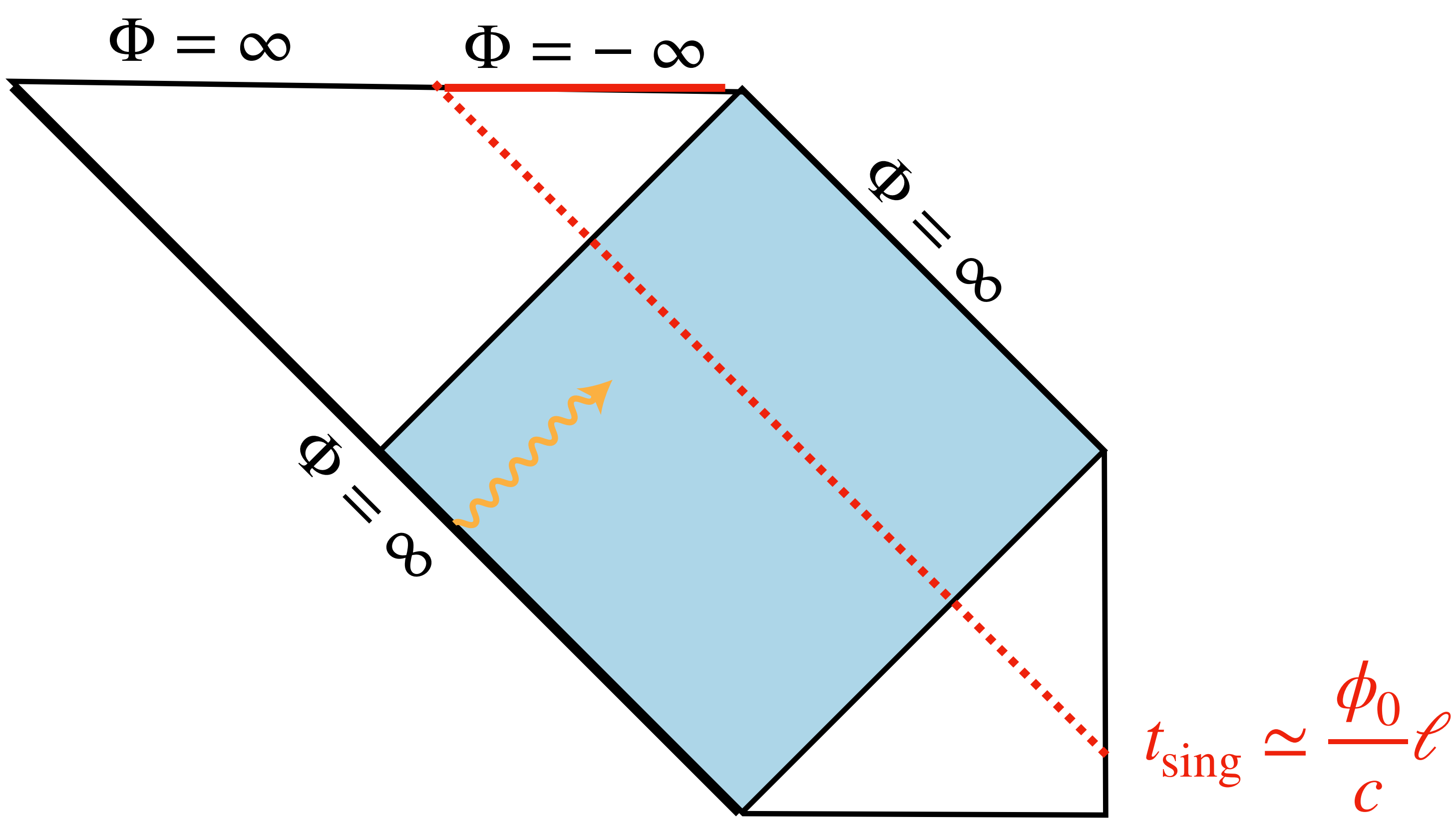
What about backreaction?

An estimate is given by the value of the dilaton at the horizon. This becomes zero when:

$$t_{\text{end}} \simeq \frac{\ell}{c}\left(\Phi_0 + \Phi(t=0)\right) \qquad \Rightarrow \qquad t_{\text{Page}} = t_{\text{end}}/3$$

Formation of Singularity

However, before the Page time formation of a singularity is unavoidable.



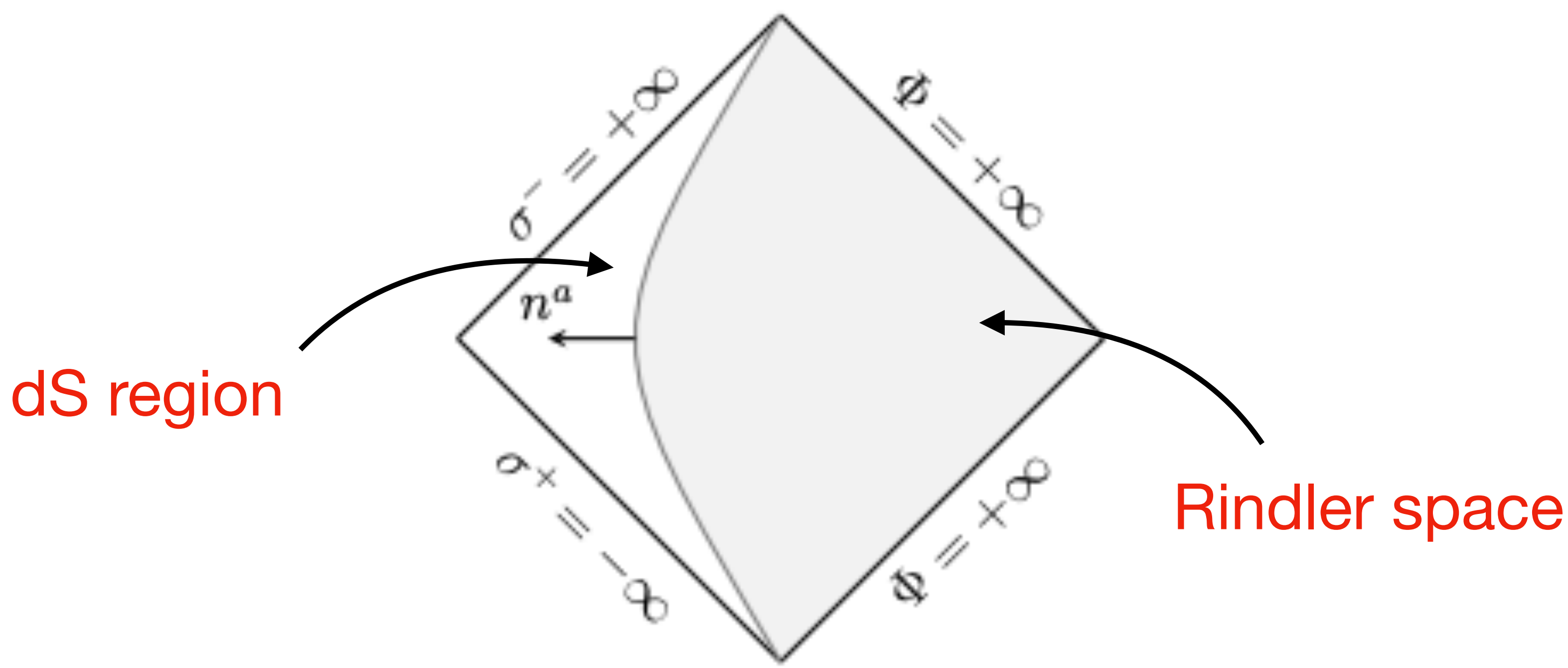
$t_{\text{sing}} \ll t_{\text{Page}} < t_{\text{end}}$

Islands allow for information recovery. However, in this state, guaranteed to lead to a singularity. The observer dies.

Finite Thermal Equilibrium

The situation is better if we break the equilibrium only for a finite time.
[LA, Aguilar-Gutierrez, Sybesma '22]

Need to introduce a bath region separated by a domain wall to have weak gravity.



For this to be a JT solution, need to satisfy junction conditions.

Junction Conditions

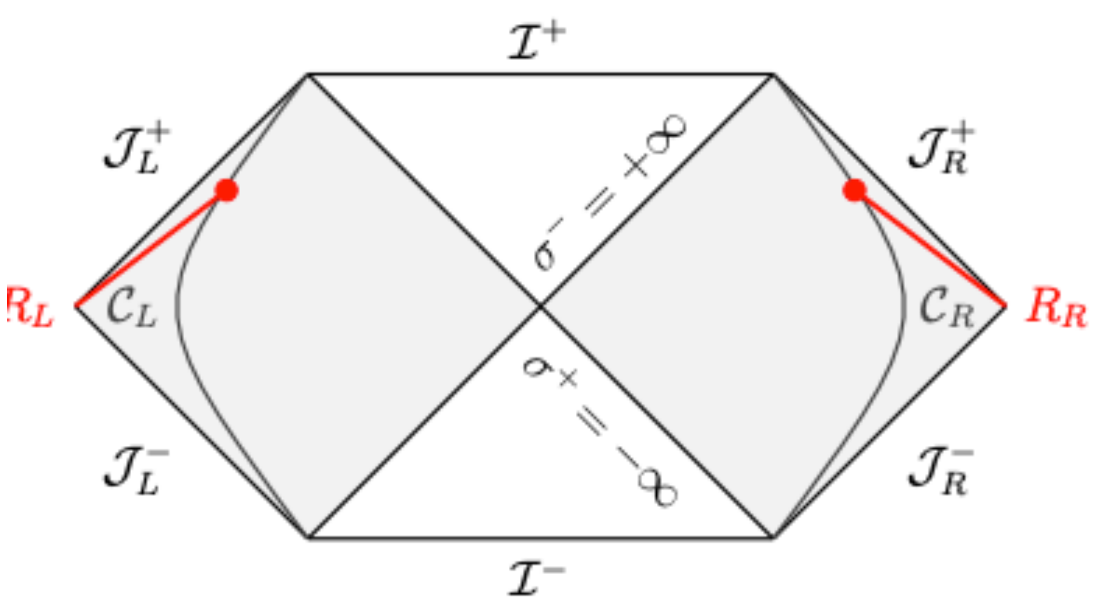
In JT gravity, the junction conditions are given by: [Engelhardt, Folkestad '22]

$$[\Phi]\Big|_{\textbf{horizon}} = 0$$

$$\kappa^2 T_{ab} l^a l^b + [l^a \nabla_a \Phi]\delta(x^-) = 0$$

These can be solved for different quantum states.

In Bunch-Davies we find that the Rindler region "eats" the static patch.

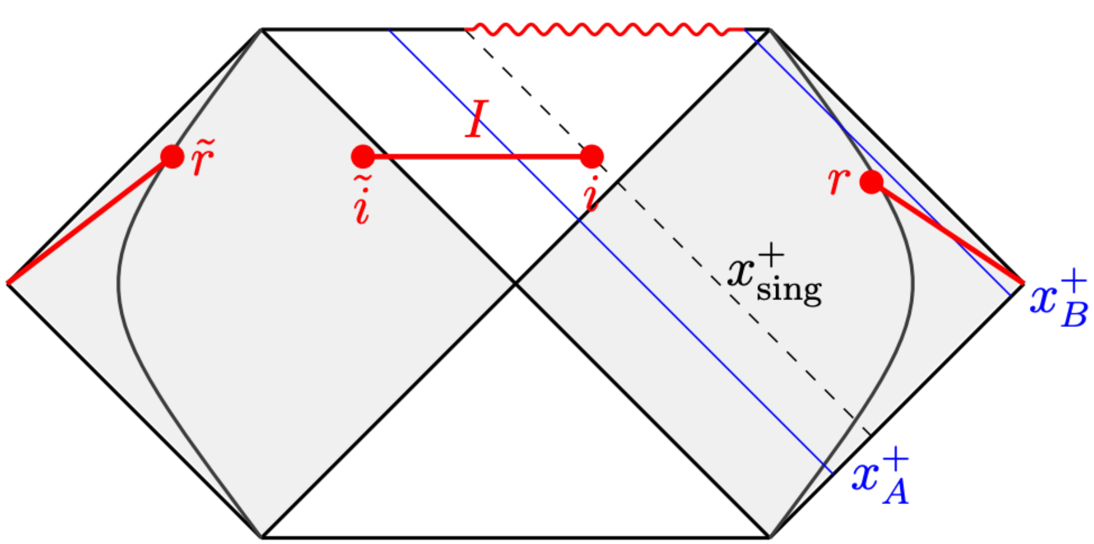


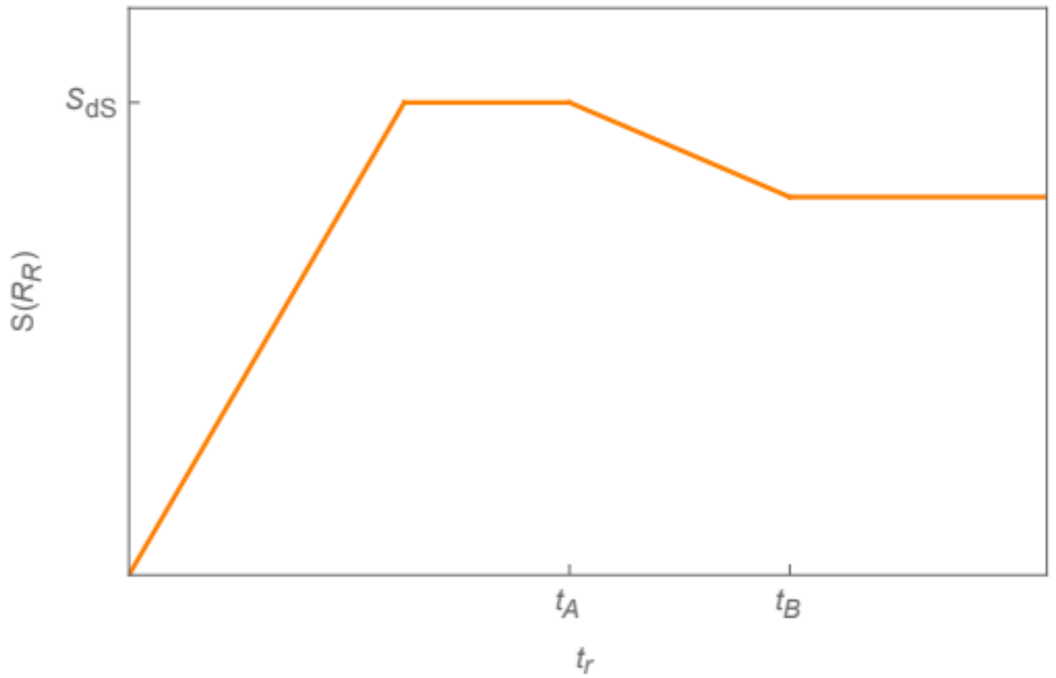
Breaking Thermal Equilibrium

[LA, Aguilar-Gutierrez, Sybesma '22]

We can now break the thermal equilibrium for a finite time.

This introduces an island and allows for information recovery.





Still, it's true that $t_{\text{sing}} \sim t_{\text{recovery}}$. Non-trivial islands in dS require large backreaction.

Layout

- Motivation
- Symmetries of the Cosmological Horizon
- JT Gravity + Matter
- Entanglement and Islands
- **Outlook**

Summary

De Sitter space has several properties that make it difficult to use the island formula to probe S_{dS}:

- **Finite entropy suggests only a subset of isometries should be preserved.**
- **Within the static patch, there is no non-gravitating region.**
- **To define a dynamic subsystem within the static patch requires breaking thermal equilibrium.**

These points are addressed in a non-standard non-equilibrium state.

Islands contribute, but a singularity is unavoidable.

Less drastic state modifications?

Some open questions

In future work, would be interesting to see:

- If we can extend the island formula for gravitating regions. [Bousso, Penington '22]
- If island effects play any role in 4d (inflationary) cosmology / eternal inflation.
- If we can better understand the role of the observer, needed to define the algebra of observables. [Chandrasekaran, Longo, Pennington, Witten '22]
- If we can study islands in microscopic dS models.

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There is lots to be explored!

Thank you!

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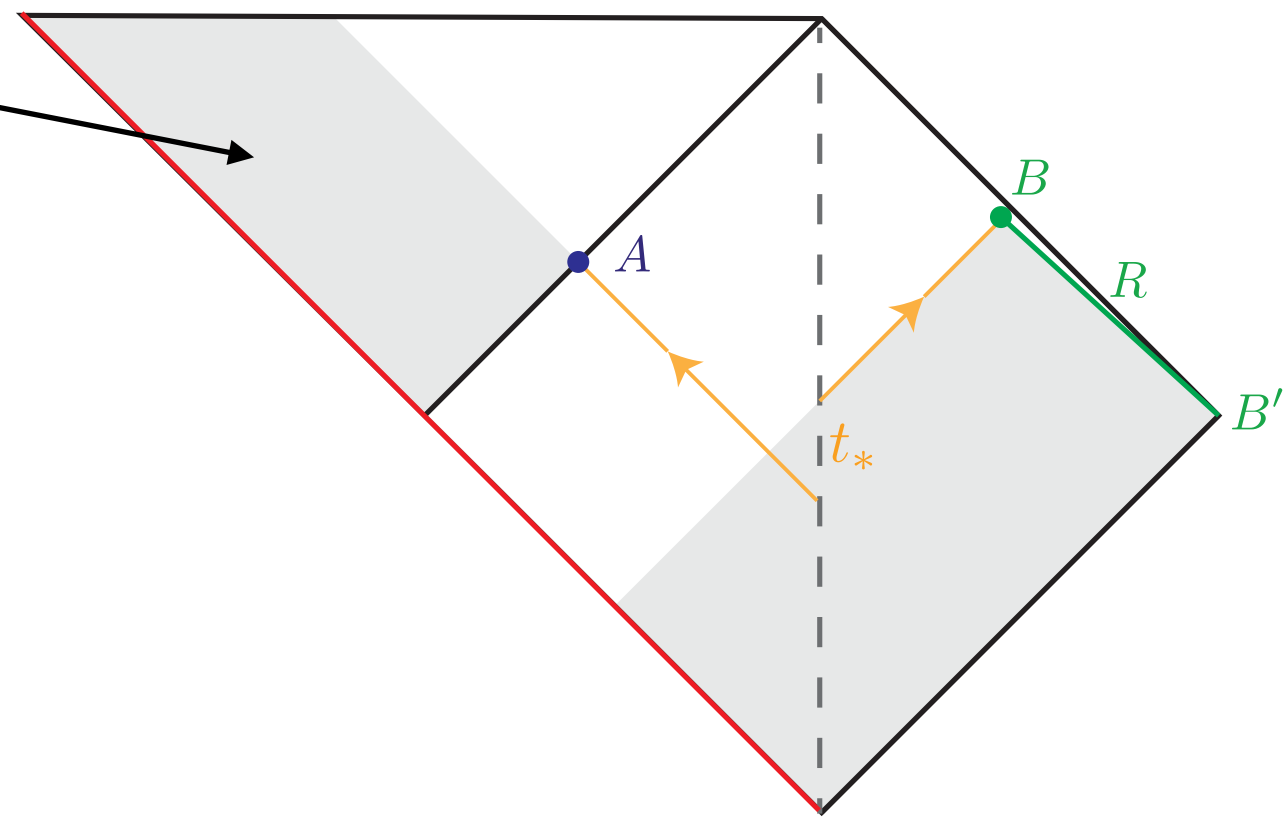
Scrambling Time

- The scrambling time is given by the time difference between sending and recovering.

Gray regions indicate entanglement wedge.



Using the location of the island, we can compute when a lightray that exits the static patch enters the entanglement wedge.

Around Page time: $t_* \simeq \ell \log(S_{\mathrm{dS}})$