Hilbert space and holography of information in de Sitter quantum gravity

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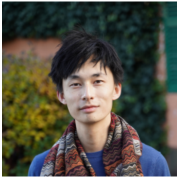
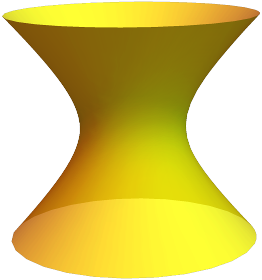


Victor Godet

- `2303.16316` and `2303.16315`, Joydeep Chakravarty, Tuneer Chakraborty, Victor Godet, Priyadarshi Paul, S.R.

Overview: Two questions.



Focus on asymptotically dS spacetime from a global perspective.

- What is the Hilbert space for gravity in such a spacetime?
- Gravity localizes information unusually. How does holography of information work in such a spacetime?

Outline

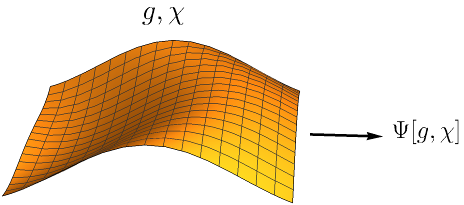
Motivation

Hilbert space

Cosmological correlators

Holography of information

Wavefunctionals



States can be represented as wavefunctionals on the late-time slice.

$\Psi[g, \chi]$ assigns an amplitude to a configuration of

metric on a spacelike slice g

and

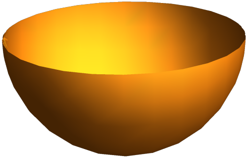
matter fields χ

Vacuum wavefunctional

- We understand the Euclidean vacuum state well.

$$|0\rangle \leftrightarrow \Psi_0[g, \chi]$$

- Computed using the Hartle-Hawking proposal



[Hartle,Hawking, 1983]

- Also computed via analytic continuation from AdS

$$Z_{\mathrm{CFT}}[g, \chi] \to \Psi_0[g, \chi]$$

[Maldacena, 2001]

Other states?

- But

$$|0\rangle$$

is one state.

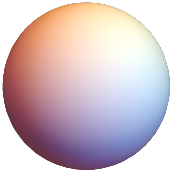
[Anninos, Denef, Monten, Sun, 2017]

- Attempt to construct other states using

$$|\Psi\rangle \stackrel{?}{=} \int \chi(x_1)\ldots\chi(x_n)f(x_1,\ldots x_n)|0\rangle$$

does not work in the presence of gravity.

dS invariance



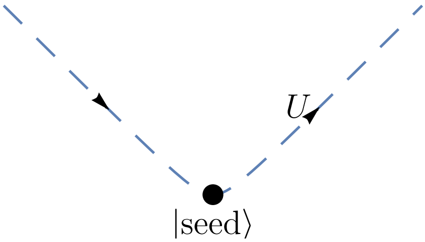
In gravity, charges can be measured at the boundary. But dS spatial slice has no boundaries.

Gauss law: Even as $G_N \to 0$, all states must have zero charges,

$$U|\Psi\rangle = |\Psi\rangle, \qquad \forall\, U \in SO(d+1,1)$$

In original Hilbert space, the only such state is $|0\rangle$!

Higuchi's solution in the nongravitational limit



Starting with a "seed state", construct

$$|\Psi_{\rm ng}\rangle = \int [dU]\, U|\mathrm{seed}\rangle$$

[Higuchi, 1991]

[Moncrief, 1975]

$$\text{Norm:} \qquad (\Psi_{\rm ng}, \Psi_{\rm ng}) = \frac{1}{\mathrm{vol}(SO(d+1,1))}\langle\Psi_{\rm ng}|\Psi_{\rm ng}\rangle_{\rm QFT}$$

There is some evidence for this.

[Marolf, Morrison, 2008]

[Chandrasekaran,Longo,Penington,Witten, 2022]

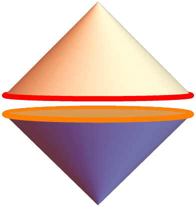
We will derive this prescription systematically and show how to correct it beyond $G_N \to 0$.

Holography of information

Gravity localizes information unusually!

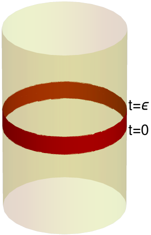
[Laddha, Prabhu, S.R., Shrivastava, 2020]

Asymptotically flat space



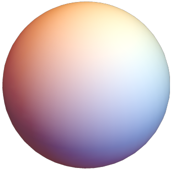
All information about massless particles is present near the past boundary of future null infinity.

Asymptotic AdS



Asymptotic correlators on an infinitesimal time band at the boundary completely fix the bulk state. (Does not assume AdS/CFT)

Holography of information in dS



How does holography of information work in dS?

Step towards: What is the holographic dual of gravity in dS?

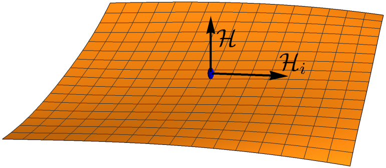
dS/CFT?

$$Z_{\mathrm{CFT}}[g, \chi] \to \Psi_0[g, \chi]$$

is not holography. It is a technique of computing the details of one particular state.

Outline

Constraints of gravity



Wavefunctionals in quantum gravity obey

$$\mathcal{H}\Psi[g,\phi] = 0; \qquad \mathcal{H}_i\Psi[g,\phi] = 0.$$

Procedure: Solve for the Hilbert space by finding a complete basis of solutions to the WDW equation.

WDW equation

Explicitly,

$$\mathcal{H} = 2\kappa^2 g^{-1}\left(g_{ik}g_{jl}\pi^{kl}\pi^{ij} - \frac{1}{d-1}(g_{ij}\pi^{ij})^2\right) - \frac{1}{2\kappa^2}(R - 2\Lambda)$$

$$+ \mathcal{H}_{\text{matter}} + \mathcal{H}_{\text{int}},$$

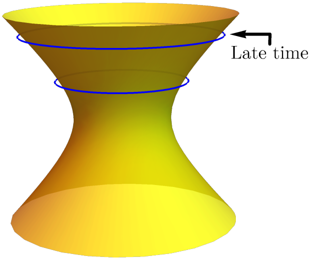
$$\mathcal{H}_i = -2g_{ij}D_k\frac{\pi^{jk}}{\sqrt{g}} + \mathcal{H}_i^{\text{matter}},$$

Simplifying the WDW equation

- In the regime

$$\Lambda \gg R; \qquad \Lambda \gg V_{\text{matter}}$$

the WDW equation turns out to be tractable.



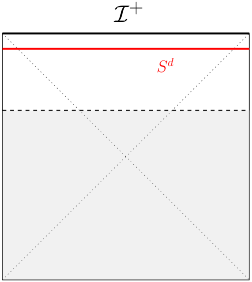
The limit $\Lambda \gg R$ focuses us on the late-time slice.

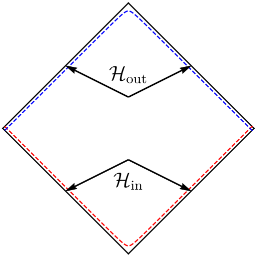
$$\log(\int \sqrt{g})$$

serves as an autonomous clock.

Late-time limit

- Solving WDW at “large volume” gives us “late time” behaviour of the state.
- Sufficient to understand Hilbert space. (cf. asymptotic quantization).
- Insufficient for bulk dynamics/“earlier-time physics”.





Solution

At large volume all solutions of the WDW equation take the form

$$\Psi \longrightarrow e^{iS[g,\chi]} Z[g,\chi]$$

see AdS solutions by Freidel (2008), Regado, Khan, Wall (2022)

1. S is a divergent universal phase factor.
2. $Z[g,\chi]$ is diff invariant and almost Weyl invariant

$$\Omega \frac{\delta Z[g,\chi]}{\delta \Omega(x)} = \mathcal{A}_d[g] Z[g,\chi].$$

   $\mathcal{A}_d$ is an imaginary local function of g in even d for dS_{d+1}.
3.

$$|Z[g,\chi]|^2$$

   is Weyl invariant.

Phase factor

The phase factor S contains terms familiar from holographic renormalization.

$$S = \frac{(d-1)}{\kappa^2}\int \sqrt{g}\,d^dx - \frac{1}{2\kappa^2(d-2)}\int \sqrt{g}\,R\,d^dx + \dots$$

[Papadimitriou, Skenderis, 2004]

It comprises integrals of local densities.

It doesn't depend on details of state.

Cancels out in $|\Psi[g,\chi]|^2$.

Expansion of $Z[g, \chi]$

After Weyl transformation to frame

$$g_{ij} = \delta_{ij} + \kappa h_{ij}$$

Expand

$$Z[g, \chi] = \exp[\sum_{n,m} \kappa^n \mathcal{G}_{n,m}]$$

with

$$\mathcal{G}_{n,m} = \int d\vec{y} d\vec{z} \, G^{\vec{i}\vec{j}}_{n,m}(\vec{y}, \vec{z}) h_{i_1 j_1}(z_1) \dots h_{i_n j_n}(z_n) \chi(y_1) \dots \chi(y_m),$$

Coefficient fns obey same Ward identities as CFT correlators.

$$G^{\vec{i}\vec{j}}_{n,m}(\vec{y}, \vec{z}) \sim \langle T^{i_1 j_1}(y_1) \dots T^{i_n j_n}(y_n) \phi(z_1) \dots \phi(z_m) \rangle^{\text{connected}}_{\text{CFT}} \, ,$$

"CFT" has imaginary central charge in even d. Not necessarily local or unitary.

Hartle-Hawking state and other states

$$\Psi_0 = e^{iS} \exp[\sum_{n,m} \kappa^n \mathcal{G}_{n,m}]$$

[Pimentel, 2013]

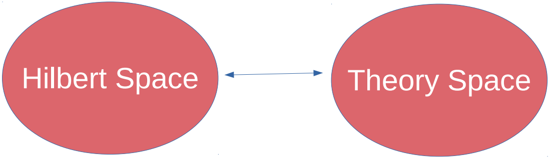
Not just the Hartle-Hawking state but all states have this form.

Interactions do not constrain precise form of $\mathcal{G}_{n,m}$ beyond conformal invariance of coefficient fns.

State space as theory space

List of correlators $\{G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z})\} \longrightarrow$ WDW solution

But list of correlators can be thought of as defining a “theory”.



(**Caution**: there might be additional constraints on allowed states beyond what we have found.)

Small fluctuations basis for states

Starting with $\mathcal{G}_{n,m}$ for H.H. state,

$$\mathcal{G}^{\lambda}_{n,m} = (1-\lambda)\mathcal{G}_{n,m} + \lambda\widetilde{\mathcal{G}}_{n,m}$$

Then

$$\frac{\partial\Psi_\lambda[g,\chi]}{\partial\lambda} = \sum_{n,m} \kappa^n \delta\mathcal{G}_{n,m}\Psi_0[g,\chi]$$
$$= \sum_{n,m} \kappa^n \int d\vec{x}\, G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z}) h_{i_1 j_1}(z_1)\dots h_{i_n j_n}(z_n)\chi(y_1)\dots\chi(y_m)\Psi_0[g,\chi]$$

The Ward identities tell us

$$\delta\mathcal{G}_{n,m} \neq 0 \Rightarrow \delta\mathcal{G}_{n+1,m} \neq 0.$$

In general we require an infinite series to satisfy the constraints.

Higuchi states

- When $\kappa \to 0$, Ward identities do not relate $\delta\mathcal{G}_{n,m}$ to $\delta\mathcal{G}_{n+1,m}$.
- Leads to a special class of states at $\kappa \to 0$

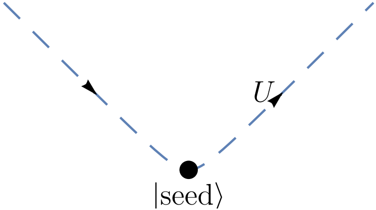
$$|\Psi_{\text{ng}}\rangle = \int d\vec{x} f(x_1, \dots x_n)\chi(x_1)\dots\chi(x_n)|0\rangle$$

where f has the symmetries of a conformal correlator.

- These states are invariant under the dS isometries!

$$U|\Psi_{\text{ng}}\rangle = |\Psi_{\text{ng}}\rangle$$

Correction to Higuchi states



Precisely Higuchi's states!

$$|\Psi_{\text{ng}}\rangle = \int dU\, U|\Psi_{\text{seed}}\rangle;$$

$$|\Psi_{\text{seed}}\rangle \propto \int d^d x_4 \dots d^d x_m f(\hat{x}_1, \hat{x}_2, \hat{x}_3, x_4, \dots, x_m) \\ \chi(\hat{x}_1)\chi(\hat{x}_2)\chi(\hat{x}_3)\chi(x_4)\dots\chi(x_m)|0\rangle$$

Away from $\kappa \to 0$,

$$|\Psi\rangle = \sum \kappa^n \delta\mathcal{G}_{n,m}|0\rangle$$

Lowest order term is Higuchi's construction.

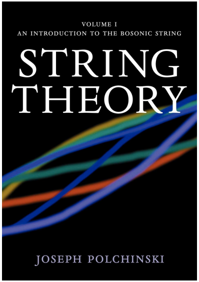
Our solution justifies Higuchi's construction and provides gravitational corrections to it.

Proposal for norm

We propose

$$(\Psi,\Psi) = \frac{1}{\mathrm{vol(diff\times Weyl)}} \int DgD\chi \sum_{n,m,n',m'} \kappa^{n+n'} \delta\mathcal{G}^*_{n,m}\delta\mathcal{G}_{n',m'}|Z_0[g,\chi]|^2$$

Proposal is not unique. But natural and simple.



Fixing gauge

$$\text{Fix gauge}: \qquad \sum_i \partial_i g_{ij} = 0; \qquad \delta^{ij} g_{ij} = d$$

Gauge choice leaves behind residual global transformations.

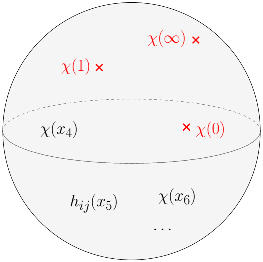
$$\begin{aligned} \text{translations}: &\quad \xi^i = \alpha^i; \\ \text{rotations}: &\quad \xi^i = M^{ij} x^j \\ \text{dilatations}: &\quad \xi^i = \lambda x^i \\ \text{SCTs}: &\quad \xi^i = (2(\beta \cdot x) x^i - x^2 \beta^i) + v^i_j \beta^j \end{aligned}$$

SCTs are corrected by a metric-dependent term.

[Hinterbichler, Hui, Khoury, 2013]

[Ghosh, Kundu, S.R., Trivedi, 2014]

Fixing residual gauge freedom



Fix residual transformations by fixing positions of "vertex operators" in $\delta\mathcal{G}_{n,m}$.

$$x_1 = 0, \qquad x_2 = 1 \qquad x_3 = \infty$$

Finally

$$
\begin{aligned}
(\Psi, \Psi) &= \sum_{n,m,n',m'} \kappa^{n+n'} \left\langle\!\!\left\langle \overline{\delta\mathcal{G}^*_{n,m}\, \delta\mathcal{G}_{n',m'}} \right\rangle\!\!\right\rangle \\
&\propto \sum_{n,m,n',m'} \kappa^{n+n'} \int DgD\chi\, \delta(\mathrm{g.f})\Delta'_{\mathrm{FP}} |Z_0[g,\chi]|^2 \overline{\delta\mathcal{G}^*_{n,m}\, \delta\mathcal{G}_{n',m'}}
\end{aligned}
$$

Normalizable states require at least two insertions $(2 + 2 > 3)$.
H.H. state is not naively normalizable.

Higuchi's norm

In nongravitational limit, instead of fixing three points we can just divide by the volume of the conformal group.

$$(\Psi_{\rm ng}, \Psi_{\rm ng}) \propto \frac{1}{\mathrm{vol}(SO(d+1,1))} \lim_{\kappa \to 0} \langle\!\langle \delta\mathcal{G}^*_{n,m}\, \delta\mathcal{G}_{n',m'} \rangle\!\rangle$$

$$= \frac{1}{\mathrm{vol}(SO(d+1,1))} \langle \Psi_{\rm ng} | \Psi_{\rm ng} \rangle_{\rm QFT}$$

Mysterious group-averaged norm emerges naturally!

Our method justifies Higuchi's proposal for the norm and provides gravitational corrections to it.

Outline

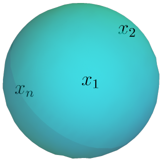
Motivation

Hilbert space

Cosmological correlators

Holography of information

Cosmological correlators



We commonly compute cosmological correlators

$$\langle \chi(x_1) \dots \chi(x_n) \rangle$$

As written, expression does not commute with the constraints.

We propose interpretation as gauge-fixed operators

$$\langle\!\langle \Psi | \chi(x_1) \dots \chi(x_n) | \Psi \rangle\!\rangle_{CC} = \int |\Psi|^2 \chi(x_1) \dots \chi(x_n) \delta(\mathrm{g.f}) \Delta'_{FP} Dg D\chi$$

Cosmological correlators

$$\langle\!\langle \Psi_1 | \chi(x_1) \ldots \chi(x_n) | \Psi_2 \rangle\!\rangle_{\mathrm{CC}} = \int \Psi_1^* \Psi_2 \chi(x_1) \ldots \chi(x_n) \delta(\mathrm{g.f}) \Delta'_{FP} Dg D\chi$$

gives unambiguous prescription for the matrix elements.

$\exists$ gauge invariant operator with the same matrix elements.

Gauge-fixing $\longrightarrow$ setting our reference frame as observers.

Symmetries of cosmological correlators

Residual gauge transformations turn into symmetries of cosmological correlators.

Translations/Dilatations:

$$\langle\!\langle \Psi | \chi(\lambda x_1 + v) \dots \chi(\lambda x_n + v) | \Psi \rangle\!\rangle_{\mathrm{CC}} = \lambda^{-n\Delta} \langle\!\langle \Psi | \chi(x_1) \dots \chi(x_n) | \Psi \rangle\!\rangle_{\mathrm{CC}}$$

Rotations:

$$\langle\!\langle \Psi | \chi(M \cdot x_1) \dots \chi(M \cdot x_n) | \Psi \rangle\!\rangle_{\mathrm{CC}} = \langle\!\langle \Psi | \chi(x_1) \dots \chi(x_n) | \Psi \rangle\!\rangle_{\mathrm{CC}}$$

SCTs relate cosmological correlators of different orders.

Symmetries of cosmological correlators

All states display these symmetries.

Conformal invariance of cosmological correlators does not require choice of specific initial conditions. Generic prediction of inflation + Q.G.

Conversely, conformal-invariance of early-Universe correlators does not provide evidence for Hartle-Hawking proposal.

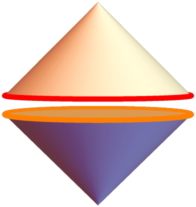
Outline

Holography of information

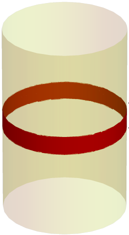
Gravity localizes information unusually!

[Laddha, Prabhu, S.R., Shrivastava, 2020]

Asymptotically flat space

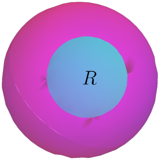


Asymptotic AdS



- Follows from analysis of gravitational constraints.
- Explains why gravitational theories are holographic.

Holography of information in dS



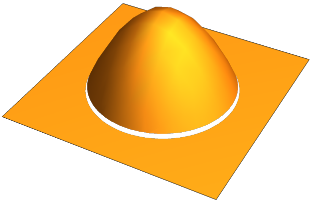
In dS, cosmological correlators in an arbitrarily small region on the asymptotic time slice are sufficient to determine them everywhere.

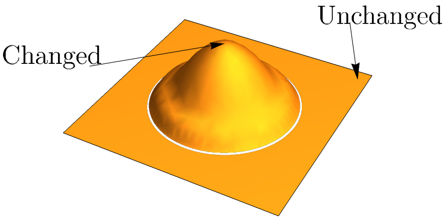
$$\langle\!\langle \Psi | \chi(\lambda x_1 + v) \ldots \chi(\lambda x_n + v) \rangle\!\rangle_{\rm CC} = \lambda^{-n\Delta} \langle\!\langle \langle \Psi | \chi(x_1) \ldots \chi(x_n) | \Psi \rangle\!\rangle_{\rm CC}$$

Holography of information and cosmological correlators

$$\langle\!\langle \Psi_1|\chi(x_1)\ldots\chi(x_n)|\Psi_1\rangle\!\rangle_{\rm CC} = \langle\!\langle \Psi_2|\chi(x_1)\ldots\chi(x_n)|\Psi_2\rangle\!\rangle_{\rm CC} \forall x_i \in \mathcal{R},$$
$$\Rightarrow \langle\!\langle \Psi_1|\chi(x_1)\ldots\chi(x_n)|\Psi_1\rangle\!\rangle_{\rm CC} = \langle\!\langle \Psi_2|\chi(x_1)\ldots\chi(x_n)|\Psi_2\rangle\!\rangle_{\rm CC} \forall x_i,$$

In sharp contrast to QFT.





Holography of information

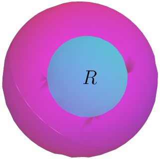
Holography of information is a consequence of the gravitational constraints

$$\Psi[g,\chi] = \sum_{n,m} \kappa^n \delta\mathcal{G}_{n,m}\Psi_0[g,\chi],$$

$$\delta\mathcal{G}_{n,m} = \int d\vec{x}\,d\vec{y}\Big[h_{i_1 j_1}(x_1)\dots h_{i_n j_n}(x_n)\chi(y_1)\dots\chi(y_m)G^{\vec{i},\vec{j}}_{n,m}(\vec{x},\vec{y})\Big]$$

The functions $G^{\vec{i},\vec{j}}_{n,m}$ are conformally covariant $\Rightarrow$ the ingredients of the state are conformally covariant.

Therefore, if we are given $G^{\vec{i},\vec{j}}_{n,m}$ in any small region, we know them everywhere.

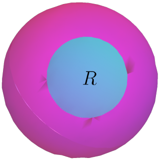


Nongravitational limit

Holography of information persists in the nongravitational limit.

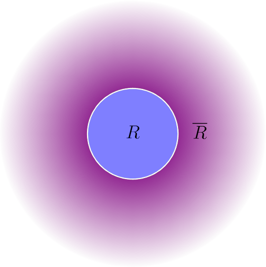
$$\text{if}\,\forall x_i \in \mathcal{R},$$
$$\langle\!\langle \Psi_{\text{ng},1} | \chi(x_1)\ldots\chi(x_n) | \Psi_{\text{ng},1} \rangle\!\rangle_{\text{CC}} = \langle\!\langle \Psi_{\text{ng},2} | \chi(x_1)\ldots\chi(x_n) | \Psi_{\text{ng},2} \rangle\!\rangle_{\text{CC}}$$
$$\Rightarrow \forall x_i$$
$$\langle\!\langle \Psi_{\text{ng},1} | \chi(x_1)\ldots\chi(x_n) | \Psi_{\text{ng},1} \rangle\!\rangle_{\text{CC}} = \langle\!\langle \Psi_{\text{ng},2} | \chi(x_1)\ldots\chi(x_n) | \Psi_{\text{ng},2} \rangle\!\rangle_{\text{CC}},$$

$$|\Psi_{\text{ng}}\rangle = \int dx_i f(x_1,\ldots x_n)\chi(x_1)\ldots\chi(x_n)|0\rangle$$



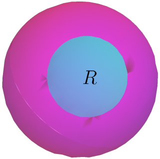
Holography of information

AdS and flat space



Whenever the complement of a region surrounds the region, it has information about the region.

dS



In dS, the complement of every region surrounds the region and vice versa!

Cautionary remarks

Holography of information $\Rightarrow$ sharp mathematical difference between QFT and QG.

Caution:

- "cosmological correlators" are secretly nonlocal since they are gauge fixed.
- Identifying the state requires all-point correlators.
- No claim that these gauge-fixed operators can all be "measured" by an "observer".

Conclusion

- **Hilbert space:** Solutions of WDW-eqn are of the form $e^{iS}Z[g,\chi]$, where $|Z[g,\chi]|^2$ is a diff and Weyl-invariant functional.
- All states are of this form, not just the Hartle-Hawking state. (HH state itself does not appear normalizable.)
- **Symmetries.** Cosmological correlators, after gauge-fixing, covariant under scaling, rotations, translations in all states, not just the HH state.
- **Holography of information:** Cosmological correlators in an arbitrarily small region suffice to determine the state. Sharp contrast with QFT.