

Who is "c" scaling weight

$$\Delta = \frac{d}{2} + C$$

$$C = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}} \begin{cases} C = i\rho \\ C \in \mathbb{R} \end{cases}$$

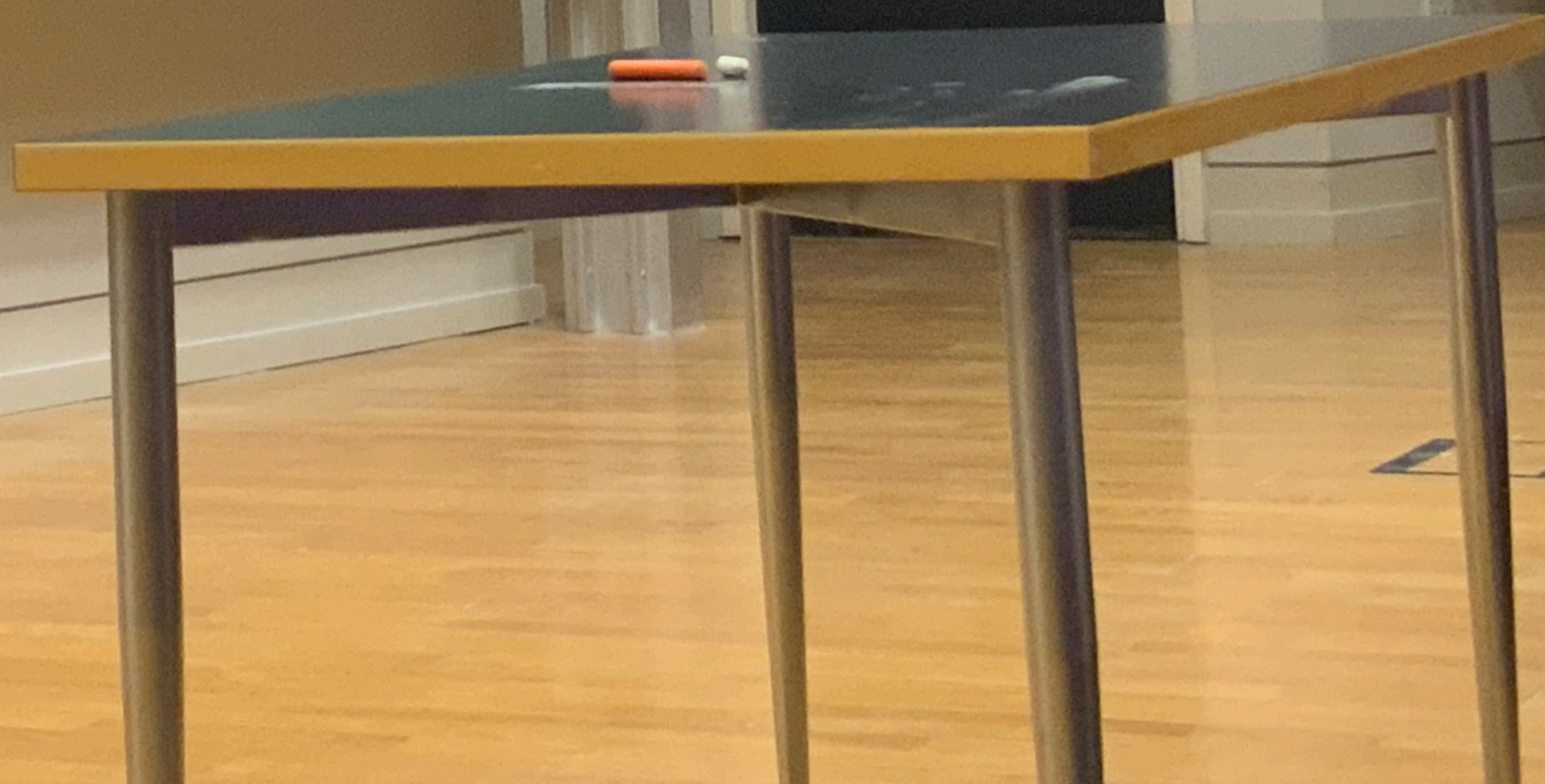
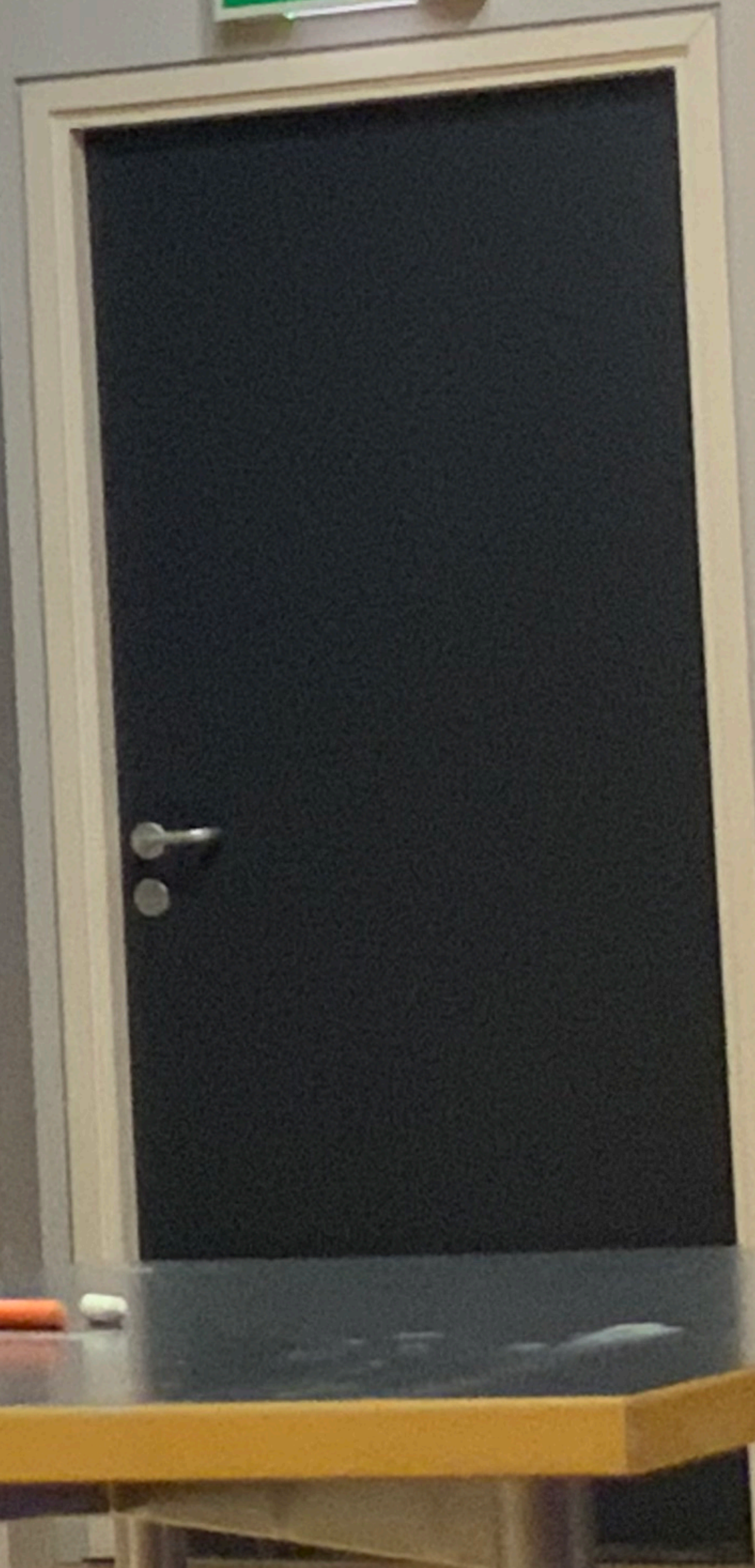
Intertwining operators

invertible, normalizable maps

$$\chi = (\rho, c) \rightarrow \tilde{\chi} = (\rho, -c)$$

$$\Delta + \tilde{\Delta} = d$$

G = (normalization) (initial)



late-time boundary of de Sitter
 ISO(d,1) symmetry of Mink - with C. Skordis
 SO(d+1,1)

Goal: UNITARITY & the

3 ways to build reps

- 1) from generators Sun, I_g^x structure preserving maps
- 2) for $g \in G$ Sun, Dobrev et al working with function spaces C_X act on C_X
- 3) analytic cont. from the plane Higuchi, et al $C_X = \{f: \text{Domain} \rightarrow V, \text{properties related to how they transform under stability subgroup}\}$

Unitary rep: $(I_g^z f_1, I_g^z f_2) = (f_1, f_2) \int |a|^{-k+c}$

In momentum space
 $\mathcal{O}(\vec{x}) = \int \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{x}} \mathcal{O}(\vec{k})$ $\mathcal{O}(E) \sim k^c$ $\langle E|E\rangle = 2\pi^d S^{d-1} (E-E)$
 $\mathcal{O}(\vec{x}, \eta) = \int \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{x}} [\Phi_E(\eta) a_{\vec{k}} + \Phi_{-E}(\eta) a_{-\vec{k}}^\dagger]$ $e^{i\vec{k}\cdot\vec{x}} \Phi_{\vec{k}} \sim \sqrt{k} G$
 $\lim_{\eta \rightarrow 0} \mathcal{O}(\vec{x}, \eta) = \int \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{x}} [|\eta|^{\frac{d}{2} + \frac{1}{2} - i\epsilon} \alpha(\vec{k}) + |\eta|^{\frac{d}{2} + \frac{1}{2} + i\epsilon} \beta(\vec{k})]$ states: $|\mathcal{O}(E)\rangle = \mathcal{O}(E)|0\rangle = k^{-1} | -E\rangle$
 Normalization? $(\mathcal{O}, \mathcal{O}) = \int \frac{d^d k}{(2\pi)^d} \langle \mathcal{O}(E) | \mathcal{O}(E) \rangle \sqrt{c=i\epsilon}$

$(c=0, d+1=4)$
 $c = i\epsilon$ $(\mathcal{O}, \mathcal{O})$ $\rho \in \mathbb{R}$ Principal Series $(\frac{1}{2})^c$

 $-\frac{3}{2} < c < \frac{3}{2}$ Compl. for $c > 0$ applies for β^L $\tilde{\mathcal{O}} = G_X^+ \mathcal{O}$
 for $c < 0$ $\mathcal{O} = G_X^+ \tilde{\mathcal{O}}$
 $c \in \mathbb{R}$ $(\mathcal{O}, \tilde{\mathcal{O}})$

 $\Delta \in \mathbb{Z} \cup \{0\}$ Discrete $c = -\frac{3}{2}$ $\tilde{\mathcal{O}} = G_X^+ \mathcal{O}$ $(\frac{k^2}{2})^c \alpha^M$
 symmetric traceless tensor (type-I) $SO(2,1), SO(4,1)$

$$\sqrt{c} = i\rho$$

$$\sqrt{4 - \frac{m^2}{H^2}} \quad c \in \mathbb{R}$$

normalizable maps
 $\chi = (\rho, c) \rightarrow \tilde{\chi} = (\rho, -c)$
 $\Delta + \tilde{\Delta} = d$

G = (normalization) (projection)

$$\alpha_N^H(\vec{k}) = \sqrt{\rho \pi \sinh(\rho \pi)} \left[-\frac{i}{\pi} \Gamma(\rho) e^{-\rho \pi} a_{\vec{k}} + \frac{1}{\sinh(\rho \pi) \Gamma(1-\rho)} a_{-\vec{k}}^{\dagger} \right] \left(\frac{k}{2}\right)^{-\rho}$$

$$\beta_N^H(\vec{k}) =$$

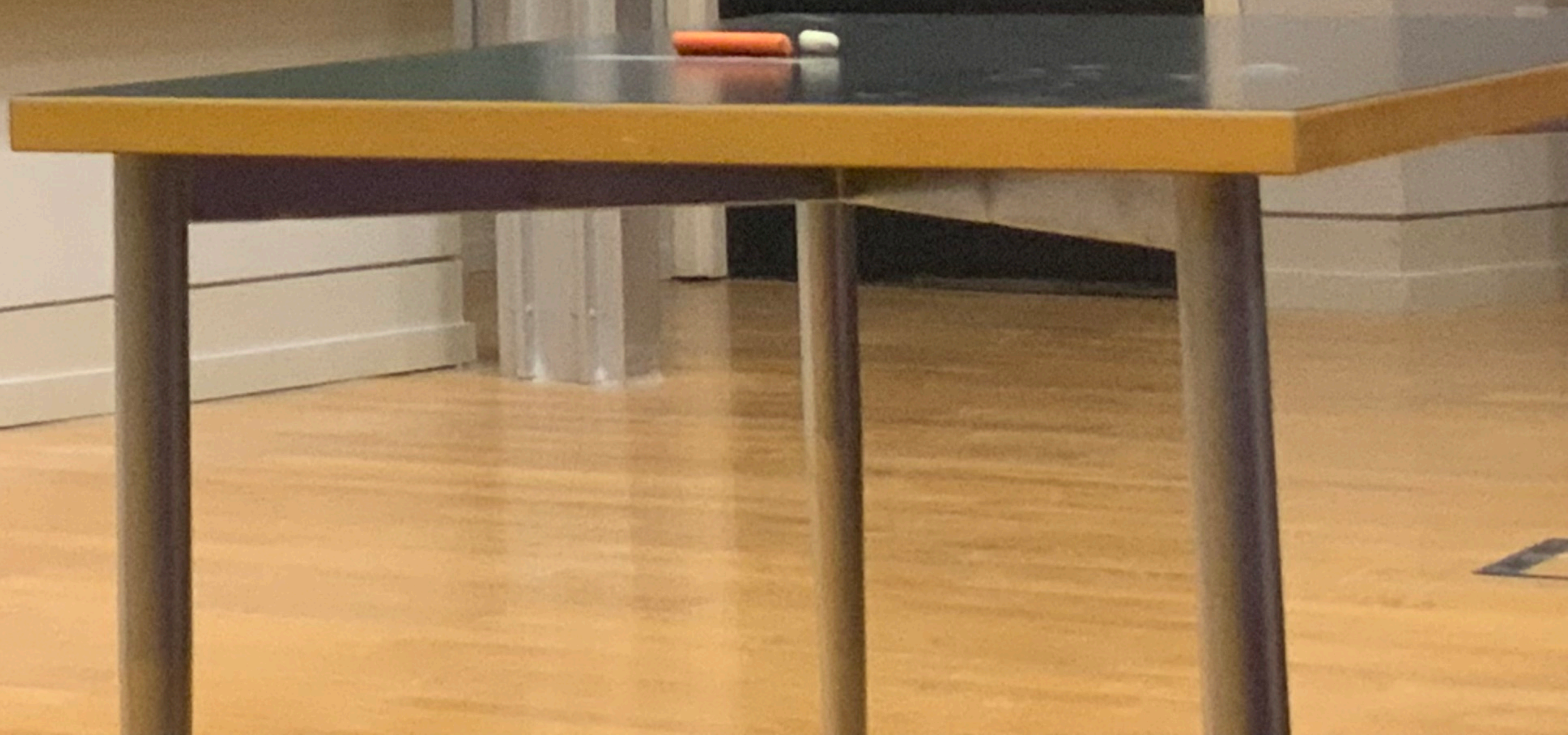
$$\beta_N^L(\vec{k}) = 2^{-\nu/2} \left[\frac{1+i\cot(\pi\nu)}{1-i\cot(\pi\nu)} a_{\vec{k}} + a_{-\vec{k}}^{\dagger} \right] k^{\nu}$$

$$\alpha_N^L(\vec{k}) = -i 2^{\nu/2} [a_{\vec{k}} - a_{-\vec{k}}^{\dagger}] k^{-\nu}$$

$$\tilde{\alpha}_N^L(\vec{k}) = -i 2^{-\nu/2} [a_{\vec{k}} - a_{-\vec{k}}^{\dagger}] k^{\nu}$$

$$[\alpha_N, \beta_N] \neq 0$$

$$\alpha_N^M(\vec{k}) = -i 2^{3\nu/4} [a_{\vec{k}} - a_{-\vec{k}}^{\dagger}] k^{-3\nu/2}$$



Scalar Particles at the late-time boundary of de Sitter

Set up: free $ds^2 = \frac{-dt^2 + dx^i{}^2}{H^2 |t|^2}$ ∇^2

Wigner
spacelike boundary

Particles \rightarrow UIR

$ISO(d,1)$

isometry of Mink

-with C. Skordis

dS_{d+1}

$\Lambda > 0$

No global ∂_t

Mink

$\Lambda = 0$

$SO(d+1,1)$

AdS

$\Lambda < 0$

timelike boundary

Goal: UNITARITY & the inner product

$SO(d+1,1)$

dS_{d+1}

\vec{x} -transl \tilde{N}

dilatations $A=SO(1,1)$

SCT N

Rotations $M=SO(d)$

maximally compact $K=SO(d+1)$

Casimir $\chi = \{e, c\}$

Invariant/stability subgroup NAM, K

$ISO(d,1)$

Mink $_{d+1}$

t -transl

\vec{x} -transl.

boosts

rotations

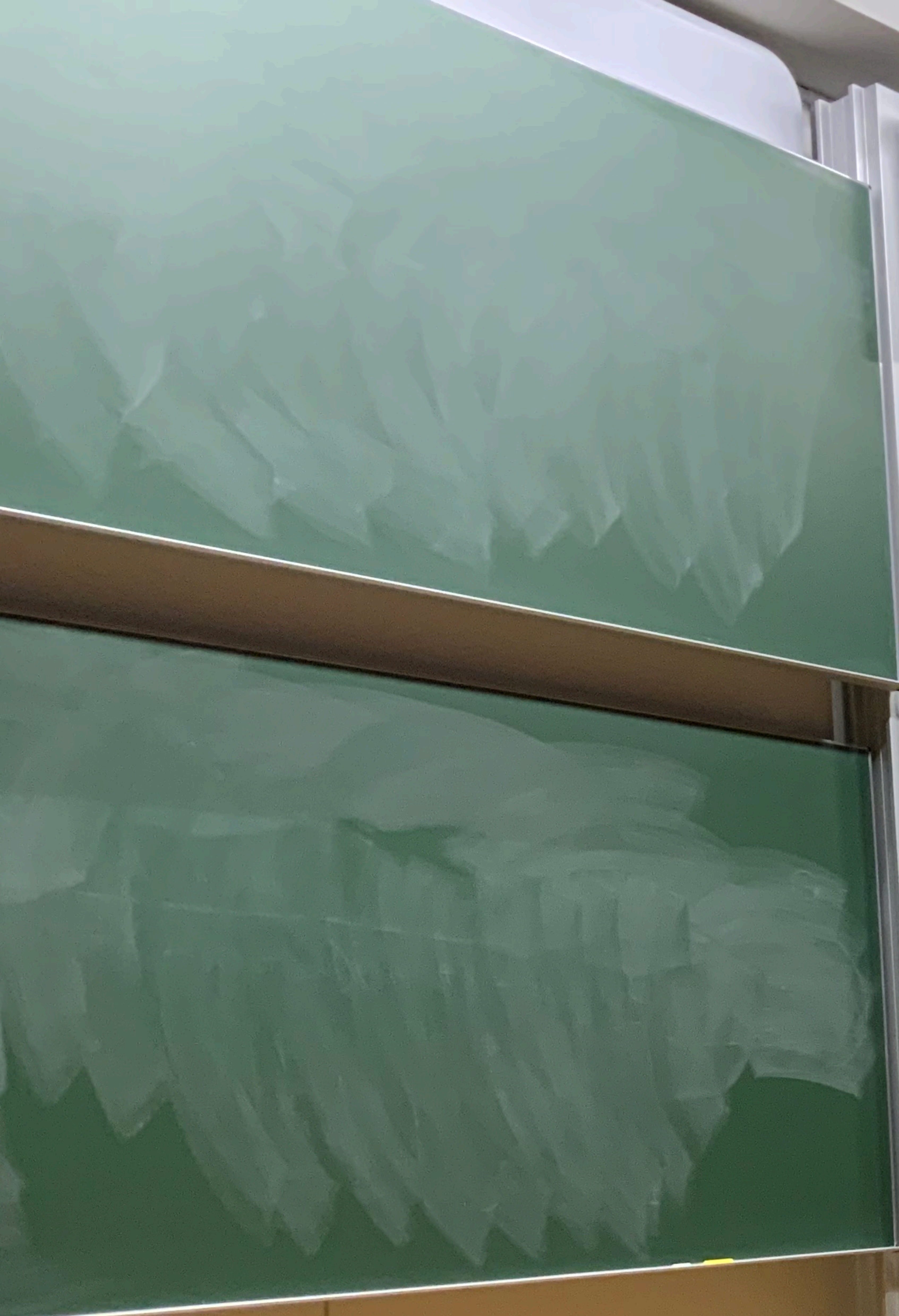
Translations categorization \rightarrow ref momenta

3 ways to build reps

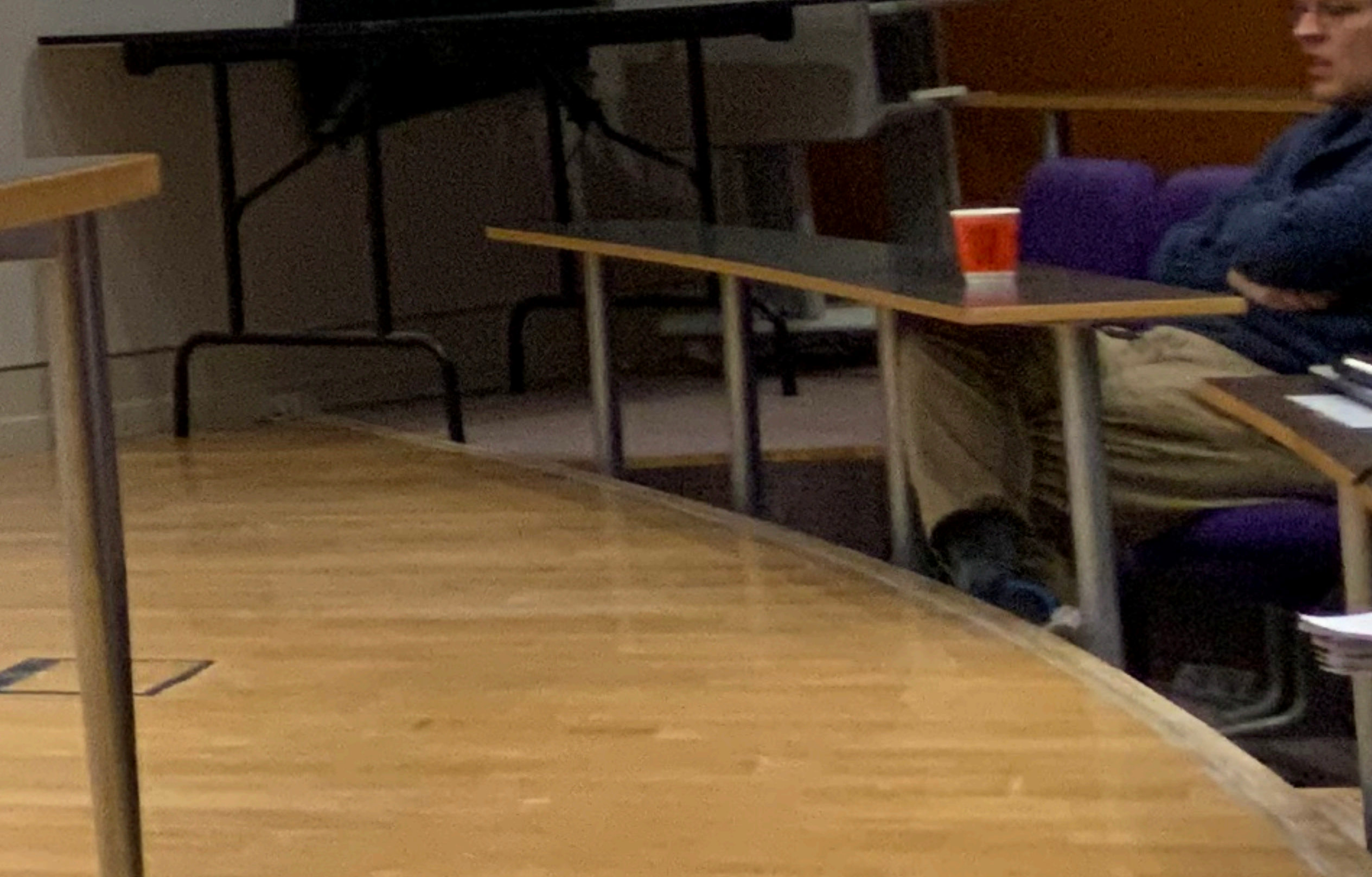
1) from generators S_{un}

2) for $g \in G$ working with function spaces S_{un} , Dobrev et al

3) analytic cont. from the sphere Higuchi, LeClair



Who is c 's scaling weight

$$\Delta = \frac{d}{2} + C$$
$$C = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}} \begin{cases} C = i\rho \\ C \in \mathbb{R} \end{cases}$$


In momentum space

$$\phi(\vec{x}) = \int \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{x}} \phi(\vec{k}) \quad \phi(\vec{k}) \sim k^c$$

$$\phi(\vec{x}, \eta) = \int \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{x}} \left[\phi_c(\eta) a_{\vec{k}} + \phi_c^*(\eta) a_{-\vec{k}}^\dagger \right]$$

$$\lim_{\eta \rightarrow 0} \phi(\vec{x}, \eta) = \int \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{x}} \left[|A| \left\langle \frac{d}{2} + \frac{1}{2} - i\rho \right\rangle \phi(\vec{k}) + |B| \left\langle \frac{d}{2} + \frac{1}{2} + i\rho \right\rangle \phi(\vec{k}) \right]$$

states: $|\phi(\vec{k})\rangle = \phi(\vec{k})|0\rangle = k^c |-\vec{k}\rangle$

Normalization? $\langle \phi, \phi \rangle = \int \frac{d^d k}{(2\pi)^d} \frac{\langle \phi(\vec{k}) | \phi(\vec{k}) \rangle}{k^{c+c^*}} \left\{ \begin{array}{l} \sqrt{c} = \rho \\ c \in \mathbb{R} \end{array} \right.$

$d=0, d+1=4$

$c = i\rho \quad (0, \rho) \quad \rho \in \mathbb{R} \quad \text{Principal Series}$

$-\frac{3}{2} < c < \frac{3}{2} \quad \text{Comp.}$

$c \in \mathbb{R} \quad (0, \tilde{0})$

$\Delta \in \mathbb{Z} \cup \{0\} \quad \text{Discrete}$

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Who is c^* scaling weight

$$\Delta = \frac{d}{2} + c$$

$$c = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}} \quad \left\{ \begin{array}{l} c = i\rho \\ c \in \mathbb{R} \end{array} \right.$$


particles at the late-time boundary of de Sitter

Set up: free $ds^2 = \frac{-dy^2 + d\vec{x}^2}{H^2 |y|^2}$

Wigner
spatial boundary

Particles \rightarrow UIR

$ISO(d,1)$ isometry of Mink

-with C. Skordis

dS_{d+1} $\Lambda > 0$
Mink $\Lambda = 0$
AdS $\Lambda < 0$

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$SO(d+1,1)$

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Rotations $M = SO(d)$

maximally compact $K = SO(d+1)$

Casimir $\chi = \{l, c\}$

Invariant/stability subgroup NAM, K
Principal & Comp \rightarrow Discrete

$ISO(d,1)$

Mink $_{d+1}$

t -transl

\vec{x} -transl.

boosts

rotations

Translations categorization \rightarrow ref momenta

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2) for $g \in G$ S_{un}

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\mathcal{I}_g^x structure preserving maps

working with function spaces C_X act on

$C_X = \{f: \text{Domain} \rightarrow V, \text{properties related to how they transform under stability subgroup}\}$

(f_1, f_2)

Unitary rep $\equiv (\mathcal{I}_g^x f_1, \mathcal{I}_g^x f_2) \stackrel{!}{=} (f_1, f_2)$
 $\int |a|^{-k+c} ||$

In momentum space

$\mathcal{O}(\vec{x}) = \int \frac{d^d k}{(2\pi)^d} e^{ik \cdot \vec{x}}$

$\Phi(\vec{x}, \eta) = \int \frac{d^d k}{(2\pi)^d} e^{ik \cdot \vec{x}}$

$\lim_{\eta \rightarrow 0} \Phi(\vec{x}, \eta) = \int \frac{d^d k}{(2\pi)^d} e^{ik \cdot \vec{x}}$

$l=0, d+1=4$

$C = ip \quad (0, \vec{0})$

$C \in \mathbb{R} \quad (0, \vec{0})$