$$\varepsilon = \begin{cases} 0 & \text{dS} \\ -1 & \text{flat} \\ -2 & \text{RD} \\ -3 & \text{MD} \end{cases}$$

$$\boxed{\Phi_A(X_1, X_2)\,]\,\Psi = 0}$$

$$dX_1 + \frac{\partial}{\partial X_2}\, dX_2$$

$n_i \in \mathbb{Z}$

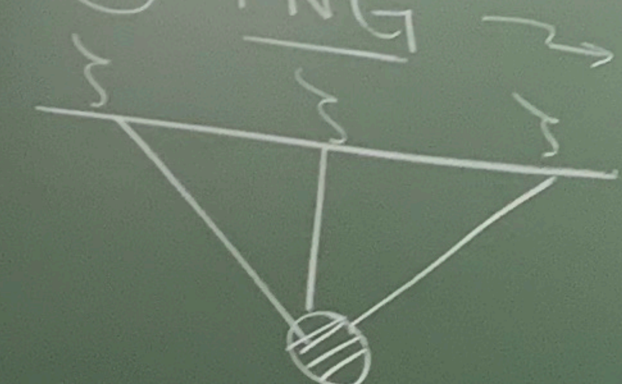
Es always exist!

Motivation:

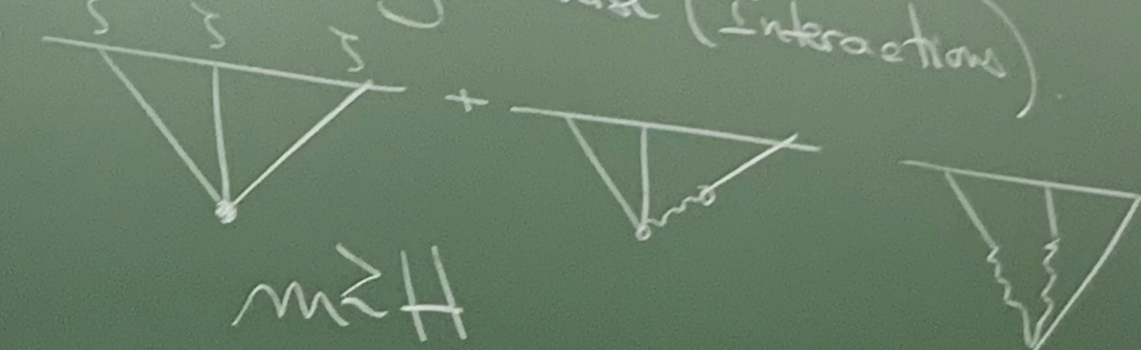
① Phenomenological (PNG in inflation).

② Conceptual $\begin{cases} \text{Scattering Amps \& Cosmology.} \\ \text{"Holography"} \end{cases}$

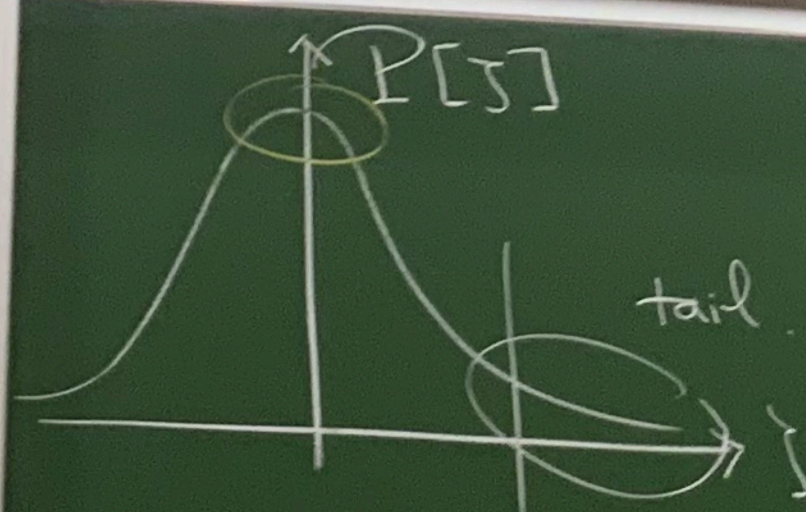
① PNG ⟶ Dynamics of Early Universe (Interactions).



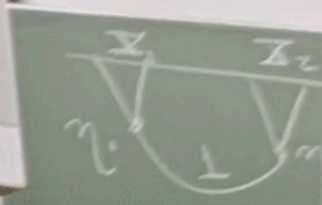
$\supset$



$m \gtrsim H$.



Stochastic Inflation



$\eta \to x$ (Cosmological Polytope)

$X_1 \equiv$

$X_2 \equiv$

$$F_\varepsilon(X_1, X_2) = \int_0^\infty \frac{(x_1 x_2)^\varepsilon\, dx_1 \wedge dx_2}{\underbrace{(x_1 + X_1 + 1)}_{L_1}\underbrace{(x_2 + X_2 + 1)}_{L_2}\underbrace{(x_1 + x_2 + X_1 + \ldots)}_{L_3}}$$

$$= {}_2F_1\left(1+\varepsilon, 1-\varepsilon, \frac{X_1 + X_2}{1 + X_1}\right) + {}_2F_1(X_1 \leftrightarrow \ldots$$

? How to find DE

$$= \int_0^\infty (x_1 x_2)^\varepsilon\, \Omega$$ ← canonical form (dlog ∧ dlog in all ∂'s).

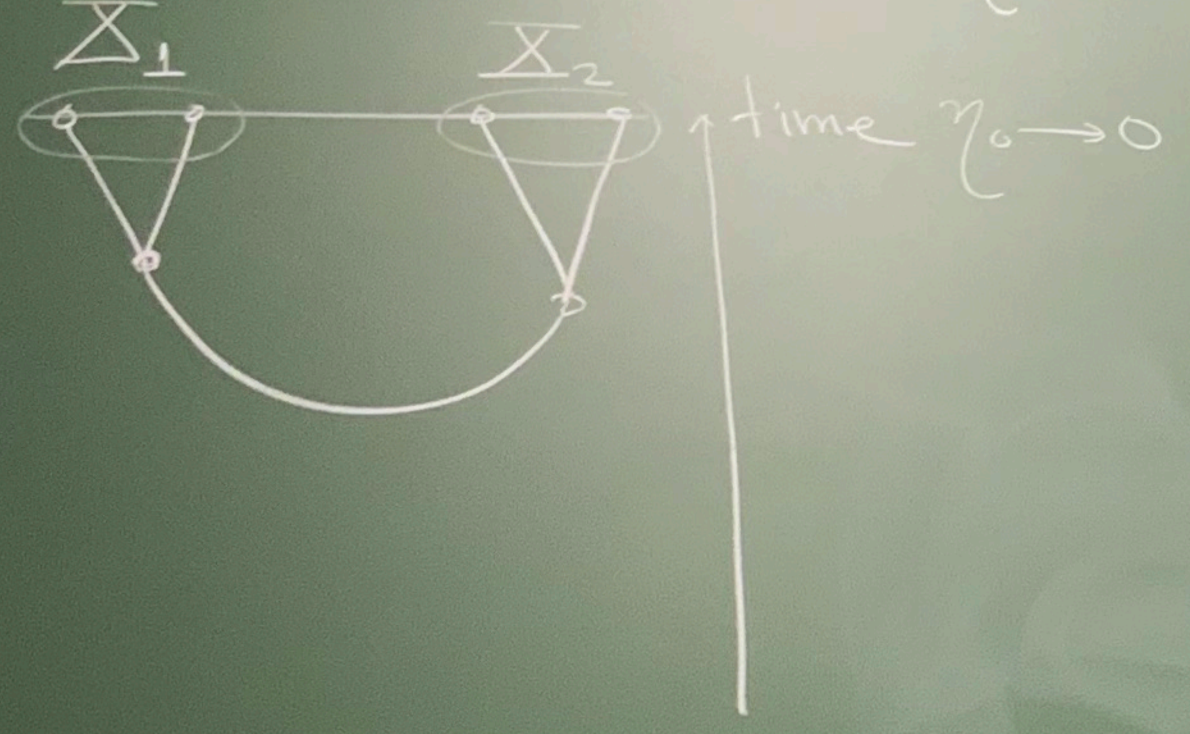
$$\Omega = [12] + [23] + [31] \qquad [ij] \equiv d\log L_i \wedge \ldots$$

enomenological (PNG in inflation)
onceptual
Scattering Amps & Cosmology
"Holography"
s of Early Universe (Interactions)
m ≳ H
Stochastic Inflation
η → x (Cosmological Polytope)
X₁ ≡ (|k₁| + |k₂|) / |k₁ + k₂|
X₂ ≡ (|k₃| + |k₄|) / |k₁ + k₂|
F_ε(X₁, X₂) = ∫₀^∞ (x₁x₂)^ε dx₁ ∧ dx₂ / ((x₁ + X₁ + 1)(x₂ + X₂ + 1)(x₁ + x₂ + X₁ + X₂))
How to find DE
canonical form
(dlog ∧ dlog in all ∂'s)
Ω = [12] + [23] + [31]
[ij] ≡ dlog L_i ∧ dlog L_j

$$\underset{\text{FLRW}}{a(\eta)} = \frac{1}{\eta}\,\eta^{\varepsilon}$$

$$m^2 = \frac{1}{6} R \qquad \varepsilon = \begin{cases} 0 & \text{dS} \\ -1 & \text{flat} \\ -2 & \text{RD} \\ -3 & \text{MD} \end{cases}$$

$$\left(\varphi_k^{cl} = \eta^{(\varepsilon+1)} e^{ik\eta}\right).$$



$$\boxed{d_{(X_1,X_2)} \Psi + \varepsilon \left[d\log \Phi_A(X_1,X_2) \right] \Psi = 0}$$

$$d = \frac{\partial}{\partial X_1} dX_1 + \frac{\partial}{\partial X_2} dX_2$$

$$F^{\varepsilon}_{n_1,n_2,n_3,n_4,n_5} = \int \frac{(x_1 x_2)^{\varepsilon}\, dx_1 \wedge dx_2}{x_1^{n_1} x_2^{n_2} L_1^{n_3} L_2^{n_4} L_5^{n_5}} \qquad n_i \in \mathbb{Z}.$$

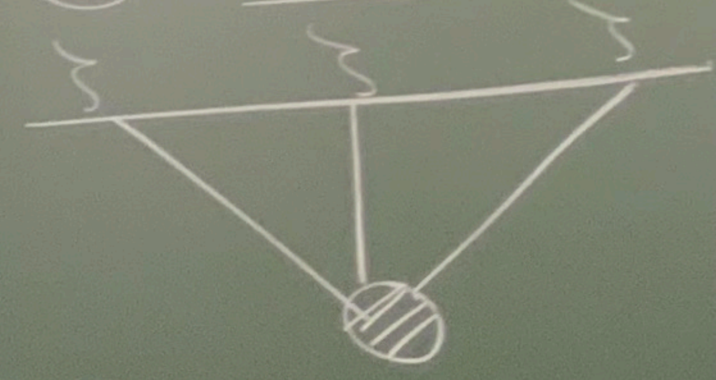
(Aomoto, ...)

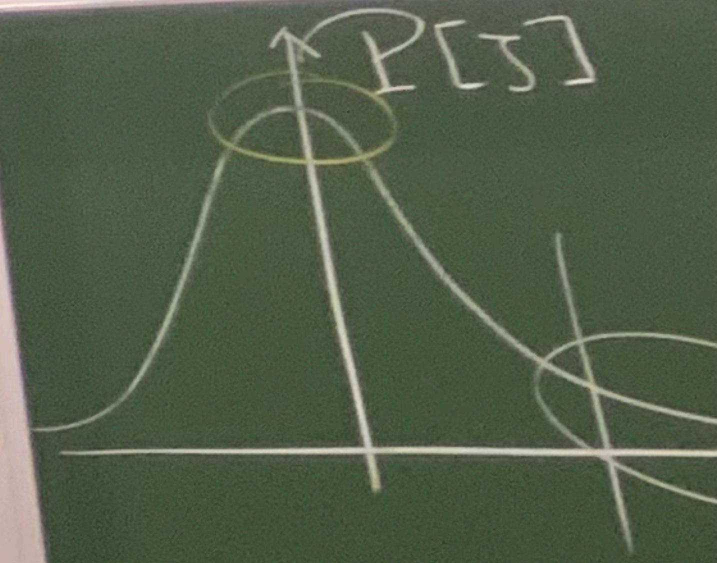
Finitely many $\int$s generate all others! $\Rightarrow$ DEs always exist!

$F = F_{00,111}$ $\begin{cases} \text{* Partial Fractions.} \\ \text{* IBPs.} \end{cases}$

Motivation:

① PNG →





$$\varepsilon = \begin{cases} 0 & dS \\ -1 & \text{flat} \\ -2 & RD \\ -3 & MD \end{cases}$$

$$\boxed{[\ldots(X_1, X_2)]\,\Psi = 0}$$

$$\ldots X_1 + \frac{\partial}{\partial X_2}\, dX_2$$

$n_i \in \mathbb{Z}$.

[illegible]ts always exist!

$$\frac{x_1^{\varepsilon_1} x_2^{\varepsilon_2} L_1^{\varepsilon_3} L_2^{\varepsilon_4} L_3^{\varepsilon_5}\, dx_1 \wedge dx_2}{x_1^{n_1} x_2^{n_2} L_1^{n_3} L_2^{n_4} L_3^{n_5}}$$

$$d\Psi + \varepsilon A \wedge \Psi = 0 \qquad d: \; 0 + dA\,\Psi + A\,\underbrace{d\Psi}_{-A\Psi} = 0$$

$$\varepsilon\, dA - \varepsilon^2 A \wedge A = 0.$$

$$A = \varepsilon\, \#$$

$\underline{dS}$: $\quad Li_2(\cdots) + \log(\ )\log(\ ) + \frac{\pi^2}{6}$.

$$\frac{\partial}{\partial X_1} F_{\varepsilon=0} = \int \frac{\partial}{\partial x_1} \frac{dx_1 \wedge dx_2}{(x_1+X_1+1)(x_2+X_2+1)(x_1+x_2+X_1+X_2)} = \underbrace{\frac{1}{X_1+1}}_{d\log(X_1+1)} \; \overbrace{\frac{dx_2}{(x_2+X_2+1)(x_2+X_1+X_2)}}^{G}$$

$$\frac{\partial}{\partial X_1} \tilde{G} = \frac{1}{X_1+X_2} = H$$

$$d \begin{bmatrix} F \\ \tilde{G} \\ H \end{bmatrix} = \begin{bmatrix} 0 & \# & 0 \\ 0 & 0 & \# \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ \tilde{G} \\ H \end{bmatrix}$$

$$\frac{\partial}{\partial X_1}(X_1+X_2)H = 0.$$

(Corre[illegible] Pol[illegible]

$$F = \int \frac{(\ldots)}{\underbrace{(x_1 + X_1 + 1)}_{L_1}\ldots}$$

$$= {}_2F_1\left(1+\varepsilon,\, 1-\varepsilon,\, \ldots\right)$$


How to find DE
canonical form
PARTIAL FRACTION
Power-law

How to find DE
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