

$$e^{ik\eta} \quad \epsilon = \begin{cases} 0 \\ -1 \\ -2 \\ -3 \end{cases} \quad \begin{matrix} dS \\ \text{flat} \\ RD \\ MD \end{matrix}$$

$$\frac{\Phi(x_1, x_2)}{A} \Psi = 0$$

$$dX_1 + \frac{\partial}{\partial X_2} dX_2$$

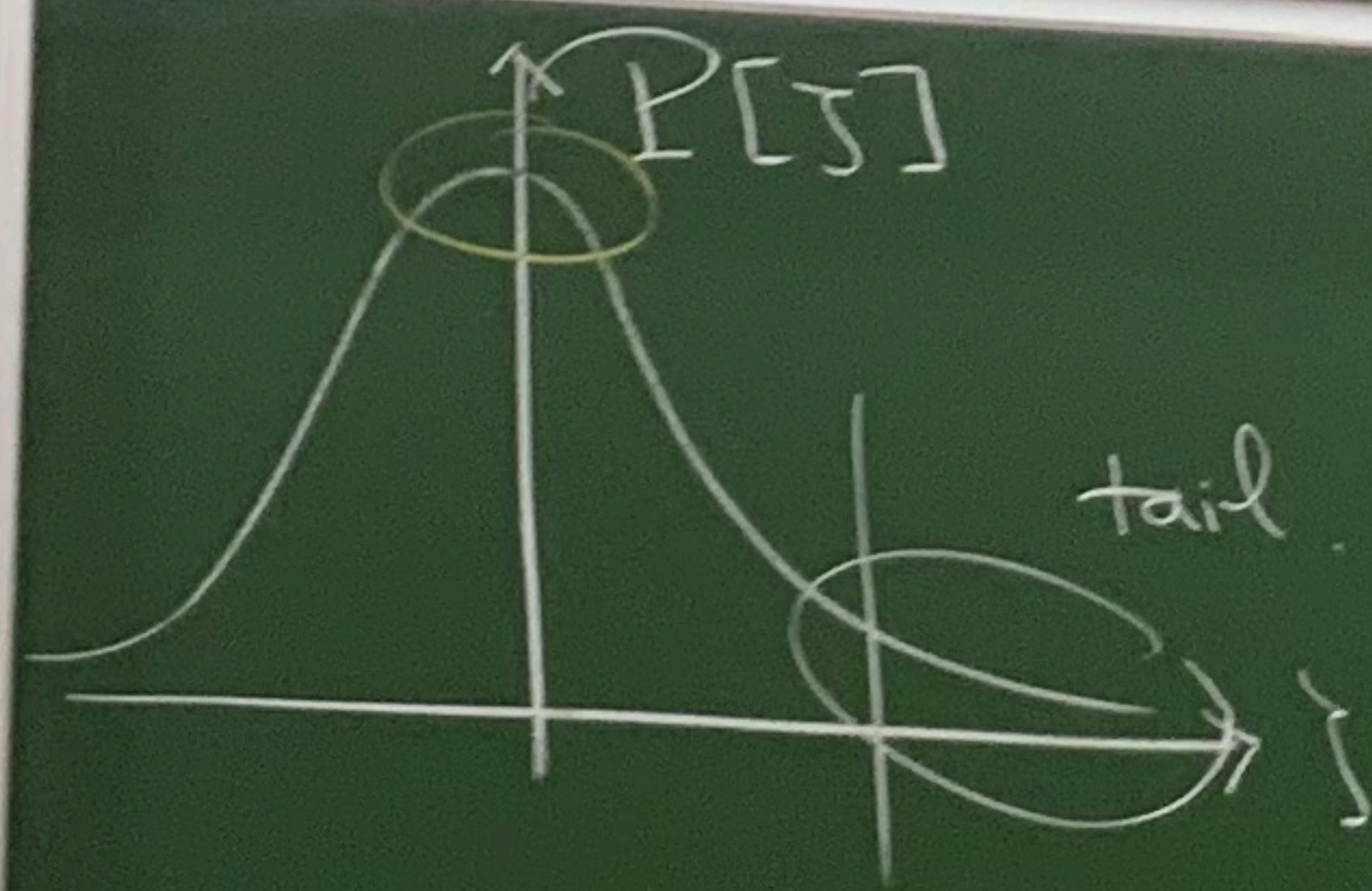
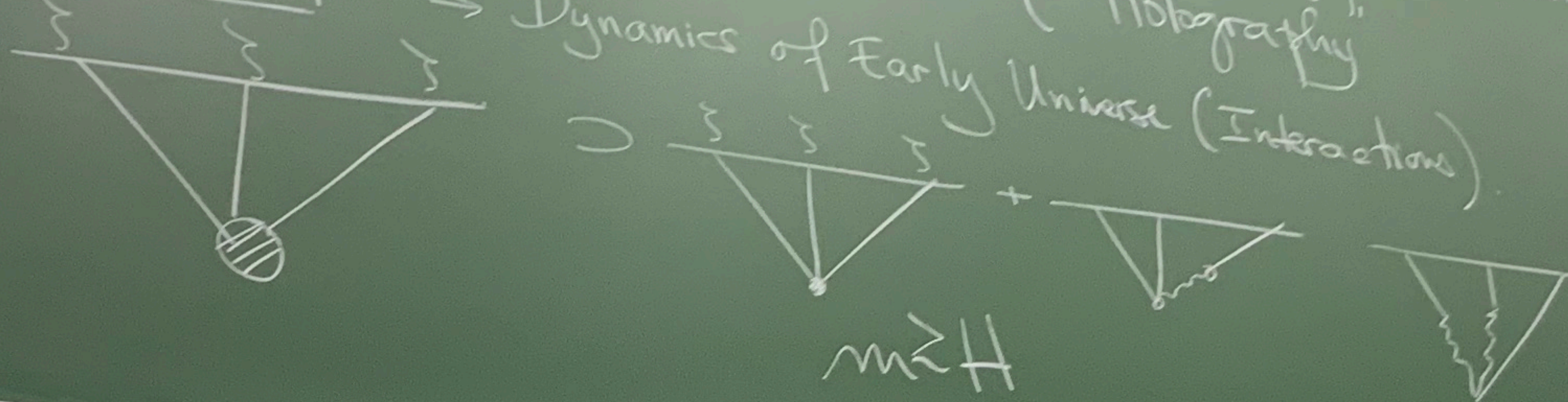
$$n_i \in \mathbb{H}$$

Es always exist!

Motivation:

- ① Phenomenological (PNG in inflation)
- ② Conceptual { Scattering Amps & Cosmology  
"Holography"

① PNG



Stochastic Inflation

$$\int \frac{dX_1 dX_2}{\sqrt{g}} \rightarrow x \text{ (Cosmological)}$$

$$F_\epsilon(X_1, X_2) = \int_0^\infty \frac{(1+z)^{\epsilon}}{(1+X_1+1)(1+X_2+1)(1+X_1+X_2+1)} dx_1 dx_2$$

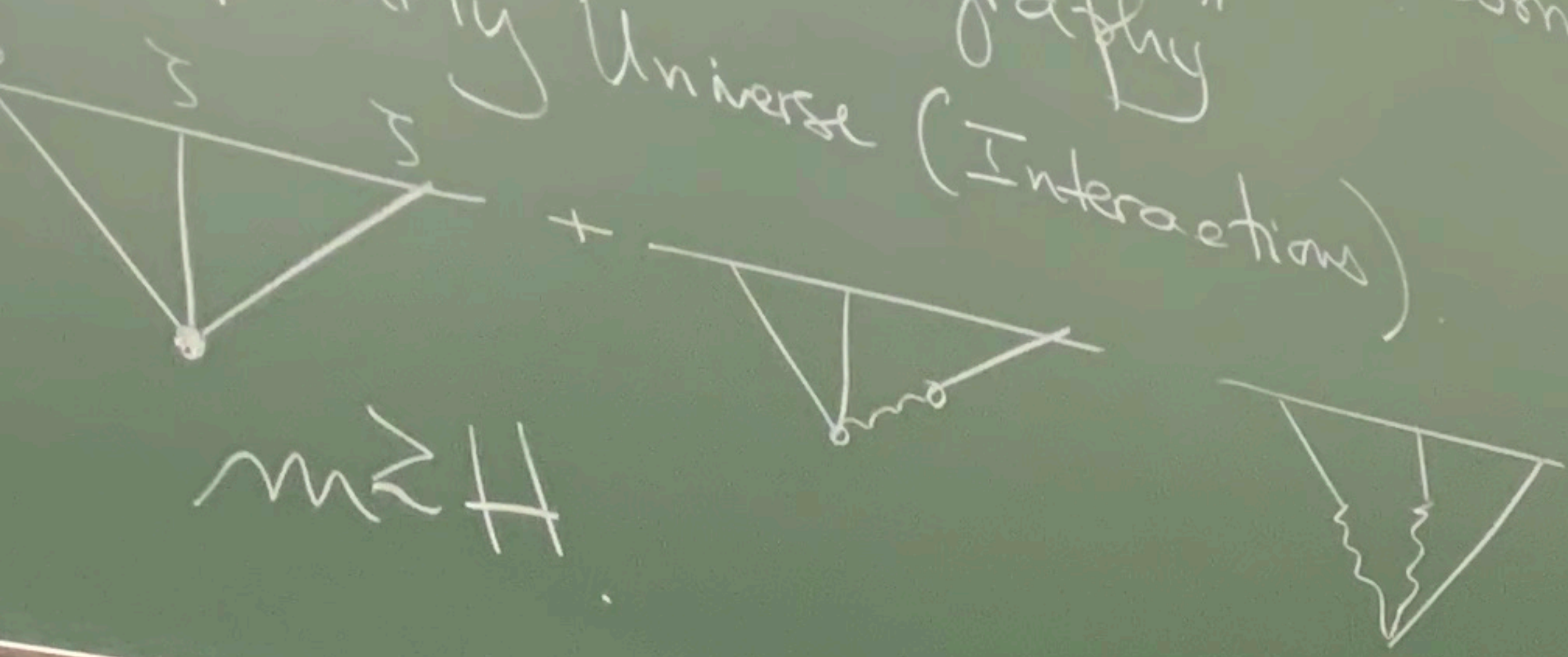
$$= {}_2F_1(1+\epsilon, 1-\epsilon, \frac{X_1+X_2}{1+X_1}) + {}_2F_1(1+\epsilon, 1-\epsilon, \frac{X_1+X_2}{1+X_2})$$

? How to find DE

$$= \int_0^\infty (1+z)^\epsilon \Omega \leftarrow \text{anomalous form (dlog, dlog in all } \partial_i)$$

$$\Omega = [12] + [23] + [31] \quad [ij] = \text{dlog } L_{ij}$$

Phenomenological (PNG in inflation)  
 Conceptual { Scattering Amps & Cosmology  
 "Holography"  
 of Early Universe (Interactions)



Stochastic Inflation

$\eta \rightarrow x$  (Cosmological  
 Polytrope)

$$F_{\epsilon}(\Delta_1, \Delta_2) = \int_0^{\infty} \frac{(\eta_1 \eta_2)^{\epsilon} dx_1 dx_2}{(x_1 + \Delta_1 + 1)(x_2 + \Delta_2 + 1)(x_1 + x_2 + \Delta_1 + \Delta_2)}$$

$$= {}_2F_1\left(1+\epsilon, 1-\epsilon, \frac{\Delta_1 + \Delta_2}{1 + \Delta_1}\right) + {}_2F_1(x_1 \leftrightarrow x_2) + (x_1 + x_2)^{\#}$$

$$\Delta_1 = \frac{|\vec{k}_1| + |\vec{k}_2|}{|\vec{k}_1 + \vec{k}_2|}$$

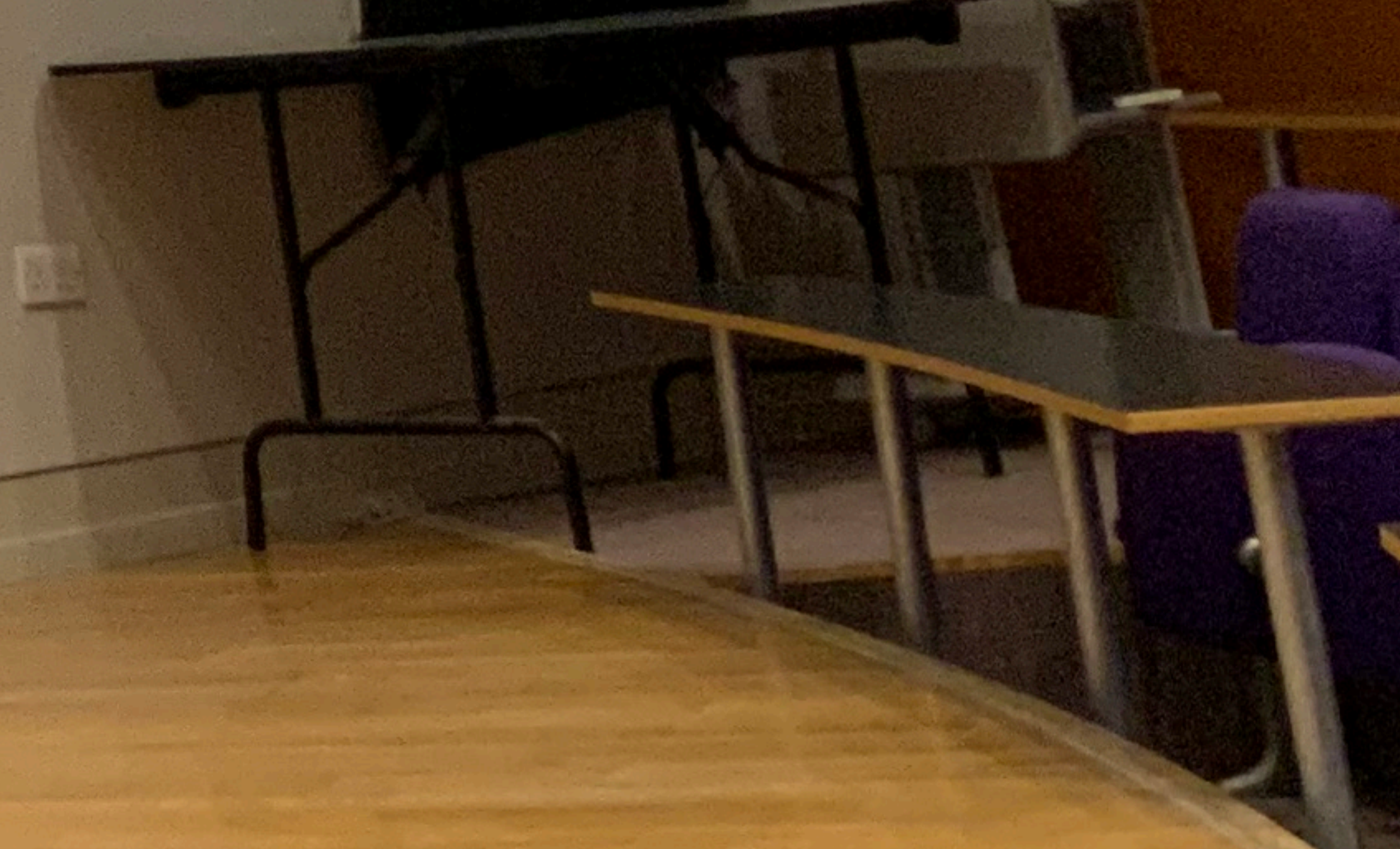
$$\Delta_2 = \frac{|\vec{k}_1 + \vec{k}_2|}{|\vec{k}_1 + \vec{k}_2|}$$

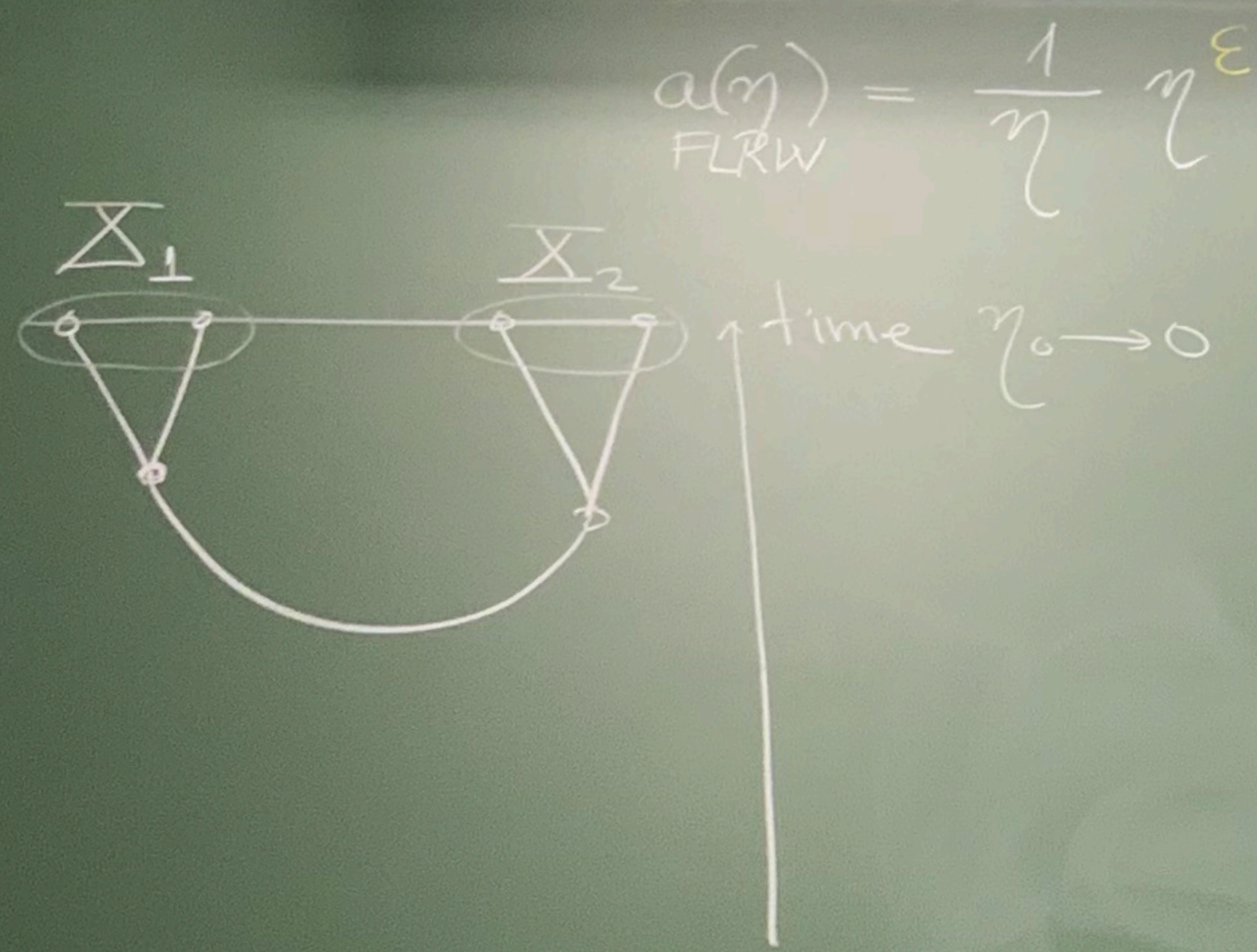
? How to find DE

$$= \int_0^{\infty} (\eta_1 \eta_2)^{\epsilon} \Omega$$

← canonical form  
 (dlog, dlog in all  $\partial_s$ )

$$\Omega = [12] + [23] + [31] \quad [ij] \equiv d\log L_i d\log L_j$$





$m^2 = \frac{1}{6} R$       $\epsilon = \begin{cases} 0 & dS \\ -1 & \text{flat} \\ -2 & RD \\ -3 & MD \end{cases}$   
 $(\psi_k^{cl} = \eta^{(\epsilon+1)} e^{ik\eta})$

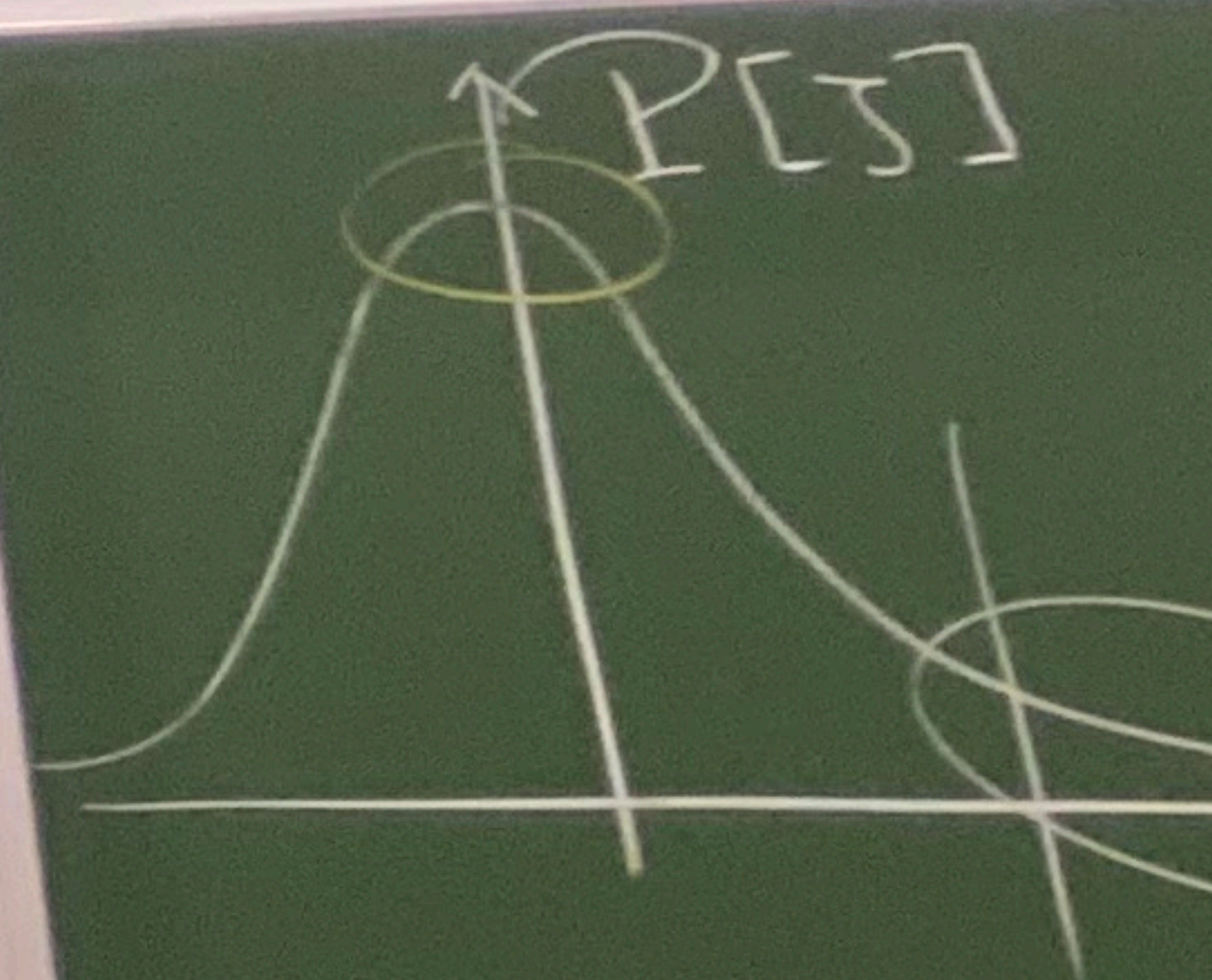
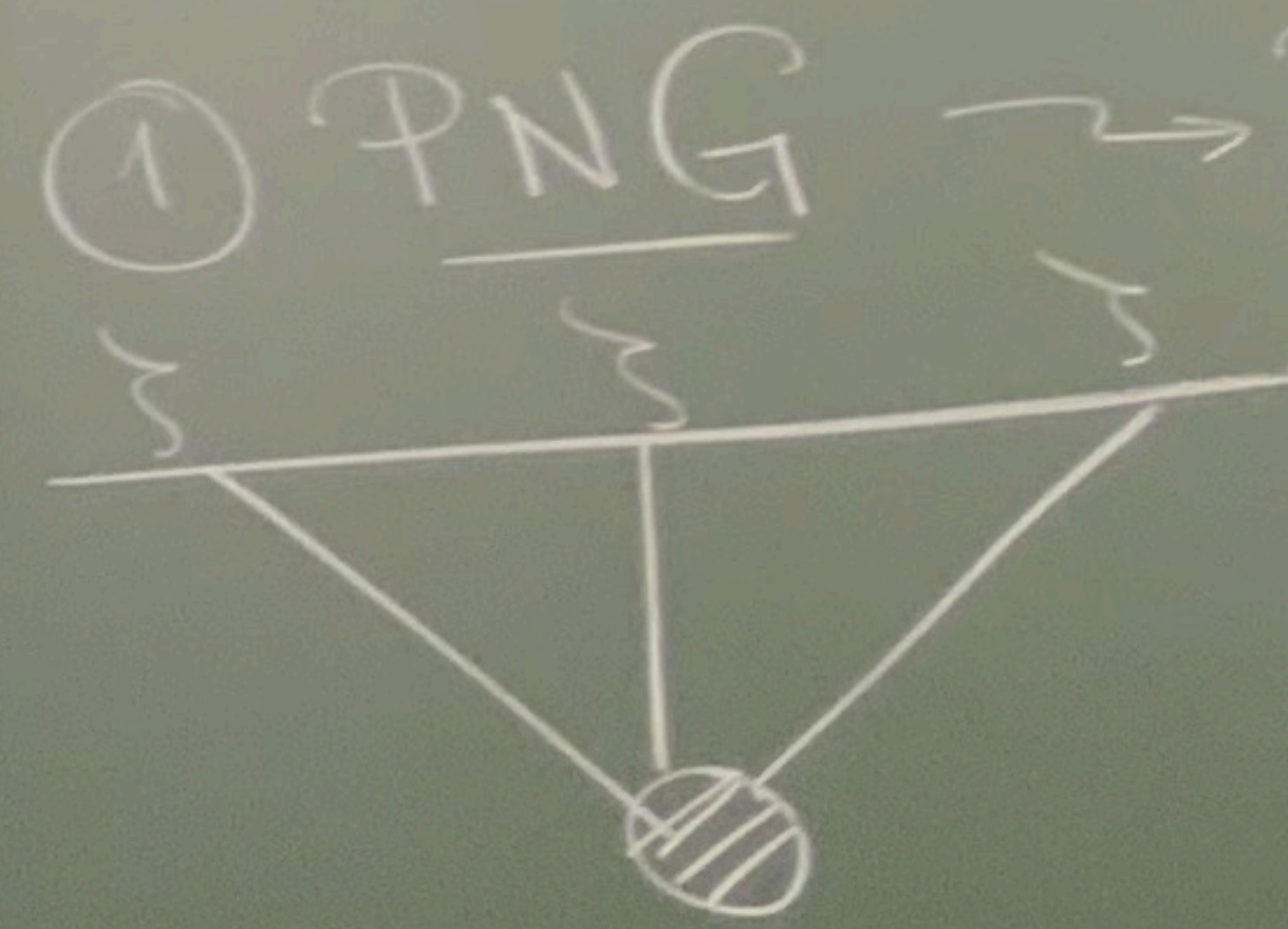
$$d_{(X, \mathbb{R})} \Psi + \epsilon [d \log \Phi_A(X_1, X_2)] \Psi = 0$$

$$d = \frac{\partial}{\partial X_1} dX_1 + \frac{\partial}{\partial X_2} dX_2$$

$F_{n_1, n_2, n_3, n_4, n_5}^\epsilon = \int \frac{(x_1, x_2)^\epsilon dx_1 \wedge dx_2}{x_1^{n_1} x_2^{n_2} L_1^{n_3} L_2^{n_4} L_5^{n_5}} \quad n_i \in \mathbb{Z}$   
 (Asymptotic, ...)  
Finitely many  $\int_S$  generate all others!  $\Rightarrow$  DEs always exist!  
 $F = F_{0,0,1,1,1}$

- \* Partial Fractions.
- \* IBPs.

Motivation:



$$\varepsilon = \begin{cases} 0 \\ -1 \\ -2 \\ -3 \end{cases} \begin{array}{l} dS \\ \text{flat} \\ RD \\ MD \end{array}$$

$$\Psi(x_1, x_2) = 0$$

$$x_1 + \frac{\partial}{\partial x_2} d x_2$$

$$x_i \in \mathbb{Z}$$

$\mathbb{F}$ s always exist!

$\varepsilon_1$	$\varepsilon_2$	$\varepsilon_3$	$\varepsilon_4$	$\varepsilon_5$
$x_1$	$x_2$	$L_1$	$L_2$	$L_3$

$$dx_1 dx_2$$


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$h_1$	$h_2$	$h_3$	$h_4$	$h_5$
$x_1$	$x_2$	$L_1$	$L_2$	$L_3$

$$d\psi + \varepsilon A \psi = 0 \quad d: 0 + dA\psi + A d\psi = 0$$

$$\varepsilon dA - \varepsilon A \cdot A = 0$$

$$A = \varepsilon \#$$

$$dS: \quad L_2(\dots) + \log(\dots) \log(\dots) + \frac{\pi^2}{6}$$

$$\frac{\partial}{\partial x_1} F_{\varepsilon=0} = \int \frac{\partial}{\partial x_1} \frac{dx_1 dx_2}{(x_1+x_1+1)(x_2+x_2+1)(x_1+x_2+1)} = \frac{d \log(x_1+1)}{x_1+1} \frac{dx_2}{(x_2+x_2+1)(x_2+x_1+1)}$$

$$\frac{\partial}{\partial x_1} \tilde{G} = \frac{1}{x_1+x_2} = H$$

$$d \begin{bmatrix} F \\ \tilde{G} \\ H \end{bmatrix} = \begin{bmatrix} 0 & \# & 0 \\ 0 & 0 & \# \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ \tilde{G} \\ H \end{bmatrix}$$

$$\frac{\partial}{\partial x_1} (x_1+x_2)H = 0$$

(Cosmological Poly)

$$F = \int \frac{dx_1 dx_2}{(x_1+x_1+1)(x_2+x_2+1)(x_1+x_2+1)}$$

$$= {}_2F_1(1+\varepsilon, 1-\varepsilon, \Delta, \dots)$$

A man in a dark sweater and khaki pants is standing on the right side of the room, pointing at the whiteboard.

$$dA\psi + A d\psi = 0$$

$$\epsilon dA - \epsilon A \cdot A = 0$$

$$A = \epsilon \#$$

$$= \int (x, x)^\epsilon \frac{\partial}{\partial x} ( ) \sim \int (x, x)^\epsilon \frac{1}{L_1 L_2 L_3}$$

$\epsilon = 0$

$$\frac{\partial}{\partial x} F = G = \int dx_2 / \dots$$

$\epsilon \neq 0$  PARTIAL FRACTIONS

$$\frac{\partial}{\partial x} F = \epsilon dx_2 F + \epsilon dx_2 \bar{G}$$


$$\epsilon A = \begin{bmatrix} \# \\ \# \\ \# \\ \# \end{bmatrix} \rightarrow F_i \quad d\psi + \epsilon A \psi = 0$$

Power-law

? How to find DE

$$= \int_0^\infty (x, x)^\epsilon \Omega$$

canonical form (diag. diag. in all  $\partial_i$ )

$$\Omega = [12] + [23] + [31] \quad [ij] \equiv d\log L_i d\log L_j$$


$$\log( ) + d\log(x, y)$$

$$= \dots + x + y$$

$$\begin{bmatrix} 0 & \# & 0 \\ 0 & 0 & \# \\ 0 & 0 & 0 \end{bmatrix}$$



$$A\psi + A d\psi = 0$$

$$\epsilon dA - \epsilon A A = 0$$

$$A = \epsilon \#$$

$$\log(x) + \frac{\pi^2}{6}$$

$$\frac{d \log(x+1)}{dx} = \frac{1}{x+1}$$

$$\frac{dx_2}{(x_2+x_2+1)(x_2+x_1+1)}$$

$$\begin{bmatrix} 0 & \# & 0 \\ 0 & 0 & \# \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ \tilde{G} \\ H \end{bmatrix}$$

$$= \int_{(x_1, x_2)}^\epsilon \frac{\partial}{\partial x_1} \left( \dots \right) \sim \int_{(x_1, x_2)}^\epsilon \frac{1}{x_1} \frac{1}{L_1 L_2 L_3}$$

PARTIAL FRACTION

$$\frac{\partial}{\partial x_1} F = G = \int dx_2 \dots$$

$$\frac{\partial}{\partial x_1} F = \epsilon \log F + \epsilon \log \tilde{G}$$

$$\epsilon A = \begin{bmatrix} \# & \# \\ \# & \# \\ \# & \# \end{bmatrix} \rightarrow F_1 \quad d\psi + \epsilon A \psi = 0$$

Power-law

How to find DE  $\mathcal{D}$

$$= \int_0^\infty \int_0^\infty \Omega$$

canonical form (dlog, dlog in all  $\partial_i$ )

$$\Omega = [12] + [23] + [31] \quad [ij] \equiv d \log L_i d \log L_j$$

Plot sp. unit  $-\log(x+1)$

