dS through holography:

3D case study

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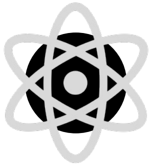
Outline



The desolate world of de Sitter in 3D

Let us start by writing down the static patch of the d-dimensional Schwarzschild-de Sitter solution,

$$ds^2 = -(1 - \frac{2m}{r^{d-3}} - r^2)\; dt^2 + \frac{1}{1 - \frac{2m}{r^{d-3}} - r^2}\; dr^2 + r^2 d\Omega_{d-2}^2,$$

where m is a mass parameter related to the black hole mass,

$$2m = 16\pi G_d \mathcal{M}/(d-2)\,\mathrm{Vol}(S^{d-2})$$

We see that there are two horizons – the black hole one and the cosmological one – and that there is a limit where these two approach one another: the Nariai limit. This limit sets the bound on the mass of the black hole that can reside in de Sitter.

Taking d = 3, we lose one of the horizons,

$$ds^2 = -(1 - 8GE - r^2)dt^2 + \frac{dr^2}{(1 - 8GE - r^2)} + r^2 d\phi^2,$$

and the one horizon that is left is given by $r_H = \sqrt{1 - 8GE}$. When E goes to zero, this horizon reduces to the usual pure dS horizon radius.

We can learn more about this solution by looking near $r = 0$,

$$ds^2 \sim -r_H^2 dt^2 + \frac{dr^2}{r_H^2} + r^2 d\phi^2.$$

We can rescale the coordinates by defining

$$t' = r_H t, \qquad r' = r/r_H, \qquad \phi' = r_H \phi$$

for which our solution becomes simply

$$ds^2 = -dt'^2 + dr'^2 + r'^2 d\phi'^2.$$

This geometry looks like flat space, but it is not quite so: the angle ϕ' is identified with the period $2\pi r_H$, which indicates we have a conical deficit.

Is there a simple way to see why we don’t get black holes in 3D flat or dS spacetimes?

It all has to do with scales.

Namely, in 3D gravity, GM = 1 and so, including a massive object does not give us a length scale we can work with. Without a length scale, we cannot make horizons, and therefore, no black hole can exist.

Adding a cosmological constant helps us since it allows for a length scale to be introduced. However, even though a length scale is enough to make a horizon, it is not enough to make a black hole: one needs an attractive potential for that.

In this sense, black holes in AdS_3 can exist, due to the gravitational potential of the AdS spacetime.

But dS spacetimes expand, and they dilute gravitational interaction – this makes it difficult to form black holes, and in 3D, the best one can do is a conical singularity. The horizon one can make instead is a cosmological one.

Cosmological constant is not the only way one can introduce a length scale: quantum physics does so as well. Namely, allowing for quantum effects, we can construct the Planck length, $L_P \sim \hbar G$.

And if we want our quantum black holes to not be of Planck size, we introduce a great deal of quantum fields, so that $L_P << cL_P \sim c\hbar G$, where $c >> 1$ is the central charge of the supposed quantum fields.

Indeed, if one computes the effect of quantum fields in a 3D dS spacetime with a defect, one obtains a horizon:

$$ds^2 = -\left(1 - 2m - \frac{r^2}{L^2} - \frac{2L_P^{(3)}F(\gamma)}{r}\right)dt^2 + \left(1 - 2m - \frac{r^2}{L^2} - \frac{2L_P^{(3)}F(\gamma)}{r}\right)^{-1}dr^2 + r^2 d\phi^2 \ ,$$

where $F(\gamma)$ is a form factor associated to the stress tensor

$$8\pi G_3 \langle T^\mu_\nu \rangle = \frac{L_{Pl}^{(3)}F(\gamma)}{r^3}\mathrm{diag}(1, 1-2)$$

and where we explicitly wrote the Planck length $L_P^{(3)} = G_3\hbar.$

Of course, this calculation can be trusted only perturbatively.

In order to put this quantum-induced horizon on a stronger footing, we must solve simultaneously the coupled system of the metric + quantum fields.

A way to obtain all orders in $\hbar$ is to put our dS_3 on a brane.

Gravity at the end of the world

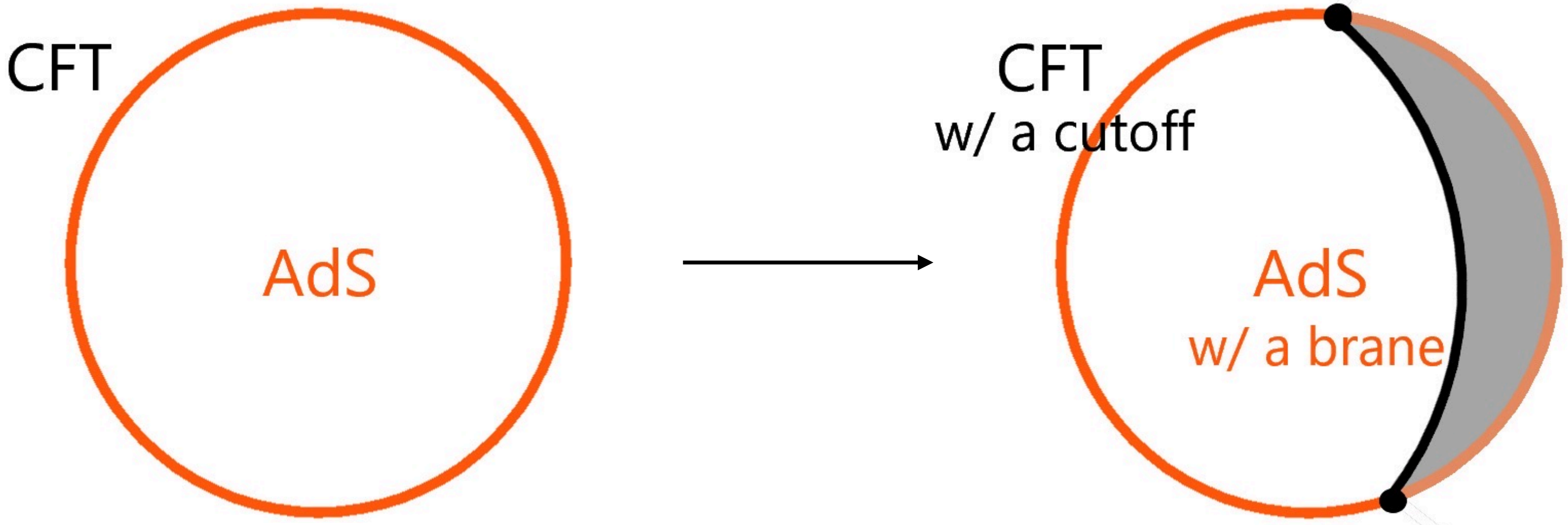
What is the general philosophy?

Classical dynamics in an AdS_{d+1} bulk with a d-dim brane holographically encodes the quantum dynamics of the dual d-dim CFT coupled to a d-dim gravitational theory on the brane.

Essentially, putting the brane in the bulk corresponds to integrating out the holographic CFT degrees of freedom above the ultraviolet cutoff.

The CFT below this cutoff gives rise to a renormalized $\langle T_{\mu\nu} \rangle$, with a large central charge $c \gg 1$ given by the AdS_{d+1} radius in (d+1)-dim Planck units.

[de Haro, Skenderis, Solodukhin '00]


CFT
AdS
CFT
w/ a cutoff
AdS
w/ a brane

The theory of the brane can be now viewed in two different ways:

1) Brane is an object in the (d+1)-dim bulk with some boundary conditions, $I + I_b$

$$I = \frac{1}{16\pi G_N}\left[\int_{\mathcal{M}} d^{d+1}x\sqrt{-G}\left(R[G] + \frac{d(d-1)}{\ell^2}\right) + 2\int_{\partial\mathcal{M}} d^dx\sqrt{-g}K\right]$$

$$I_b = -T\int_{\rho=\varepsilon} d^dx\sqrt{-g}$$

2) Higher curvature gravity in d dims coupled to some cutoff CFT, $I_{bgrav} + I_{CFT}$

$$I = \frac{\ell}{16\pi G_N}\int d^dx\sqrt{-\hat{g}^{(0)}}\left(\varepsilon^{-d/2}\hat{\mathcal{L}}_{(0)} + \cdots + \varepsilon^{-1}\hat{\mathcal{L}}_{(\lceil d/2\rceil-1)} - \log(\varepsilon)\hat{\mathcal{L}}_{(d/2)}\right) + \mathcal{O}(\varepsilon^0)$$

$$I + I_b = I_{\rm bgrav} + I_{\rm CFT}$$

Higher curvature gravity doesn't sound too appealing

However, the closer we are to the boundary, the less relevant higher curvature terms become, and in the perturbative limit they are completely suppressed

The crucial point is that our classical bulk encodes the quantum dynamics of the cutoff CFT which resides on the brane

In the rest of this talk, we will focus on the d-dim view of the brane physics, but to construct this theory, we start from the (d+1)-dim bulk

We will take as our higher-dimensional bulk the AdS_4 C-metric

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left(-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right)$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r},$$

$$G(x) = 1 - \kappa x^2 - \mu x^3$$

$$R_4 = -12\left(\frac{1}{\ell^2} + \frac{1}{\ell_3^2}\right) = -\frac{12}{\ell_4^2}$$

Emparan, Horowitz, Myers '99
Emparan, Frassino, Way '20

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We can slice this metric along x=0 to obtain our brane

Why x=0? We get a simple solution with $K_{ab} = -\frac{1}{\ell}h_{ab}$ and crucially, we obtain black holes on the brane

$$ds^2 = -H(r)dt^2 + \frac{dr^2}{H(r)} + r^2 d\phi^2$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}$$

μ is a parameter set by bulk regularity – it sets the stress tensor of the brane CFT – this is why it’s a *quantum correction* to the metric

ℓ determines the brane tension, and so, the distance of the brane to the would-be boundary

Depending on the brane tension, we can obtain three kinds of branes: AdS, flat and dS

Recall

$$R_4 = -12\left(\frac{1}{\ell^2} + \frac{1}{\ell_3^2}\right) = -\frac{12}{\ell_4^2}$$

$$-\lambda_4 = \frac{1}{\ell^2} - \lambda_3, \qquad \frac{1}{\ell_i^2} = -\lambda_i$$

We want to keep the bulk spacetime AdS, and so

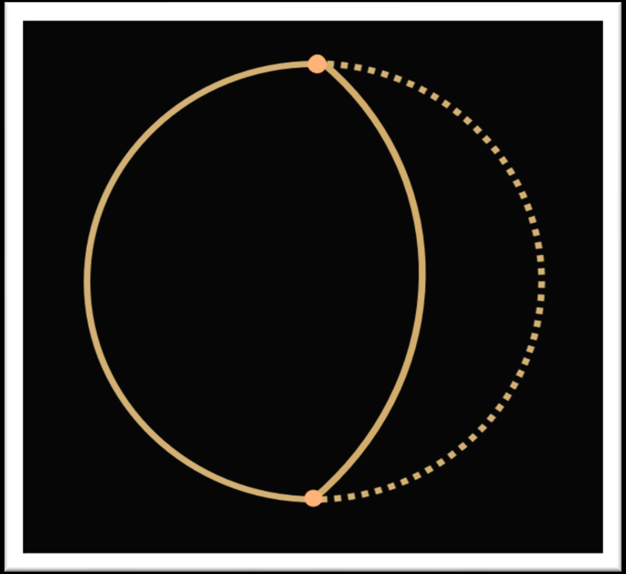
$$-\lambda_4 > 0 \longrightarrow \frac{1}{\ell^2} > \lambda_3$$

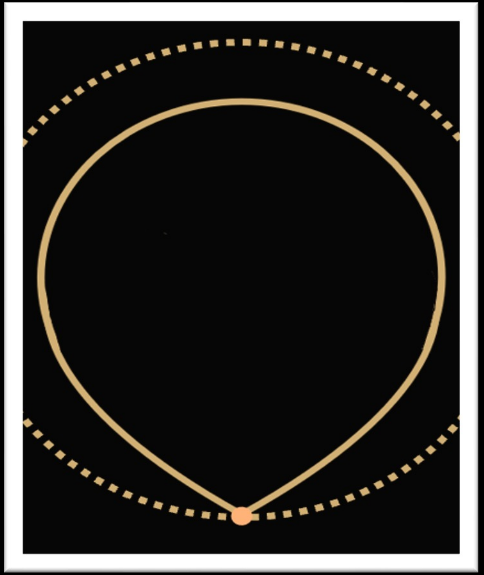
This gives us three possibilities

$$\lambda_3 < 0 \rightarrow \text{AdS brane} \rightarrow 0 \leq \ell < \infty$$

$$\lambda_3 = 0 \rightarrow \text{flat brane} \rightarrow \ell = \ell_4$$

$$\lambda_3 > 0 \rightarrow \text{dS brane} \rightarrow \ell \text{ bounded}$$





dS brane: quantum black hole in dS_3

$$ds^2|_{x=0} = -\left(1 - \frac{\mu\ell}{r} - \frac{r^2}{R_3^2}\right) dt^2 + \left(1 - \frac{\mu\ell}{r} - \frac{r^2}{R_3^2}\right)^{-1} dr^2 + r^2 d\phi^2 \, .$$

We see that we have obtained a 3D Sch-dS-like metric, since we can clearly see there are two horizons. It very much looks like the 4D solution, which is fine: our solution is obtained as a section of such a 4D black hole system.

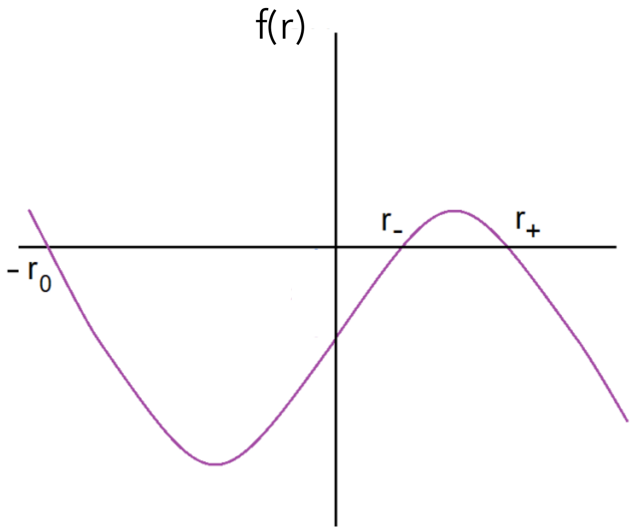
Note that the form also agrees with the one found by a perturbative calculation.
The difference is in the validity of the solution: given that the setup is for strongly coupled fields, the method used in the perturbative calculation cannot be applied here.

Nevertheless, the holographic calculation yields simple ‘resummed’ quantities.

Quantum black holes in dS_3

What are some of the properties of these quantum black holes?

For instance, we can study the Nariai limit now:
when r+ --> r- so that the black hole barely fits inside the cosmological horizon



What does this tell us about the CFT?

The Nariai limit places a bound on the mass, which is connected to the backreaction of the CFT: the backreaction cannot be too strong, otherwise we'll make a black hole that does not fit.

Another interesting question is regarding the probability for a dS horizon to pop-up a black hole as a fluctuation.

This calculation was done by Susskind, and he used the results of this calculation to propose a conjecture regarding the holographic dual of de Sitter spacetimes.

The calculation is pretty straightforward. The entropy of pure dS in e.g., 4D, is given by the area of the cosmological horizon,

$$S_0 = \frac{4\pi R_4^2}{4G_4} \ ,$$

where R_4 is the radius of dS_4. Fluctuations may occur in which the entropy is decreased to a smaller value S_1. The probability for such a fluctuation is given in terms of the entropy deficit

$$\text{Probability} = e^{-\Delta S} \ , \quad \Delta S \equiv S_0 - S_1$$

Small black holes constitute such fluctuations,

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_2^2 \,, \quad f(r) = 1 - \frac{r^2}{R_4^2} - \frac{2MG_4}{r} \,,$$

where by a small black hole we mean that its radius is much smaller than R_4 .

The shrunk entropy of the dS spacetime is now given by the shrunk area, and so, to leading order in the black hole mass, we obtain

$$S = \frac{\pi}{G_4}(R_4^2 - 2R_4 M G_4) = S_0 - 2\pi R_4 M = S_1$$

The entropy deficit is then simply given by

$$\Delta S = 2\pi R_4 M = \sqrt{Ss} = \sqrt{S_0 s} \,,$$

where $s = 4\pi M^2 G_4$ is entropy of the 4D AF Schwarzschild black hole.

What can we extract from this simple exercise?

Susskind argued that this entropy deficit can be reproduced by a tentative holographic dual to dS which is described in terms of a matrix model.

Namely, let's assume that the horizon degrees of freedom are a collection of N × N Hermitian matrices,

$$A_{m,n} = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,N} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,N} \end{pmatrix}$$

We can also assume that the total entropy in thermal equilibrium is proportional to the number of degrees of freedom, $S_0 = \sigma N^2$ where σ is the entropy per dof.

Now, consider a state with a black hole of entropy $s = \sigma m^2$.

Motivated by M(atrix) theory, we assume that in a state composed of two well-separated components – in this case the small black hole and the large cosmological horizon – the off-diagonal degrees of freedom are constrained to be in their ground states, and therefore carry no entropy.

Therefore, the dofs split into block-diagonal forms, with the cosmological horizon dofs forming an (N − m) × (N − m) block, and the black hole dofs forming an m × m block.

Calculating the entropy of this state, we obtain $S_1 = \sigma(N-m)^2 + \sigma m^2$

Assuming m < < N, we obtain a very similar formula for the entropy deficit,

$$\Delta S = 2\sqrt{Ss}$$

One can generalize the formula for general dimensions, in which case we obtain

$$\Delta S = \left(\frac{d-2}{2}\right) \; S_0^{\frac{1}{d-2}} \; s^{\frac{d-3}{d-2}}.$$

This formula, derived from the d-dimensional Schwarzschild solution, can be reproduced with matrix degrees of freedom but at a cost: one has to allow the entropy per degree of freedom to depend on N according to

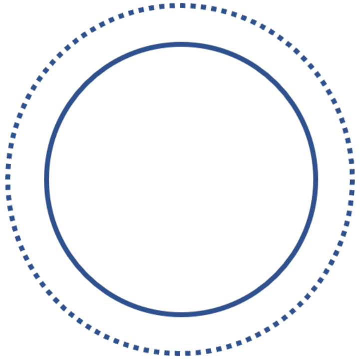
$$\sigma(N) \sim \frac{1}{N^{\left(\frac{d-4}{d-3}\right)}}.$$

What about 3D?

In 3D, this bound fails because *classically* bhs don't exist in 3D dS!

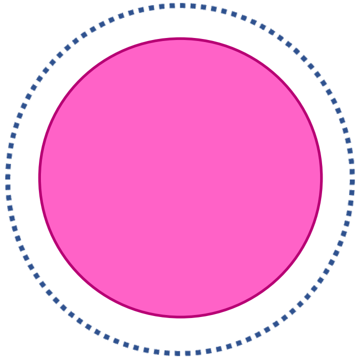
However, we can do these calculations in our setup with quantum corrections

Instead of going through the detailed form of the entropies, let us try and see what result should we expect.
To do so, note that we can use the doubly-holographic perspective in order to get the entropies



$$S_4 = S_3 + S_{vN}$$

$$S_i = \frac{A^i_{BH}}{4G} + \frac{A^i_{CH}}{4G}$$



Entropy in the 4D is fully captured by the *generalized entropy* on the brane.

Therefore, it's clear our calculation of the entropy will essentially give 4D results which we can then interpret as 3D area entropies 'corrected' by the entropy of quantum fields.

This indicates that the original proposal by Susskind regarding the possible matrix description needs to be modified to include the effects of quantum fields, at least for those that are strongly coupled and for a very large number of them.

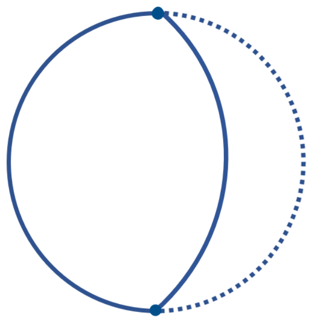
In essence, one could conjecture that because $\Delta S_4 \sim \sqrt{Ss} \longrightarrow \Delta S_3 \sim \sqrt{S_{\text{gen}} s_{\text{gen}}}$

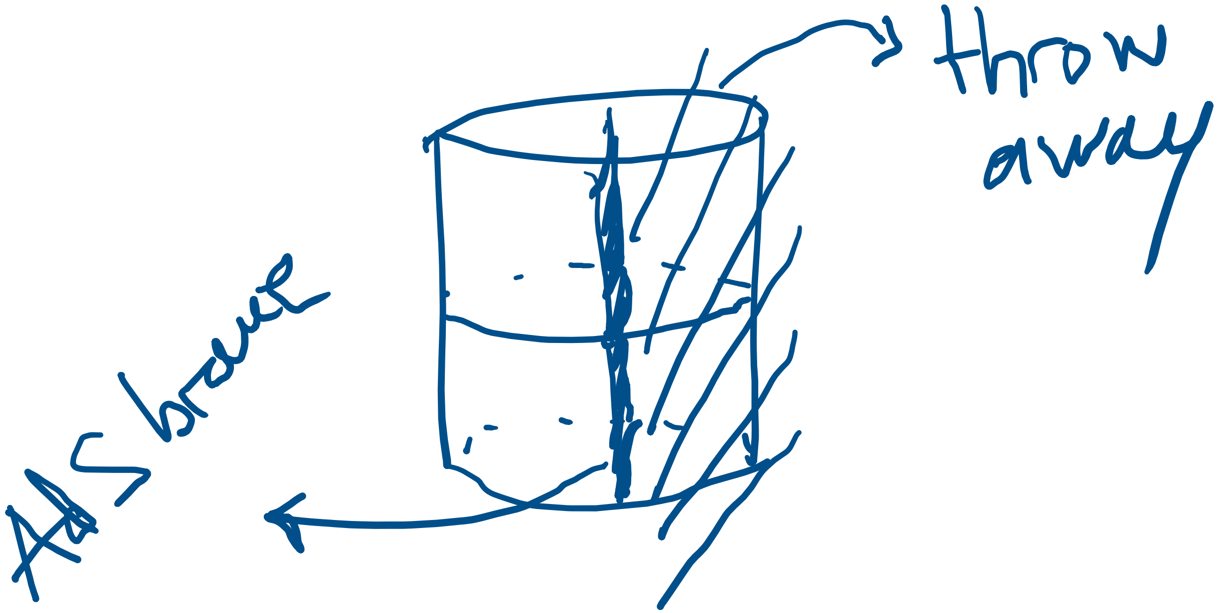
$$\Delta S = \left(\frac{d-2}{2}\right) S^{\frac{1}{d-2}} s^{\frac{d-3}{d-2}} \longrightarrow \Delta S \sim S_{\text{gen}}^{f(d)} s_{\text{gen}}^{g(d)}$$

Unfolding the layers of holography

Can this braneworld picture indicate what would be the correct dual to de Sitter spacetime?

In order to see some hints, let us go back to the case of an AdS brane. In this case we know that there are (timelike) defect/boundary CFTs sourcing the brane. In other words, the defect CFT is dual to the brane.

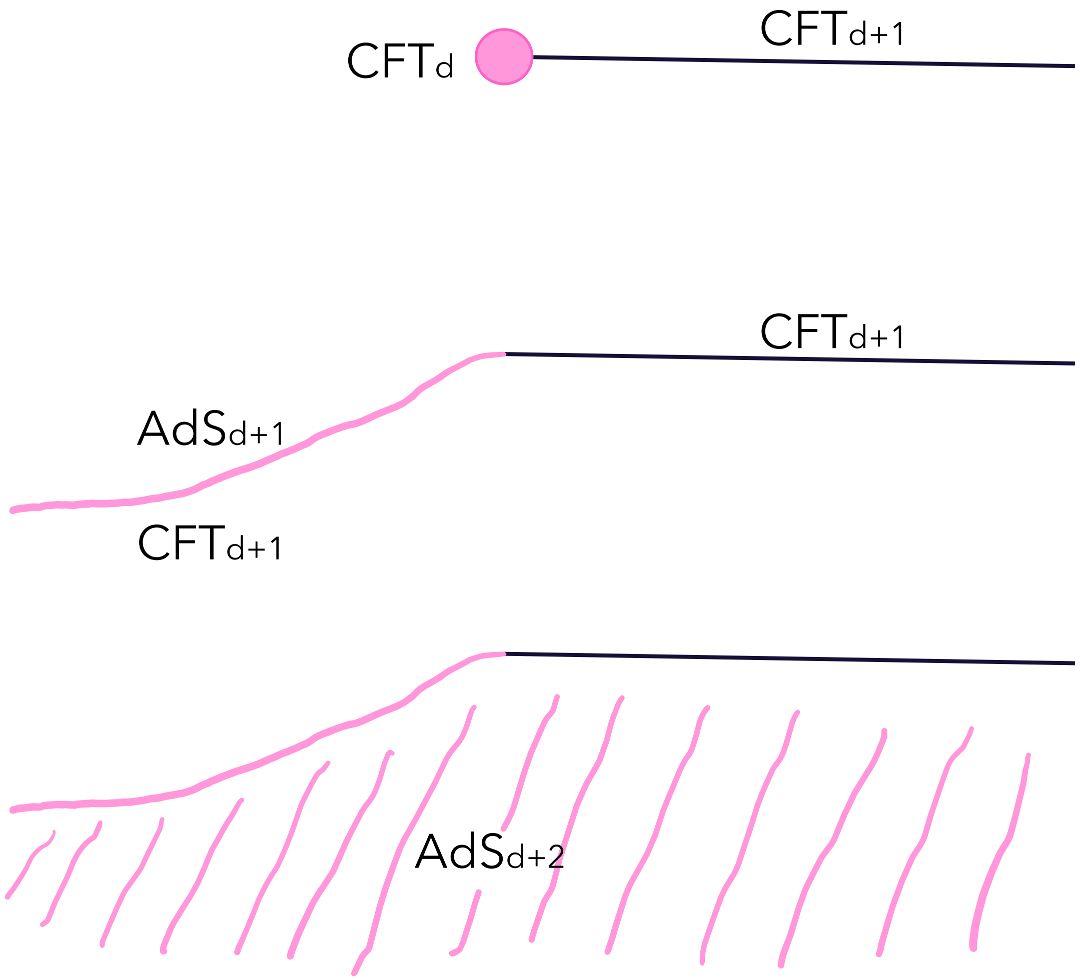




To be more precise, the basic idea is the following:

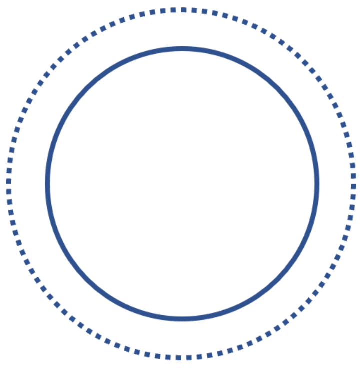
connect three levels of holography

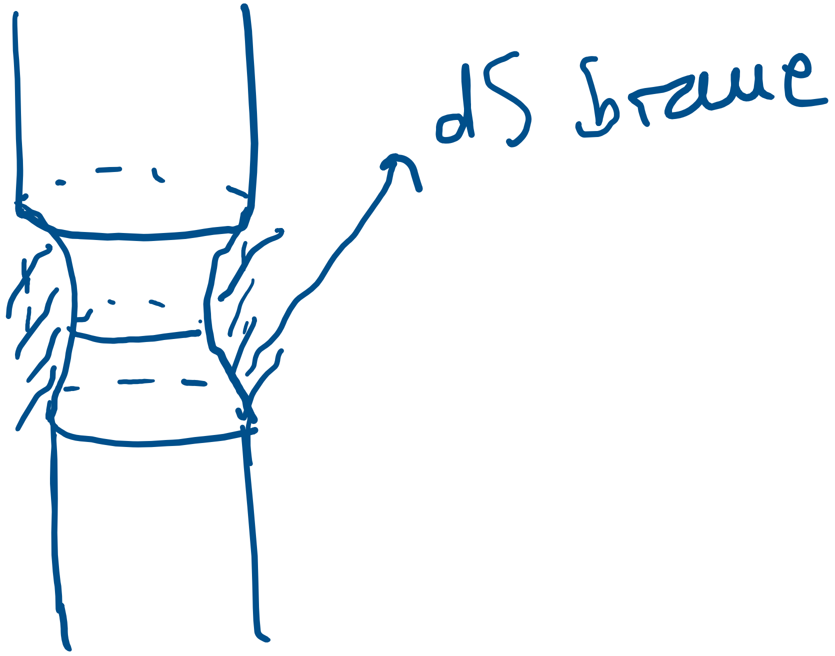
- Couple a defect CFT_a to a bath
- Represent the CFT_a as AdS_b+CFT_b
- Represent CFT_b as AdS_c



Let's take a leap of faith and say that in braneworld models, for any branes, the defect CFTs are dual to them.

What does this mean for our solution, where is the defect CFT?





It turns out that the hyperboloid hits the boundary at some finite time and so, the defect CFT is the Euclidean CFT living at the asymptotic infinity of dS (both past and future).

From this picture, we see that one gets a hint that something like dS/CFT should be the dual picture for de Sitter spacetimes.

Summary

- We constructed a black hole inside 3D dS spacetime using the braneworld formalism;
- We can compute all the usual thermodynamic properties of such objects and answer potentially interesting questions;
- One can also try to understand what would be the tentative dual of dS spacetimes.




Thank you!