The discreet Charm of the discrete series in dS_2

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The goal of my talk today is to introduce you to a few peculiarities about QFT in de Sitter in a setting where we should be able to compute to our heart's desire. That way there is nowhere for us to escape.

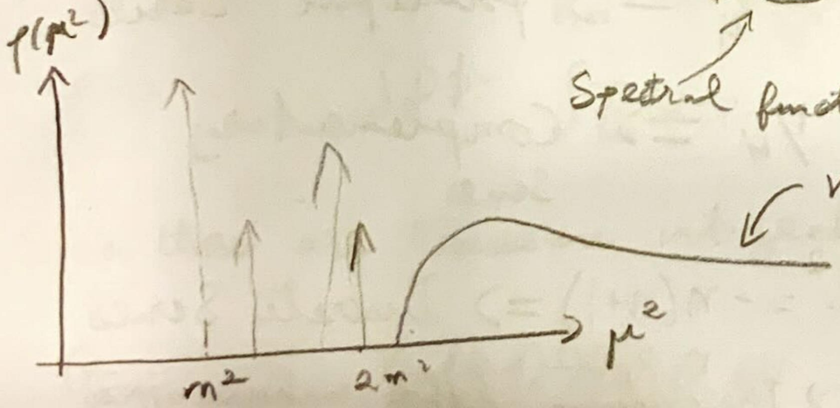
Now, Let me set the stage: (See also Kamran's talk)

I want you to recall a fact about interacting QFT in flat space.

Say we have a scalar field $\phi(x)$.

Its two point function, generally, can be written as:

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int d\mu^2 \underbrace{\rho(\mu^2)}_{\text{Spectral function}} \underbrace{G_F(x-y,\mu^2)}_{\text{Free two-point function}} \quad (*)$$



↳ tells us about 1-particle states (delta functions) & multiparticle states, which have a continuous spectrum

The fact that ρ is a function of μ^2 is because μ^2 is the eigenvalue of H in the rest frame, so it is a good quantum number

Now, a basic question about interacting QFT in de Sitter is how to generalize (*) to the de Sitter case & what does ρ tell us about the spectrum of bound and multiparticle states

dS_2

I'm going to work in fixed dS_2

$$-(X^0)^2+(X^1)^2+(X^2)^2 = L^2$$

→ has one rotation & 2 boosts

→ $SL(2,\mathbb{R})$ symmetry

The generalization of $H \Longrightarrow C_2$ of $SL(2,\mathbb{R})$ and these C_2 eig's are: $\Delta(\Delta-1) = -m^2L^2 \Longrightarrow \Delta = \frac{1}{2}\left(1 \pm \sqrt{1-4m^2L^2}\right)$

<u>Unitary Irreps of $SL(2,\mathbb{R})$</u>: See Dobrev, Lang, or Zimo Sun for recent review

$\Delta = \frac{1}{2}(1+i\nu) \Longleftrightarrow m^2L^2 > \frac{1}{4} \Longrightarrow$ Principal Series

$\Delta \in [0,1] \Longleftrightarrow m^2L^2 \leq \frac{1}{4} \Longrightarrow$ Complementary Series

$\Delta = 1, 2, 3 \ldots \in \mathbb{Z}^+ \quad m^2L^2 = -n(n+1) \Longrightarrow$ Discrete Series

$n = 0, 1, 2$

→ The existence of these tachyons appear weird but is standard.

<u>Källén-Lehmann For dS_2</u>

dS inv distance

$$G_F^{\Delta}(x,y) = \frac{\Gamma(\Delta)\Gamma(1-\Delta)}{4\pi}\, {}_2F_1\left(\Delta, 1-\Delta, 1, 1-\chi/2\right)$$

$$\boxed{\langle 0|\, \phi(x)\, \phi(y)\, |0\rangle \overset{?}{=} \int d\Delta\, \rho(\Delta)\, G_F^{\Delta}}$$

⇝ what are the characteristics of $\rho(\Delta)$ in a healthy QFT on dS_2?

Immediate Problem Repka 1978 Tensor products $SL(2,\mathbb{R})$

Principal ⊗ Principal = $\int$ Principal ⊕ $\sum$ discrete

↳ The multiparticle Hilbert space of a heavy particle in dS must contain these tachyonic states

Does this suggest something about $\rho(\Delta)$? No! In fact

$$\Delta = 1+n \Rightarrow G_F^{\Delta} = \frac{\Gamma(1+n)\,\Gamma(-n)}{4\pi}\, {}_2F_1\left(1+n, n, 1, 1-\tfrac{u}{2}\right)$$

divergent!

The naive free propagator itself seems to have a problem on these states.

We spent some time being confused about this fact so let me dwell on it: Let me work in Euclidean signature (Folacci)

$$G_F(x,y) = \int D\phi\, \phi(x)\, \phi(y)\, e^{-\int_{S^2} \phi(-\nabla^2+m^2)\phi}$$

Euclidean: $\phi = \sum_{l,m} Y_{lm}(\theta,\phi)\, c_{lm}$ $\quad (-\nabla^2+m^2)\, Y_{lm} = \{l(l+1)+m^2\}\, Y_{lm}$

$$\int D\phi = \int \prod_{l,m} dc_{lm}$$

Since these are Gaussian integrals; we can compute exactly

$$G_F = \sum_{l,m} \frac{Y_{lm}(\theta,\varphi)\, Y_{lm}(\theta',\varphi')}{l(l+1)+m^2}$$

⇝ to get dS answer, rotate $\theta \to iT + \pi/2$

This function is analytic for any m^2 but is divergent if $m^2 = -n(n+1)$

⇒ These are additional zeromodes that must be removed to make the problem well-defined.

These zeromodes are related to emergent shift symmetry for the scalar for these particular values of the masses.

How do we define G_F^n ?

$$\tilde{G}_F^n = \sum_{\ell \neq n} \frac{Y_{\ell m} Y'_{\ell n}}{\ell(\ell+1) - n(n+1)} \quad ?$$

It is hard to imagine this removal of zeromodes as arising from a local QFT, but This suggests at least the following:

$$\boxed{\langle 0 | \phi(x) \phi(y) | 0 \rangle = \int_{\text{Principal}} d\Delta \, \rho(\Delta) \, G_F^\Delta + \sum_{n=0}^{\infty} \tilde{\rho}(n) \, \tilde{G}_F^n} \qquad (**)$$

Caveat

$$\left\{ -\nabla^2 - n(n+1) \right\} \tilde{G}_F^n = \sum_{\ell \neq n} Y_{\ell m} Y'_{\ell m} = \delta(X-Y) - \underbrace{\sum_{m=-n}^{n} Y_{nm} Y'_{n,m}}_{\text{Inhomogeneous}}$$

Argument against

because of inhomogeneous RHS, it is hard to imagine reproducing $\tilde{G}_T^n$ from a Lorentzian Fock space of discrete series states

Argument for

$\tilde{G}_F^n$ naturally induces a positive inner product on discrete series states, see: Epstein, Moschella, Zimo Sun, Bros, Epstein Moschella

→ Really suggests that $\tilde{G}_F^n$ should be on RHS of $(**)$

Because we're confused we should work out an example:

$n=0$ is the massless free boson and the zero-mode is the shift of $\phi(x) \to \phi(x) + c$

We can either compactify the boson $\phi \simeq \phi + R$ or we can gauge the shift symmetry

$$S = -\frac{1}{2}\int d^2x\sqrt{g}\,(\partial_\mu\phi - A_\mu)(\partial^\mu\phi - A^\mu) + \frac{k}{2\pi}\int d^2x\sqrt{g}\,B\epsilon^{\mu\nu}F_{\mu\nu}$$

$$\phi \to \phi + \omega(x)\ , \quad A_\mu \to A_\mu + \partial_\mu\omega$$

$\rightsquigarrow$ Canonically Quantizing we see that the gauging procedure removes the zeromode but at the expense of ϕ no longer being Gauge invariant

Should instead consider:

$\langle(\partial_\mu\phi - A_\mu)(\partial_\nu\phi - A'_\nu)\rangle \rightsquigarrow$ like in the Free boson CFT $\langle\partial_+ X\,\partial_+ X\rangle$

For higher n, there should be a similar story

Then perhaps the appearance of $\tilde{G}_T^{\,n}$ on the RHS of the Källen-Lehmann decomposition is suggesting that we should always have to apply projectors onto some Gauge-invariant subspace, but how to do this if the sum includes higher Δ higher n.

Final remarks

The EoM for the $\Delta = 1+n$ UIR makes an appearance in the pre-Hilbert space of $SL(n,\mathbb{R})$ BF Theory in a particular gauge.

$\rightsquigarrow$ But this theory is topological & doesn't have the Hilbert space associated w/ a local QFT

I wonder if this suggests something deep about QFT in dS:

Namely, when we want to turn on G_N, we will have to project out all dS irreps except the trivial rep. I think the appearance of the discrete series is confronting us with that fact sooner than expected