De Sitter space semi-classical: wavefunctions, wormholes, swampland,…

Thomas Van Riet, Leuven (Belgium)

‘Quantum de Sitter Universe’, Cambridge 2023

1. Status dS space in string theory. (5%)

2. Dark bubble scenario and Vilenkin's wavefunction (30%)

3. Axion wormholes in dS space (30%)

4. Festina Lente (30%)

5. Summary (5%)

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$$ds^2_{10} = ds^2_4 + ds^2_6$$

Metric on compact space.

$$L_{KK} = \text{Volume}^{1/6} = \frac{1}{M_{KK}}$$

$$L_{\text{Hubble}} = \frac{1}{M_\Lambda}$$

Vacuum is perceived as 4D if

$$L_{\text{Hubble}} \gg L_{KK} \leftrightarrow M_\Lambda \ll M_{KK}$$

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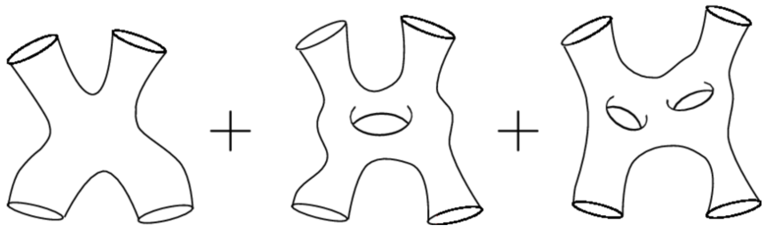
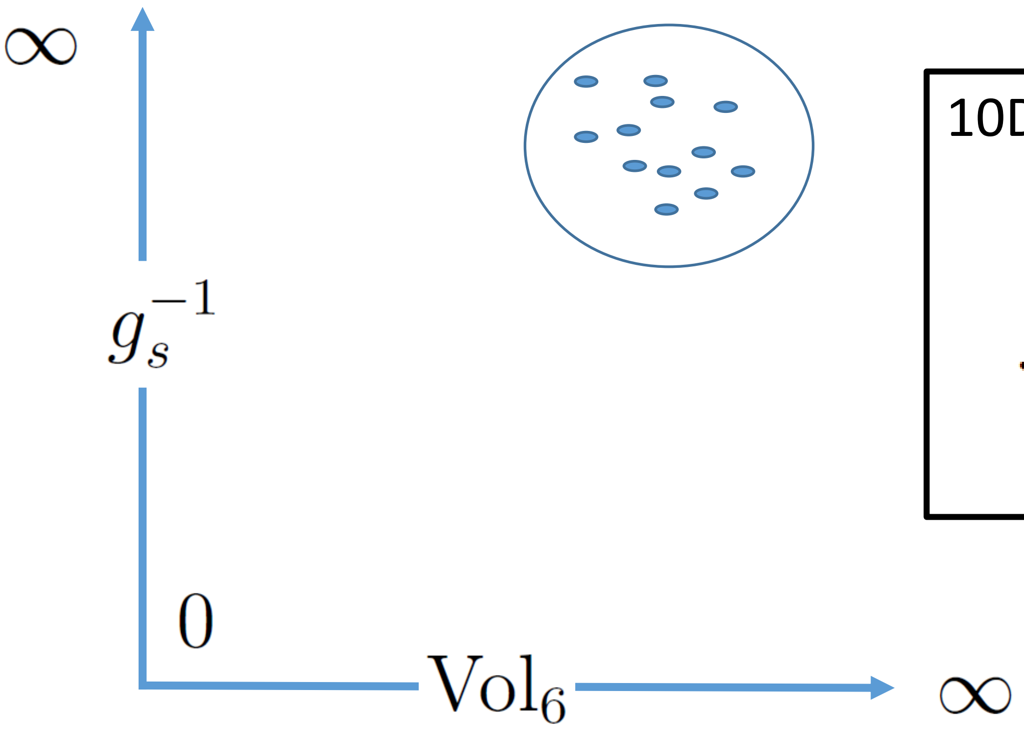
String theory reduces to classical 10D SUGRA if

1) g_s is small (g_s << 1):



2) All field gradients are small with respect to $1/l_s$ to control higher derivative expansion.

boundary of string moduli space:



10D sugra, possibly with some leading quantum corrections

$$\int \sqrt{g}\left\{R - \tfrac{1}{2}(\partial\phi)^2 - \sum_n \frac{1}{2\,n!} e^{a_n\phi} F_n^2\right\} + S_{loc}\,,$$

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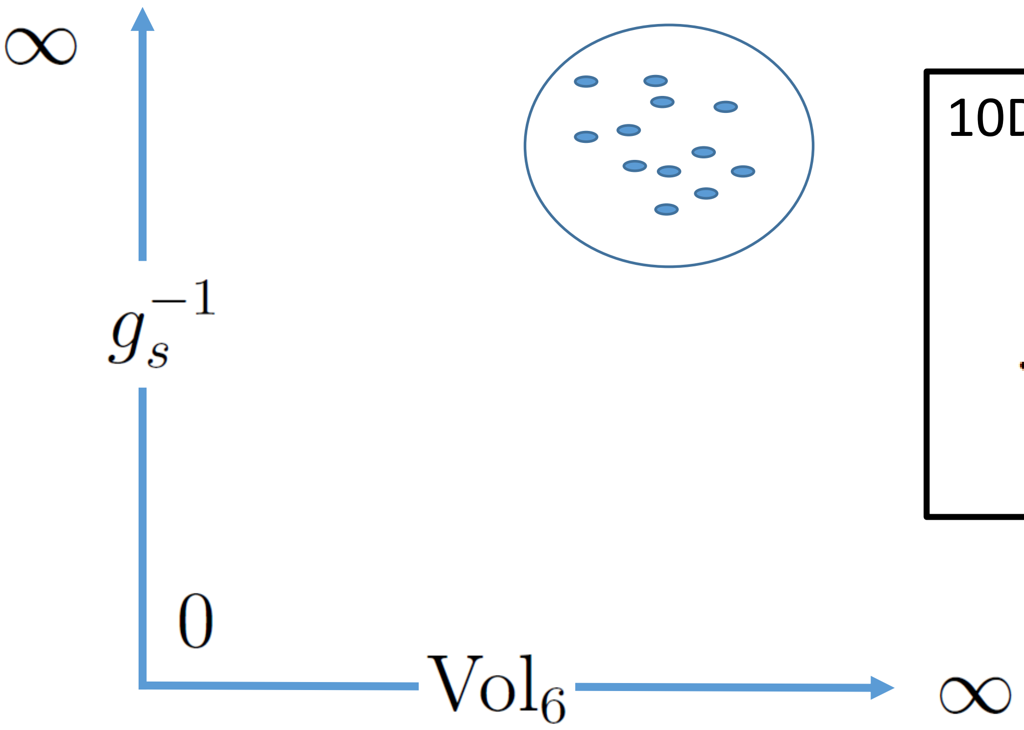

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E = Fluxes + Branes + Curvature

'Arrange' solutions such that quantum corrections are negligible or not.

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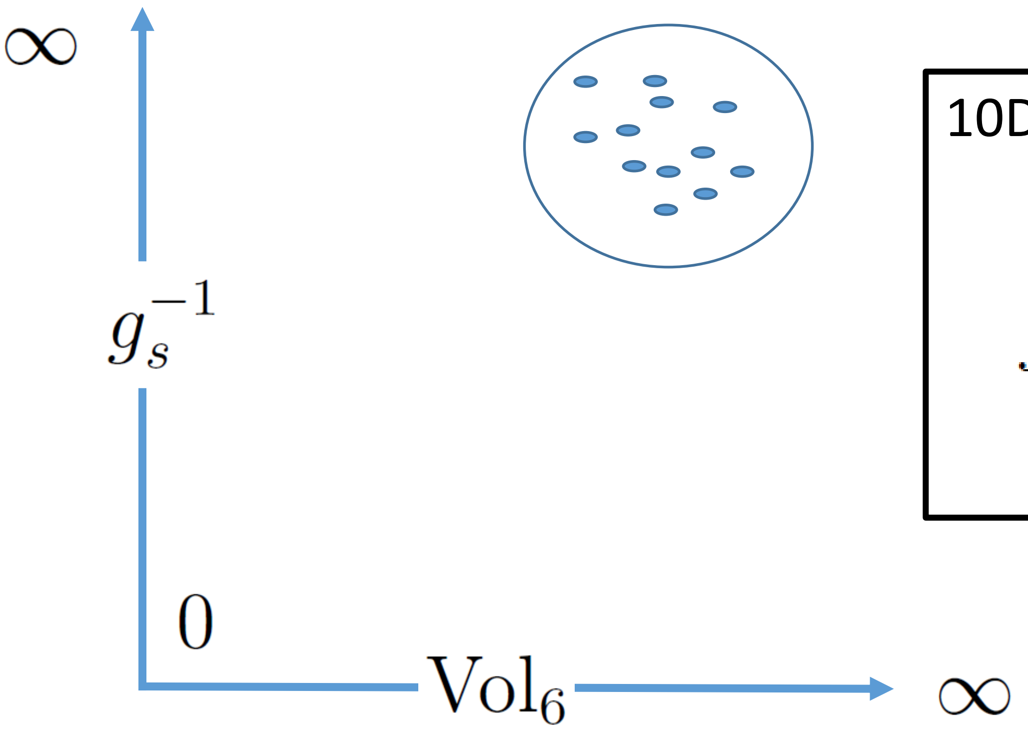

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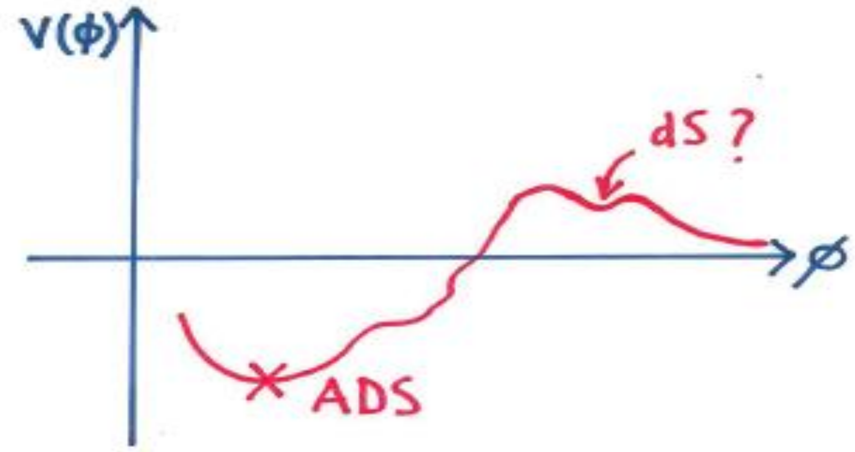
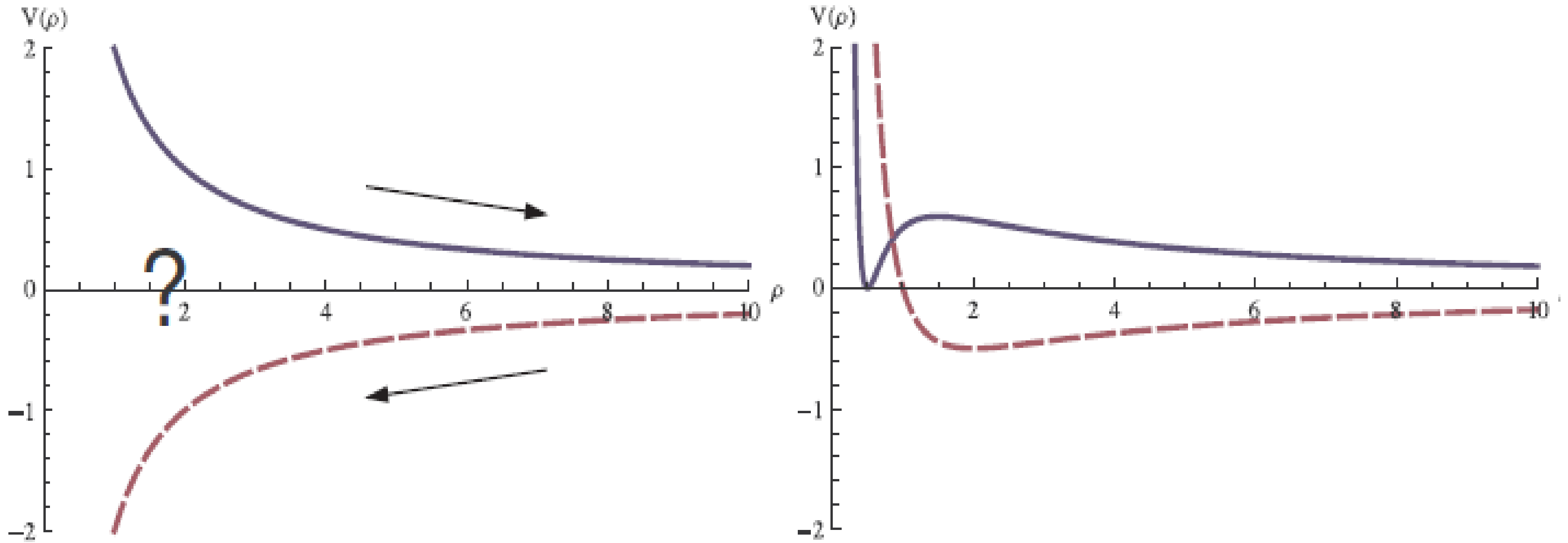


Vacuum energy:
E = Fluxes + Branes + Curvature

‘Arrange’ solutions such that quantum corrections are negligible or not.

Then the computed result is the full result (up to small corrections.) Nice virtue of string theory. We can compute vacuum energies in certain corners of the theory!

Fluxes are a way out of Dine-Seiberg problem: vacua are typically “non-calculable” [Denef review 2008]

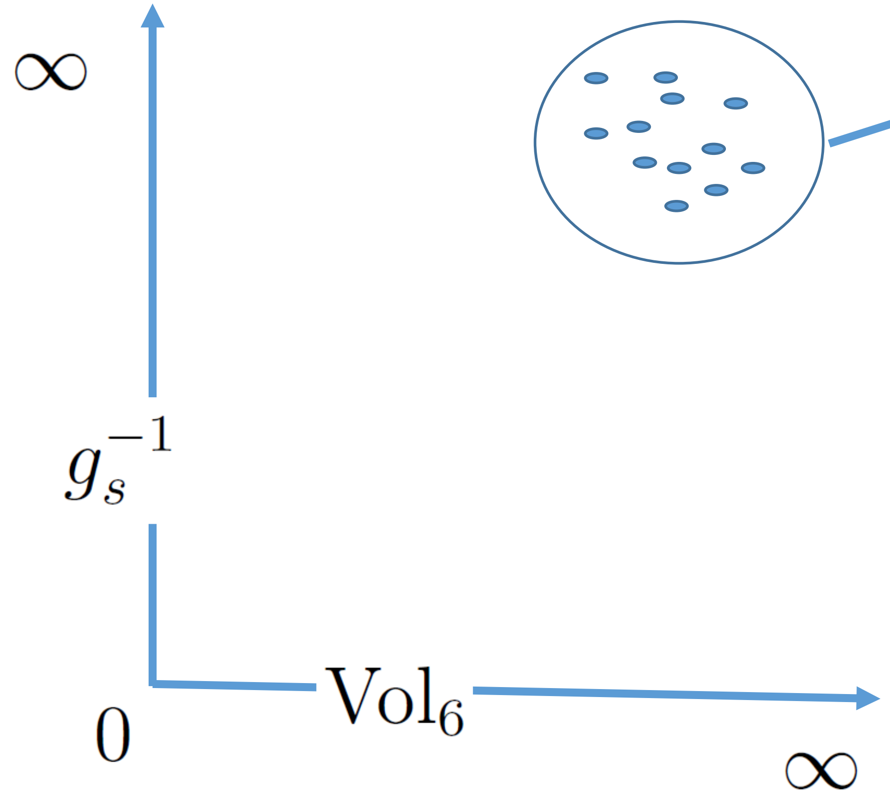


Aim of flux compactification program is to construct ***calculable vacua.*** Solutions “under control”. We can stabilize at the boundary of moduli space?

Recent developments have crushed this hope

boundary of string moduli space:

Only anti-de Sitter space here?



- Example $AdS_5 \times S_5$. As you crank up flux to infinity all length scales go to infinity, coupling is free parameter and can be dialed small. We trust it.
- Such a ``cranking up" never gives dS solutions. So no number that can be dialed. [Junghans 2018, Banlaki-Showdury-Roupec-Wrase, 2018]
- Consistent with heuristic (and more general) Swampland arguments. [Ooguri-Palti-Shiu-Vafa 2018, Wrase-Hebecker 2018]

Status of dS space -Developments last 5 years?

- Existing models, (anti-brane uplifts): new problems found, older problems resolved.

- New models; eg [De Luca, Silverstein, Torroba 2021]: hyperboloids with Casimir energy

- Sort of consensus on at least the (Swampland) arguments that parametrically controlled dS is impossible?

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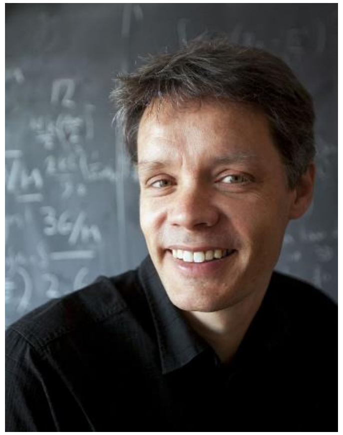
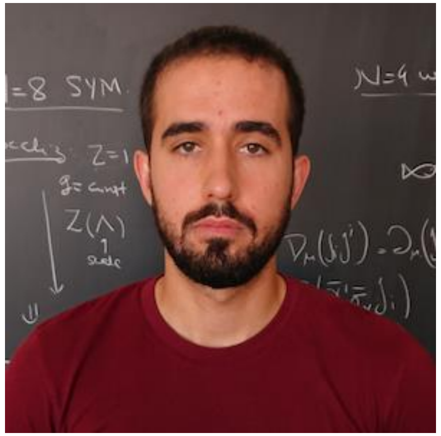
Based on

A higher-dimensional view on quantum cosmology, arXiv:2105.03253 with Ulf Danielsson & Daniel Panizo & Rob Tielemans.





All things non-SUSY are at best meta-stable?

- **For black holes**: extremal (non-SUSY) black holes should be able to decay [A-Hamed, Motl, Nicolis, Vafa 2006]: Weak Gravity Conjecture

- **For AdS space**: Any non-SUSY AdS space should be able to decay [Ooguri, Vafa 2016].

(If perturbatively stable, there must exists a domain wall so that Coleman De Luccia tunneling happens.)

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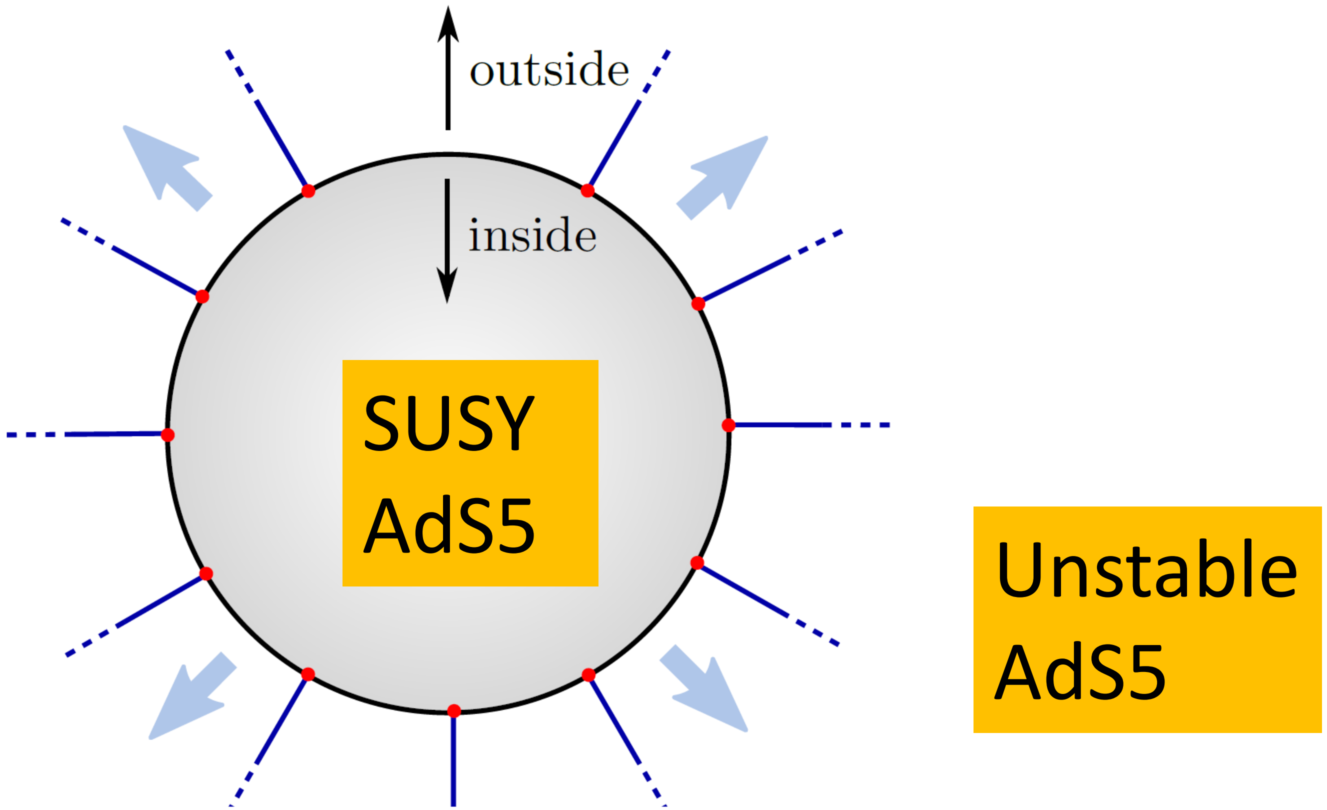
Idea Uppsala group [Banerjee, Danielsson, Dibitetto, Giri, Schillo 2018 & many follow-ups]:
there is a natural string theory embedding of brane worlds with de Sitter geometry exactly inspired by the Swampland logic!
Bubble worlds:



$$\kappa_5 = 8\pi G_5.$$

Denote the cc of the true and false vacuum as

$$\Lambda_{\pm} = -6k_{\pm}^2 \qquad \Lambda_- < \Lambda_+ < 0.$$

Shell metric: $ds^2_{\rm shell} = -d\tau^2 + a(\tau)^2 d\Omega_3^2$

The Israel Junction condition gives the cosmological dynamics:

$$\sigma = \frac{3}{\kappa_5}\left(\sqrt{k_-^2 + \frac{1+\dot{a}^2}{a^2}} - \sqrt{k_+^2 + \frac{1+\dot{a}^2}{a^2}}\right) \Rightarrow \boxed{\dot{a}^2 = -1 + \frac{a^2}{R^2}}$$

Where brane tension is σ &

$$\kappa_4 = \frac{2k_-k_+}{k_- - k_+}\kappa_5 \,.$$

Physical picture

In the limit of large enough k the vacuum energy takes a simple expression

$$\rho_{\Lambda_4} \equiv \frac{3}{\kappa_4} R^{-2} = \frac{3\,(k_- - k_+)}{\kappa_5} - \sigma.$$

A bubble can only nucleate if its tension is smaller than $\sigma_{cr} = \frac{3}{k_5}(k_- - k_+)$

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Explicit dS construction in string theory? Main difficulty; find an unstable AdS5 that decays primarily through Coleman de Luccia bubbles. [Basile, Lanza 2020] Clear embeddings recently of early universe acceleration (inflation?) and suggestion of why unavoidable late time acceleration [Danielsson, Henriksson, Panizo, 2211.10191]

Recent criticism of not being spin two gravity, but spin 0 [Mirbabayi 2210.14276], yet criticism refuted in [Banerjee, Danielsson, Giri 2212.14004]

Resolution of the Big Bang singularity and the boundary choice problem [arXiv:2105.03253].

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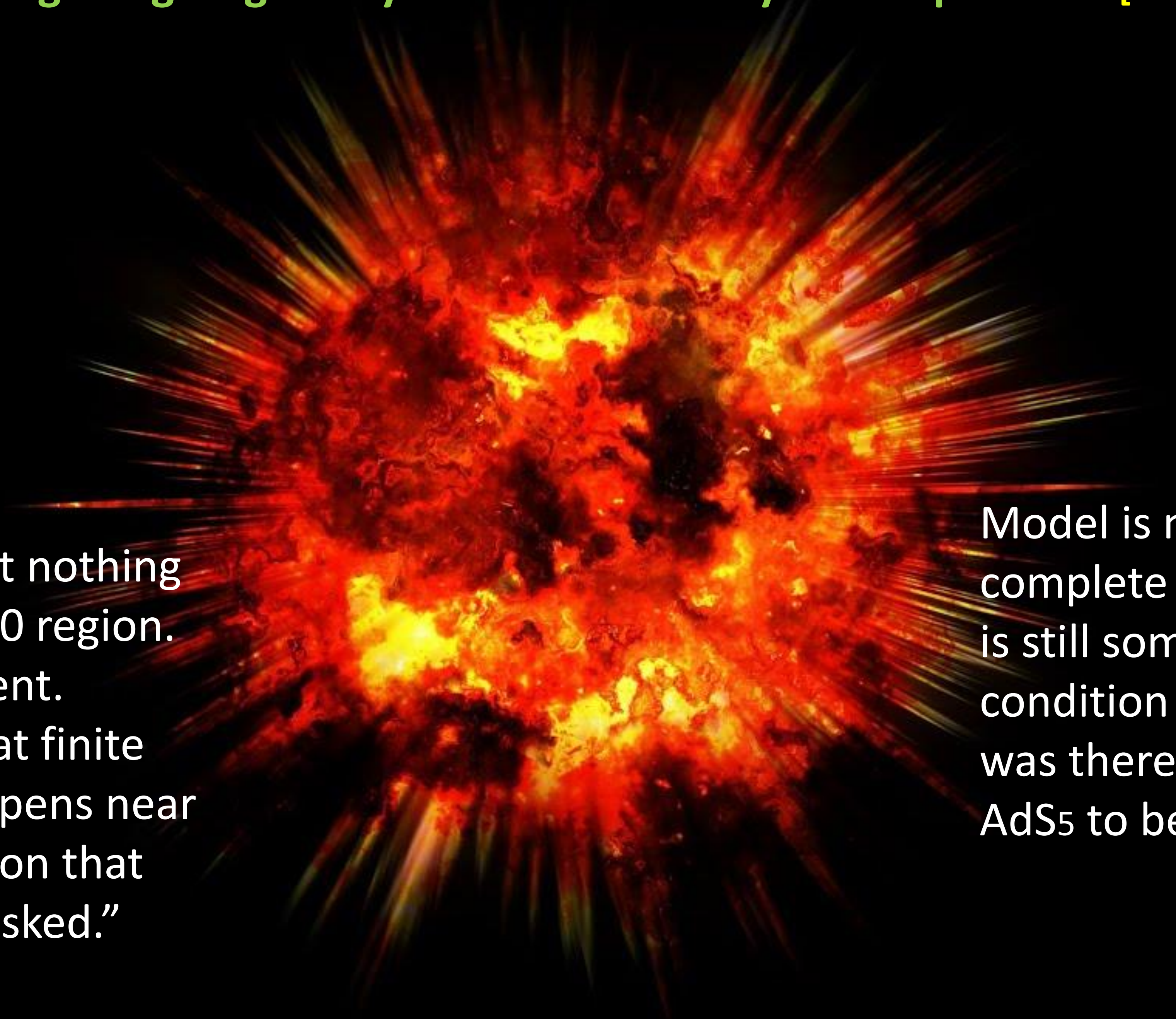

Big Bang?

From 5D viewpoint nothing is singular bout a=0 region. It is just non-existent. Bubble nucleates at finite radius. "What happens near a=0 is not a question that can or should be asked."

Model is not past complete though, there is still some initial condition problem. Why was there an unstable AdS_5 to begin with?

What is the boundary condition of ψ at "the Big Bang"?

→ heuristic: answer requires UV description, so our model should fix it.

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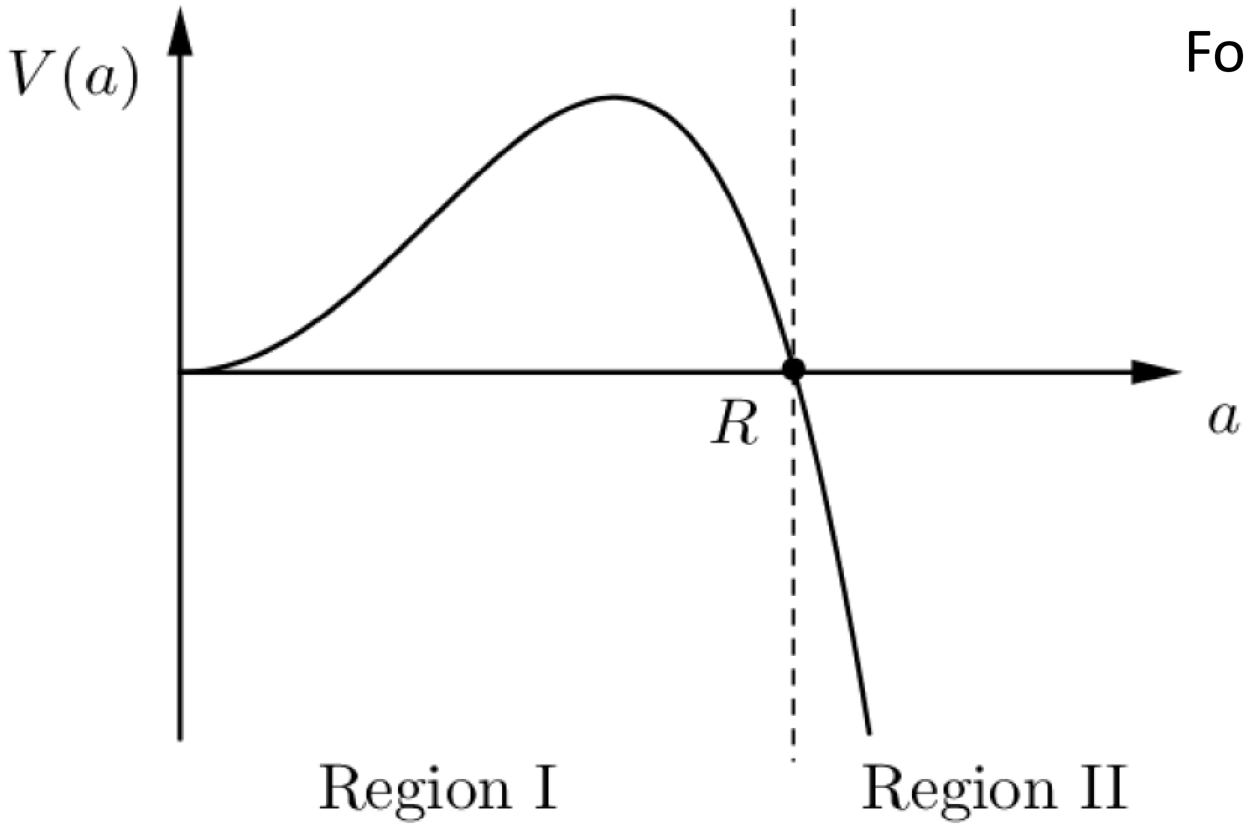
→ We recap the issue in the simplified mini-superspace picture.

A quantisation of the effective one-dimensional action (mini-superspace)

$$ds^2 = -N^2(\tau)\mathrm{d}\tau^2 + a(\tau)^2\mathrm{d}\Omega_3^2 \qquad S = \frac{6\pi^2}{\kappa_4}\int \mathrm{d}\tau N\left(-\frac{a\dot{a}^2}{N^2} + a - \frac{a^3}{R^2}\right)$$

gives the following Wheeler deWitt equation:

$$\frac{N}{a}\left(-\frac{1}{24\pi^2}\frac{d^2}{da^2} + 6\pi^2 V(a)\right)\Psi(a) = 0\,, \qquad V(a) = a^2 - \frac{a^4}{R^2}$$



For simplicity we discuss WKB solution

$$\Psi_{\rm I}(a) = \frac{1}{|V(a)|^{1/4}} \left(c e^{S(a,0)} + d e^{-S(a,0)} \right) ,$$
$$\Psi_{\rm II}(a) = \frac{1}{|V(a)|^{1/4}} \left(A e^{iS(a,R)} + B e^{-iS(a,R)} \right) ,$$

$$S(a, a_i) \equiv \frac{12\pi^2}{\kappa_4} \int_{a_i}^{a} \sqrt{|V(a')|} da' , \qquad S_0 \equiv S(R,0) = \frac{4\pi^2 R^2}{\kappa_4}$$

Normalisation $\lim_{a\to 0} |V(a)|^{1/4} \Psi(a) = 1 \, .$

→ Still need to fix boundary conditions to pick a wavefunction uniquely.

$$S_0 \equiv S(R,0) = \frac{4\pi^2 R^2}{\kappa_4}$$

- Hartle Hawking (no boundary) $(c,d) = (1,0)$

$$\Psi_{\rm HH}(a) = \frac{1}{|V(a)|^{1/4}} \begin{cases} e^{S(a,0)} & \text{Region I} \\ 2e^{S_0}\cos\left(S(a,R) - \frac{\pi}{4}\right) & \text{Region II} \end{cases}.$$

$$P_{\rm HH} \propto e^{2S_0}.$$

- Vilenkin (tunneling) $(A,B) = (0,B)$

$$\Psi_{\rm V}(a) \approx \frac{1}{|V(a)|^{1/4}} \begin{cases} e^{S_0} e^{-S(a,0)+i\frac{\pi}{4}} & \text{Region I} \\ e^{-iS(a,R)} & \text{Region II} \end{cases},$$

$$P_{\rm V} \propto e^{-2S_0}$$

Dark bubble scenario

the physics is that of decay through bubble nucleation (CDL). Natural expectation is tunneling wave function. We verified this is correct by checking that **CDL amplitude is Vilenkin's amplitude**;

CDL in 5D (after a longer story where ref [19] was useful):

$$P = e^{-B} \qquad B = \frac{24\pi^2}{\kappa_4} \int_0^R da \sqrt{a^2 - \frac{a^4}{R^2}} = \frac{8\pi^2 R^2}{\kappa_4}$$

We verified further by using the expressions of Brown-Teitelboim

[19] S. Ansoldi, A. Aurilia, R. Balbinot, and E. Spallucci, "Classical and quantum shell dynamics, and vacuum decay," *Class. Quant. Grav.* **14** (1997) 2727–2755, arXiv:gr-qc/9706081.

Take away message?

- Vilenkin vs Hartle Hawking can be answered in explicit UV complete models.

- Not unlikely that models also exist that give HH.

- It is not a matter of mathematical consistency,…or religion.

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Axion wormholes in dS space, in progress, with Sergio Aguilar & Thomas Hertog & Rob Tielemans & Jan Pieter van der Schaar .

Quantum cosmology, beyond mini-superspace?: a path integral treatment using low energy variables and saddle point expansion.

→ Into full blown path integral definition using dS/CFT? [Anninos, Hawking , Hertog, Maldacena, Silverstein, Skenderis, Strominger,….]

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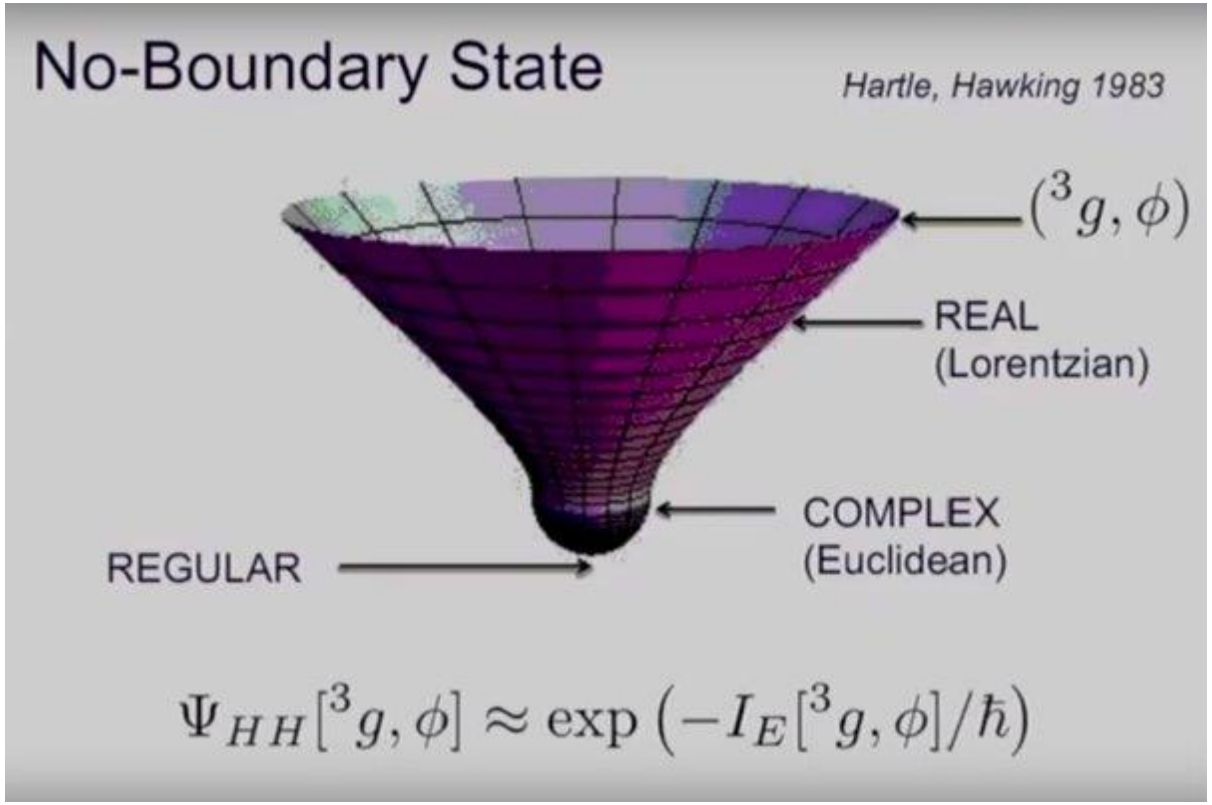
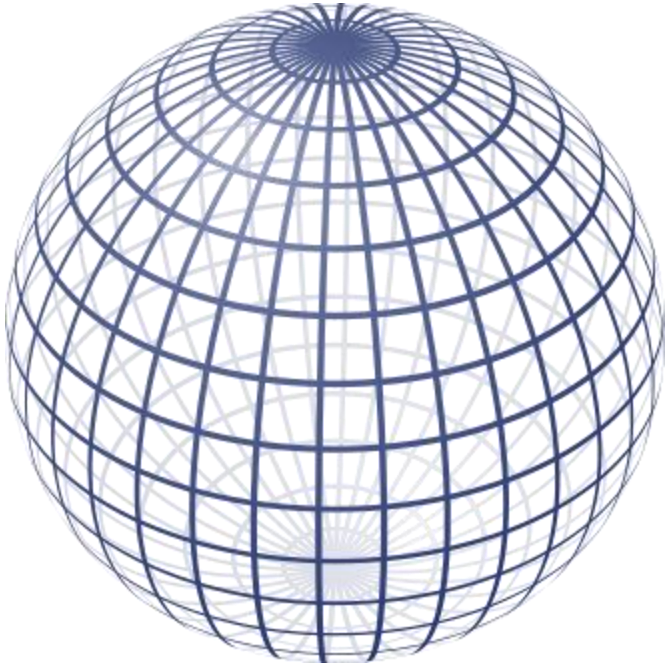
Hartle-Hawking condition using 4-sphere description in Euclidean signature:

(selects BD vacuum for the quantum fields. But so does Vilenkin boundary condition.)



Relies on Gibbons-Hawking instanton, reproduces HH weighting



$$P_{\mathrm{HH}} \propto e^{S_0} \propto e^{1/\Lambda}$$

Our goal:

1) extension of GH instanton with handles.
2) Resolve conceptual issues with wormholes?

Quick recap Giddings-Strominger wormholes

$$\mathcal{L}_{\text{matter}} = \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) \quad \Rightarrow \quad \mathcal{L}_{\text{matter}} = \frac{1}{12}(F_{\mu\nu\rho}F^{\mu\nu\rho})$$

Ansatz:

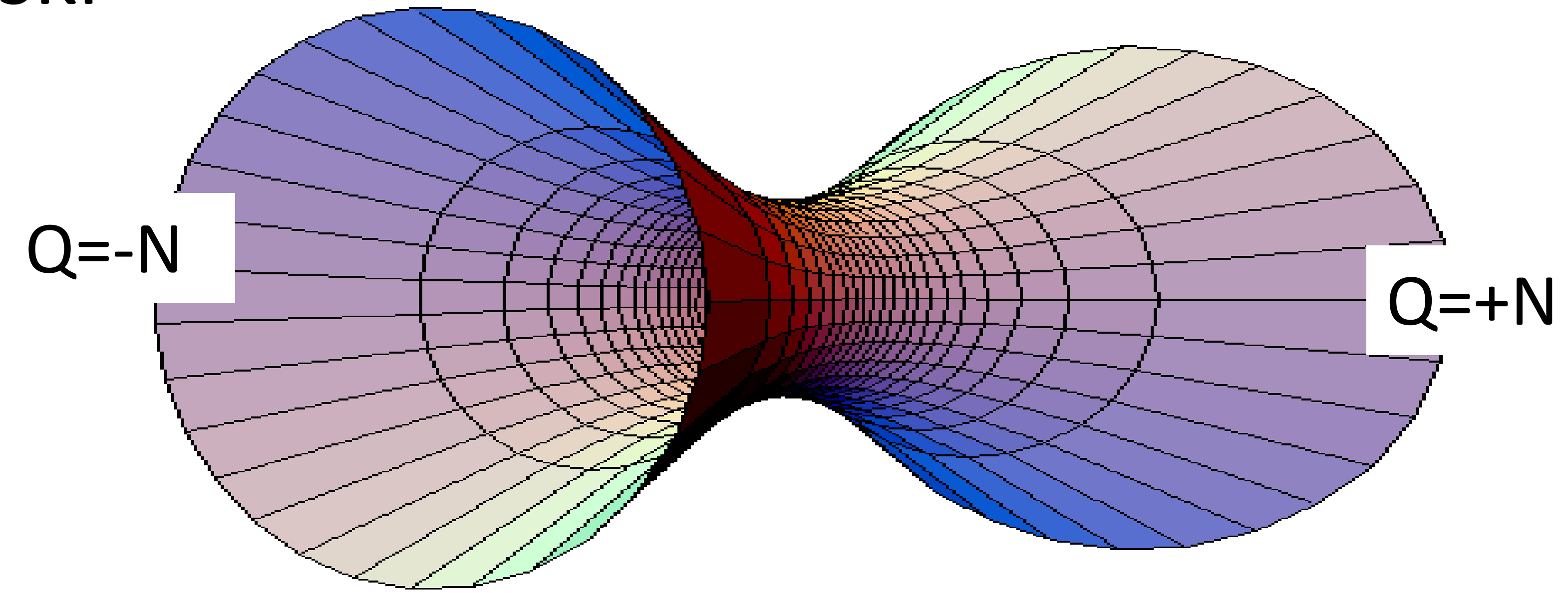
$$ds^2 = f(\tau)^2 d\tau^2 + a^2(\tau)^2 d\Omega_3^2$$
$$F_3 = Q\epsilon_3$$

Wormhole? In gauge f=1, a(t) should grow, reach a minimum and then grow again.
Other gauge is easier:

$$ds^2 = \frac{1}{1 \mp \frac{\tau^2}{\ell^2} - \frac{Q^2}{6\tau^4}} + \tau^2 d\Omega_3^2$$

$$\Lambda = \pm\frac{6}{\ell^2}$$

In AdS &Minkowski

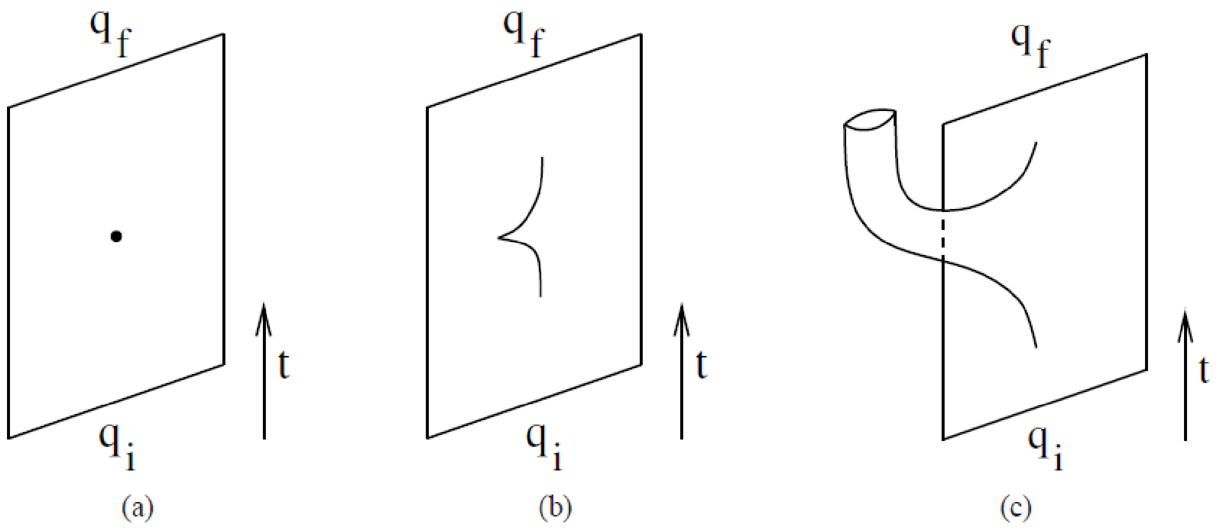


Wormhole is a dipole. There is no monopole axion charge, only locally at one side.

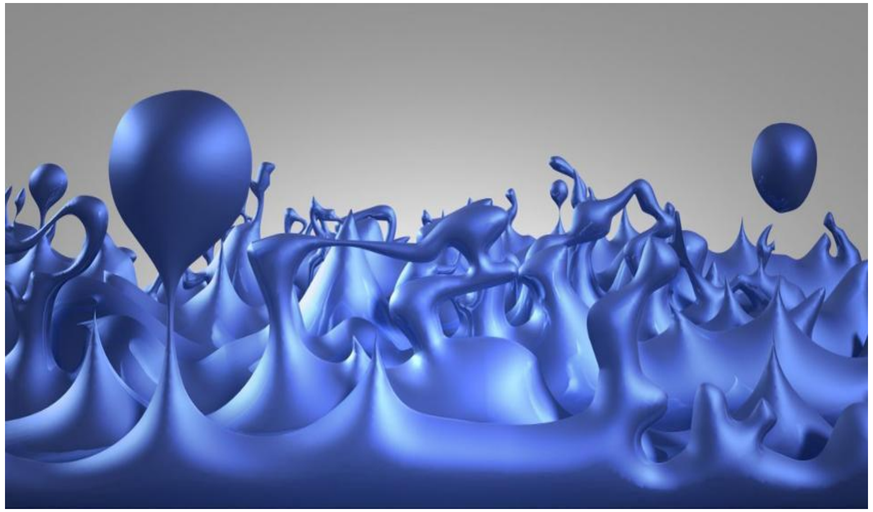
Finite action: $\boxed{S \sim |Q|}$

Very rich and long history in quantum gravity, prior to string theory. See [Hebecker, Mikhail, Soler 2018] for comprehensive review

Interpretation as instantons describing nucleation of baby universes → only if cut in half:



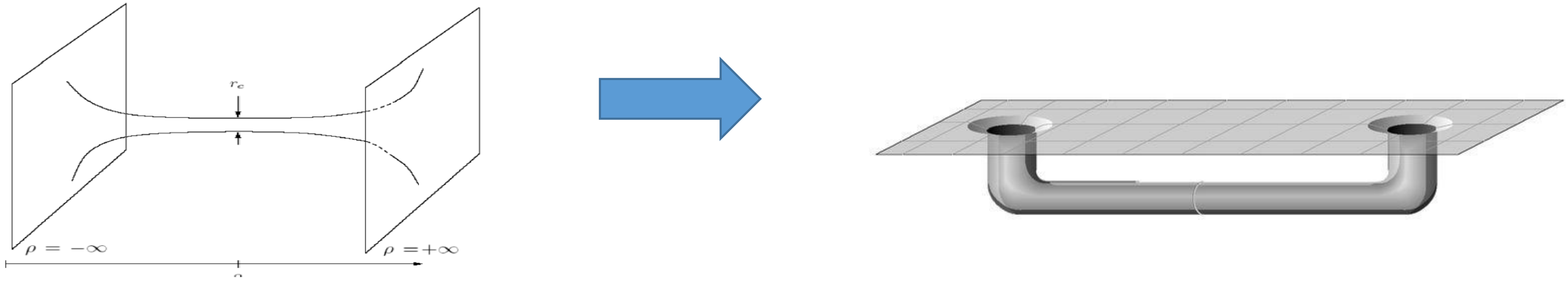
[Giddings/Strominger 1987, Lavrelashvili/Tinyakov/Rubakov 1998, Hawking 1987, …]



→Full wormhole describes emission *and* subsequent absorption of baby universe. Tunneling probability Planckian suppressed. (Planckian sized universes only)

An observer detects a violation of axion charge conservation, apparent *non-unitarity*.

If one glues the two boundaries into one space-time:



then wormholes represent a breakdown of (macroscopic) locality : the effective action would include operators of the form

$$S_{WH} = -\frac{1}{2}\sum_{IJ}\int d^D x\, d^D y\, \mathcal{O}_I(x) C_{IJ} \mathcal{O}_J(y)\ ;$$

[Coleman 1998]: Not really since

$$e^{-S_{WH}} = \int d\alpha_I\, e^{-\frac{1}{2}\alpha_I (C^{-1})_{IJ}\alpha_J} e^{-\int d^D x \sum_I \alpha_I \mathcal{O}_I(x)}\ .$$

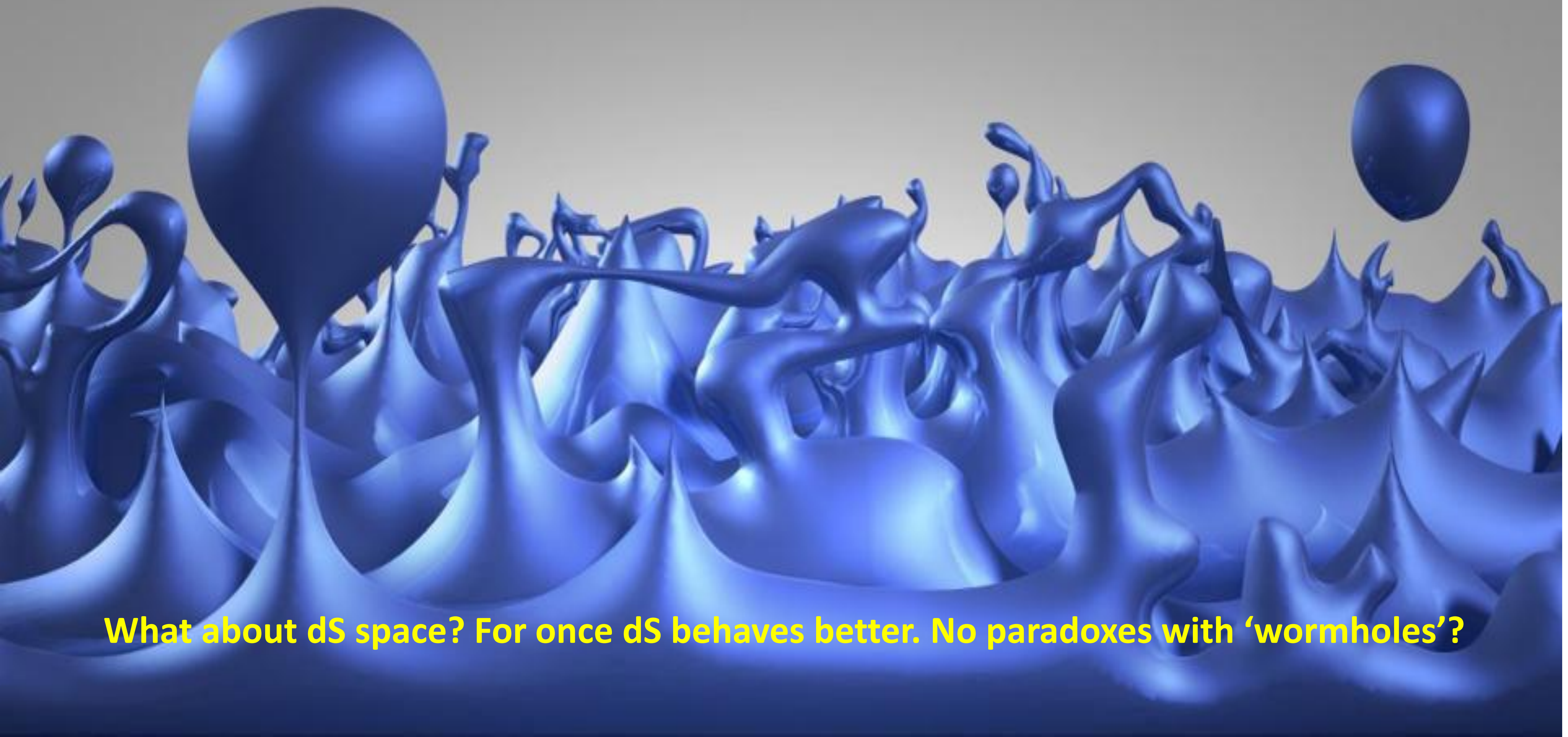
ENSEMBLES

Very clean embedding in AdS compactifications now [Hertog, Trigiante VR, 2017], [Loges, Shiu, VR 2022]

Problematic features, not fully understood: Factorisation problem, no dual alpha parameters [Maldacena, Maoz] [Arkani-Hamed, Orgera, Polchinski],
+ violation of operator positivity in dual CFT [Katmadas, Ruggeri, Trigiante, VR, 2018], [Loges, Shiu, VR 2022] :

$$|\mathrm{Tr}[F_\alpha^2]| < |\mathrm{Tr}[F_\alpha \wedge F_\alpha]| \,.$$

Giddings-Strominger wormholes in dS space



What about dS space? For once dS behaves better. No paradoxes with ‘wormholes’?

How to interpret metric? As a dumbbell?

$$ds^2 = \frac{1}{1 - \frac{\tau^2}{\ell^2} - \frac{Q^2}{6\tau^4}} + \tau^2 d\Omega_3^2$$

Would just be squashed sphere…

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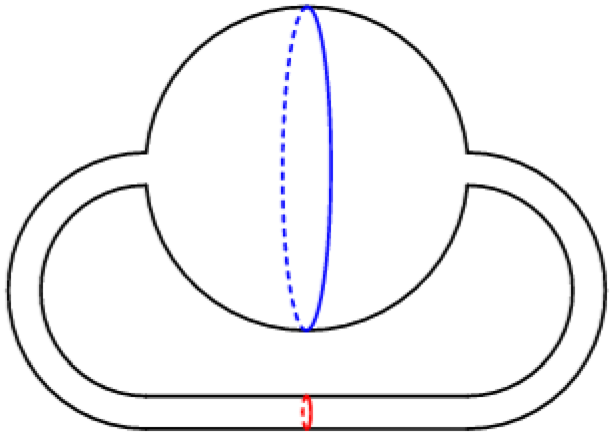
It helps to change coordinates. Explicit in D=3:

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 + \frac{\ell^2}{2}\left(1 + \sin\left(2\frac{\tau}{\ell}\right)\sqrt{1 - \frac{2\kappa^2 Q^2}{\ell^2}}\right)\mathrm{d}\Omega_2^2 \ .$$

→ kettlebell!

A one-handle extension of the GH instanton

- Unlike AdS, Minkowski, the explicit solution has the wormhole glued back to the same space (4-sphere.)



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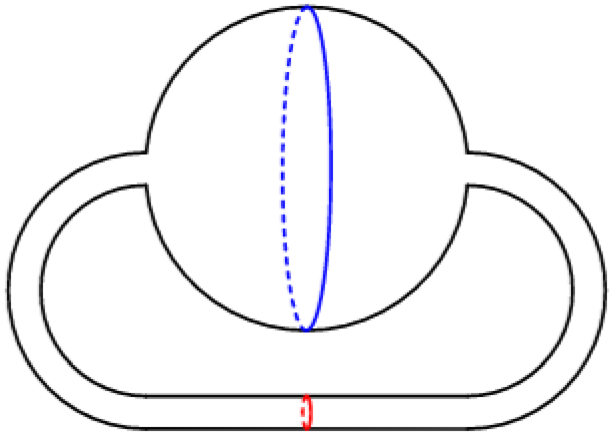

- There is a max radius and a minimal radius

$$a^2_{\min,\max} = \frac{\ell^2}{2}\left(1 \pm \sqrt{1 - \frac{2\kappa^2 Q^2}{\ell^2}}\right),$$

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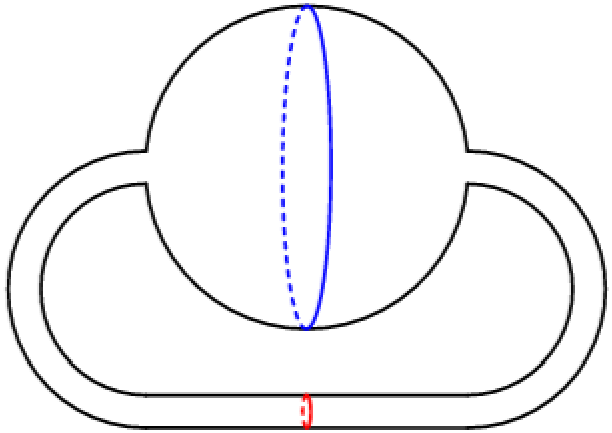

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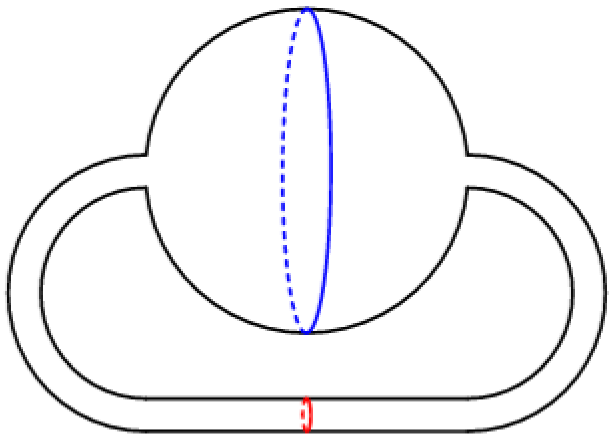

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- The on-shell action ranges between the HH instanton (Q=0) where I ~ - Vol(S^4) all the way to exactly zero where Q=Q_{max}.

$$I_E^{(d=3)} = -\frac{4\pi^2 \ell}{\kappa_3^2}\left(1 - \frac{\sqrt{2}\kappa_3 Q}{\ell}\right),$$

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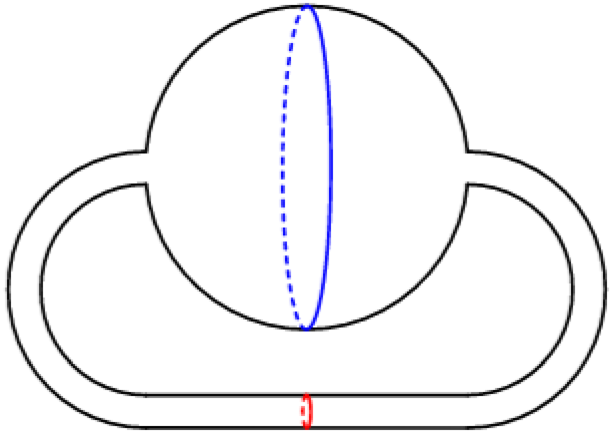

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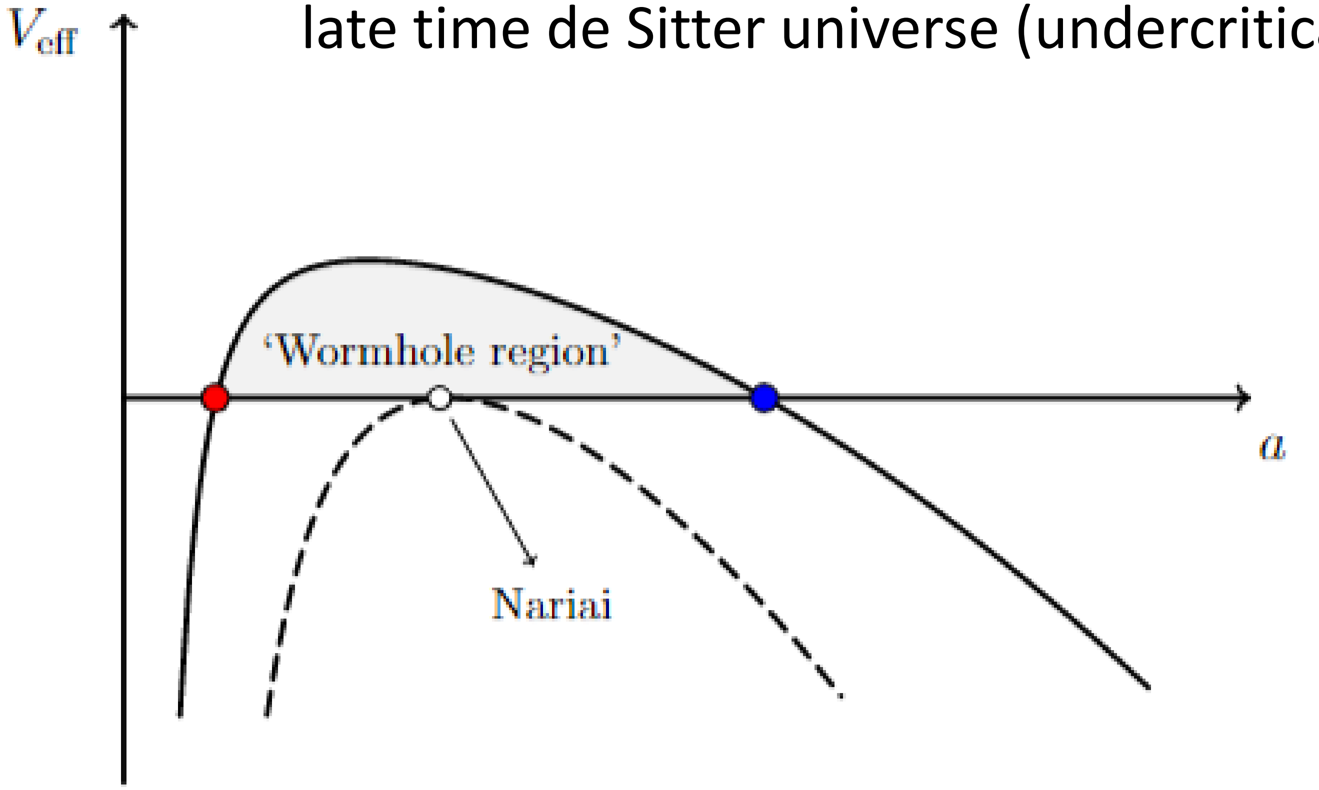
$$I_E^{(d=3)} = -\frac{4\pi^2 \ell}{\kappa_3^2}\left(1 - \frac{\sqrt{2}\kappa_3 Q}{\ell}\right),$$

- The wormholes are stable for fluctuations that preserve axion flux. (Long story, based on [Loges, Shiu, Sudhir 2203.01956], [Hertog, Truijen, VR 1811.12690])

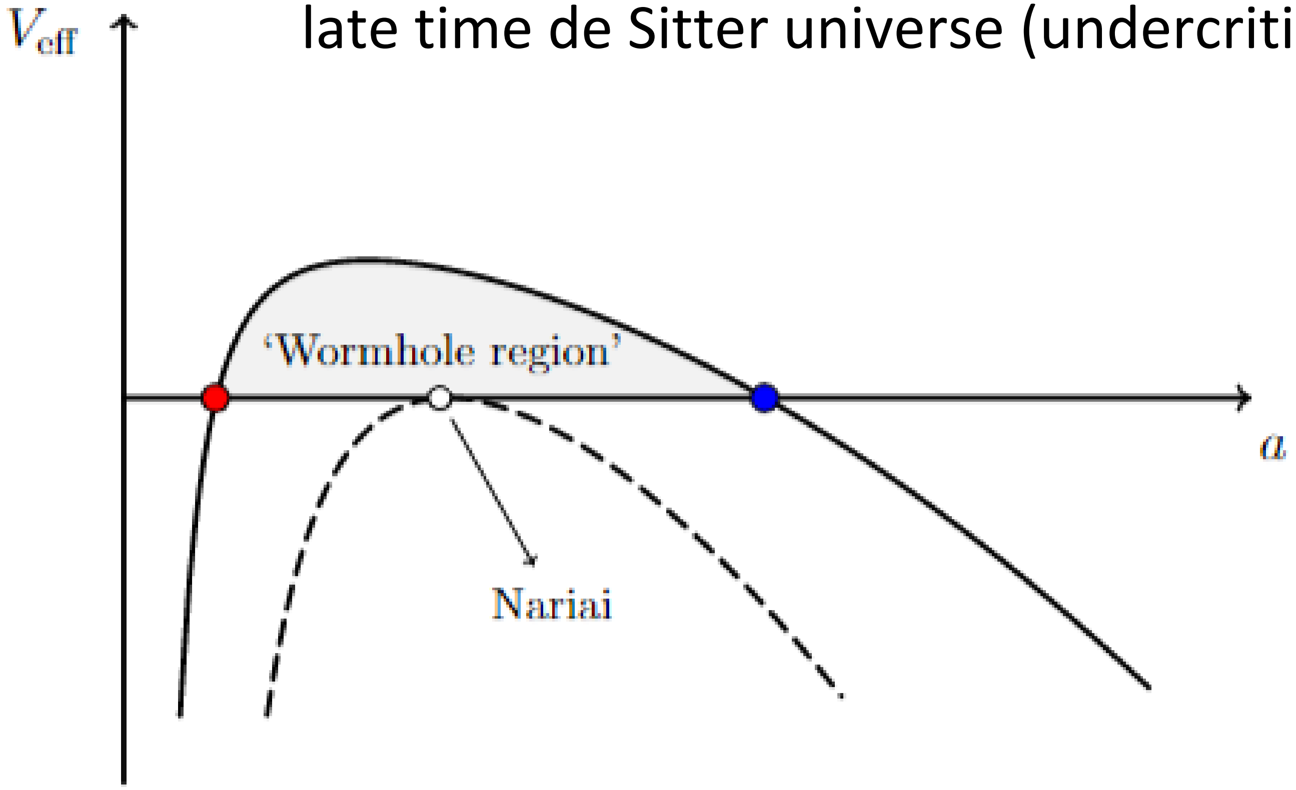
Because of quantum cosmological context interpretation is ‘easy’

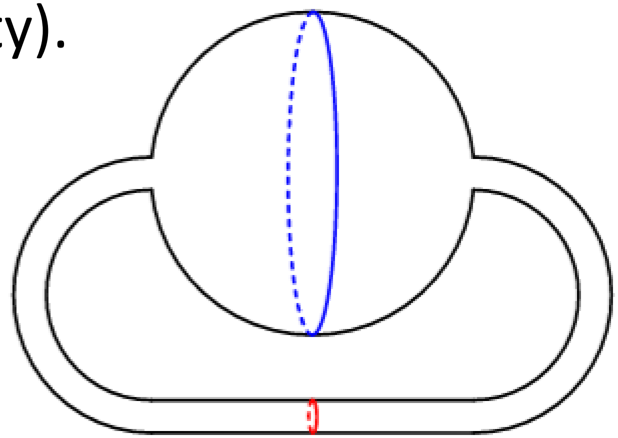
- Lorentzian continuation possible at the two turning points (red blue.)
- Axion density is like stiff fluid.
- We either nucleate a collapsing baby universe (overcritical axion density) or a late time de Sitter universe (undercritical axion density).



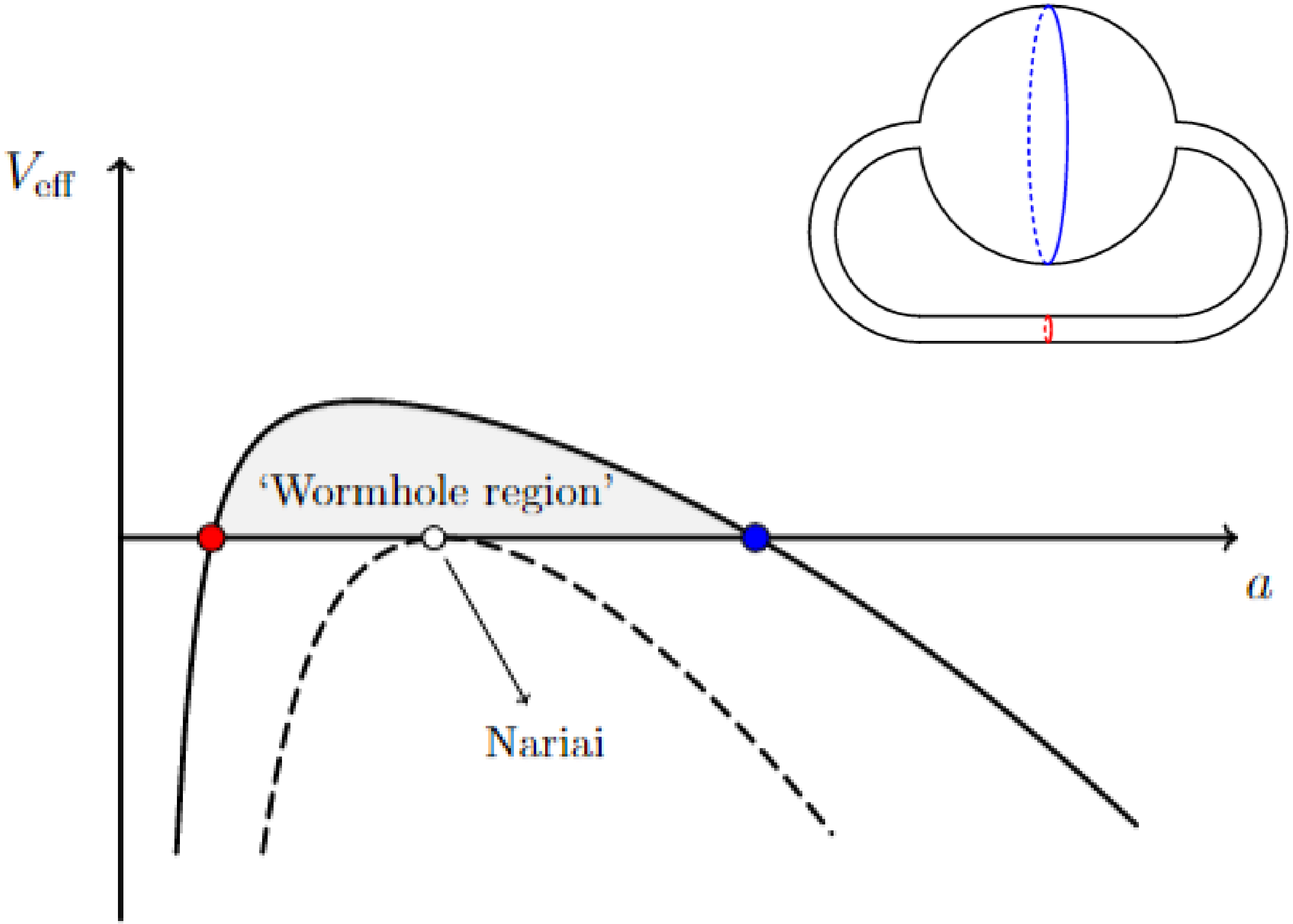
Because of quantum cosmological context interpretation is ‘easy’

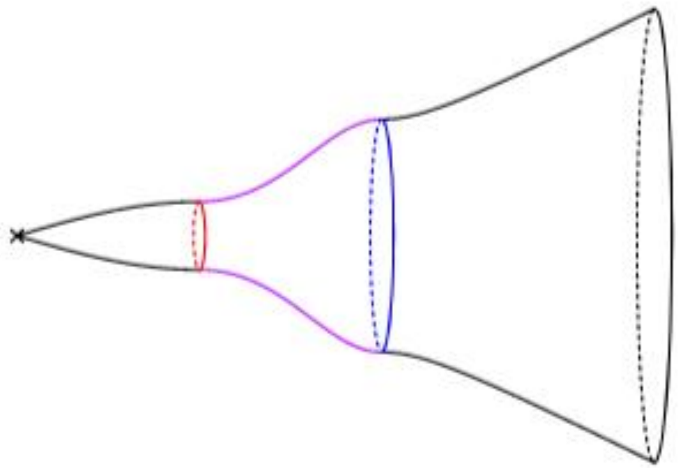
- Lorentzian continuation possible at the two turning points (red blue.)
- Axion density is like stiff fluid.
- We either nucleate a collapsing baby universe (overcritical axion density) or a late time de Sitter universe (undercritical axion density).





Our on-shell action is the log of the probability to nucleate a universe with positive cc and a certain axion density Q.





(a) Big bang to inflationary cosmology. The opposite evolution, i.e. collapsing universe to a big crunch cosmology, is also possible for the opposite time orientation for the instanton.

(blue region is Euclidean)

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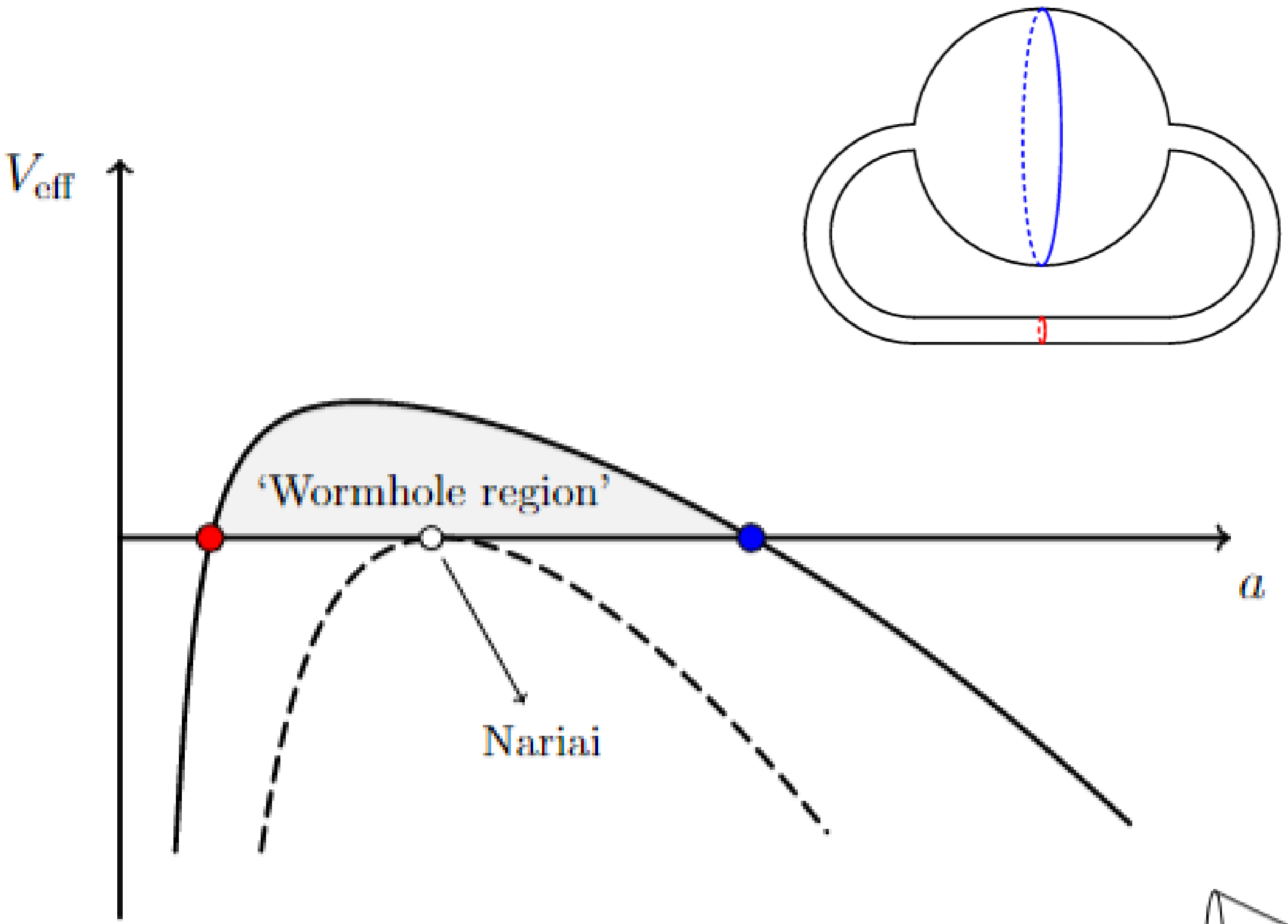

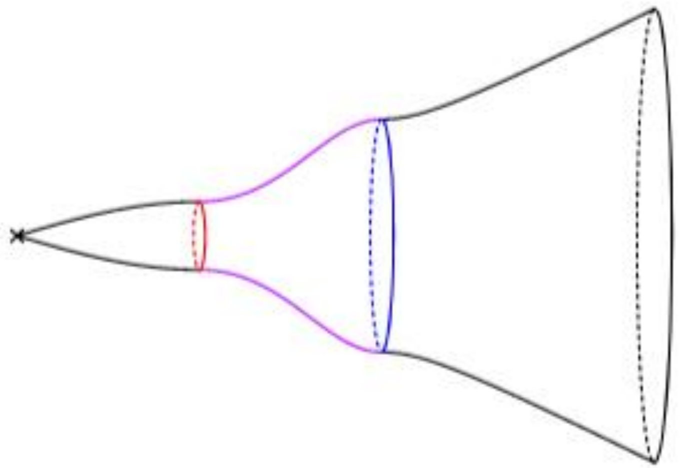

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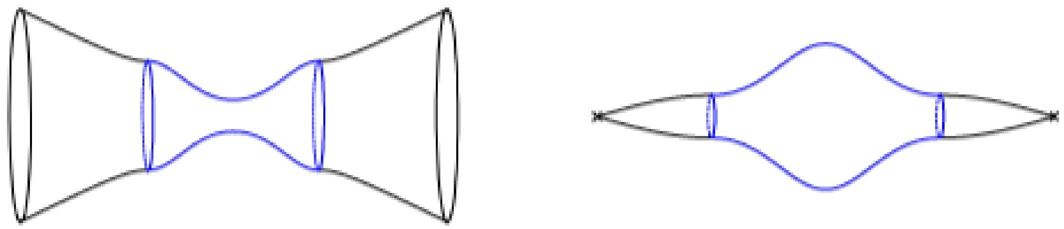

(b) *Left*: Collapsing universe tunneling to a phase of accelerated expansion. *Right*: big bang to big crunch phases meditated by the wormhole instanton.

the probability to nucleate a universe with positive cc and a certain axion density Q.

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2. Dark bubble scenario and Vilenkin’s wavefunction (30%)

3. Axion wormholes in dS space (30%)

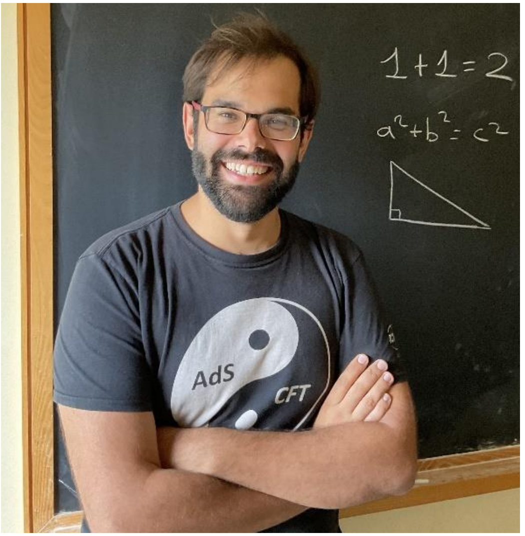
4. Festina Lente (30%)

5. Summary (5%)

Based on

Festina Lente, EFT constraints from charged black hole evaporation in dS space
arXiv:1910.01648, with Miguel Montero, & Gerben Venken .

The FL bound and its phenomenological applications, arXiv: 2106.07650, with Miguel Montero, & Cumrun Vafa & Gerben Venken



Change of perspective:

Assume we have a dS background, what are the conditions on the matter content of the universe if we want it to be consistent with its background?

Consider Einstein-Maxwell theory with some potential V

$$S = \int d^d x \sqrt{-g} \left[\frac{1}{2} M_p^{d-2} \mathcal{R} - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - V \right].$$

For constant V, the Hubble radius is then fixed by

$$\frac{(d-1)(d-2)}{2\ell_d^2} = M_p^{2-d} V.$$

- The Electric Weak Gravity bound is: $$\frac{gq}{m} \geq \sqrt{\frac{d-3}{d-2}} M_p^{-\frac{(d-2)}{2}} \quad \text{for some charged state}$$

- The Festina Lente bound is: $$m^4 \gtrsim (gq)^2 V \quad \text{for every charged state}$$

In 4D, in terms of fine structure constant, we have a window:

$$(8\pi\alpha V)^{1/4} < m < (8\pi\alpha)^{1/2} M_{\mathrm{P}}$$

$$\alpha = \frac{g^2}{4\pi}$$

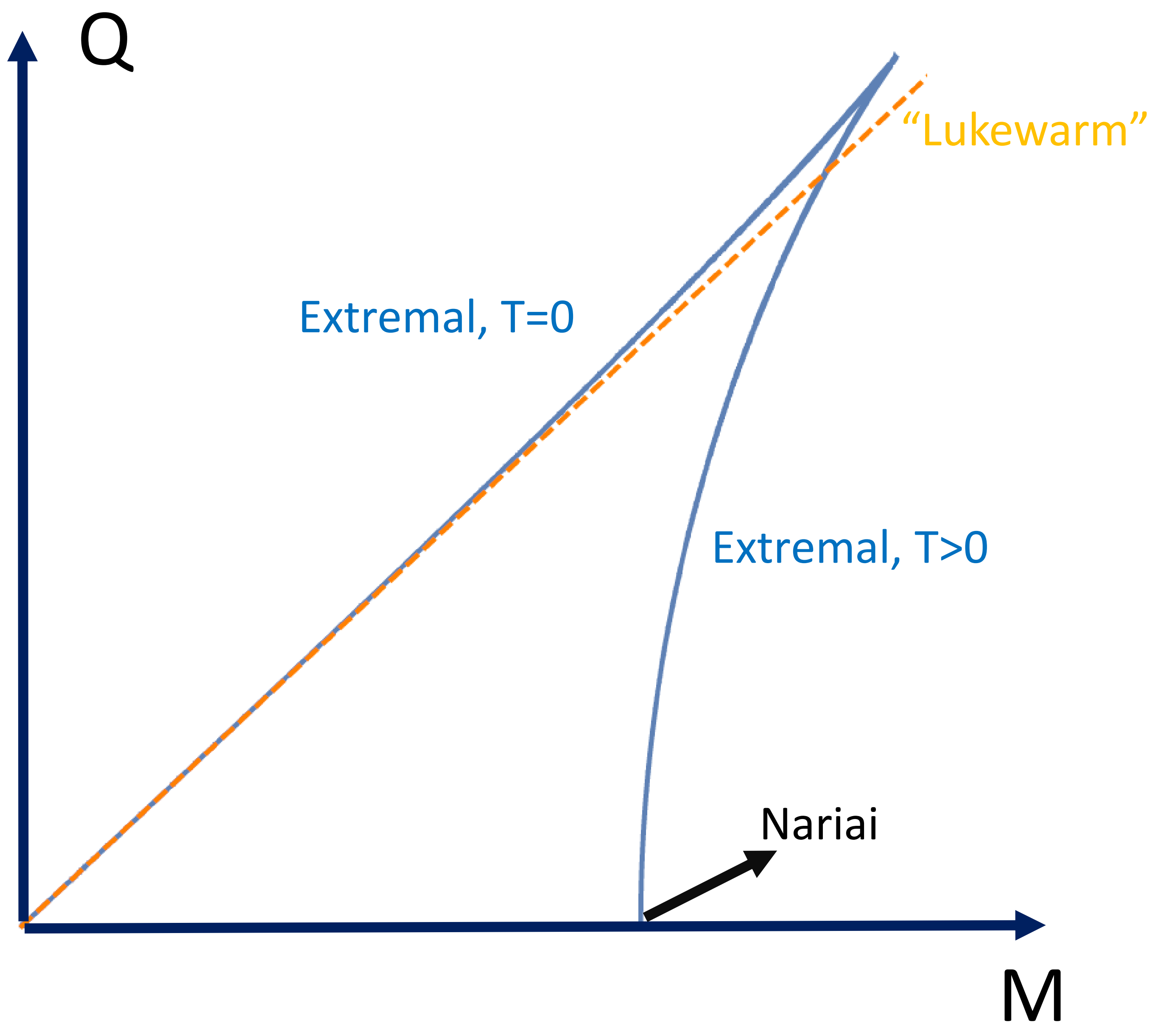
Argument 1: Quantum dynamics of charged black holes in de Sitter space

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2 d\Omega,$$

$$U(r) \equiv 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - r^2$$

$$M \equiv \frac{GM}{\ell}, \qquad Q^2 \equiv \frac{Gg^2Q_r^2}{4\pi\ell^2}.$$

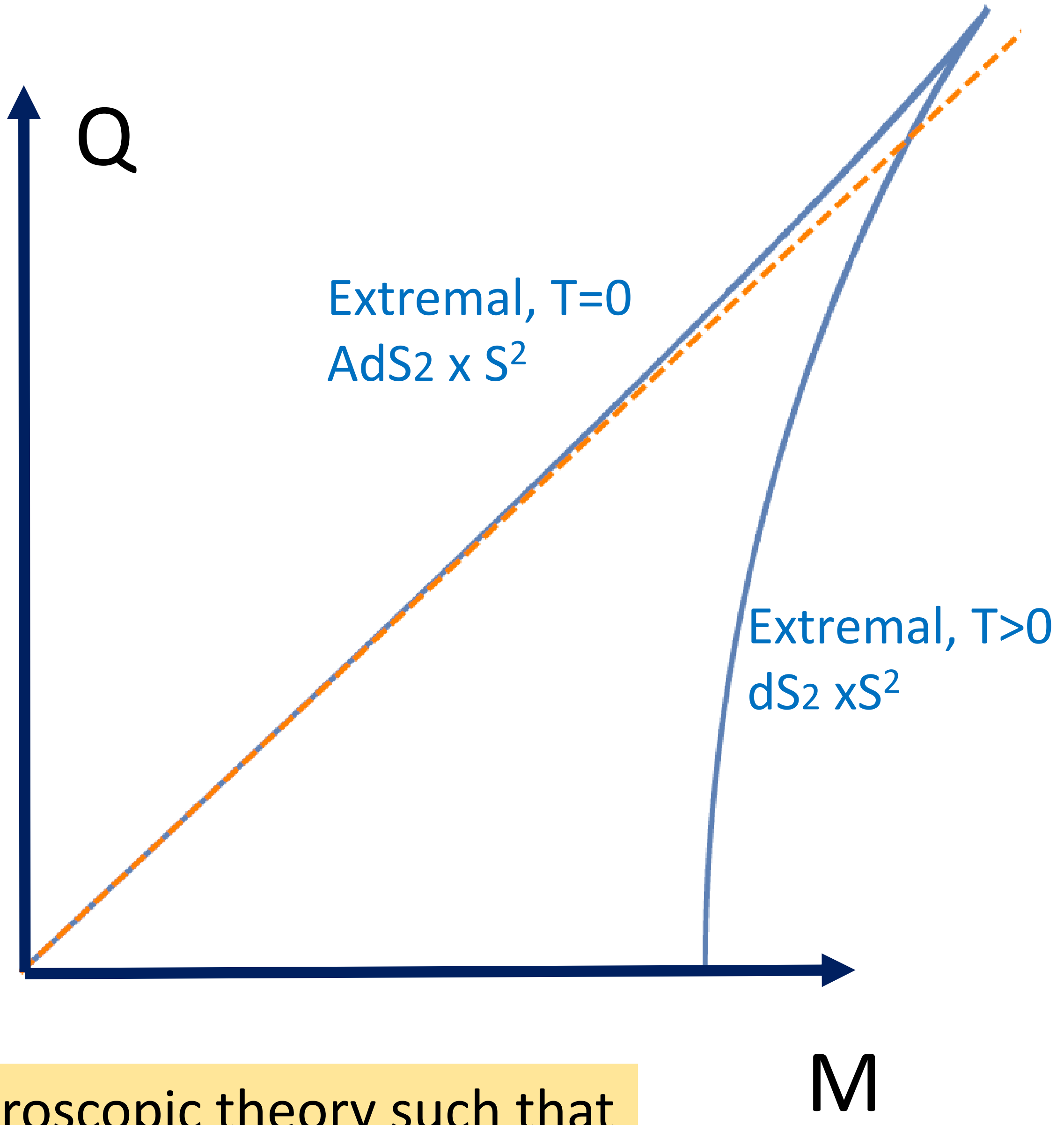
$$S = \frac{\pi}{4G}\left(r_{BH}^2 + r_{CH}^2\right)$$



Weak gravity principles for extremal black holes?

- **Left extremal branch**. Like in flat space. But now black holes unstable without even requiring weak gravity. → de Sitter expansion helps the Schwinger effect. Always unstable. Need time scales?

- **Right extremal branch. Charged Nariai.** Gigantic black holes probing cosmic horizon. Super-extremal if black hole horizon catches up with cosmic horizon. Should be forbidden = *Cosmic censorship*.

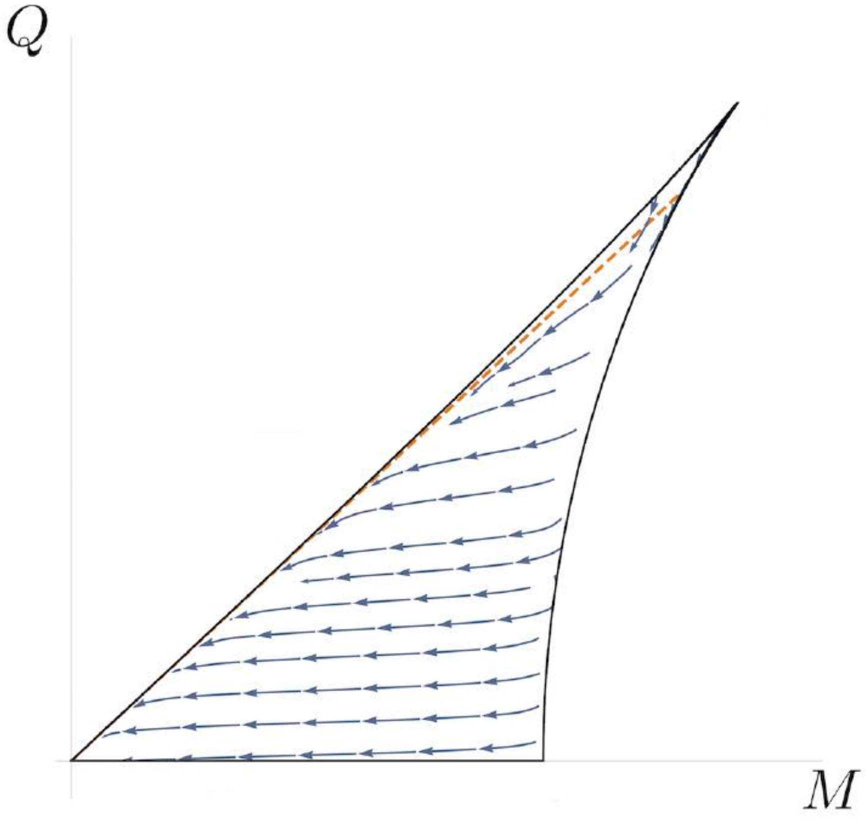


Guiding principle: constrain microscopic theory such that black holes *do not decay to the super-extremal side*.

Adiabatic motion in Q,M plane. Semi-classical analysis of Hawking&Schwinger radiation:

$$\dot{Q} = -4\pi\mathcal{J}, \qquad \frac{4r(r\dot{M} - Q\dot{Q})}{-2Mr + Q^2 - r^4 + r^2} = -16\pi r^4 G\mathcal{T}.$$

[Montero & Venken & VR 2019 , Lüben& Lüst & Ribes Metidieri 2020]



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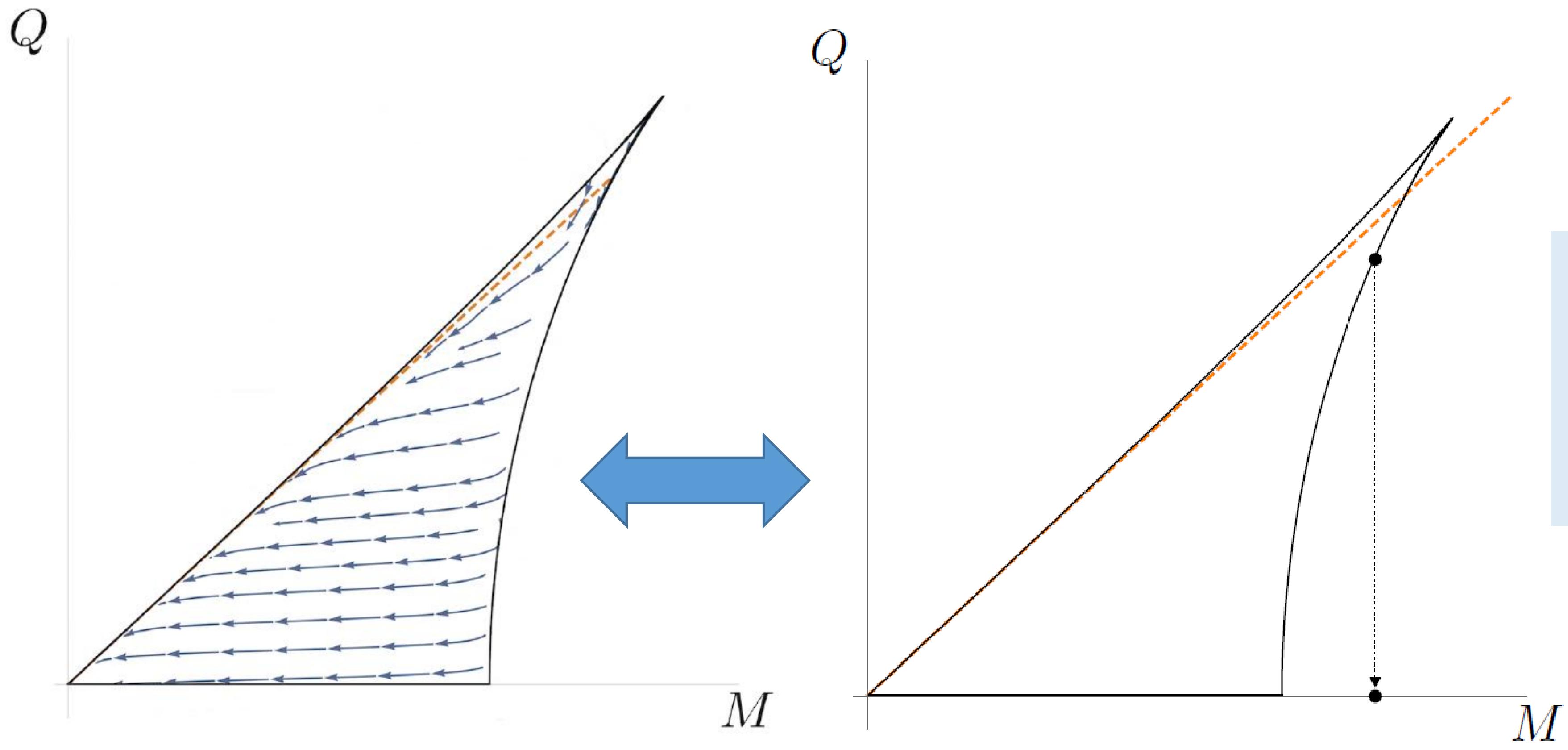

Details J and T are such that evolution brings you to super-extremal branch unless you obey FL bound.

Argument 2: Magnetic Weak Gravity & Completeness

The magnetic WGC: $\Lambda_{EFT} \leq g M_P$

Can be found from demanding that a monopole of unit charge is larger than its corresponding black hole solution. In dS space we must also demand that the monopole is smaller than the charged Nariai solution, ie, it fits in dS space [Huang & Li &Son 2006]. That leads to

$$g^2 \geq \frac{3}{2}\left(\frac{H}{M_P}\right)^2$$

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$$g^2 \geq \frac{3}{2} \left(\frac{H}{M_P} \right)^2$$

This is the same as demanding that the dark energy scale is below cut-off scale. Makes sense. Now we get logical triangle with Festina Lente; applying FL to the (electric) WGC particle :

$$m \lesssim g M_P \quad \text{and} \quad m^2 \gtrsim g M_P H \quad \Rightarrow \quad g \gtrsim \frac{H}{M_P},$$

This allows us to fix the unknown constant in FL bound

$$m^2 \geq \sqrt{6}\, g M_P H.$$

Note how the inequality

$$g^2 \geq \frac{3}{2}\left(\frac{H}{M_P}\right)^2$$

Resonates with the Swampland bounds that forbid dS vacua at parametric weak coupling!

Even when you are a Swampland critic, you surely appreciate the inner consistency of this all!

Wormholes meet Festina Lente?: Axion FL bound [Guidetti, Righi, Venken, Westphal 2022]

A fundamental axion in a consistent 4d EFT with a quasi-dS background should satisfy

$$Sf \gtrsim \sqrt{M_P H} \sim V^{1/4} .$$

Where:

$$S_4 \supset \frac{f^2}{2} \int_{M_4} F_1 \wedge \star F_1$$

Whereas standard WGC for axion is $fS \leq M_p$.

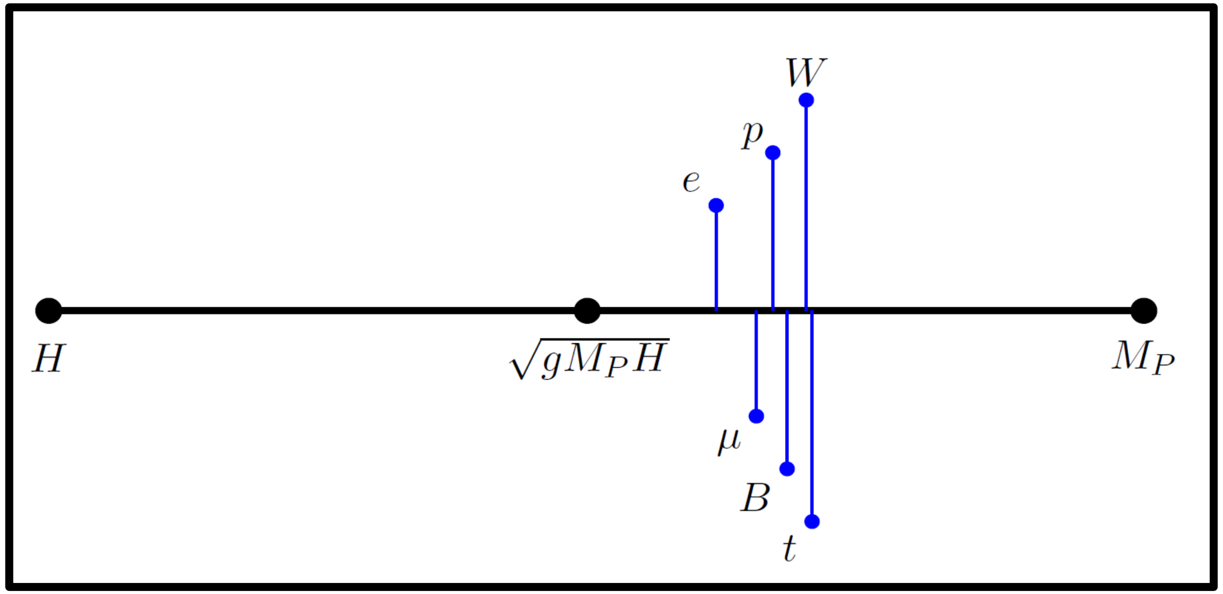
Leads to constraints on axion inflation models.

Pure Pheno applications

- All charged fields in the SM obey FL ☺ $\sqrt{gM_P H} \sim 10^{-3}\, eV$;
- Can FL help with explaining hierarchy problems?

→ CC hierarchy (Planck units): $\Lambda \lesssim \frac{m^4}{4\pi\alpha}$; Electron ⟹ $\Lambda \lesssim 3 \cdot 10^{-89}$;

→ Electro weak hierarchy: $v^2 \gtrsim \frac{1}{g} M_P H = \frac{V^{1/2}}{g}$ (FL applied to W-boson)



Logarithmic scale

A non-abelian gauge theory automatically contains massless charged states: the gluons. Nariai black holes? → Use the Cartan of the gauge group. So massless non-abelian gauge fields are in contradiction with FL.

→There cannot be a phase of the Standard Model where the weak interaction is long range → no local minimum at Phi = 0 for the Higgs potential.

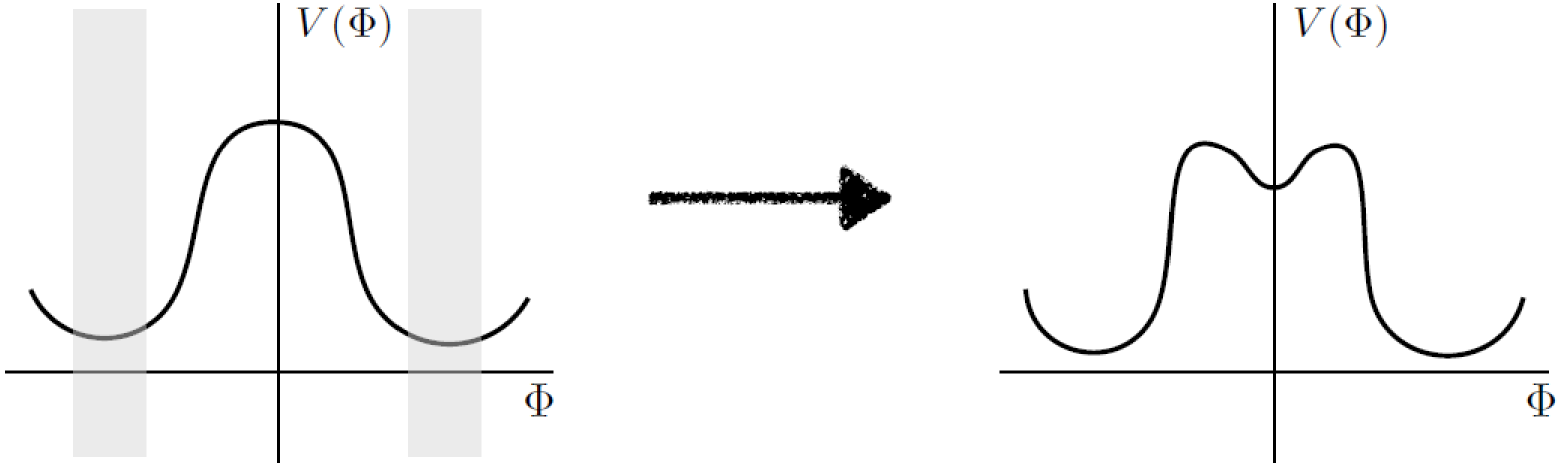


Figure 2. On the left, we show the usual shape of the "Mexican hat" Higgs potential, which arises from equation (4.5). However, only the region shaded in gray has been accessed experimentally. It is conceivable that the region near $\Phi \approx 0$ has a different shape, for instance, that of the "cowboy hat"

See also [Mook Lee et al 2111.04010]

→The other possibility consistent with non-abelian gauge fields and FL is confinement. Is realized by the gluons in the SM.

FL predicts that in a de Sitter background non-abelian gauge fields must confine or be spontaneously broken, at a scale above H.

$$m_{\text{Gauge field}} \gtrsim H, \quad \text{or} \quad \Lambda_{\text{Confinement}} \gtrsim H$$

More pheno constraints

- Constraints on charged dark matter [Montero, Munoz, Obied, 2207.09448]
- Very constraining for inflationary models.

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4. Assuming a dS-like state, one can find Swampland conditions from contemplating about charged Nariai black holes. This is how one finds the Festina Lente bound.

$$m^4 \gtrsim (gq)^2 V \quad \text{for every charged state}$$

EXTRA SLIDES

$$\mathcal{L} \supset -f^2 (\partial a)^2 + \Lambda^4 \sum_q e^{-qS} (1 - \cos qa) \ ,$$

TCC $$T \leq \frac{1}{H} \log \frac{M_p}{H}$$

Neutrino's?

- Suggestive numerology $\sqrt{g M_P H} \sim 10^{-3}\, eV$
- If B-L is weakly gauged instead of spontaneously broken at high E, then lightest neutrino cannot be massless.

FL with runaway quintessence

$$S = \int \sqrt{|g|}\Big(\frac{1}{2}M_p^2\mathcal{R} - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}f(\phi)F_{\mu\nu}F^{\mu\nu} - V(\phi)\Big) + \text{matter}\,.$$

The Festina Lente (FL) bound: If the signs of V' and f' are the same, and the inequalities

$$\frac{V'}{V} \leq (d-3)\frac{f'}{f} \quad \text{and} \quad V'' \geq \frac{V'}{f'}\left(f'' - 2\frac{f'^2}{f}\right) \tag{2.16}$$

are satisfied, there exist classically stable electric Nariai solutions to which we can apply the bound that **every** particle of charge q and mass m must satisfy the inequality

$$m^4 \gtrsim 6(gqM_PH)^2 = 2(gq)^2V. \tag{2.17}$$

For general d, the existence condition comes from analyzing an electric Nariai. In d=4 one can make the FL argument with both the electric and magnetic U(1)'s. With magnetic charge, we can ensure there is always a Nariai solution, for all relative signs of V' and f' if:

$$\left|\frac{V'}{V}\right| < \left|\frac{f'}{f}\right|$$

In [Montero, Venken, VR 2020] we used **the opposite** of this inequality (when viewed as a no-dS2 requirement) to bound f' to many orders to *explain* hierarchy problem for time-dependence constraints on alpha in a quintessence universe:

$$\left|\frac{\dot{\alpha}}{\alpha}\right| \lesssim \frac{c}{10^{10}\ \text{years}} \qquad c \equiv |V'/V|$$

Oklo nuclear reactor: $\dfrac{\dot{\alpha}}{\alpha} < 10^{-15}(yrs)^{-1}$

FL with multiple fields

Lagrangian (4D) $e^{-1}\mathcal{L} = \frac{M^2}{2}\mathcal{R} - \frac{1}{2}G_{ij}(\phi)\partial\phi^i\partial\phi^j - \frac{1}{4}f_{AB}(\phi)F^A F^B - V(\phi)$

FL bound $m^4 >> \left((f^{-1})^{AB} q_A q_B + f_{AB} p^A p^B\right) (M_p H)^2$

Can be applied if $G^{ij}\frac{\partial_i V \partial_j V}{V^2} \leq -G^{ij}\partial_i f_{AB} \partial_j f^{AB}$

FL and dimensional reduction

1. Assume higher d theory without gauge fields and look at KK gauge field:

$$S = \frac{1}{8\pi G_{d+1}} \int \sqrt{|g|}\,(R - 2\Lambda_{d+1}) \Longrightarrow ds^2_{d+1} = e^{-2\alpha\phi} ds^2_d + R_0^2 e^{2(d-2)\alpha\phi}(dy - A)^2.$$

Then lower-d action is

$$S = \frac{1}{16\pi G_d} \int \sqrt{|g|}\Big(\mathcal{R} - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{R_0^2}{2} e^{\gamma\phi} F_{\mu\nu}F^{\mu\nu} - 2\Lambda_{d+1} e^{-\delta\phi} - (16\pi G_d) V_{\rm Casimir}(\phi)\Big)$$

With charged KK states $m^2_{\rm KK} \sim \frac{n^2}{R_0^2} e^{-\gamma\phi_{\rm min}}.$

FL implies $M_{\rm KK} \gtrsim V^{1/2}.$

2. Assume U(1) gauge field in higher dimensions obeying FL, then reduce over circle:

$$m^4 \gtrsim e^{2\alpha\phi} g_{d+1}^2 \frac{V_d}{2\pi R_0} = g_{d+1}^2 V_{d+1} \left(e^{2\alpha\phi} \frac{V_d}{2\pi R_0 V_{d+1}} \right)$$

If the radion is simply runaway (no extra stabilization), we have an exact preservation of FL under dim reduction, since parenthesis becomes 1.

If however, there is a stabilization, say due to Casimir energies as in [Gonzalo-Ibáñez-Valenzuela, 2021 & ArkaniHamed-Dubovsky-Nicolis-Villadoro2007] . New constraints on light fields?

→ However no Nariai in 3D…

3. Assume higher d theory p-forms, giving vectors upon reduction:

Example 2-form→ 1-form. Constraint on string tension from reduction

$$T^4 e^{-2(d-4)\alpha\phi} \gtrsim g_{d+1}^2 V_{d+1}.$$

Phi dependence cancels in D=4.

Similarly we can get constraints on brane tensions in higher dimensions. (By looking at n-branes in n+4 dimensions).

→ AI very non-trivial it works out so nice. Similar to other Swampland constraints under dimensional reduction [eg Heidenreich & Reece & Rudelius 2015, Rudelius 2021]

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To do this more precise we demand that the function

$$P(r, \tilde{M}, \tilde{Q}) \equiv -r^2 U(r) = \frac{r^4}{\ell_4^2} + 2\tilde{M}r - \tilde{Q}^2 n^2 - r^2$$

with $\tilde{M} = GM, \quad \tilde{Q}^2 n^2 = \frac{G}{4\pi}\frac{Q_m^2}{g^2} = \frac{G\pi}{g^2} n^2$

Is negative when evaluated at the monopole radius $r = \Lambda_{EFT}^{-1}$

$$\frac{r_+}{\ell_4} = \sqrt{\frac{1}{6}\left(1 - \sqrt{1 - 12\tilde{Q}^2/\ell_4^2}\right)} \leq \frac{1}{\Lambda_{EFT}\ell_4} \leq \sqrt{\frac{1}{6}\left(1 + \sqrt{1 - 12\tilde{Q}^2/\ell_4^2}\right)} = \frac{r_c^{\text{Nariai}}}{\ell_4}.$$

Two inequalities instead of 1. At small Q we approach the standard magnetic WGC. At the max value for Q:

$$\frac{\tilde{Q}^2}{\ell_4^2} = \frac{1}{12},$$

We cannot obey this anymore. Demanding that Q is smaller then this for n=1 leads to

$$g^2 \geq \frac{3}{2}\left(\frac{H}{M_P}\right)^2$$

This is the same as demanding that the dark energy scale is below cut-off scale. Makes sense. Now we get logical triangle with Festina Lente; applying FL to the (electric) WGC particle :

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