The Matter with $T\bar{T} + \Lambda_2$

Based on

work in progress/to appear w/Batra, De Luca, Shyam, Soni, Torroba, Yang,…

Anninos Denef Law Sun '20 Quantum de Sitter horizon entropy from quasicanonical bulk, edge, sphere and topological string partition functions

de Sitter Microstates from $T\bar{T} + \Lambda_2$ and the Hawking-Page Transition
arXiv:2110.14670, *JHEP* 07 (2022) 140
w/ Evan Coleman, Edward A. Mazenc,
Vasudev Shyam, Ronak M Soni, Gonzalo Torroba, Sungyeon Yang

Black hole to cosmic horizon microstates in string/M theory: timelike boundaries and internal averaging 2212.00588 [hep-th], *JHEP*.

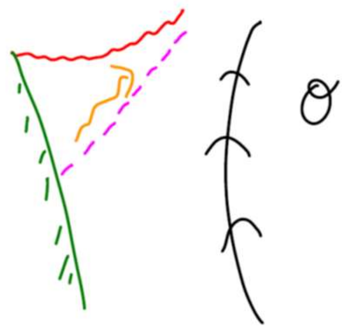
+others w/ Alishahiha et al, …, De Luca, Dong, Gorbenko, Lewkowycz, Liu, Torroba…

Connections with many other works…

General relativity predicts horizons, observed in nature

Black Hole horizon

$$ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \frac{dr^2}{1-\frac{r_s}{r}} + r^2 d\Omega^2$$

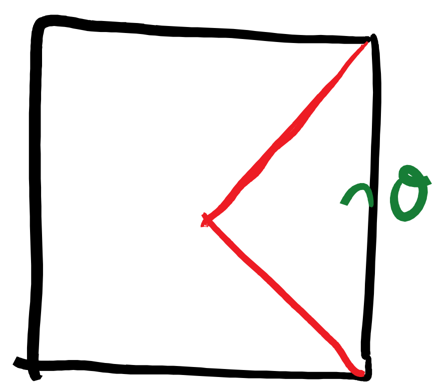


Cosmological horizon

exponentially expanding universe

$$ds^2 = -dt^2 + e^{2Ht}(dx^2+dy^2+dz^2)$$

Cosmological constant $\Lambda > 0$



At semiclassical level, the gravitational black hole system behaves as if it has entropy

Beckenstein, Gibbons, Hawking, …

S = $Area_{classical\ horizon} / (4G_{Newton})$+ entropy of quantum field fluctuations

and a temperature, energy and angular momentum like a thermal system (e.g. a box of gas):

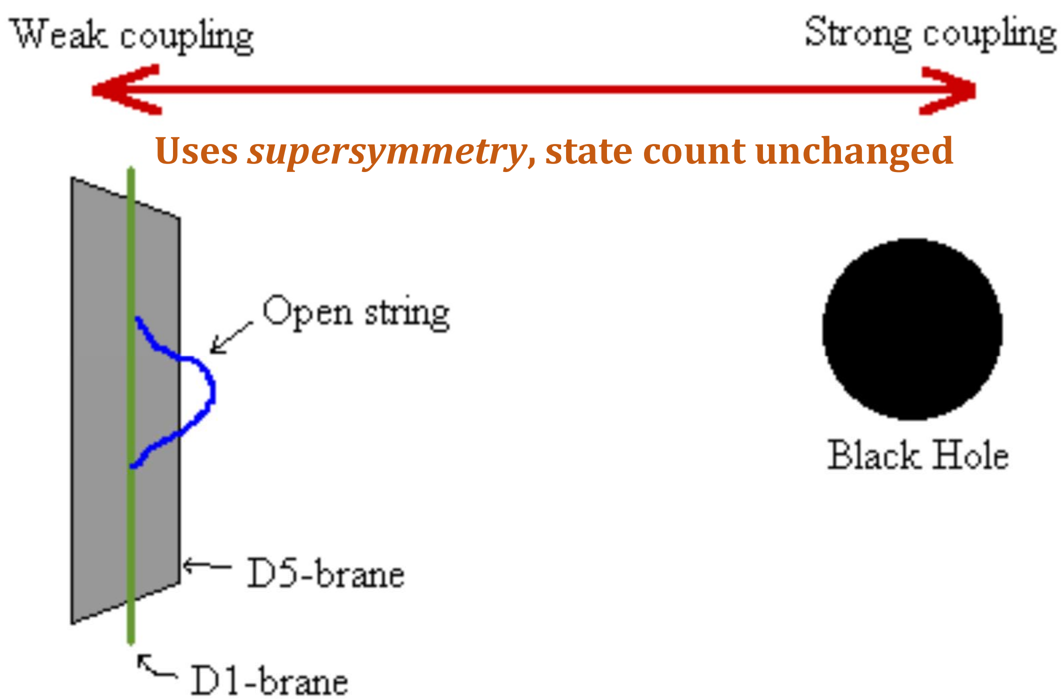
$$dM = \kappa\ dA + \mu\ dQ + \Omega\ dJ \qquad\qquad dE = TdS + \mu dN + \cdots$$

$$dA \geq 0 \qquad\qquad dS \geq 0$$

Finite number of available states, discrete quantization of energy levels?

Finite number of available states, discrete quantization of energy levels, with S = log(number of available states)? **Yes** (computed in some cases)

Strominger-Vafa, Callan-Maldacena,…Sen

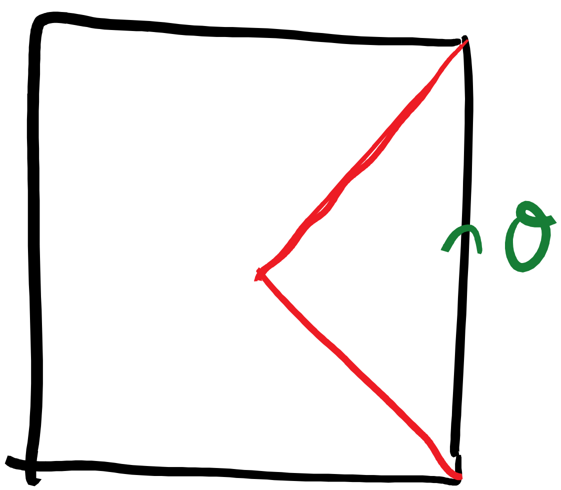


We generalized this to *integrable deformations* to preserve state count for cosmo case.

Gravitational calculations suggest an entropic interpretation of the de Sitter observer horizon area, somewhat analogous to black hole thermodynamics

Gibbons-Hawking '70s ... **Anninos Denef Law Sun '19 (logarithmic corrections), ...**

Banihashemi Jacobson Svesko Visser '22 1st law via Brown-York energy

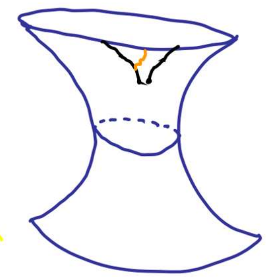


$$S = S_{GH} - 3\log(S_{GH}) + \ldots$$

$S_{GH} = \frac{A}{4G_N}$ (A) dS_3 case

Suggests that finite Hilbert space captures observer patch. (cf ...Anninos/Hartnoll/Hofman '11...Banks et al...Susskind)

Cosmological case:



Naively much harder because of the absence of timelike boundary for global dS => fluctuating lower-d gravity, no notion of energy

But we *can* consider **a timelike boundary** for finite (A)dS patches in GR, giving us a notion of energy (e.g. Brown-York,..) and non-fluctuating boundary gravity.

Moreover, $E_{\{Brown-York\}} = E_{\{dressed\ by\ T\bar{T}+\Lambda_2\ deformation\}}$

Gives a holographic dual description of 3d gravity as a deformed CFT passing nontrivial tests. Bulk matter, M theory? today's talk

$$S = S_{GH} - 3\log(S_{GH}) + \dots$$

Suggests that finite Hilbert space captures observer patch.

(cf ...Anninos/Hartnoll/Hofman '11...Banks et al...Susskind)

Coleman et al '21: *Real dressed spectrum* of the universal & solvable

$T\bar{T} + \Lambda_2$ deformation

Zamalodchikov et al, Dubovsky et al, Cavaglia et al ... Gorbenko ES Torroba (GST) '18, LLST '19

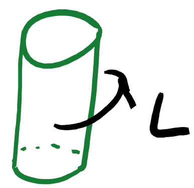
of a CFT on a cylinder captures the leading+log microstates and the radial geometry of the dS_3 observer patch.

Does *not* by itself capture all details of local bulk matter. Additional specifications required for that (note subleading for S_{GH}=area/4G and geometry). **Today's talk: incorporate the local bulk matter**

$$T\bar{T} \equiv \frac{1}{8}\left(T_{ab}T^{ab} - (T^a_a)^2\right)$$

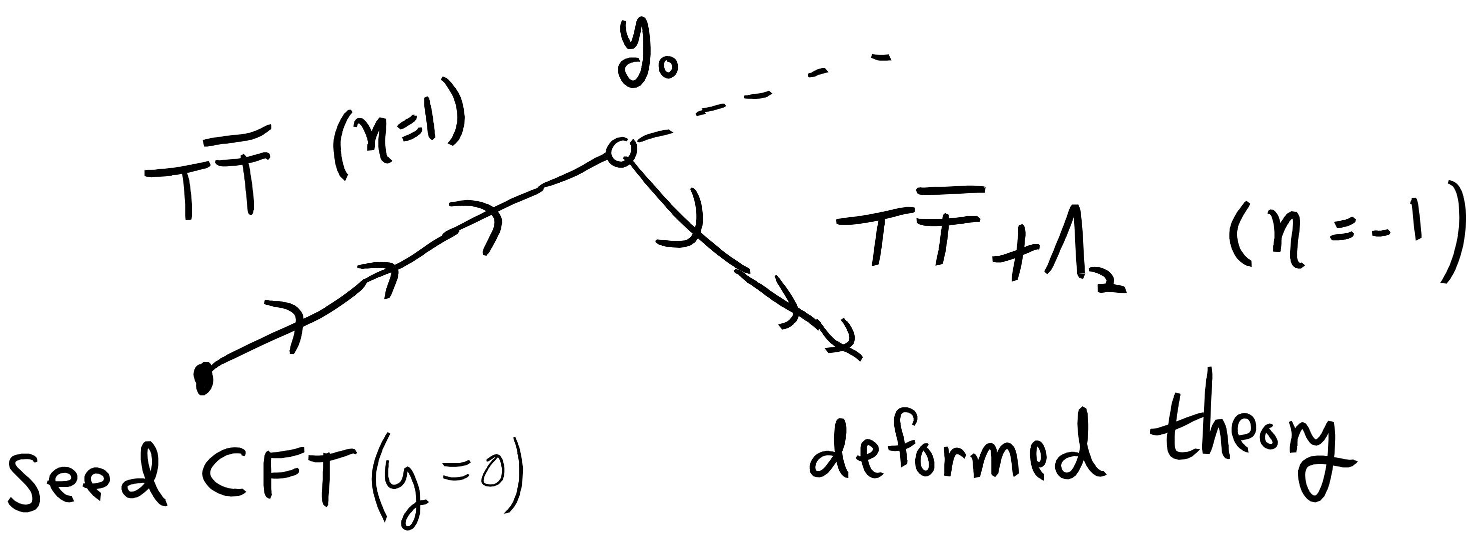
$$\frac{\partial}{\partial \lambda} \log Z = -2\pi \int d^2x \sqrt{g}\; T\bar{T} + \frac{1-\eta}{2\pi\lambda^2} \int d^2x \sqrt{g}$$

e.g. cylinder



Λ_2

$y \equiv \frac{\lambda}{L^2}$



Precise deformed energy spectrum:

Smirnov-Zamolodchikov, Cavaglia et al, Dubovsky et al…Gorbenko et al…

$$\frac{\partial}{\partial\lambda}\log Z = -2\pi\int d^2x\sqrt{g}\;T\bar{T} + \frac{1-\eta}{2\pi\lambda^2}\int d^2x\sqrt{g}$$

$\rightarrow \partial_\lambda\langle H\rangle \sim \langle T\bar{T} + \Lambda_2\rangle,\quad T_x^x \sim \frac{\partial E}{\partial L}$ (pressure), … $\rightarrow$ $\mathcal{E} = EL$

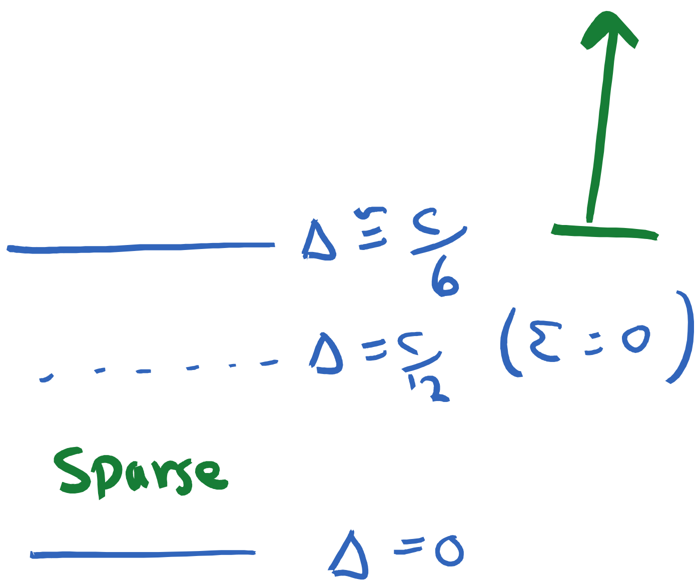
$$\pi y\mathcal{E}(y)\mathcal{E}'(y) - \mathcal{E}'(y) + \frac{\pi}{2}\mathcal{E}(y)^2 = \frac{1-\eta}{2\pi y^2} + 2\pi^3 J^2$$

$$\mathcal{E}\Big|_{y=0,\eta=1} = \mathcal{E}_{CFT} = 2\pi\left(\Delta - \frac{c}{12}\right)$$

$$\mathcal{E}\Big|_{y=0,\eta=1} = \mathcal{E}_{CFT} = 2\pi\left(\Delta - \frac{c}{12}\right)$$

We will be interested in a seed CFT with a *sparse light spectrum* (in particular a holographic CFT)

Hartman Keller Stoica et al



$$S = S_{Cardy} = 2\pi\sqrt{\frac{c}{3}\left(\Delta - \frac{c}{12}\right)}$$

$\simeq \frac{A}{4G_N}$ for holographic CFTs, with BTZ Black Holes for $\Delta > \frac{c}{12}$

General solution:

$$\mathcal{E}(y) = \frac{1}{\pi y}\left(1 \pm \sqrt{\eta - 4C_1 y + 4\pi^4 J^2 y^2}\right)$$

Fix integration constant and branch via appropriate boundary conditions for a given trajectory in theory space.

We will do two key examples, where the deformed energy matches the *Brown-York energy* for a given patch of dS, via a trajectory which is continuous for the corresponding band of energies.

Brown-York (quasilocal) stress-energy

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta I_{\text{on-shell}}}{\delta g^{\mu\nu}} = \frac{1}{8\pi G_N}\left(K_{\mu\nu} - g_{\mu\nu}K + \frac{1}{\ell}g_{\mu\nu}\right)$$

$\sqrt{\dots}$ part of $\mathcal{E}$

← Dirichlet* condition

$G_{ij}|_\partial = g_{ij}$ fixed

$\Pi^{\text{radial}}_{ij}|_\partial = K_{ij}$ free

With cylinder slices, a subset of the Einstein equations imply the above differential equation for $\mathcal{E} = L\int_0^L dx\, T^t_t$

with dictionary $c = \dfrac{3\ell}{2G_N}, \quad \lambda = 8G_N\ell\,. \quad \Lambda_3 = -\dfrac{\eta}{\ell^2}$

McGough, Mezei, Verlinde; Kraus, Liu, Marolf, Kraus, Monten, Roumpedakis, Ebert, Hijano, Caputa, Datta, Jian, Myers,… , Gorbenko ES Torroba,… Banihashemi, Jacobson, Svesko, Visser…1[st] law of thermodynamics wrt $E_{Brown-York}$.

*Dirichlet boundaries not fully understood:

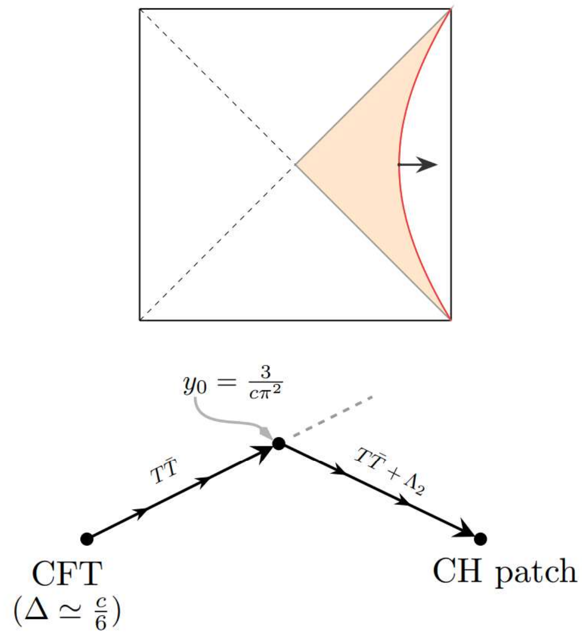
- Certain potential instabilities Marolf/Santos et al
- Difficulties defining perturbation theory Anderson, Witten et al
  exceptions for definite-sign extrinsic curvature, also UV-sensitive
  Solutions of IBVP not guaranteed, yet BY energy defined on-shell
- Generalization to UV complete theory? In progress (below)

*$T\bar{T}$ theories not fully understood

Other boundary conditions also possible in this framework Coleman-Shyam, different ensembles. Bounded regions can be building blocks for joined system.

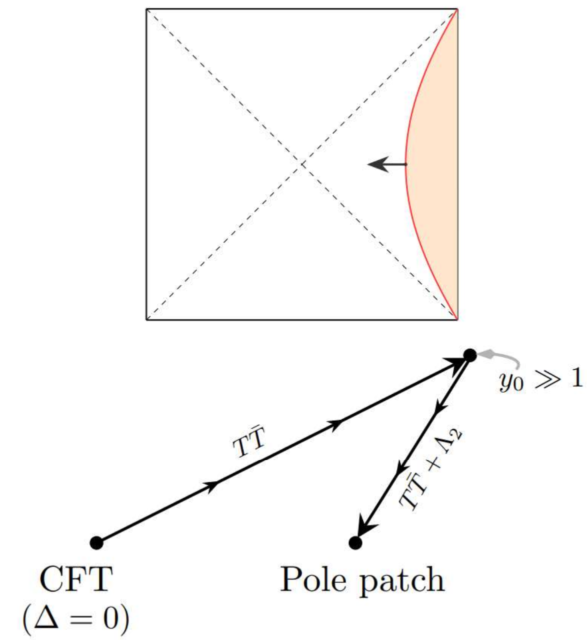
Cosmic horizon patch

(Dressed $\Delta \simeq \frac{c}{6}$ black hole microstates)



Pole patch

(Dressed $\Delta = 0$ vacuum)



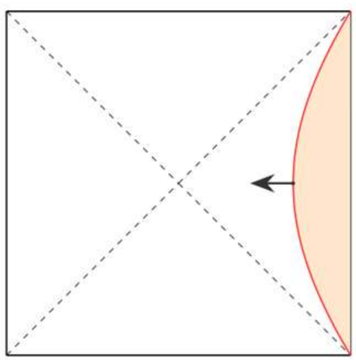
$\mathcal{E} = \frac{1}{\pi y}\left(1 + \sqrt{\eta + \ldots}\right)$ $\longleftarrow$ related by $\pm\sqrt{}$ $\longrightarrow$ $\mathcal{E} = \frac{1}{\pi y}\left(1 - \sqrt{\eta + \ldots}\right)$

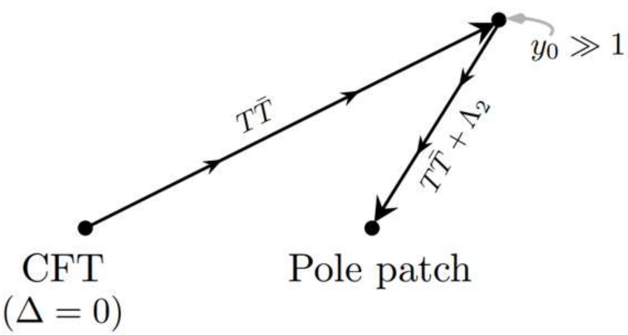
$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

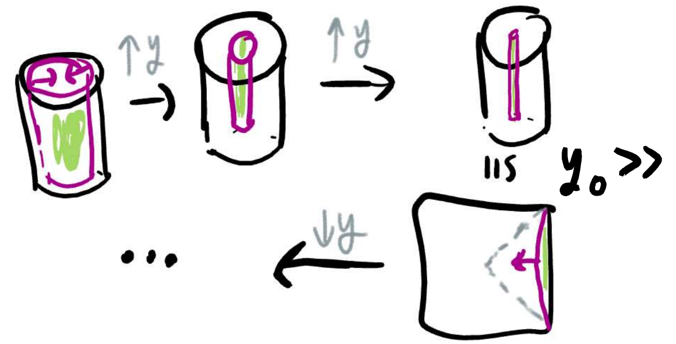
Pole patch

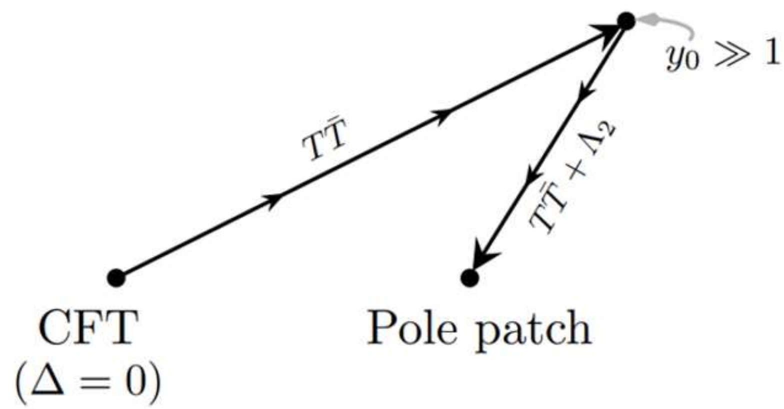
(Dressed $\Delta = 0$ vacuum)

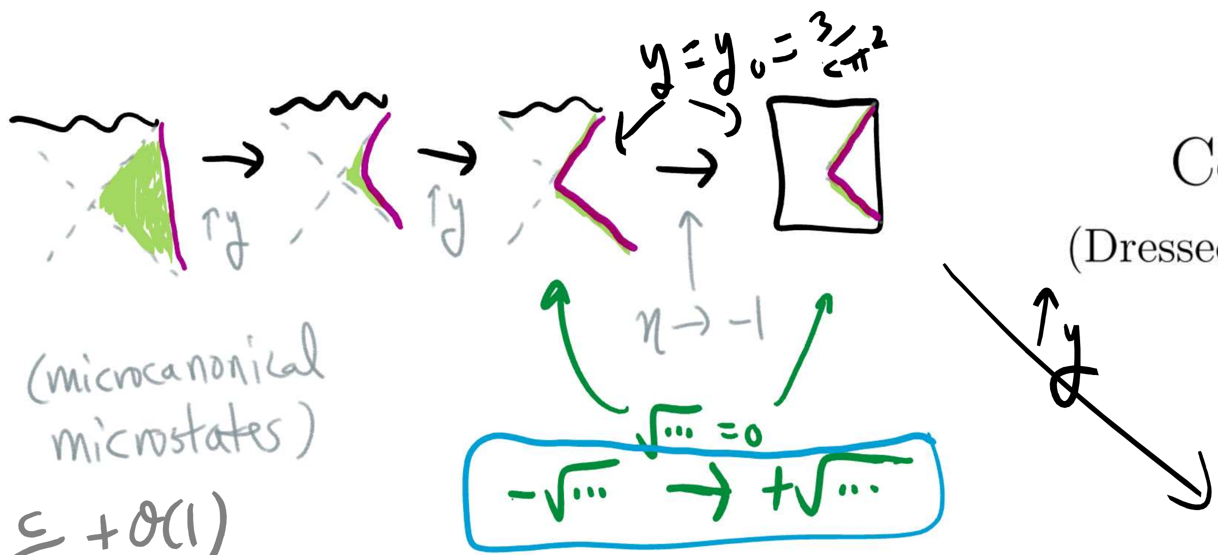
Gorbenko ES Torroba '18
Lewkowycz Liu ES Torroba '19







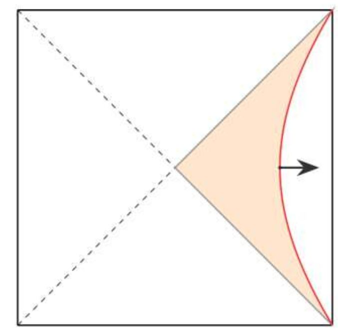




$\Delta \leq \frac{c}{6} + \mathcal{O}(1)$

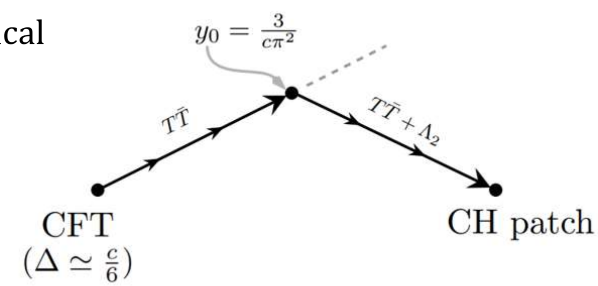
Cosmic horizon patch

(Dressed $\Delta \simeq \frac{c}{6}$ black hole microstates)



$$ds_3^2 = -\frac{w^2}{\ell^2}d\tau^2 + dw^2 + (\ell^2 + \eta w^2)d\phi^2$$

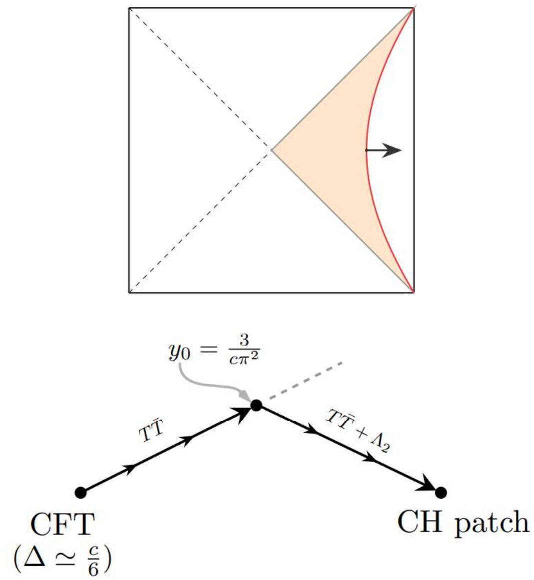
At $y = y_0$, the near horizon patches are identical



$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

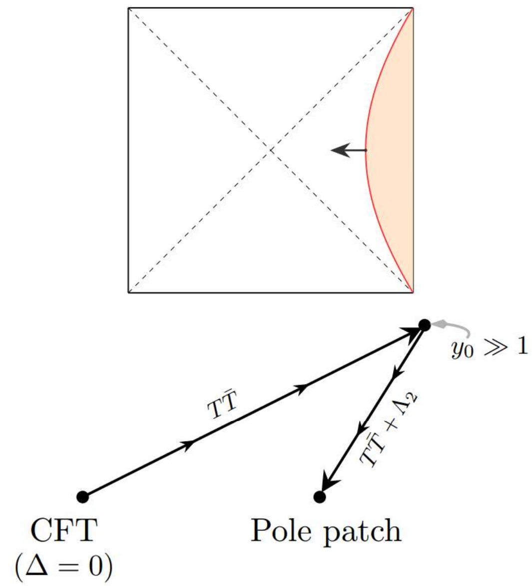
Cosmic horizon patch

(Dressed $\Delta \simeq \frac{c}{6}$ black hole microstates)



Pole patch

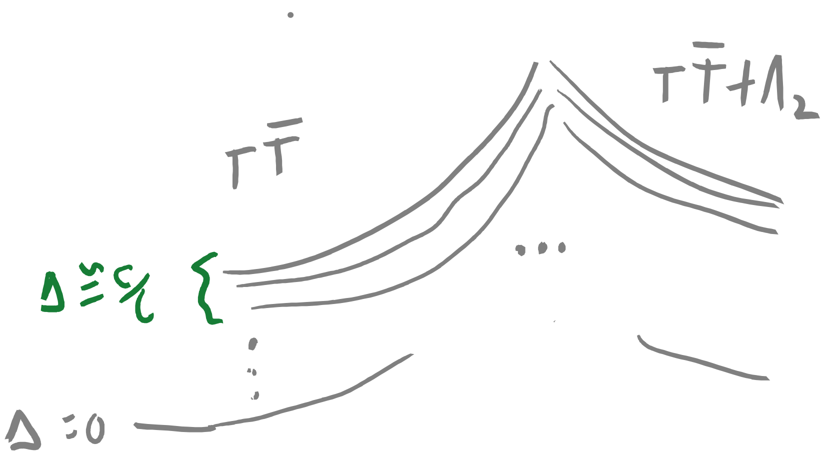
(Dressed $\Delta = 0$ vacuum)



$\mathcal{E} = \frac{1}{\pi y}\left(1 + \sqrt{\eta + \ldots}\right)$ $\longleftarrow$ related by $\pm\sqrt{\ }$ $\longrightarrow$ $\mathcal{E} = \frac{1}{\pi y}\left(1 - \sqrt{\eta + \ldots}\right)$

As we vary y, we capture precisely the geometry of the dS patch

$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$



Count of states goes along for the ride (`integrable deformation').
Note: follow states w/o using SUSY BPS arguments

$$\Delta < \frac{c}{12} : \text{ sparse spectrum} \quad \text{(particle states)}$$

$$\Delta \geq \frac{c}{6} : S \simeq S_{Cardy} = 2\pi\sqrt{\frac{c}{3}\left(\Delta - \frac{c}{12}\right)}$$

Other states: $$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

$\Delta > \frac{c}{6}$ states: dressed energies formally become complex, discarded in a unitary version of the theory => Finite dimensional Hilbert space with count of states agreeing with Gibbons-Hawking

Also: related to resurgence https://vasushyam.quarto.pub/ttb-branches/ , Griguolo, Luca, Rodolfo Panerai, Jacopo Papalini, and Domenico Seminara. 2022. "Nonperturbative Effects and Resurgence in Jackiw-Teitelboim Gravity at Finite Cutoff." *Physical Review D* 105 (4).

$\Delta < \frac{c}{6}$ states: subdominant at large c, model-dependent (details require additions to the deformation). Includes interesting landscape states.

=> Real spectrum of the $T\bar{T} + \Lambda_2$ deformation captures the finite dimensional Hilbert space (i) agreeing with Gibbons-Hawking and (ii) building up the geometry of the dS observer patch

Count of states goes along for the ride (`integrable deformation'), subleading check agrees at `pure gravity' level:

$$S = A/4G_N - 3\log(A/4G_N)$$

Sen (AdS BTZ case) … Anninos Denef Law Sun (dS)

Cf Bousso-Maloney-Strominger '01 Kerr-dS entropy from Cardy formula;
In some sense this explains that (self-described) `numerology'.
Cf van Leuven, E. Verlinde, Visser '18, DST '18

Summary: At the `pure gravity' level, the *real dressed spectrum* of the universal and solvable

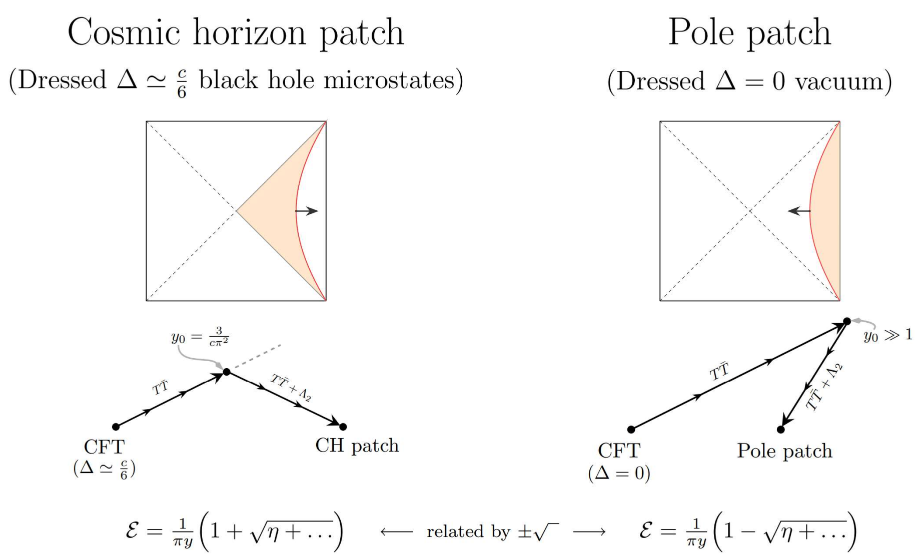
$$\frac{\partial}{\partial\lambda}\log Z = -2\pi\int d^2x\sqrt{g}\,\langle T\bar{T}\rangle + \frac{1-\eta}{2\pi\lambda^2}\int d^2x\sqrt{g}$$

$T\bar{T} + \Lambda_2$ deformation

Zamalodchikov et al, Dubovsky et al, Cavaglia et al … Gorbenko ES Torroba '18

of a CFT on a cylinder captures (only) the microstates and the radial geometry of the dS_3 observer patch Shyam, Coleman et al '21

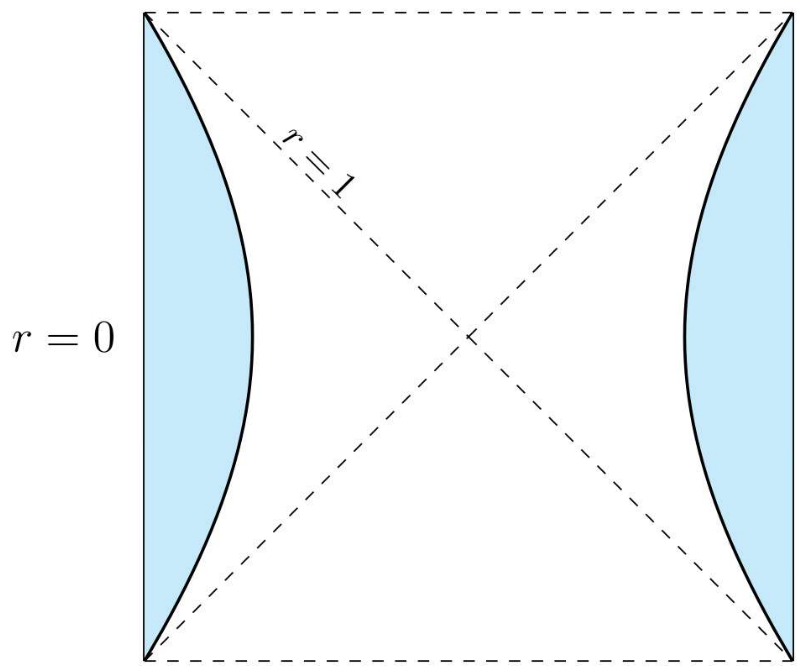
$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

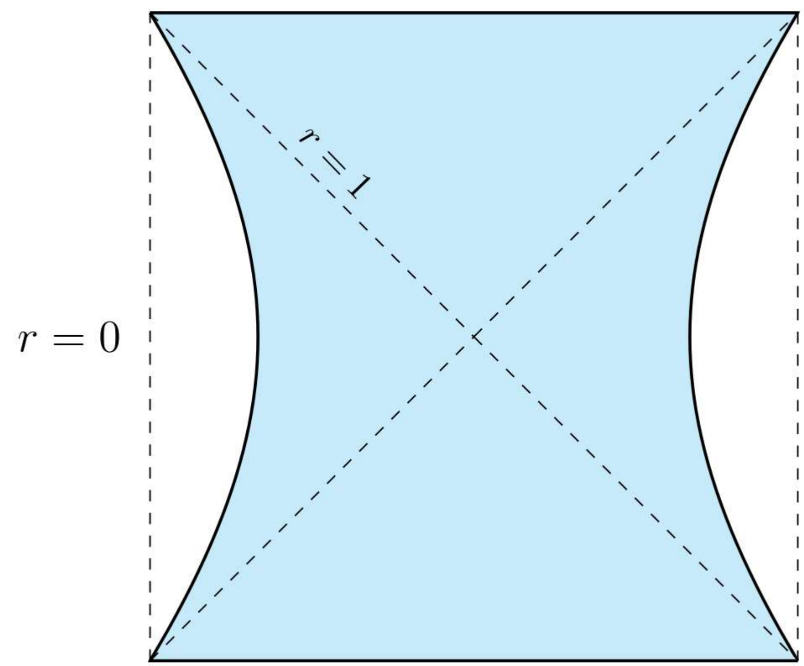


BPS black hole state counting (Strominger/Vafa), used extended SUSY to control weak → strong coupling deformations preserving state count. Here we have a **new type of controlled deformation** applicable to dS, again preserving state count: 'integrable deformation' of non-integrable seed theory.

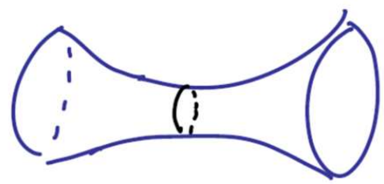
Can in principle double and glue the two patches together to get global dS:

cf Coleman, Soni, Yang '22 spread of entanglement





One recent theme is that entanglement and other properties of the quantum state are tied to the knitting together of spacetime.



Thermal state of two CFTs (entangled at thermal scale) dual to joined spacetime

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2}|n\rangle|n\rangle\,.$$

``ER=EPR"

Van Raamsdonk, Maldacena/Susskind

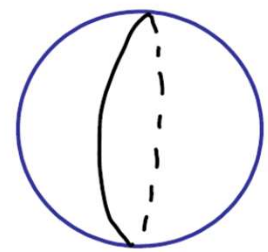
Realistic de Sitter case: Entangled state, with flat entanglement spectrum, of low energy part of 2 CFTs deformed appropriately to produce the dual of dS.

$T_{effective} \to \infty$

Dong ES Torroba '18, Gorbenko ES Torroba '19, Shyam '21, ... also Banks et al

$\Rightarrow$ $type \leq II$ operator algebra

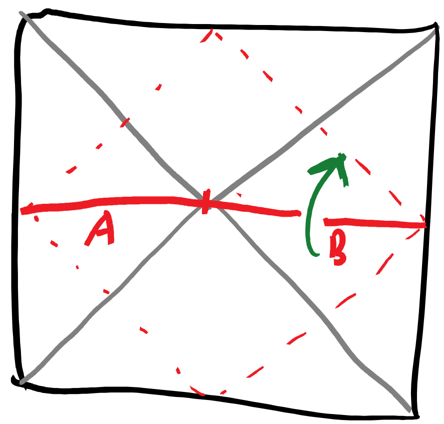
Chandrasekharan Longo Penington Witten

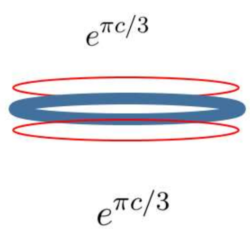


The static patch Hamiltonian is the Modular Hamiltonian K for dS/dS

$$K = -\log(\rho_A)$$

with ρ_A the reduced density matrix for 1 of the 2 dS/dS warped throats





$$S_{GH} = -Tr\rho_A \log(\rho_A) = S_{EE} = \log(\dim H_{T\bar{T}+\Lambda_2})$$

Dong ES Torroba '18

Coleman Mazenc ES Shyam Soni Torroba Yang '21

Flat entanglement spectrum

Next: The Matter of $T\bar{T} + \Lambda_2$

- It is known how to generalize the prescription $\partial_y S = \int T\bar{T} + \Lambda_2$+ `OO'... to define a boundary theory which encodes also local Dirichlet conditions for matter fields, perturbatively in 1/c. Accommodates matter dynamics => additional probes of geometry.

- We can go beyond large c by defining a finite theory via an initial application of pure $T\bar{T}$. Even though `OO' doesn't factorize beyond large c, we don't need that for a well defined theory. Enough that its matrix elements $\langle n|OO|m\rangle$ are finite.

- In M/String theory, the prescription of deforming from AdS/CFT to dS/deformed-CFT survives despite the large internal difference between metastable dS and AdS solutions. This internal difference washes out at the matching point between trajectories ($y = y_0$, boundary at horizon) because of high-temperature thermal mixing there.

Details of these points in the remainder of the talk.

Generalizations of $T\bar{T} + \Lambda_d$ **Matter**:

M. Taylor; Guica et al, Hartman Kruthoff Shaghoulian Tajdini '18, Shyam et al …

(1) Local bulk matter (model-dependent, subleading in entropy) requires similar term for each low-energy field: (large c regime). Use for $\Delta < c/6$ states and local probes.

$\pi_{\phi,\text{radial}} \sim \mathcal{O} \rightarrow$

$$T_i^i = -4\pi\lambda T\bar{T} - \frac{1}{\pi\lambda}\left(\frac{c\lambda}{L^2}\right)^{\Delta}\tilde{\mathcal{O}}^2 - \frac{cR^{(2)}}{24\pi} + \Lambda_2$$

(2) Higher dimensions & curvature (large c regime):

$$\tilde{T}^\mu_\mu = -4\pi G\ell\left(\tilde{T}_{\mu\nu}\tilde{T}^{\mu\nu} - \frac{1}{d-1}(\tilde{T}^\mu_\mu)^2\right) - \frac{\ell}{16\pi G}R^{(d)} - \frac{d(d-1)}{16\pi G\ell}(\eta - 1)$$

$$\tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + a_d C_{\mu\nu}\,. \qquad C_{\mu\nu} = G_{\mu\nu} \text{ for } d \leq 4$$

(3) Bulk gauge field matter:

$$T\bar{T} + \frac{J\bar{J}}{\lambda} + \frac{1-\eta}{\lambda^2}$$

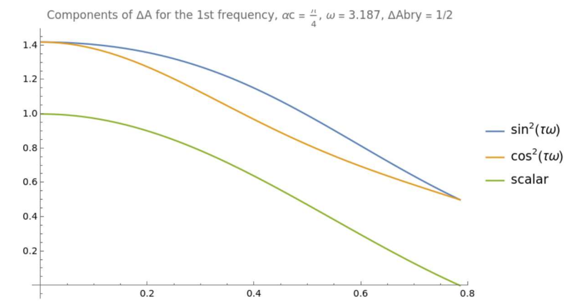
T^2 deformation also interesting for spacelike (Cauchy slice) approaches to holography

Araujo-Regado, Khan, Wall

Scalar $\phi(t, radial)$ (dual to vector in 3d bulk) + gravity solutions:

Explicit order $E_{scalar} G_N \ll 1$ t-dependent back reacted solutions singularity-free, with Dirichlet condition for both matter and gravity.

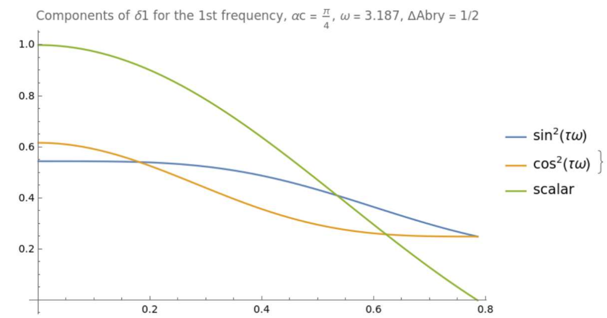
$$ds^2 = \frac{\ell^2}{\cos^2 x}\left(-Ae^{-2\delta}dt^2 + A^{-1}dx^2 + \sin^2 x d\theta^2\right)$$



Perturbations around vacuum given by

$$A_2(x,t) = 8\pi G \cos^2 x \int_{x_A=0}^{x} (\dot{\phi}_1^2 + \phi_1'^2)\tan y\, dy,$$

$$\delta(x,t) = -8\pi G \int_{x_\delta(t)}^{x} (\dot{\phi}_1^2 + \phi_1'^2)\sin y \cos y\, dy.$$



Simple calculable effect on dressed energies: preserves finiteness of real spectrum, ability to continuously connect AdS to dS.

$$T^{(BY)}{}^t_t = \frac{1}{\pi y}\left(1 - \sqrt{1 - 4\pi^2 y\left(\frac{c}{12}A_2 - \frac{c}{12}\right) - A_2}\right)$$

Charged black holes and $T\bar{T} + \frac{J\bar{J}}{\lambda} + \frac{1-\eta}{\lambda^2}$

$$-\partial_y \mathcal{E}_n + \pi y \mathcal{E}_n \partial_y \mathcal{E}_n + \frac{\pi}{2}\mathcal{E}_n^2 = 2\pi^3 J_n^2 + \frac{1-\eta}{2\pi y^2} \mp \frac{Q^2}{y}.$$

$$\mathcal{E}_n(y) = \frac{1}{\pi y}\left[1 \pm \sqrt{\eta + C_1 y + 4\pi^4 J_n^2 y^2 \mp 2\pi Q^2 y \log(y)}\right]$$

Matches charged black hole Brown-York energy in AdS, charged cosmic horizon in dS (sourced by charge outside the patch).

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\phi^2 - \frac{4GJ}{r^2}dt\right)^2$$

$$f(r) = -8GM + \eta\frac{r^2}{\ell^2} + \frac{16G^2J^2}{r^2} - 8\pi G Q^2 \log\frac{r}{\ell}$$

$$F_{tr} = \frac{Q}{r}\,,\ A_t = Q\left(1 - \log\frac{r}{\ell}\right).$$

Finiteness of the real Hilbert space with the local Matter:

Proposal steps:

1) Pure $T\bar{T}$ from seed finite-c CFT for y $= 0$ to $y_{-1} \ll 1$
2) $T\bar{T} + OO$ for y $= y_{-1}$ to y_0
3) $T\bar{T} + OO + \Lambda_2$ for y $= y_0$ to y_{final}

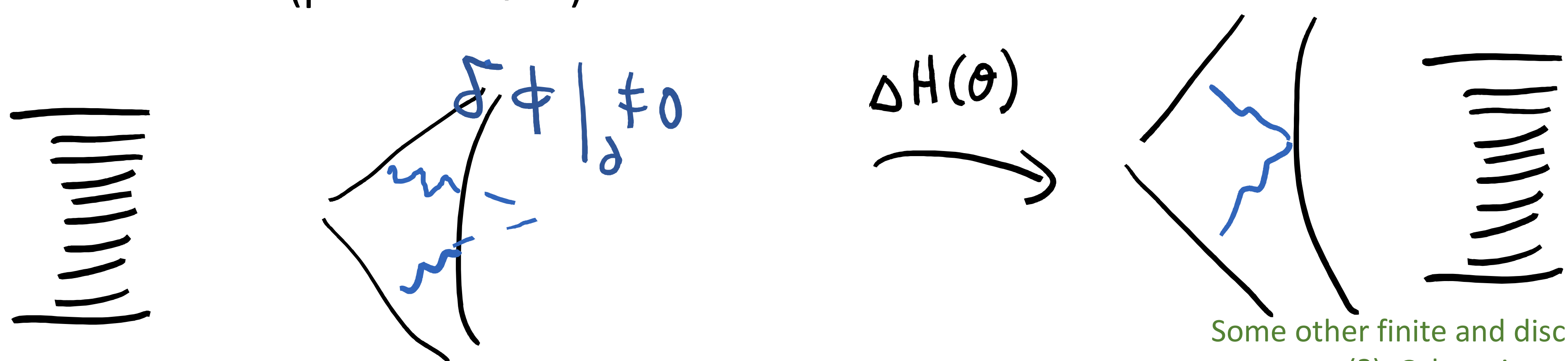
After (1), the real spectrum is finite. Given a compact target space for the 2d theory (as in D1-D5 CFT), no room for an infinite spectrum to develop (states can't come in from infinity, similar to index calculations). This doesn't require large-c factorization, in general just need well-defined $\langle n|OO|m\rangle$. For O= J (current) we can define J all along the trajectory using the path integral with dressed classical action, whose corresponding Hamiltonian exhibits complexification of levels.

So far, at least without "OO" deformation, we have a discrete, finite quantum mechanics system. Type I algebra Cf type II in Chandrashekharan, Longo, Penington, Witten.

With the extra deformation needed for local bulk matter, say in the Q.M. version where the composite ops are easy to define, do these properties continue? Not known for sure, we might expect so (?): Work at 1 << c < ∞.
Pure $T\bar{T}$ first => finite Hilbert space

Without OO's (pure $T\bar{T} + \Lambda$)

$T\bar{T} + \Lambda + OO$



Finite and discrete spectrum, spacings ~ exp(-S)

Gravity side picture: ΔH effects a change of ϕ boundary conditions to achieve ~bulk locality.

Some other finite and discrete spectrum (?) Otherwise, ΔH would need to bring in an infinite number of states from somewhere to fill in a continuum.

- t0mperature $\sim \frac{1}{\beta}$ from $\partial_c E_{dressed}$ Lin/Susskind: from above dictionary and

$$ds_3^2 = -\left(\frac{\ell^2 - r^2}{\ell^2}\right) d\tau^2 + \left(\frac{\ell^2}{\ell^2 - r^2}\right) dr^2 + r^2 d\phi^2$$ we get $(\beta/L)^2 = \frac{y}{y_0} - 1$

$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

-1

$\partial_c E_{dressed} \propto 1/\beta$. Change in energy upon changing # d.o.f.

- Complexity: note that we cannot expand the square root once $\eta = -1$.

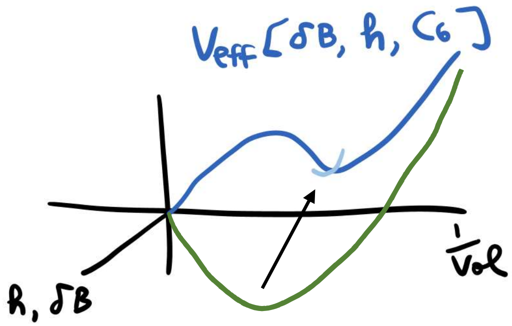
If we take the seed CFT as proxy for the `fundamental' degrees of freedom and $E_{dressed}$ as proxy for the Hamiltonian, it means we have fully nonlinear all to all interactions in the case (dS) where complexity blows up.

This is encouraging, but raises many questions

- Generalization to 4-dimensional dS? cf Hartman et al, Shyam,…

- Relation to string theoretic de Sitter (=dS quantum gravity)?
Late time physics (metastable decay)?

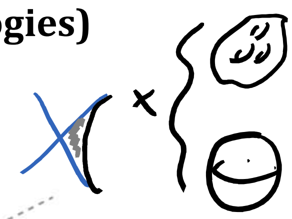
M/String theory includes direct uplifts from AdS/CFT Dong et al ‘10, De Luca et al ‘21, again connecting the dS case to a CFT. e.g. recent example:

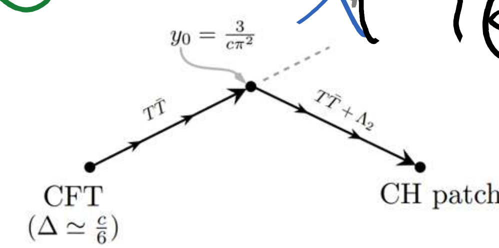


dS x hyperbolic space

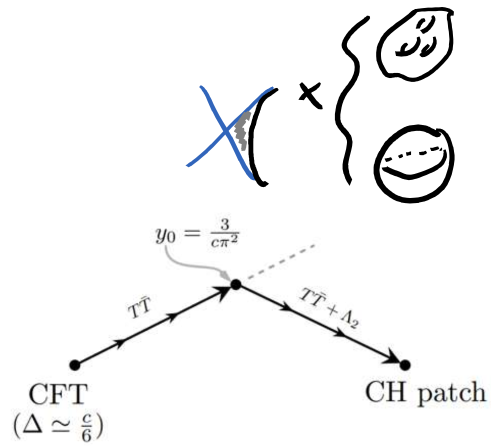
(Very different internal topologies)

AdS x sphere





Nonetheless, the matching between BH and dS horizons may extend to the full UV theory



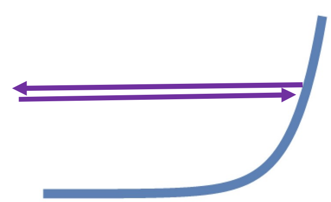
The matching happens at large `tOmperature' of the boundary theory

=> Mixing among all internal configurations consistent with the horizon. Even in the full string/M theory, can't distinguish the AdS/BH and dS horizons.

This relies on the existence of the bounding walls => question:
do they exist in full quantum gravity (string/M theory)? In terms of embedding of the fundamental degrees of freedom into the target spacetime:

$$Matrix\ Theory\ Hamiltonian = \sum Tr(\dot{X}^2) + Tr[X^M, X^N]^2 + \mathrm{Tr}\ \mathrm{O}_\kappa \exp(\kappa X^{(10)})$$

$$\text{String theory worldsheet action} = tension * \int (G_{MN}\partial X^M \partial X^N + O_\kappa \exp(\kappa X^{(9)}))$$



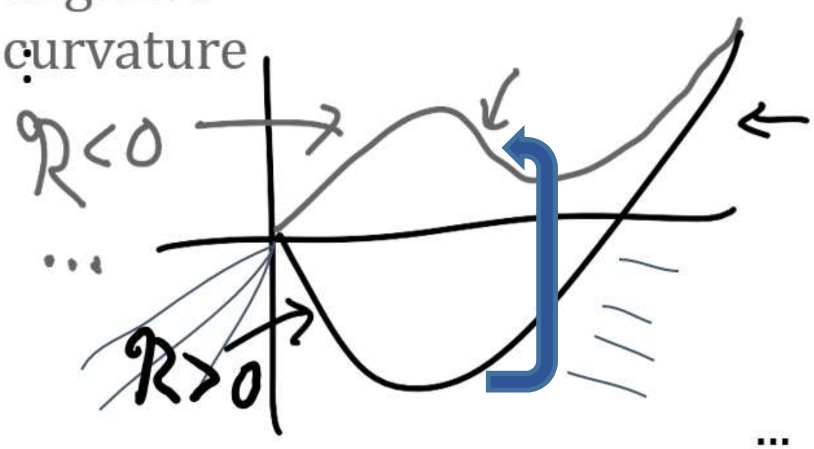
The problem in string/M theory:

Again, the matching occurs at the horizon in the **external** dimensions
(AdS BH horizon ≃dS cosmic horizon)

Uplift from AdS/CFT to dS: nontrivial **internal topology change

D-10
anti D3 brane
negative
curvature
⋮

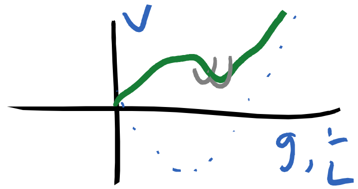
Orientifolds
Quantum



Generalized
flux

1/Volume
coupling

…



4d effective potential

Douglas '09

Mostly positive:
$D - D_c, -R^{D-4}, (Q_1 + a\,Q_2)^2, \ldots$
Intermediate negative:
O-planes, quantum

$$V_{eff}[g^{(D-4)},\ldots] = \frac{\ell_D^{D-2}}{2G_N^2}\,\frac{\int d^{D-4}y\,\sqrt{g^{(D-4)}}e^{-2\Phi}u^2|_c\left(-R^{(D-4)} - \frac{1}{4}\ell_D^{D-2}T^\mu_\mu - 3\left(\frac{\nabla u}{u}\right)^2\Big|_c\right)}{\left(\int d^{D-4}y\sqrt{g^{(D-4)}}e^{-2\Phi}u|_c\right)^2}$$

Net curvature

$u(y) = e^{2A(y)}$

$$ds^2 = e^{2A(y)}ds^2_{dS_4} + e^{2B(y)}(g_{\mathbb{H}ij} + h_{ij})dy^i dy^j$$

$R_{sec} < 0$ rigid
(cf Trodden et al, Saltman-ES, DLST)

$R_{ij} = 0$ CY
(cf KKLT, LVS…)

u(y) satisfies GR constraint (its eq. of motion):

$$\left(-\nabla^2 - \frac{1}{3}\left(-R^{(D-4)} - \frac{1}{4}\ell_D^{D-2}T^\mu_\mu\right)\right)u = -\frac{C}{6}$$

Like a Schrodinger problem for
$C\ell^2 \sim H^2\ell^2 \ll 1$

$$\Rightarrow\quad V_{eff} = \frac{C}{4G_N} = \frac{R^{(4)}_{\text{symm}}}{4G_N}\,.$$

Warp factor stabilizes runaway negativity (e.g. $-B'^2$)

dS examples:

Reviews of various aspects: Snowmass '22 `Cosmology at the Theory Frontier', Polchinski, Baumann/McAllister, Douglas/Kachru, Denef, Frey, Hebecker; ES TASI '16, Cicoli et al '23,...

- Non-perturbative stabilization on Calabi-Yau manifolds

--GKP '01/KKLT '03 and many followups, e.g.
--large volume scenario
--Antoniadis et al '22-'23

Sub-KK scale SUSY breaking

- Power-law stabilization

--(D-Dc), O-planes, flux, asymmetric orbifold (large-D expansion) '01-'02
(...other examples...)
--hyperbolic space, Casimir, flux '21
-- RG logs & powers Burgess/Quevedo '22

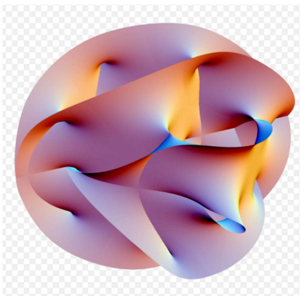
--including explicit uplifts of AdS/CFT
[D1-D5 theory -> dS3 '10,
M2 brane theory -> dS4 '21]

≥KK scale SUSY breaking

Both of these are controlled, but importantly **not** small perturbations of each other.

(Weak-coupling EFT control. Ongoing studies of internal equations of motion in various cases & models, including ones with significant gradients e.g. Cordova et al, ...)

Calabi-Yau geometry:

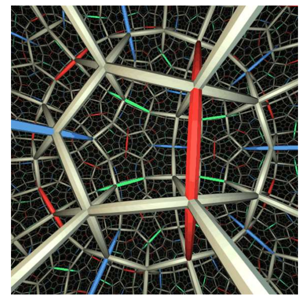


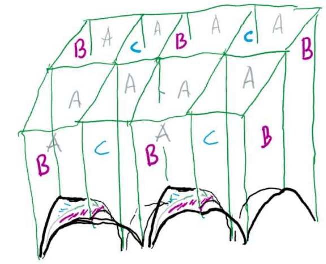
Non-constructive existence proof of Ricci flat metric*, algebraic geometry constructions of some quantities. Highly non-generic. Useful toy for interpolation from weak to strong coupling. Would be required *if* supersymmetry were observed in nature.

Starts with flat moduli, requires non-Perturbative potential. Much heroic work here!

*analytic K3 metric: Kachru, Tripathy, Zimet

Hyperbolic geometry:

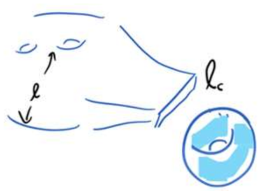




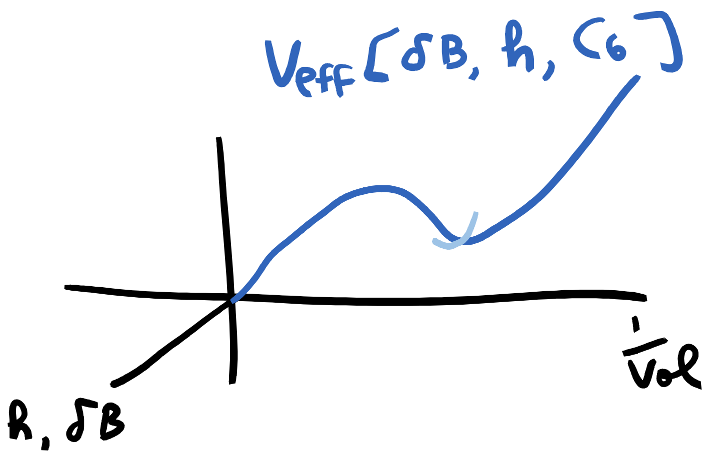
Metric known, constructions explicit via gluing polygons. Group theory governs key quantities. Dehn/Anderson filling of cusps. Surgery and topology change. Rigidity (no flat moduli for dimension n>2).

Physics +: Positive potential energy from negative internal curvature, genericity, rigidity (deformations massive). Supersymmetry neither required for control nor observed in nature.

Curved internal dim's: recent mechanism for Λ from string/M theory



M theory (EFT: 11d SUGRA) on explicit infinite discrete family of finite-volume hyperbolic spaces with $\int -R - 3u'^2 \ll -\int R$ **parametrically**, automatically-generated Casimir energy, 7-form flux yields immediate volume stabilization and approximate piecewise solution dressed with warp & conformal variations, small residual shifts in other directions.



Strong positive Hessian contributions from **hyperbolic rigidity** and from **warping** (redshifting) effects on conformal factor and on Casimir energy.

Douglas '09

4d effective potential

net curvature term

$\ell_{11}^9 \rho_c(R_c) \sim -\frac{\ell_{11}^9}{R_c^{11}}$

$$V_{eff}[g^{(7)}, C_6] = \frac{\ell_{11}^9}{2G_N^2} \frac{\int d^7y \sqrt{g^{(7)}} u^2|_c \left(\left[-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 \Big|_c\right] - \frac{1}{4}\ell_{11}^9 T^{(Cas)\mu}{}_{\mu} + \frac{1}{2}|F_7|^2 \right)}{\left(\int d^7y \sqrt{g^{(7)}} u|_c\right)^2}$$

$$ds^2 = e^{2A(y)} ds^2_{dS_4} + e^{2B(y)} (g_{\mathbb{H}ij} + h_{ij}) dy^i dy^j \qquad u(y) = e^{2A(y)}$$

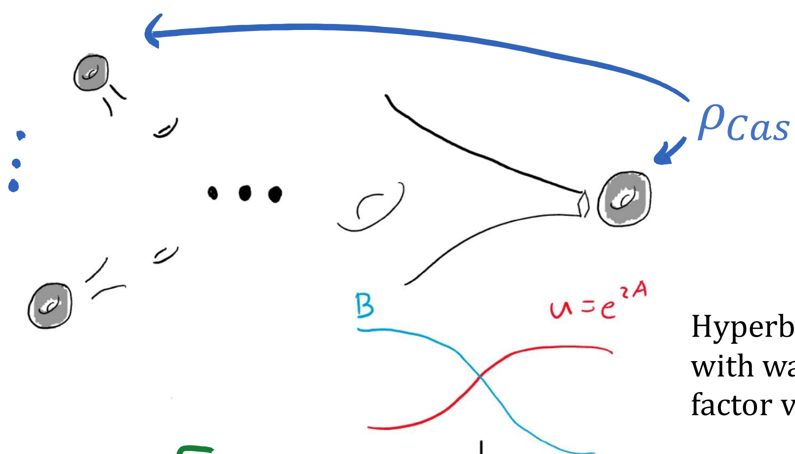
u(y) satisfies GR constraint (its equation of motion):

$$\left(-\nabla^2 - \frac{1}{3}\left(-R^{(7)} - \frac{1}{4}\ell_{11}^9 T^{(\mathrm{Cas})\mu}{}_{\mu} + \frac{1}{2}|F_7|^2\right)\right) u = -\frac{C}{6}$$

Like a Schrodinger problem for

$C\ell^2 \sim H^2\ell^2 \ll 1$

$$\Rightarrow \quad V_{eff} = \frac{C}{4G_N} = \frac{R^{(4)}_{\mathrm{symm}}}{4G_N}.$$



Hyperbolic manifold dressed with warp and conformal factor variations

Tune small to compete with Casimir with $\ell_{11} \ll R_c \ll \ell$

$$\left[-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 < 0 \;\middle|\; -R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 > 0 \right]$$

warp & conformal factor eoms $\Rightarrow$

$$-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 = 4\ell_{11}^9 |\rho_C| - \frac{C}{u} - \frac{5}{2}F_7^2$$

Douglas Kallosh '10

$$a = \frac{\int \sqrt{g^{(7)}} u^2|_c[-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2\Big|_c]}{\int \sqrt{g^{(7)}} u^2|_c\, 42/\ell^2} << 1 \;:$$

$$-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 = 4\ell_{11}^9|\rho_C| - \frac{C}{u} - \frac{5}{2}F_7^2$$

Balance Terms in U => $\hat{\ell}^4 \sim \frac{1}{\hat{\ell}_c^5}\frac{n_c}{v_7}\frac{Vol(T^6)}{\lambda_c^6} \cdot \frac{1}{a}$

If a sufficiently small, then all length scales large:

$$\ell \gg \ell_c \gg \ell_{11}$$

- If a is too large, increase volume of non-Casimir regions (e.g. via short filled cusps or covers k-fold -> (k+1)-fold)
- If a is too small, reduce flux quantum number

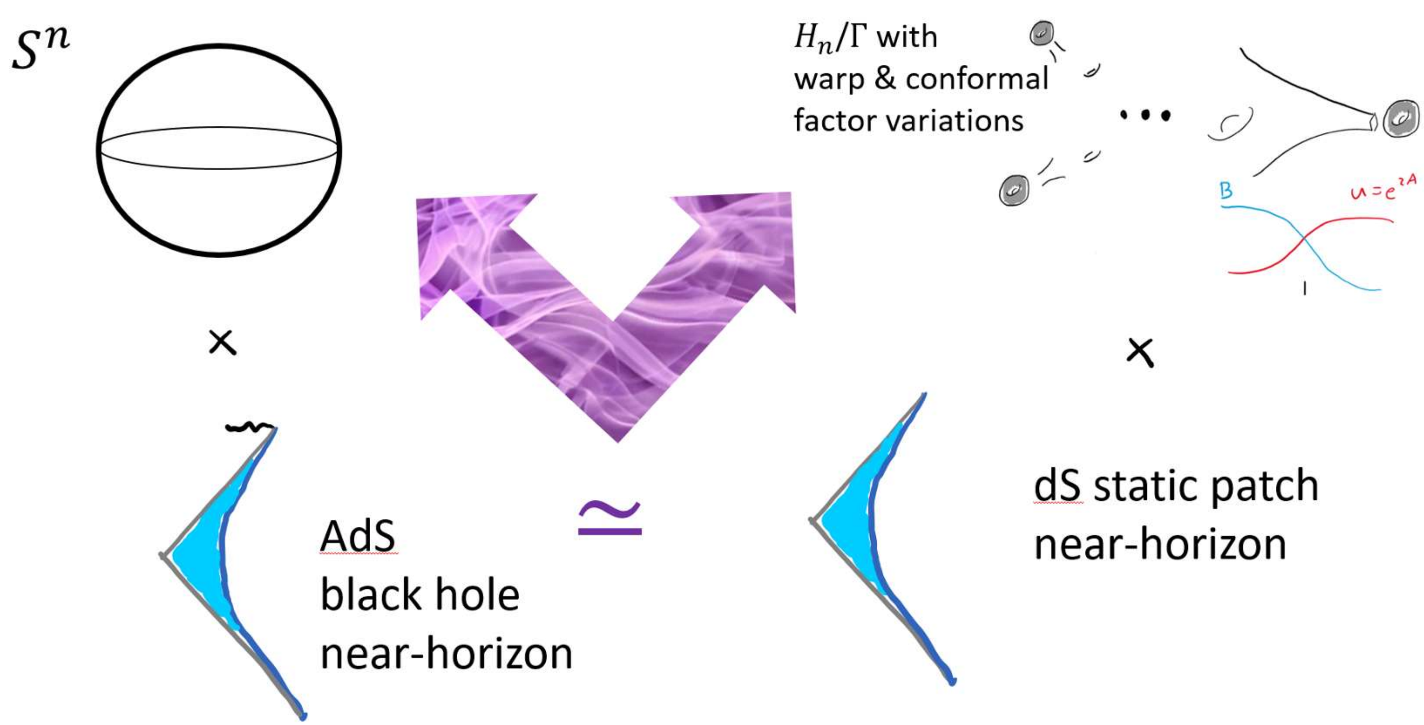
Work with simple concrete hyperbolic manifolds with comparable cusp and bulk volumes Italiano et al ‘20. Explicit radial solution illustrates $a << 1$.

Parametric suppression of residual shifts needed in other directions.

...Coming back to our problem:

The matching point corresponds to a high-temperature boundary (canonically)
=> fluctuates among all internal configurations consistent with the horizon,

So also can’t tell the difference between dS and AdS/BH **internally**!

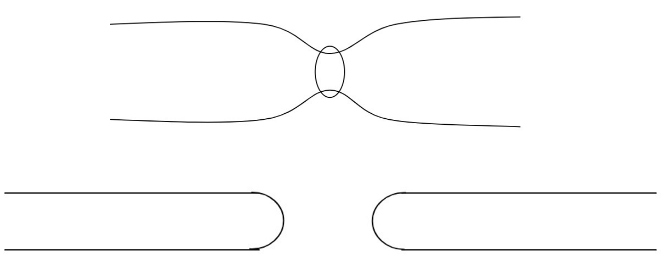


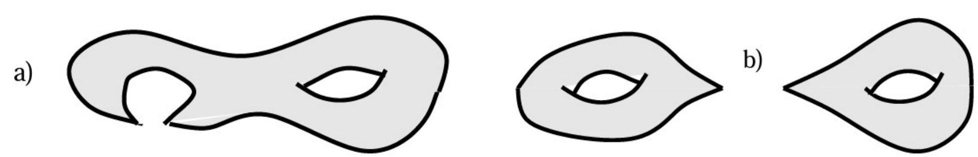
This assumes:

(1) The requisite (topology, D, …)-changing processes are possible

Many precedents:

Conifold transitions, change of Riemann surface genus, chirality-changes, dimension-changes via condensation of wrapped branes/strings:





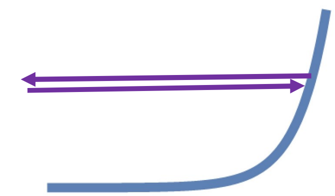
(2) The timelike boundaries exist in M/string theory

(2) Do the timelike boundaries exist in M/string theory?

Let's take the approach of generalizing Liouville walls:

$$ds^2 = e^{2A(y)}\left\{d\chi^2 + f_1(\chi)(-dt_{||}^2 + f_2(\chi)(d\vec{x}_{||})^2)\right\} + g_{mn}(y)dy^m dy^n$$

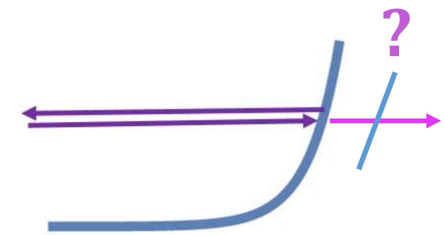
$$S_{ws} = S_{ws}^{(0)} + \int_\Sigma [\hat{O}_\Delta \Phi]_r \sim \int d^2\sigma \hat{O} e^{\kappa\chi} \qquad \chi \gg 1/\kappa$$



$\Delta > 2$

Deform the semiclassical worldsheet action by marginal, i.e. dimension (1,1), massive vertex operator. Then check that the worldsheet path integral has no support at $\chi \to \infty$. Here this is nontrivial, depending on $\hat{O}$.

Want to check that worldsheet cannot ooze out to ∞



Flat target:

$$ds^2 = -dX^{0\,2} + \sum_{i=1}^{D-1} dX^{i\,2}$$

$$S_{ws} = S_{ws}^{(0)} - \lambda_W \int_\Sigma [\{(\partial_- X_\parallel)^2 (\partial_+ X_\parallel)^2\}^2 e^{\kappa X_{D-1}}]_r$$

Marginal => $\kappa = \sqrt{\frac{2\Delta - 4}{\alpha'}} = \frac{2\sqrt{3}}{\sqrt{\alpha'}}$

from $S_{ws}^0 - \frac{1}{4}\lambda_W \int d^2\sigma \sqrt{-h}\{(J_\parallel^2)_{\alpha\beta}(J_\parallel^2)_{\gamma\delta}(h^{\alpha\gamma}h^{\beta\delta} - \frac{1}{2}h^{\alpha\beta}h^{\gamma\delta})\}^2 e^{\kappa X^{D-1}}$

We show: $\hat{O}_8 = [(\partial_- X_\parallel)^2 (\partial_+ X_\parallel)^2]^2 \not\to 0 \quad as \quad X^{D-1} \to \infty$

We see that there is indeed no transmission, as follows: $(\partial_+ X)_\parallel^2 \neq 0$, it can't vanish: it's related by the worldsheet constraints $\delta_h S_{ws} = 0$ to $(\partial_+ X)_{\{D-1\}}^2$. The latter can't vanish for a string propagating to $X_{\{D-1\}} \to \infty$.

Similarly, vacuum NS-NS $AdS_3 \times S^3 \times X_4$

Maldacena/Ooguri, Kutasov Seiberg et al, …

$$ds^2 = d\rho^2 - \cosh^2\frac{\rho}{\ell}dt^2 + \sinh^2\frac{\rho}{\ell}d\phi^2 + internal$$

$$\Delta S_{ws} = -\lambda_W \int d^2\sigma (\hat{O}_R + T_{++,internal})^2 (\hat{O}_L + T_{--,internal})^2 \Phi_{m^2}(\rho, t, \phi)$$

$u, v = \frac{1}{2}(t \pm \phi)$ parallel to boundary, Φ_{m^2} solution to massive wave equation

$$\hat{O}_R = -(J_R^3)^2 = -k^2(\partial_+ u + \cosh(2\rho)\partial_+ v)^2$$

Again here the string cannot ooze out to infinity for any path integral configuration, implying a wall.

Moreover, there is a net Brown-York energy and string charge (NS-NS flux), suggesting consistent with an effective Dirichlet condition at the wall.

The internal $S^3 \times X_4$ is constant radially, consistent with the possibility of a fluctuating boundary condition for them (required for the melting at the matching point).

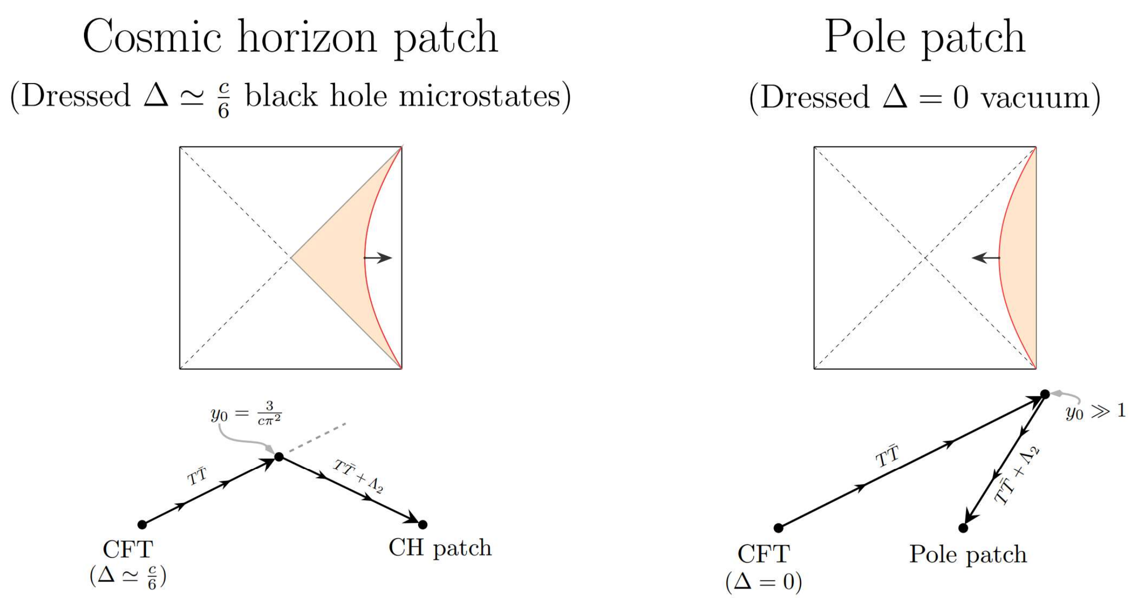
Our problem of interest requires generalization to:

- AdS black holes: fewer symmetry currents J
- dS: Fischler-Susskind worldsheet
- M theory (11d) case: M(atrix) theory interpolation between 11d and 10d M(atrix) string theory, e.g. in flat space

$$H = H_0 + Tr[\{(D_- X_\parallel - \frac{1}{4}\psi_\parallel \kappa \psi^D - 1)^2 (D_- X_\parallel - \frac{1}{4}\psi_\parallel \kappa \psi^D - 1)^2\}^2 e^{\kappa X^{10}}]$$

In general, the open question of finite (non-asymptotically AdS) timelike boundaries in general spacetimes is key for holography.

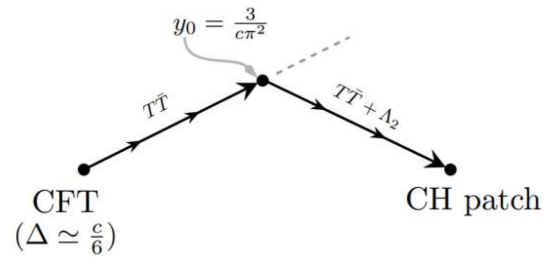
cf Ahmadain/Wall and question of boundary terms from the worldsheet



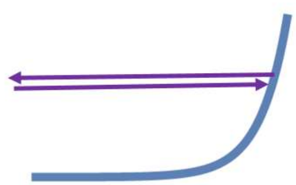
$$\mathcal{E} = \frac{1}{\pi y}\left(1 + \sqrt{\eta + \ldots}\right) \quad \longleftarrow \text{ related by } \pm\sqrt{\ } \longrightarrow \quad \mathcal{E} = \frac{1}{\pi y}\left(1 - \sqrt{\eta + \ldots}\right)$$

Summary:

- Solvable deformations capture the geometry and microstate count of the dS static patch, via integrability of the deformation. Extends to local bulk matter including $T\bar{T} + \frac{J\bar{J}}{\lambda} + \frac{1-\eta}{\lambda^2}$
- Raised the question of how this could embed in string/M theory, given the enormous difference between the internal spaces for AdS and dS. This is answered automatically by the fact that the matching between the $\eta = \pm 1$ trajectories ($y = y_0$) occurs at the horizon, where internal thermal averaging is compulsory.

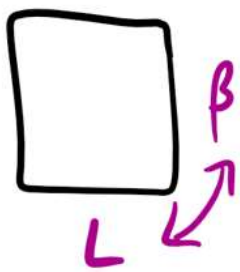


- Timelike boundaries in string/M theory may arise from generalizing Liouville walls to (super-)critical D with $\Delta S_{ws} \sim \int \hat{O} \exp(\kappa X_{radial})$



Extra Slides

In the canonical ensemble (fixed `tOmperature' ~ $1/\beta$ and L: Euclidean torus boundary), our system exhibits an intriguing remnant of modular invariance



A 2d theory on a torus is invariant under $\beta \leftrightarrow L$ $Z_\lambda(L,\beta) = Z_\lambda(\beta, L)$

But our theory, without the complex levels, is a 1d (quantum mechanics) theory, unitary but not fully local. Nonetheless, we find a remnant of modular invariance:

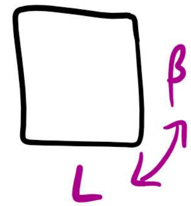
Seed CFT for c >> 1: $\log Z \simeq \max\{\overbrace{-\beta E_{vac}(L)}^{\beta > L}\ ,\ \overbrace{-L E_{vac}(\beta)}^{\beta < L}\}$ Hartman Keller Stoica et al

Deformation (β<L) : $\log Z|_{\beta<L} \simeq -L E_{vac}(\beta) = S_{Cardy}(\Delta = c/6) - \beta E_{\Delta = c/6}(L)$

The deformed $\Delta \simeq c/6$ levels propagate in the direct channel.

Shyam '21: this modular transformation starting from the pole patch spectrum yields S_{GH}

Relation/analogy to Hawking - Page :

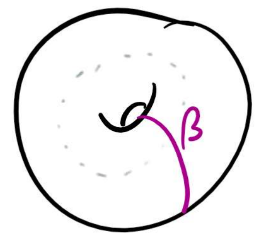


AdS :

$T < T_{HP}$

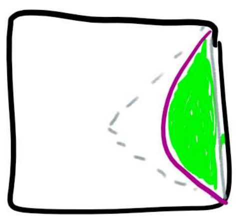
$\hat{L} \to 0$ in interior

$T > T_{HP}$

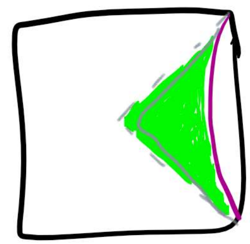


$\check{\beta} \to 0$ $(\hat{T} \to \infty)$ in interior

dS :

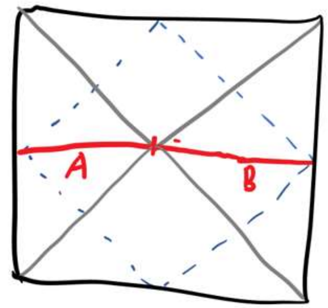


$\hat{L} \to 0$ in interior



$\check{\beta} \to 0$ $(\hat{T} \to \infty)$ in interior

Next: Maximal Mixing review



Black: global dS
Blue: dS/dS patch
Grey: static patches

It is interesting to consider global dS as a purification of the static patch. There is a path integral saddle corresponding to the VN and Renyi entropies for a division into two halves A and B (sensible at least near large c, semiclassically on the gravity side). This results in maximal mixing, confirmed by several independent calculations. DST, LLST. dS/dS gives a natural physical interpretation of this division so we'll also review it.

*General theory (Lewkowyzc/Maldacena '13, Dong '16): $Tr\ \rho_A^n$: replicate the spacetime according to the desired division. With gravity, smooth out the surfeit angles. Orbifold by Z_n, introducing a cosmic brane C_n which back reacts (including gravity where dynamical). The area of the cosmic brane in the saddle gives the Renyi entropy:

$$-n^2\partial_n\left(\frac{1}{n}\log \mathrm{Tr}\,\rho_1^n\right) = \tilde{S}_n = \frac{A(C_n)}{4G_{d+1}}$$

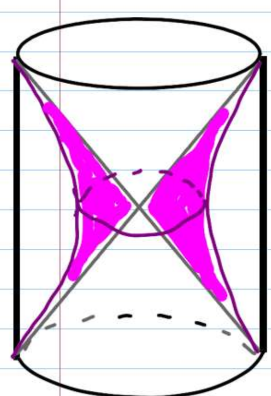
*Donnelly/Shyam '18, LLST: Entropies (up to shift) from dressed stress-energy.

$$L\frac{d}{dL}\log Z_n = -\int d^2x\,\sqrt{g}\,\langle \mathrm{tr}\,T\rangle\ ,\ L\frac{d}{dL}\tilde{S}_n = -(1-n\partial_n)\int d^2x\,\sqrt{g}\,\langle \mathrm{tr}\,T\rangle\ .$$

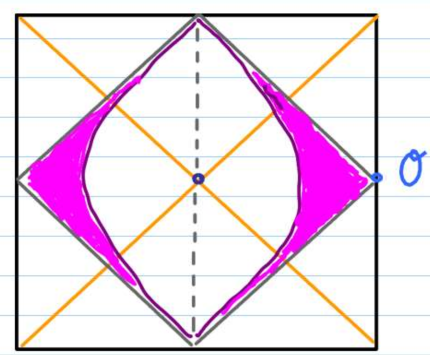
dS/dS:

Alishahiha et al ‘04, …, Dong ES Torroba ‘18, …
Gorbenko ES Torroba ’18, Shyam ‘21

$$
\begin{aligned}
ds^2_{(A)dS_{d+1}} &= dw^2 + \sin(h)^2\left(\frac{w}{\ell_{dS}}\right) ds^2_{dS_d} \\
&= dw^2 + \sin(h)^2\left(\frac{w}{\ell_{dS}}\right)\left[-d\tau^2 + \ell_{dS}^2 \cosh^2 \frac{\tau}{\ell_{dS}} d\Omega^2_{d-1}\right].
\end{aligned}
$$



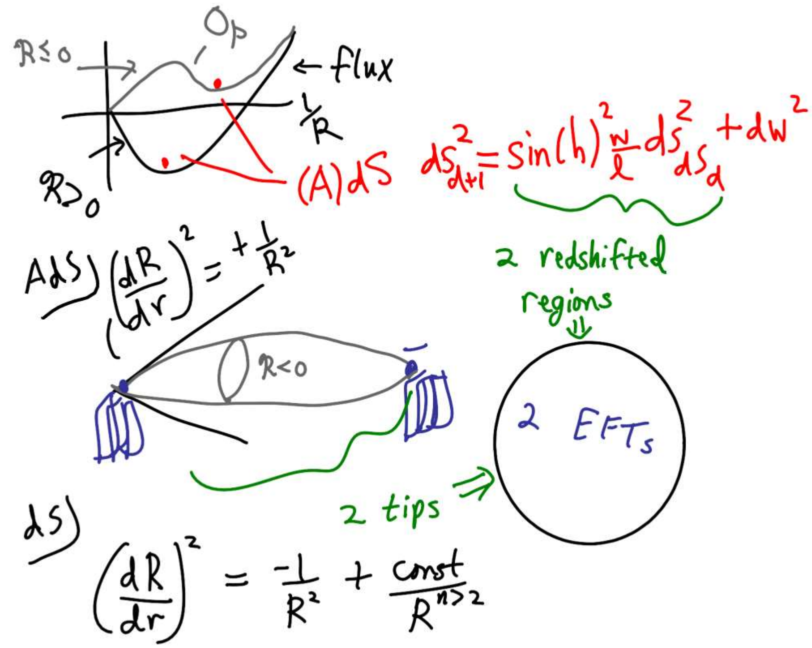
AdS/dS



dS/dS (each point is (d-1)-sphere)

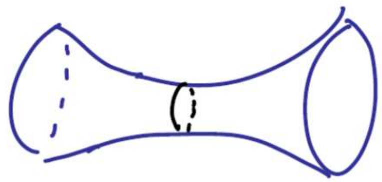
Uplifting AdS/CFT => 2 sectors

Dong Horn ES Torroba ‘10



dS vs AdS brane construction:
independent derivation of the two sectors because of metastability.
Also true in dS/CFT

General idea: entanglement and other properties of the quantum state are tied to the knitting together of spacetime.

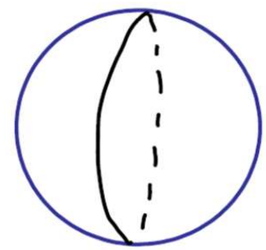


Thermal state of two CFTs (entangled at thermal scale) dual to joined spacetime

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2}|n\rangle|n\rangle\,.$$

``ER=EPR”
Van Raamsdonk, Maldacena/Susskind

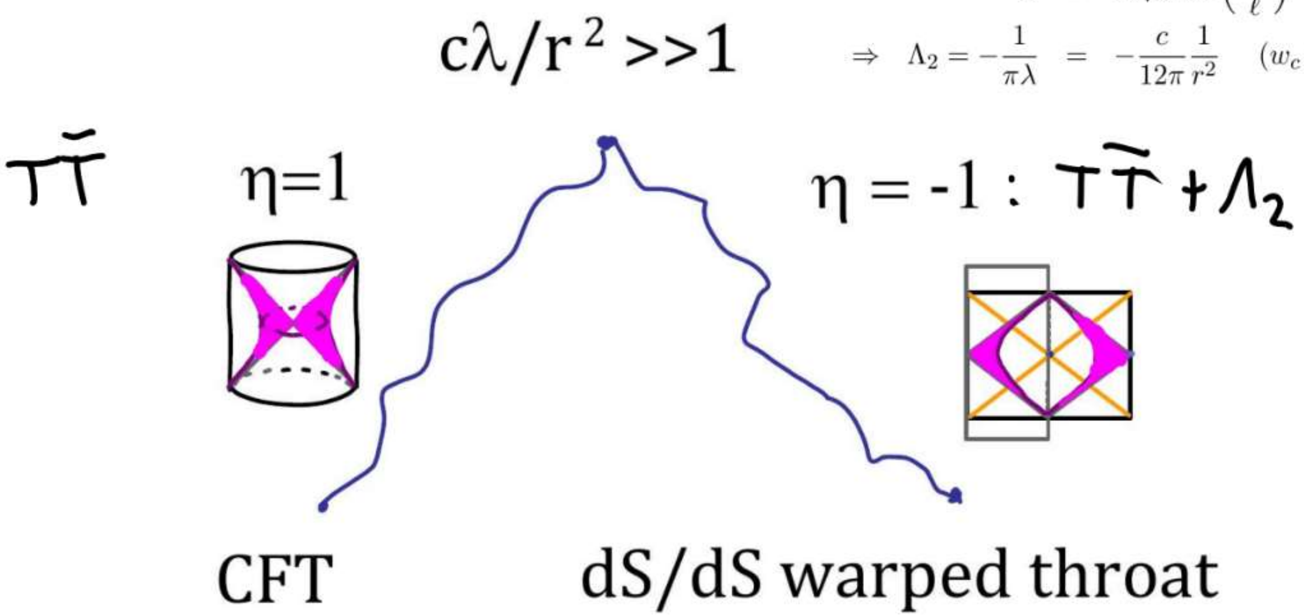
dS case: Entangled state of 2 deformed CFTs, dominated at the most UV scale ($\Delta \simeq c/6$). Strong interactions between them suggested that this state could be highly mixed.



Dong ES Torroba ’18, Gorbenko ES Torroba ’19, Shyam ‘21, …

dS/dS warped throat from the
$T\bar{T} + \Lambda_2$ deformation (GST)

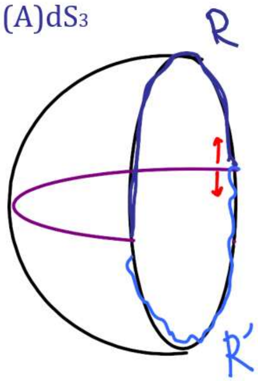
$$\begin{aligned} c &= \frac{3\ell}{2G} \\ \lambda &= 8G\ell \\ r &= \ell \sin\left(\frac{w_c}{\ell}\right) \\ L &= 2\pi\mu\ell \sin\left(\frac{w_c}{\ell}\right) \\ \Rightarrow \Lambda_2 = -\frac{1}{\pi\lambda} &= -\frac{c}{12\pi}\frac{1}{r^2} \quad (w_c = \ell\pi/2) \end{aligned}$$



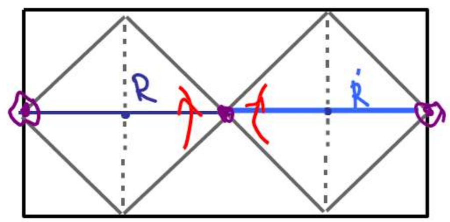
We can analyze the mixing in 2-3 ways: (i) split each throat, then join to obtain full gravity as the last step (a) $T\bar{T} + \Lambda$ analysis (b) gravity path integral or (ii) divide full neck (gravity present from start). Flat spectrum, all.

Rotated calculation: Consider first ½ of a dS/dS warped throat: ¼ of the dS neck

spatial slice of bulk (A)dS_3



Boundary dS_2

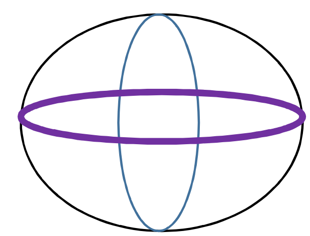


($T\bar{T}$ method: Calculation of Renyis via dressed stress energy => max mixing: LLST)

$$S_0(r) = S_1(r) = \frac{\pi c}{6}$$

Here we have a frozen boundary (D wall), so can calculate as as in Lewkowyzc/Maldacena Dong. Replicate Euclidean throat (w/boundary $EdS_2 = S^2$), getting surfeit angles which smooth out in bulk. Orbifold, giving a cosmic brane which sources deficit angles in bulk.

Now join two such throats and integrate over the shared metric to recover the global dS neck (incorporating the dynamical gravity of the joined system). The integral over the shared metric yields a simple saddle containing the fully back reacted **cosmic brane**. The back reaction is the orbifold-induced deficit angle, leaving the area of the cosmic brane independent of n.   (Contrast AdS case of fixed area states: there the area integral => n-dependence Dong Harlow Marolf, Akers Rath)



$$|z_1|^2 + |z_2|^2 = \ell_{dS}^2$$ $EdS_{d+1=3}$

$$(z_1, z_2) \simeq (e^{2\pi i/n} z_1, z_2)\,.$$

Cosmic brane = fixed locus

$$z_1 = 0, \qquad |z_2|^2 = \ell_{dS}^2\,.$$

=> $S_n = S_{VN}$ at large c

Original calculation DST:

Divide the global dS neck in 2 (to calculate ρ_A). Replicate, smooth out surfeits (everywhere, since gravity is dynamical). This gives the original EdS_3. Orbifold => same cosmic brane (just rotated by dS symmetry).

$$|z_1|^2 + |z_2|^2 = \ell_{dS}^2$$

EdS$_{d+1=3}$

$$(z_1, z_2) \simeq (e^{2\pi i/n} z_1, z_2)\,.$$

Cosmic brane = fixed locus

$$z_1 = 0, \qquad |z_2|^2 = \ell_{dS}^2\,.$$

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