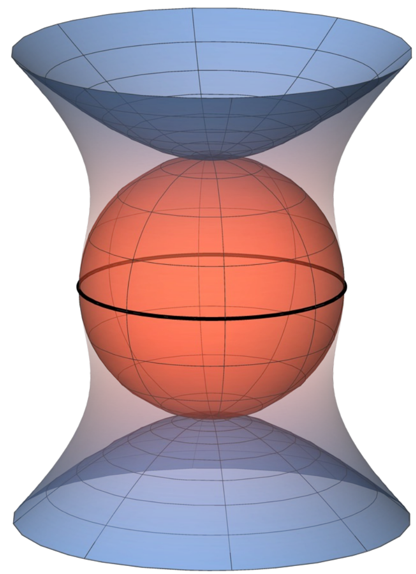
Microscopics of de Sitter entropy from precision holography



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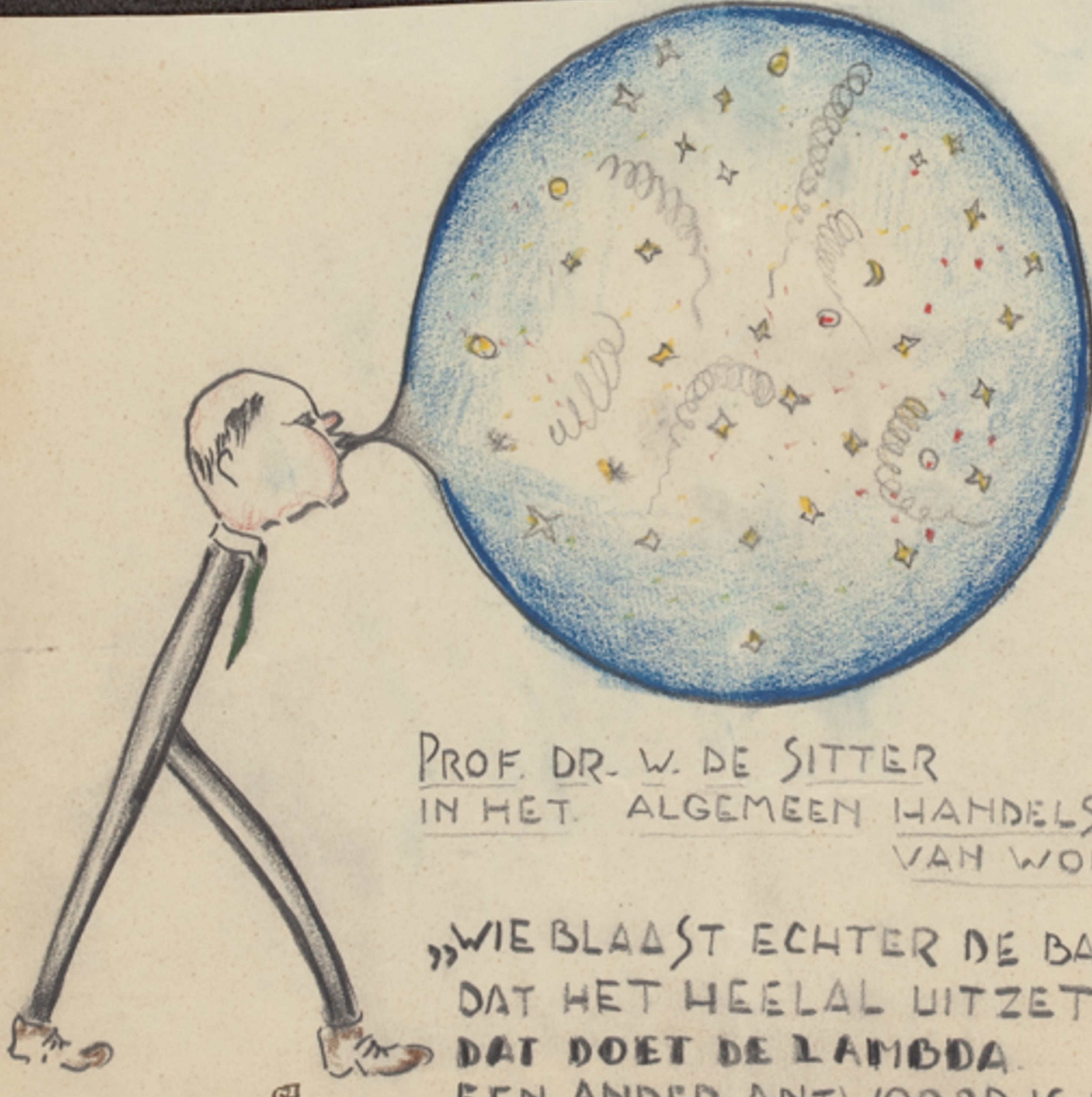


PROF. DR. W. DE SITTER
IN HET ALGEMEEN HANDELSBLAD
VAN WOENSDAG 9 JULI 1930

„WIE BLAAST ECHTER DE BAL OP? WAT MAAKT
DAT HET HEELAL UITZET, OF OPZWELT?
DAT DOET DE LAMBDA.
EEN ANDER ANTWOORD IS NIET TE GEVEN"

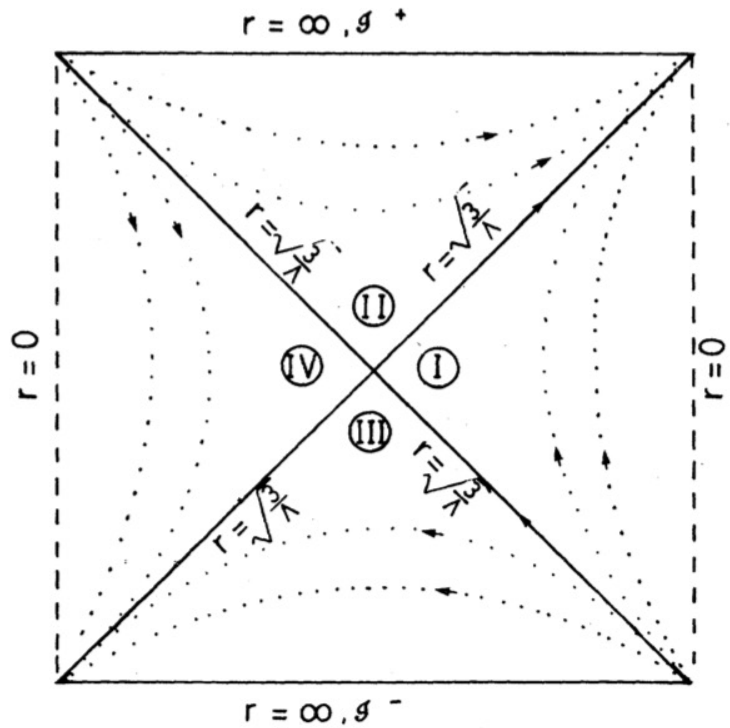
Intro

[Gibbons Hawking '77]

- An observer is surrounded by a horizon
- .... with a temperature and an entropy

$$S_{\mathrm{dS}} = \frac{A_H}{4G_N}$$

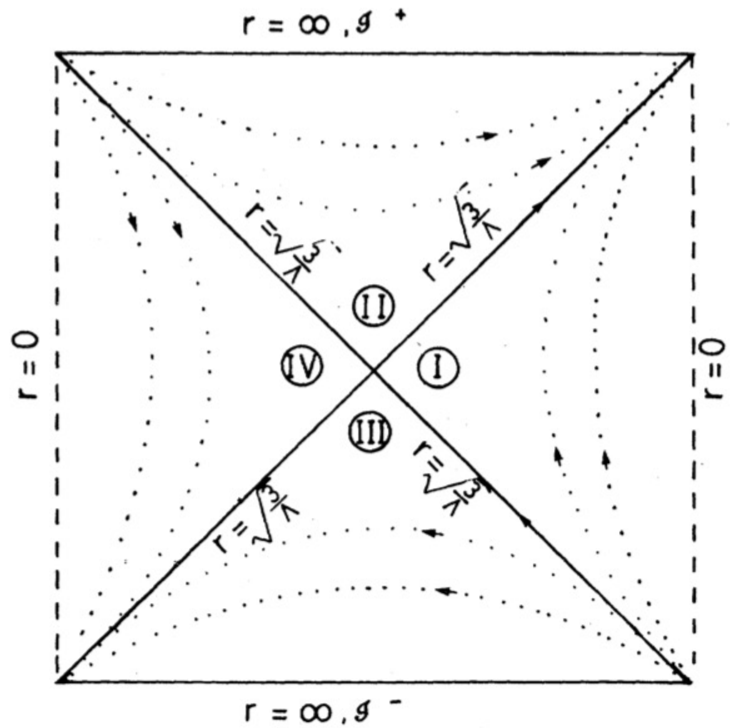
- What does S_{dS} count?



THIS TALK: 1) A concrete proposal for the microscopic d.o.f behind S_{dS}
2) A test of it by computing quantum corrections

Intro

- No spacelike boundary
- … thermodynamics tricky
- Two approaches:
  - On-shell action of S^4
    [Gibbons Hawking '77]
  - ``Wald entropy"
    [Wald '93; Iyer, Wald '94]



$$ds^2 = \left[-dt^2 + L^2 \cosh^2(t/L)\, d\Omega_3^2\,\right]$$

Wald entropy

[Wald '93; Iyer, Wald '94]

$$\mathcal{S}_H = -2\pi \int_H d^2x \sqrt{\gamma}\, \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}}\, \epsilon_{\mu\nu}\epsilon_{\rho\sigma} \qquad \epsilon_{\mu\nu}\epsilon^{\mu\nu} = -2$$

Einstein gravity:

$$\mathcal{L} = \frac{1}{16\pi G_N} \left(R_{\mu\nu\rho\sigma} g^{\mu\rho} g^{\nu\sigma} - 2\Lambda \right)$$

Wald applied to de Sitter horizon in Einstein gravity yields:

$$\mathcal{S}_{\mathrm{dS}} = \frac{A_H}{4G_N} = \frac{\pi L^2}{G_N}$$

Note: used to compute higher-derivative corrections to entropy of black holes in AdS [Bobev, Charles, Hristov, Reys '21]

Euclidean on-shell action

[Gibbons Hawking '77]

$$S_{dS} = -I_{EdS}$$

Euclidean Einstein gravity:

$$S^E = \int d^4x \sqrt{g} \Big[-\frac{1}{16\pi G_N} \Big(R - \frac{6}{L^2} \Big) \Big]$$

Euclidean dS_4:

$$ds^2 = \big[d\tau^2 + L^2 \cos^2(\tau/L)\, d\Omega_3^2 \big], \qquad \tau \in \left(\frac{\pi L}{2}, \frac{\pi L}{2} \right)$$

On-shell action:

$$I_{\rm EdS} = -\frac{\pi L^2}{G_N} = -\frac{A_H}{4G_N}$$

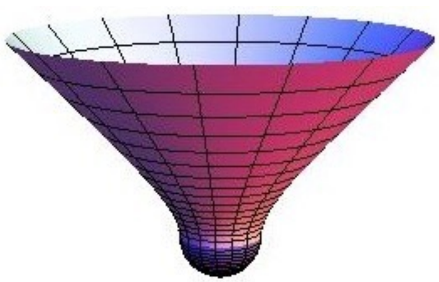
Euclidean on-shell action II

$$ds^2 = \left[-dt^2 + L^2 \cosh^2(t/L)\, d\Omega_3^2\right]$$

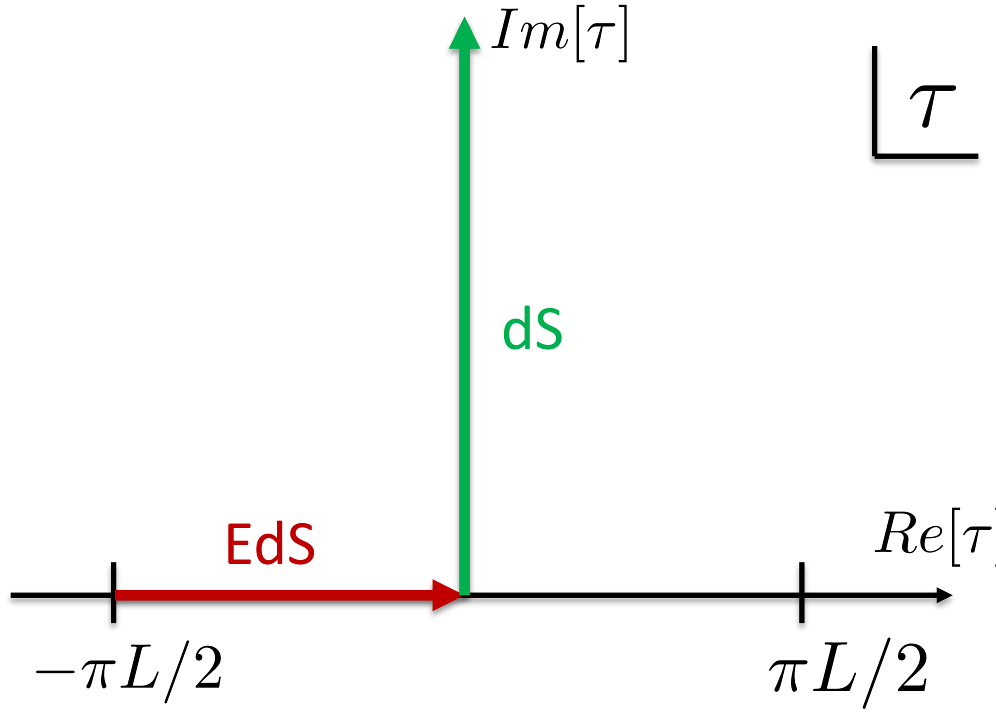
$$t \longrightarrow i\tau$$

$$ds^2 = \left[d\tau^2 + L^2 \cos^2(\tau/L)\, d\Omega_3^2\right]$$

[Hartle Hawking '83]



$$\Psi \sim e^{-I_{EdS}/2} e^{iI_{dS}}$$



Euclidean on-shell action II

$$ds^2 = \left[-dt^2 + L^2 \cosh^2(t/L)\, d\Omega_3^2\right]$$

$$ds^2 = \left[d\tau^2 + L^2 \cos^2(\tau/L)\, d\Omega_3^2\right]$$

$t \longrightarrow i\tau$

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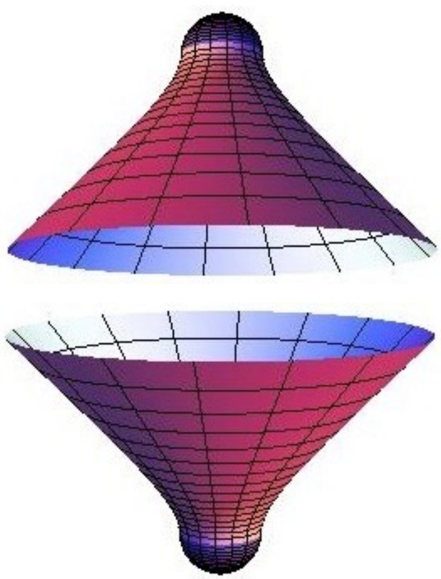
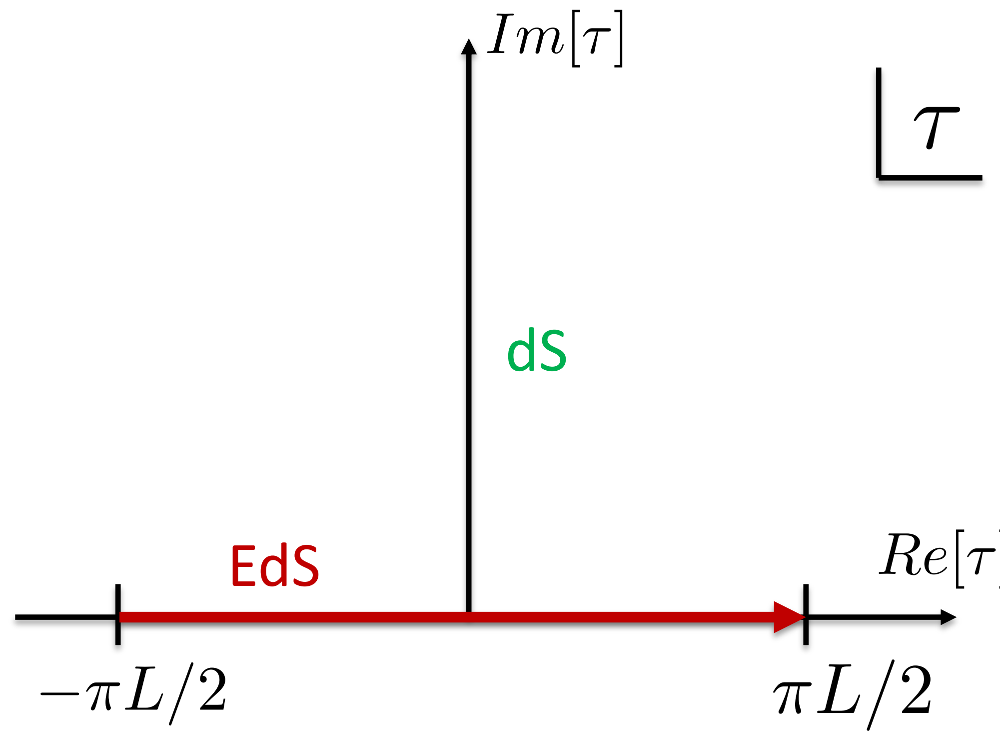


$$\Psi^* \Psi \sim e^{-I_{EdS}}$$



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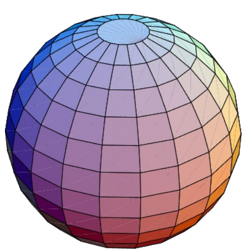
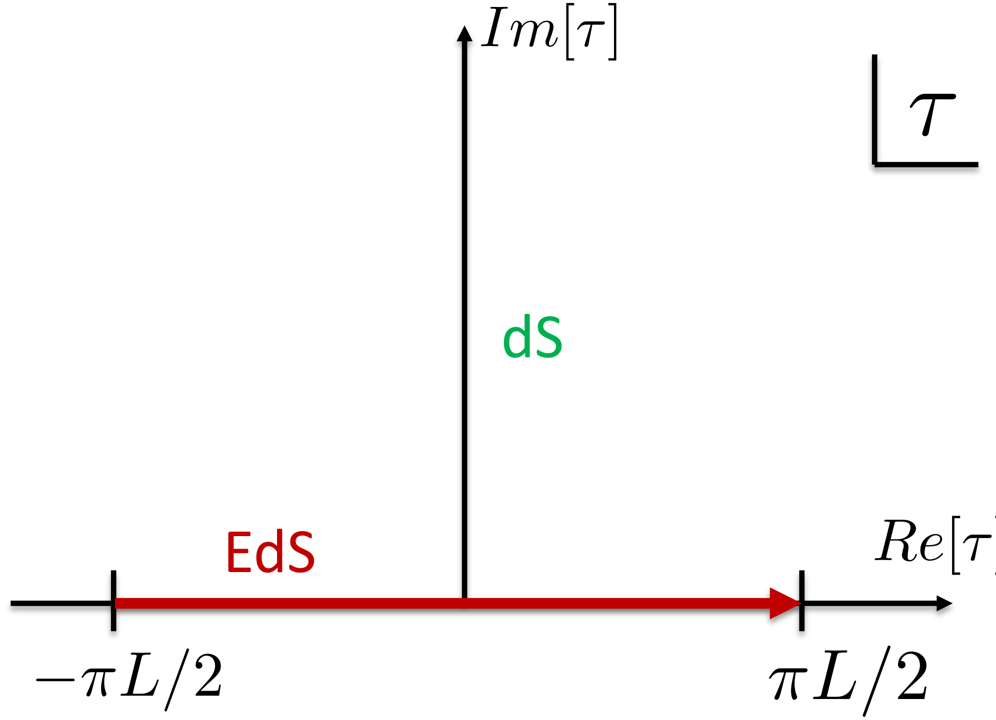


$$\Psi^*\Psi \sim e^{-I_{EdS}}$$

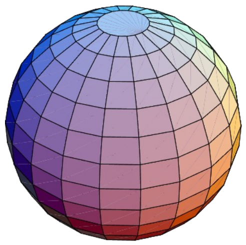


Euclidean on-shell action II

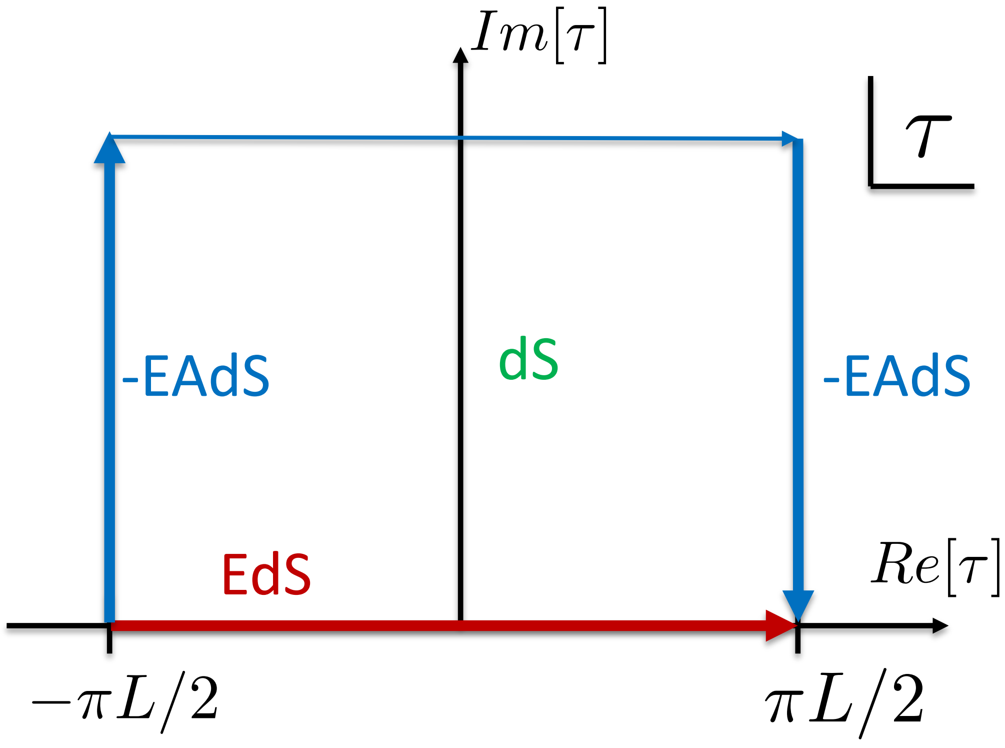
$$ds^2 = \left[d\tau^2 + L^2\cos^2(\tau/L)\, d\Omega_3^2\right]$$

$$\tau = -\pi L/2 + ir$$

$$ds^2 = \left[-dr^2 - L^2\sinh^2(r/L)\, d\Omega_3^2\right]$$



$$\Psi^*\Psi \sim e^{-I_{EdS}}$$



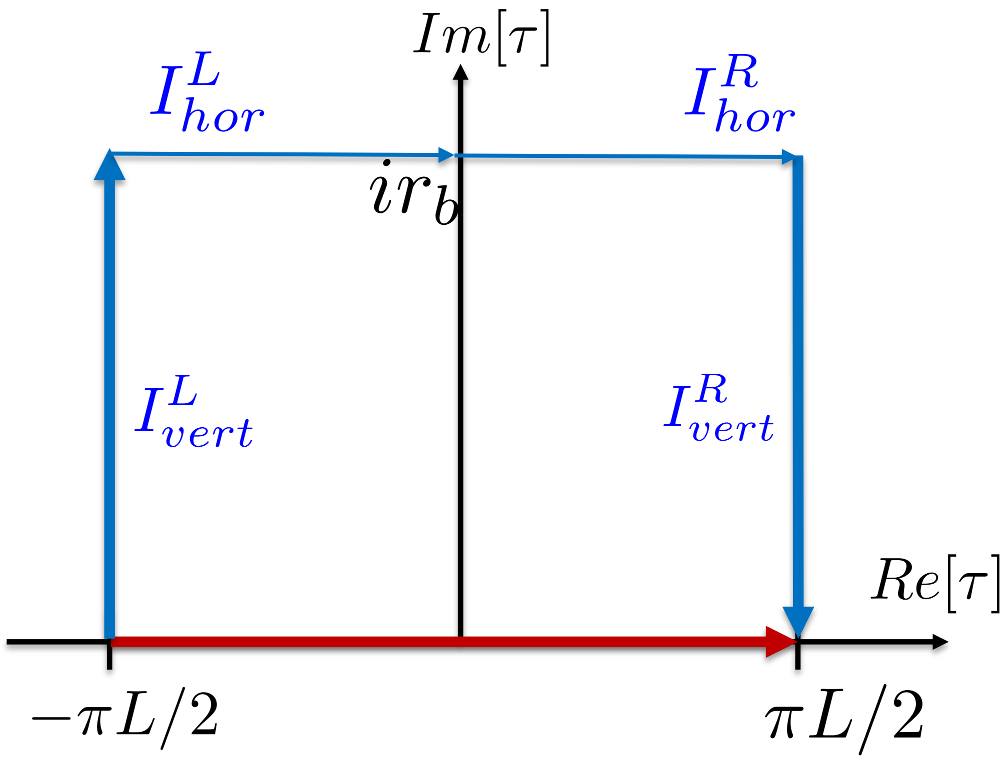
Euclidean on-shell action II

$$I^L_{vert} = -I^{\text{reg}}_{\text{EAdS}} - I_{ct} + \mathcal{O}(e^{-r_b/L})$$

$$I^L_{hor} = +I_{ct} - iI_{ct} + \mathcal{O}(e^{-r_b/L})$$

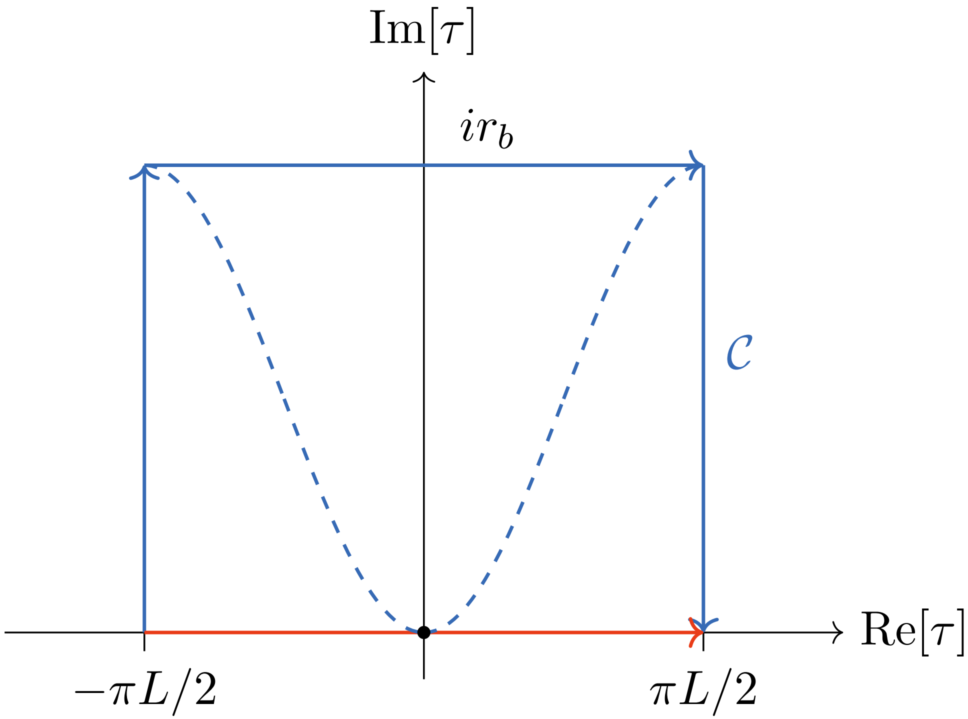
$$I^R = (I^L)^*$$

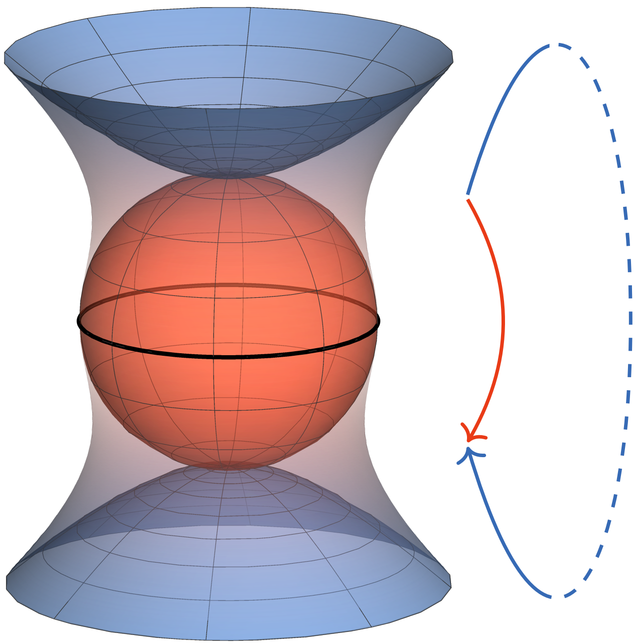
$$I_{EdS} = -2\, I^{\text{reg}}_{\text{EAdS}}$$



de Sitter entropy: microscopics

$$S_{dS} = -I_{EdS} = 2I^{\rm reg}_{\rm EAdS}$$





de Sitter entropy: microscopics

$$S_{dS} = -I_{EdS} = 2I^{\mathrm{reg}}_{\mathrm{EAdS}}$$

- AdS/CFT: $$-I^{\mathrm{reg}}_{\mathrm{EAdS}} + \cdots = \log Z^{\mathrm{CFT}}_{S^3}$$
- Conjecture: $$\boxed{S_{dS} = -2\log Z^{\mathrm{CFT}}_{S^3}}$$
- Microscopic theories: ABJM, …

de Sitter entropy: microscopics

- Consider M-theory on $EAdS_4 \times S^7/\mathbb{Z}_k$
- EAdS/CFT: dual ABJM theory on S^3
- Susy localization: [Marino, Putrov '11; Fuji, Hirano, Moriyama '11]

$$Z_{S^3}^{\mathrm{ABJM}}(N,k) = \left(\frac{2}{\pi^2 k}\right)^{-1/3} e^{\mathcal{A}(k)}\,\mathrm{Ai}\left[\left(\frac{2}{\pi^2 k}\right)^{-1/3}\left(N - \frac{k}{24} - \frac{1}{3k}\right)\right] + \mathcal{O}(e^{-\sqrt{Nk}})$$

- Proposal:

$$S_{dS} = -2\log Z_{S^3}^{\mathrm{ABJM}}(N.k) = \frac{2\pi\sqrt{2k}}{3} N^{3/2} + \mathcal{O}(N^{1/2})$$

- Holographic dictionary:

$$(2\pi\ell_{\mathrm{P}})^6 N = 6\,(2L)^6\,\mathrm{vol}(X_7)\,, \qquad \frac{1}{G_N} = \frac{16\pi(2L)^7\mathrm{vol}(X_7)}{(2\pi)^8\ell_P^9}$$

- Leading term $\mathcal{S}_{\mathrm{dS}} = \dfrac{\pi L^2}{G_N}$

de Sitter entropy: microscopics

$$S_{dS} \overset{?}{=} -I_{EdS} \overset{?}{=} 2I^{\text{reg}}_{\text{EAdS}} \overset{\text{v}}{=} -2\log Z^{\text{ABJM}}_{S^3}$$

- Leading term matches

- **Q:** What about corrections?

THIS TALK: 1) A concrete proposal for the microscopic d.o.f behind S_{dS}
2) A test of it by computing quantum corrections

- What could go wrong?

- Check subleading terms
  → consider higher-derivative terms in gravity
  [Bobev, Charles, Hristov, Reys ‘21]

Four-derivative corrections: de Sitter entropy

- $$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G_N}\Big(R - \frac{6}{L^2}\Big) + (c_1 - c_2)\, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$$
$$+c_2\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right) + c_3 R^2\Big]\,.$$

- Wald entropy

$$\mathcal{S}_{\rm dS} = \frac{\pi L^2}{G_N} + 64\pi^2(c_2 + 6c_3)$$

- Euclidean on-shell action:

$$I_{\rm EdS} = -\frac{\pi L^2}{G_N} - 64\pi^2(c_2 + 6c_3)$$

$$S_{dS} \overset{v}{=} -I_{EdS} \overset{?}{=} 2I^{\rm reg}_{\rm EAdS} \overset{v}{=} -2\log Z^{\rm ABJM}_{S^3}$$

Four-derivative corrections: Holographic Renormalization

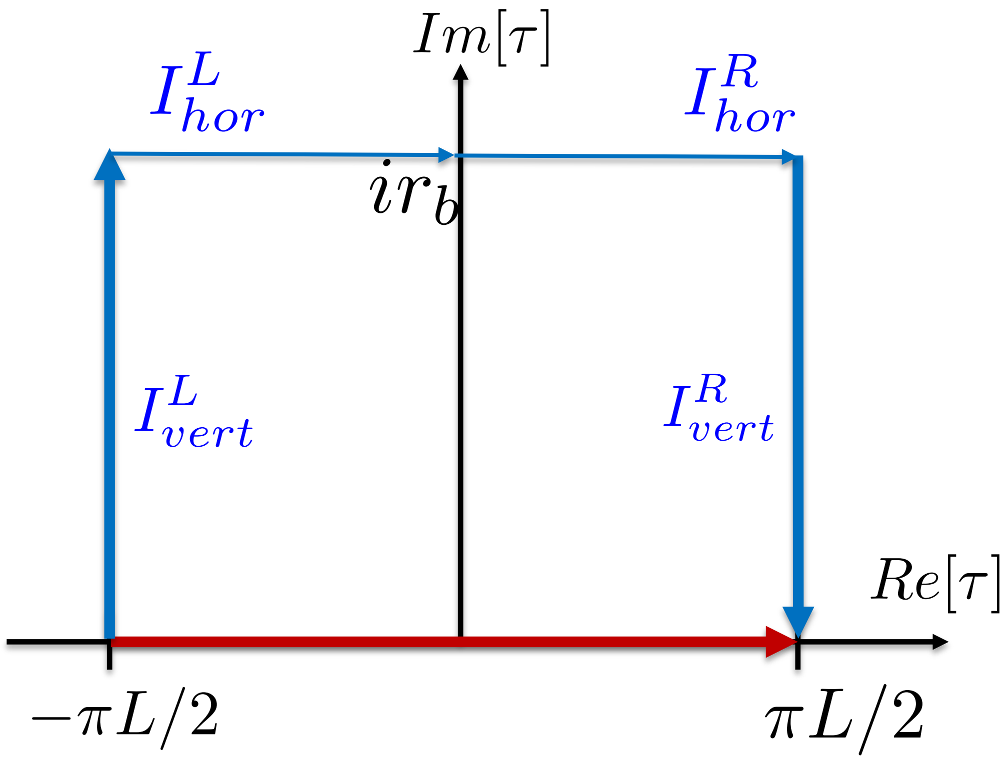
$$I^L_{vert} = -I^{\text{reg}}_{\text{EAdS}} - I_{ct} + \mathcal{O}(e^{-r_b/L})$$

$$I^L_{hor} = +I_{ct} - iI_{ct} + \mathcal{O}(e^{-r_b/L})$$

$$I^R = (I^L)^*$$

$$I_{EdS} = -2\, I^{\text{reg}}_{\text{EAdS}}$$

Extends to corrections!



Four-derivative corrections: Holographic Renormalization

$$S_E = \int d^4x\sqrt{g}\Big[-\frac{1}{16\pi G_N}\Big(R-\frac{6}{L^2}\Big)-(c_1-c_2)\mathcal{L}_{W^2}-c_2\mathcal{L}_{GB}\Big]$$

Counterterms:

$$S_{ct} = \int d^3x\sqrt{h}\Big[\frac{1}{8\pi G_N}\Big(-K+\frac{2}{L}+\frac{L}{2}\mathcal{R}\Big)+(c_1-c_2)\mathcal{L}^{\text{CT}}_{C^2}+4c_2\big(\mathcal{J}-2\mathcal{G}_{ab}K^{ab}\big)\Big]$$

with

$$\mathcal{J}_{ab} = \frac{1}{3}\Big(2KK_{ac}K^{c}{}_{b}+K_{ab}K_{cd}K^{cd}-2K_{ac}K^{cd}K_{db}-K^2K_{ab}\Big)$$

$$S_{dS} \overset{\text{v}}{=} -I_{EdS} \overset{\text{v}}{=} 2I^{\text{reg}}_{\text{EAdS}} \overset{\text{v}}{=} -2\log Z^{\text{ABJM}}_{S^3}$$

de Sitter entropy: more microscopics

- Susy localization ABJM: [Marino, Putrov '11; Fuji, Hirano, Moriyama '11]

$$Z_{S^3}^{\mathrm{ABJM}}(N,k) = \left(\frac{2}{\pi^2 k}\right)^{-1/3} e^{\mathcal{A}(k)}\, \mathrm{Ai}\left[\left(\frac{2}{\pi^2 k}\right)^{-1/3}\left(N - \frac{k}{24} - \frac{1}{3k}\right)\right] + \mathcal{O}(e^{-\sqrt{Nk}})$$

- Expansion in *N*:

$$F_{S^3}^{\mathrm{ABJM}}(N,k) = \frac{\pi\sqrt{2k}}{3}\, N^{3/2} - \frac{\pi(k^2+8)}{24\sqrt{2k}}\, N^{1/2} + \frac{1}{4}\log N + \mathcal{O}(N^0)$$

- De Sitter entropy: $\mathcal{S}_{\mathrm{dS}} = \dfrac{\pi L^2}{G_N} + 64\pi^2(c_2 + 6c_3)$

- Holographic dictionary (subleading order): [Bobev, Charles, Hristov, Reys '21]

$$\frac{L^2}{G_N} + 64\pi\, c_2 = \frac{2\sqrt{2k}}{3}\, N^{3/2} - \frac{k^2+8}{12\sqrt{2k}}\, N^{1/2}, \qquad c_3 = 0$$

de Sitter entropy: more microscopics

$$\mathcal{S}_{\mathrm{dS}} = -2\log Z_{S^3}^{\mathrm{ABJM}}$$

$$\mathcal{S}_{\mathrm{dS}} = \frac{2\pi\sqrt{2k}}{3}N^{3/2} - \frac{\pi(k^2+8)}{12\sqrt{2k}}N^{1/2} \boxed{+\frac{1}{2}\log N} + \mathcal{O}(N^0)$$

Dual predicts $\Delta\mathcal{S}_{\mathrm{dS}} = \frac{1}{2}\log N = \frac{1}{3}\log\frac{L^2}{G_N} + \cdots = 3\log\frac{L}{l_P} + \cdots$

Quid bulk?

No log from the holographic dictionary: [Bobev, Hong, Reys '22]

$$\frac{L^2}{G_N} = \frac{2\sqrt{2k}}{3}\left(N - \frac{k}{24}\right)^{3/2}, \qquad c_2 = -\frac{1}{96\pi\sqrt{2k}}\left(N - \frac{k}{24}\right)^{1/2}$$

`11d quantm supergravity'

[Bhattacharyya, Grassi, Marino, Sen '12]

- Consider 11d Euclidean SUGRA on $-S^4 \times S^7/\mathbb{Z}_k$
- One-loop determinants generate log corrections to the free energy
- Odd dimensions: only zero modes contribute
- Massless 11d fields: metric, gravitino and three-form
- Ghosts are important!
- Metric and gravitino have no zero mode because S^4 is compact.
- Logarithmic correction due to a p-form:

$$\Delta F = \sum_j (-1)^j \left(\beta_{p-j} - j - 1\right) n^0_{\Delta_{p-j}} \log L/l_P , \qquad \beta_k = \frac{D-2k}{2}$$

- → $\Delta S_{dS} = 3 \log L/l_P$ $\mathcal{S}_{\rm dS} \overset{\checkmark}{=} -2 \log Z^{\rm ABJM}_{S^3}$

Summary: Much Ado About Nothing

de Sitter microscopics

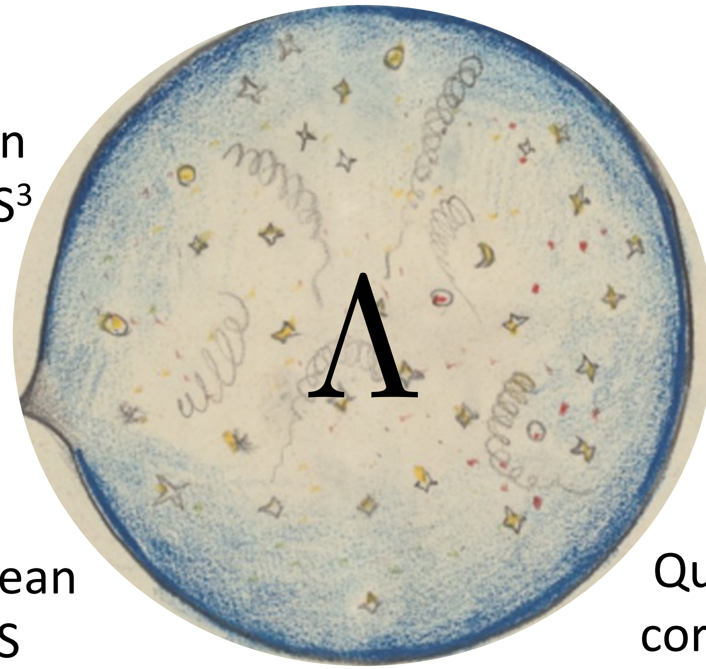
de Sitter Entropy $\mathbf{S_{ds}}$: What does it count?

Quantum corrections

S^4

Euclidean AdS

Euclidean dual on S^3



$$S_{dS} = -I_{\mathrm{EdS}} = 2I^{\mathrm{reg}}_{\mathrm{EAdS}} = -2\log Z^{\mathrm{ABJM}}_{S^3}(N,k)$$

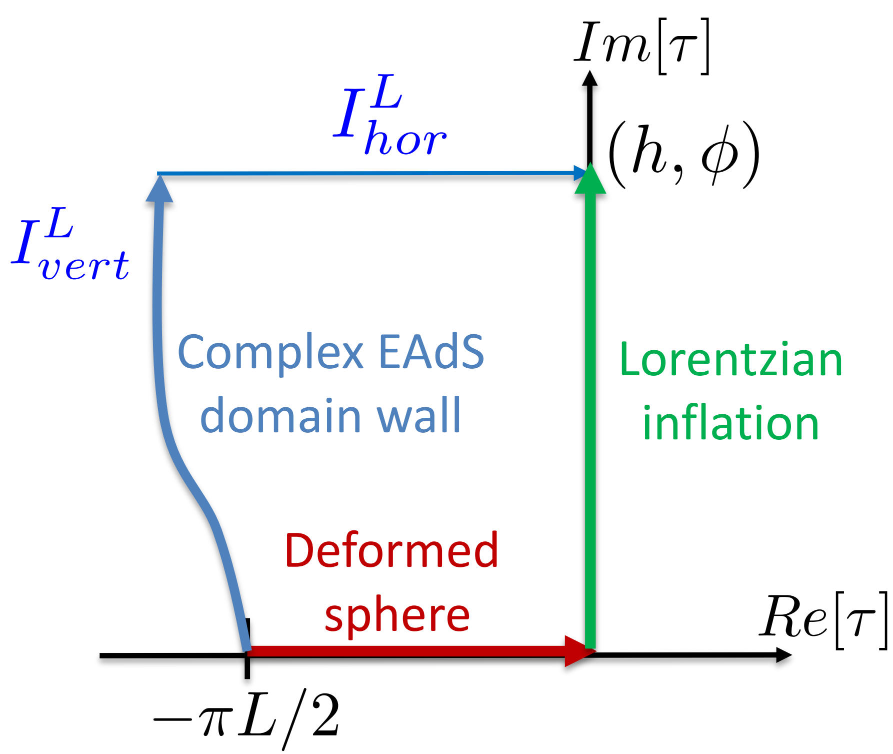
So.. what does S_{dS} count?

- The microscopic de Sitter entropy in our proposal does not quite count microstates, since there is no time, and hence no Hamiltonian.
- In effect, exp (S_{dS}) is not an integer for low (N,k)
- But, being given by a path integral, the entropy does represent some sort of measure of degrees of freedom.
- What about more tangible cosmological observables?
  → add sources

Complex saddles with sources

[Hartle, TH '11]

$$\Psi_{HH} = \mathcal{A}_{sp} e^{iS} \Psi_{fl} \qquad \log \mathcal{A}_{sp} = I^{\text{reg}}_{as\text{EAdS}}$$



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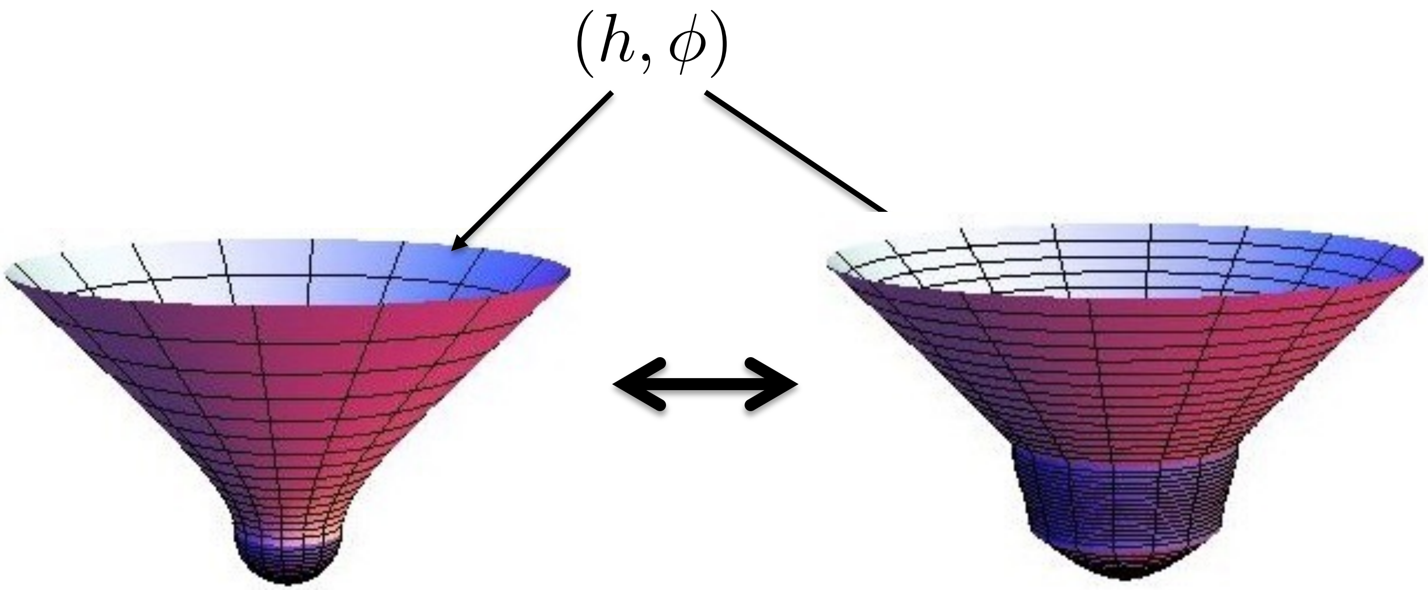
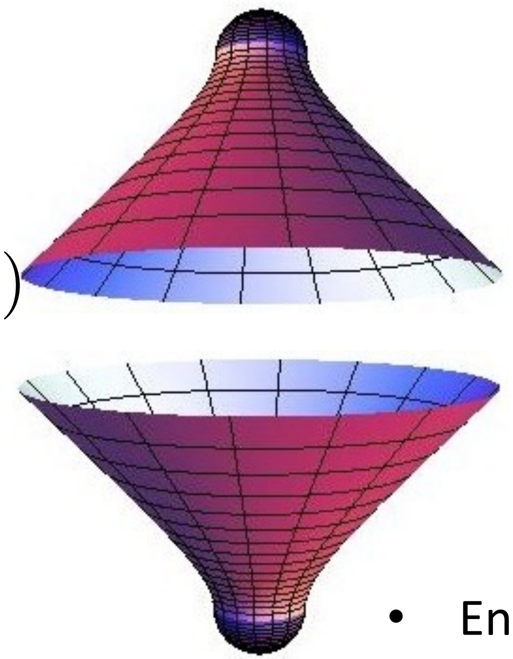


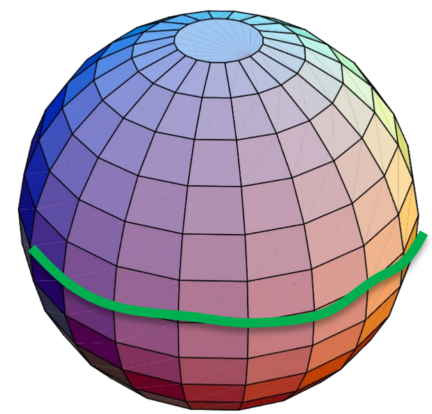
Quantum cosmology on the (deformed) sphere?

Observables ~ $\int \Psi^* \mathcal{O} \Psi$

(h, ϕ)



?



$(\tilde{h}, \tilde{\phi})$

- Entropy of inflationary universes
- Integrated 2-pt functions $< h_{ij}(x) h_{i'j'}(x') >$
  [McFadden, Skenderis; Pimentel;..]
- …

Dual example: mass-deformed ABJM on squashed S^3

[Bobev, Hong, Reys; Nosaka; Hatsuda; Hristov,...]

$$Z_{S^3}(N,k,\Delta) = e^{\mathcal{A}(k,\Delta)} C_k^{-\frac{1}{3}} \mathrm{Ai}[C_k^{-\frac{1}{3}}(N-B_k)] + \mathcal{O}(e^{-\sqrt{N}})$$

with

$$C_k = \frac{2}{\pi^2 k} \frac{(b+b^{-1})^{-4}}{\prod_{a=1}^4 \Delta_a}, \qquad B_k = \frac{k}{24} - \frac{1}{12k} \sum_{a=1}^{4} \frac{1}{\Delta_a} + \frac{1 - \frac{1}{4}\sum_a \Delta_a^2}{3k(b+b^{-1})^2 \prod_{a=1}^4 \Delta_a},$$

and

$$\Delta_1 = \frac{1}{2} - \mathrm{i}\,\frac{m_1+m_2+m_3}{b+b^{-1}}, \; \Delta_2 = \frac{1}{2} - \mathrm{i}\,\frac{m_1-m_2-m_3}{b+b^{-1}},$$
$$\Delta_3 = \frac{1}{2} + \mathrm{i}\,\frac{m_1+m_2-m_3}{b+b^{-1}}, \; \Delta_4 = \frac{1}{2} + \mathrm{i}\,\frac{m_1-m_2+m_3}{b+b^{-1}},$$

such that $\sum_a \Delta_a = 2$.

Dual example: mass-deformed ABJM on squashed S^3

[Freedman, Pufu]

$$S_{\text{bulk}} = \int \mathrm{d}^4x\, \sqrt{g} \left[-\frac{1}{2} R + \sum_{\alpha=1}^{3} \frac{\partial_\mu z^\alpha \partial^\mu \tilde{z}^\alpha}{(1 - z^\alpha \tilde{z}^\alpha)^2} + \frac{1}{L^2} \left(3 - \sum_{\alpha=1}^{3} \frac{2}{1 - z^\alpha \tilde{z}^\alpha} \right) \right]$$

$$V_{eff}(\phi) = \frac{1}{L^2} \cosh(2\phi)$$

Possibly:

- $Z_{S^3}(N, k, \Delta) \rightarrow \Psi_{HH}(h, \phi) \rightarrow \text{entropy}$
- $\partial^2 F_{S^3} \rightarrow \langle \zeta(x) \zeta(x') \rangle$