Is SMEFT Enough?

based on 2008.08597 with T. Cohen, N. Craig and X. Lu

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Why study EFTs?

1) Because they parameterise all measurable effects

EFT: Given some field content $(\psi, \bar{\psi})$ and some symmetry assumptions (Poincaré, $\psi \to e^{i\alpha}\psi$, $\bar{\psi} \to \bar{\psi}e^{-i\alpha}$), write down all invariant local operators

$$\mathcal{L} = \ldots + c_1\,(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) + \ldots + c_2\,(\bar{\psi}\gamma^\mu\psi)\Box(\bar{\psi}\gamma_\mu\psi) + \ldots$$

At the **amplitude** level, a *basis* of EFT operators spans all possible contact interactions among the known states

$$\mathcal{A}\left(\psi(1)\bar{\psi}(2)\psi(3)\bar{\psi}(4)\right) = \bar{v}(p_2)\gamma^\mu u(p_1)\bar{v}(p_4)\gamma_\mu u(p_3)\left(c_1 + c_2 s + \ldots\right) + \text{perms}$$

which can be joined together by light propagators to make the **most general perturbative amplitude consistent with locality**.

Why study EFTs?

2) Because they encode the low energy vestiges of all heavy new physics

Top down: *Matching* a UV theory with a heavy vector onto the EFT by Taylor expanding amplitudes in $\frac{1}{M}$

$$\begin{aligned}
&\mathcal{A}\left(\psi(1)\bar{\psi}(2)\psi(3)\bar{\psi}(4)\right) \\
&= \bar{v}(p_2)\gamma^{\mu}u(p_1)\frac{-e^2\,g_{\mu\nu}}{s-M^2}\bar{v}(p_4)\gamma^{\nu}u(p_3) + \text{perms} \\
&= \bar{v}(p_2)\gamma^{\mu}u(p_1)\bar{v}(p_4)\gamma_{\mu}u(p_3)\left(-\frac{e^2}{M^2}-\frac{e^2}{M^4}s+\ldots\right) + \text{perms} \\
&\overset{!}{=} \bar{v}(p_2)\gamma^{\mu}u(p_1)\bar{v}(p_4)\gamma_{\mu}u(p_3)\left(c_1+c_2 s+\ldots\right) + \text{perms}
\end{aligned}$$

Bottom up: All EFT operators lead to unitarity violation at some energy E scale

$$\begin{aligned}
&\mathcal{A}\left(\psi(1)\bar{\psi}(2)\psi(3)\bar{\psi}(4)\right) \\
&= \bar{v}(p_2)\gamma^{\mu}u(p_1)\bar{v}(p_4)\gamma_{\mu}u(p_3)\left(c_1+c_2 s+\ldots\right) + \text{perms} \\
&\sim \left(c_1 E^2 + c_2 E^4 + \ldots\right)
\end{aligned}$$

Study the (simplified) EFT of the SM scalar sector

We observe four scalar degrees of freedom in high energy collisions: the Higgs boson and the three longitudinal components of the W^+, W^- and Z. What field theory should we use to parameterise their interactions?

On first principles, what is the difference between scalar sectors of:

- **SMEFT**: built about the electroweak preserving vacuum, out of fields $\vec{\phi}$ that linearly realise electroweak symmetry, and
- **HEFT**: built about our low energy vacuum, out of fields h, $\vec{\pi}$ that don't?

Experimentally, this is slightly moot

Experimentally, the operators of SMEFT and HEFT are just different bases for parametrising the Lorentz structures in the non-factorisable pieces of amplitudes.

This means that any finite set of data (in the absence of light new particles) can be fit by either a subset of SMEFT operators or a subset of HEFT operators.

It is rather a question of the sizes of various operators' coefficients with respect to phenomenologically motivated UV completions. I.e. **is the subset of SMEFT operators at dimension 6 enough to describe any deviations we may measure in experiment?** These latter imply correlations, e.g.,

$$|H|^6 = \frac{1}{8}(v_0 + h)^6 \supset \frac{5}{2}v_0^3 h^3 + \frac{15}{8}v_0^2 h^4$$

This talk

Use the simplifications of

- *ignoring fermions & vectors;*
- *(global) custodial symmetry;*
- *no lagrangian terms with more than two derivatives.*

We draw **heavily** on recent work in the literature

- (Alonso, Jenkins, and Manohar 2016)
- (Chang and Luty 2019)
- (Falkowski and Rattazzi 2019)
- (Abu-Ajamieh, Chang, Chen, and Luty 2020)

to identify features of the field-space manifold that are unique to HEFT, and present perturbative UV completions that generate these features.

I remark on more practical (finite order) issues of convergence at the end.

SMEFT (Standard Model Effective Field Theory)

see also (Alonso, Jenkins, and Manohar 2016) for details

Comprising four equivalent real scalars

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}, \qquad \vec{\phi} \to O\vec{\phi}, \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{pmatrix}$$

where $O \in O(4) \supset SU(2) \times U(1)$. Electroweak symmetry is *linearly realised* on the $\vec{\phi}$.

Then the terms in the Lagrangian are

$$\begin{aligned}
\mathcal{L}_{\mathrm{SM}} &= \frac{1}{2}(\partial\vec{\phi} \cdot \partial\vec{\phi}) - \frac{1}{4}\lambda(\vec{\phi} \cdot \vec{\phi} - v^2)^2 \\
\mathcal{L}_{\mathrm{SMEFT}} &= \frac{1}{2}\tilde{A}(\vec{\phi} \cdot \vec{\phi})(\partial\vec{\phi} \cdot \partial\vec{\phi}) + \frac{1}{2}\tilde{B}\left(\vec{\phi} \cdot \vec{\phi}\right)(\vec{\phi} \cdot \partial\vec{\phi})^2 - \tilde{V}\left(\vec{\phi} \cdot \vec{\phi}\right) + \mathcal{O}\left(\partial^4\right) , \\
&\to \frac{1}{2}\partial\vec{\phi} \cdot \partial\vec{\phi} + \frac{1}{2}B\left(\vec{\phi} \cdot \vec{\phi}\right)(\vec{\phi} \cdot \partial\vec{\phi})^2 - V\left(\vec{\phi} \cdot \vec{\phi}\right) + \mathcal{O}\left(\partial^4\right) ,
\end{aligned}$$

HEFT (Higgs Effective Field Theory)

see also (Alonso, Jenkins, and Manohar 2016) for details

Built from a real h and a unit vector $\vec{n}$ comprising 3 Goldstones π^i

$$h\,,\qquad \vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_1^2 - n_2^2 - n_3^2} \end{pmatrix},$$

upon which the electroweak symmetry is *non-linearly realised*

$$h \to h \quad , \quad \vec{n} \to O\vec{n}\,, O \in O(4)\,.$$

The lagrangian is

$$\mathcal{L}_{\rm SM} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(v+h)^2(\partial\vec{n})^2 - \frac{1}{4}\lambda(h^2 + 2vh)^2$$

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}\left[\tilde{K}(h)\right]^2(\partial h)^2 + \frac{1}{2}\left[v\tilde{F}(h)\right]^2(\partial\vec{n})^2 - \tilde{V}(h) + \mathcal{O}\left(\partial^4\right)$$

$$\to \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h) + \mathcal{O}\left(\partial^4\right)\,.$$

[Canonically $F(0) = 1,\ V'(0) = 0$]

Do SMEFT and HEFT describe the same physics?

In other words, can we write SMEFT as HEFT, and HEFT as SMEFT?

$$\mathcal{L}_{\text{SMEFT}} = \frac{1}{2}A(\vec{\phi}\cdot\vec{\phi})(\partial\vec{\phi}\cdot\partial\vec{\phi}) + \frac{1}{2}B\left(\vec{\phi}\cdot\vec{\phi}\right)(\vec{\phi}\cdot\partial\vec{\phi})^2 - V\left(\vec{\phi}\cdot\vec{\phi}\right)$$

Writing

$$\vec{\phi} = (v_0 + h)\vec{n}(\pi)$$

always transforms a SMEFT into HEFT.

$$\mathcal{L}_{\text{SMEFT}\to\text{HEFT}} = \frac{1}{2}\left[A + (v_0+h)^2 B\right](\partial h)^2 + \frac{1}{2}\left[(v_0+h)^2 A\right](\partial\vec{n})^2 - V$$

with A, B and V even functions of $v_0 + h$.

HEFT $\to$ SMEFT?

Consider

$$\mathcal{L} = \frac{1}{2}\left(1 + \frac{h_A}{2v_A}\right)^2 (\partial h_A)^2 + \frac{1}{2}(v_A + h_A)^2 \left(\frac{3}{4} + \frac{h_A}{4v_A}\right)^2 (\partial \vec{n})^2 - V$$

With the Higgs redefinition

$$\begin{aligned} h_A &= 2v_A \sum_{n=1}^{\infty} \frac{\left(\frac{1}{2}\right)_n}{n!} \left(\frac{h_B}{v_A}\right)^n \\ &= h_B - \frac{1}{4}\frac{h_B^2}{v_A} + \dots \end{aligned}$$

(and $v_B = \frac{3}{4}v_A$) is actually the Standard Model

$$\mathcal{L} = \frac{1}{2}(\partial h_B)^2 + \frac{1}{2}(v_B + h_B)^2 (\partial \vec{n})^2 - V = |\partial H|^2 - V\,.$$

HEFT $\to$ SMEFT?

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}[K(h)]^2(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial \vec{n})^2 - V(h)$$

Writing

$$(v_0 + h) = \sqrt{\vec{\phi}\cdot\vec{\phi}}; \qquad \vec{n} = \frac{\vec{\phi}}{\sqrt{\vec{\phi}\cdot\vec{\phi}}}$$

might transform a HEFT into a SMEFT?

$$\mathcal{L}_{\text{HEFT}\to\text{SMEFT}} = \frac{1}{2}\frac{v^2F^2}{\vec{\phi}\cdot\vec{\phi}}(\partial\vec{\phi}\cdot\partial\vec{\phi}) + \frac{1}{2}\left(\frac{K^2}{\vec{\phi}\cdot\vec{\phi}} - \frac{v^2F^2}{(\vec{\phi}\cdot\vec{\phi})^2}\right)(\vec{\phi}\cdot\partial\vec{\phi})^2 - V$$

Field redefinitions are coordinate transf. in field space

(Coleman, Wess, and Zumino 1969)

Field redefinitions that leave the S-matrix unchanged are smooth and invertible[1]

$$\phi = \phi(\eta) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\mathrm{d}^n \phi}{\mathrm{d}\eta^n} \eta^n = c_0 + c_1 \eta + c_2 \eta^2 + c_3 \eta^3 + \dots$$

...because any field interpolating single particle states will do to extract amplitudes from correlators

$$\prod_i \lim_{p_i^2 \to m^2, p^0 > 0} (p_i^2 - m^2) \prod_j \lim_{q_j^2 \to m^2, q^0 > 0} (q_j^2 - m^2)$$

$$\times \prod_j \int \mathrm{d}^4 y_j e^{i q_j \cdot y_j} \prod_i \int \mathrm{d}^4 x_i e^{-i p_i \cdot x_i} \langle 0 | T\{\phi(y_1) \dots \phi(x_1) \dots\} | 0 \rangle$$

$$= \prod_i |\langle 0 | \phi | p_i \rangle| \prod_j |\langle 0 | \phi | q_i \rangle| \langle q_1 \dots | p_1 \dots \rangle \propto \prod_i |\langle 0 | \eta | p_i \rangle| \prod_j |\langle 0 | \eta | q_i \rangle| \langle q_1 \dots | p_1 \dots \rangle$$

Restrict to $\phi(\eta)$ real analytic, so each field choice corresponds to a choice of coordinates on a real analytic manifold. *Can probably relax this assumption.*

[1] **Note:** No derivatives as working to fixed derivative order in $\mathcal{L}$

Geometric picture of scalar effective field theory

On a real analytic manifold, lagrangian defines a metric and a potential function

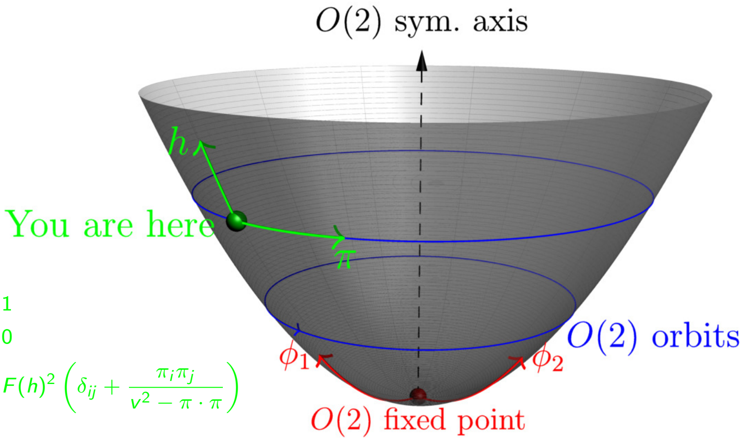
$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\partial_\mu \phi^i \partial^\mu \phi^j - V(\phi) = \frac{1}{2}\left(g_{ij}(\phi(\eta)) \frac{\partial \phi^i}{\partial \eta^m} \frac{\partial \phi^j}{\partial \eta^n} \right) \partial_\mu \eta^m \partial^\mu \eta^n - V(\phi(\eta))$$

We want the lagrangian to be analytic, so we can expand in a series of local EFT operators

$$\mathcal{L} = \frac{1}{2}\left(\sum \frac{1}{n!} g_{ij,k_1\ldots k_n}(0) \phi^{k_1} \ldots \phi^{k_n} \right) \partial_\mu \phi^i \partial^\mu \phi^j - \left(\sum \frac{1}{n!} V_{,k_1\ldots k_n}(0) \phi^{k_1} \ldots \phi^{k_n} \right)$$

Geometric picture of SMEFT & HEFT

Cartesian vs. polar coordinates, see (Alonso, Jenkins, and Manohar 2016)



$$g_{hh} = 1$$

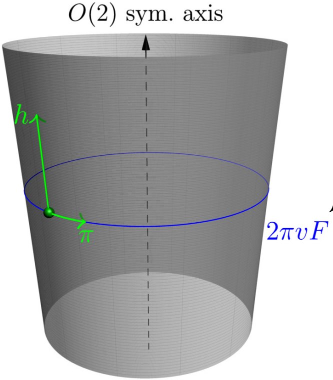
$$g_{h\pi_i} = 0$$

$$g_{\pi_i\pi_j} = F(h)^2 \left(\delta_{ij} + \frac{\pi_i\pi_j}{v^2 - \pi\cdot\pi}\right)$$

$$g_{\phi_i\phi_j} = A(\phi\cdot\phi)\delta_{ij} + B(\phi\cdot\phi)\phi_i\phi_j$$

When is a HEFT not a SMEFT?

1) When it's a funnel (Alonso, Jenkins, and Manohar 2016)



$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h)$$

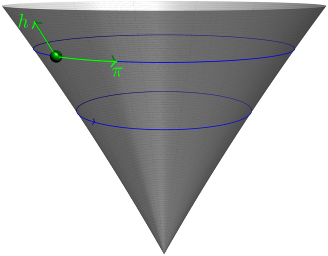
Require geodesic distance of closed $O(2)$ orbits to be non-zero everywhere

$$F(h) \neq 0$$

then there's no fixed point about which to expand in SMEFT coordinates.

When is a HEFT not a SMEFT?

2) When it's a cone



$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h)$$

Suppose $F(-v_0) = 0$ for some v_0. The HEFT chart is degenerate, and the HEFT lagrangian may hide non-analyticities.

To diagnose non-analyticites, can use **curvature invariants**

$$R = -\frac{6F''}{F} + \frac{6}{v^2F^2}\left[1 - (vF')^2\right]$$

$$\nabla^2 V = V'' + \frac{3V'}{F}$$

As $h \to -v_0$ and $F \to 0$, $R \to \infty$ (a **conical singularity**) unless $F'(-v_0) = \frac{1}{v}$ and $F''(-v_0) = 0$.

If $\exists n$, $(\nabla^2)^n R \to \infty$ and/or $(\nabla^2)^{n+1} V \to \infty$ at the fixed point, it's HEFT. Otherwise, it's SMEFT.

HEFT from extended scalar sectors

Using functional tree-level matching

Functional matching means evaluating the partition function

$$\exp(iS_{\mathsf{EFT}}[\phi_{\mathsf{SM}}]) = \int \mathcal{D}\Phi_{\mathsf{BSM}} \exp(iS_{\mathsf{UV}}[\phi_{\mathsf{SM}}, \Phi_{\mathsf{BSM}}])$$

At tree-level (mean field level)

$$S^{\mathsf{tree}}_{\mathsf{EFT}}[\phi_{\mathsf{SM}}] = S_{\mathsf{UV}}[\phi_{\mathsf{SM}}, \Phi^{\mathbf{c}}_{\mathsf{BSM}}], \text{ where } \Phi^{\mathbf{c}}_{\mathsf{BSM}} \text{ solves } \frac{\delta S_{\mathsf{UV}}}{\delta \Phi_{\mathsf{BSM}}} = 0$$

We will show that S_{EFT} is HEFT and looks like

- a **funnel** when Φ_{BSM} has an electroweak breaking vev;[2]
- a **cone** when Φ_{BSM} gets all its mass from the Higgs vev.

[2]that remains non-zero when the Higgs vev is turned off

$\mathbb{Z}_2$ singlet example

$$\mathcal{L}_{\rm UV} = |\partial H|^2 + \frac{1}{2}(\partial S)^2 - \left(-\mu_H^2 |H|^2 + \lambda_H |H|^4 + \frac{1}{2}\left(m^2 + \kappa |H|^2\right) S^2 + \frac{1}{4}\lambda_S S^4\right)$$

$$\frac{\delta S_{\rm UV}}{\delta S} = (\partial^2 + m^2 + \kappa |H|^2 + \lambda_S S^2) S = 0 \implies S^{\rm c} = 0, \pm\sqrt{-\frac{m^2 + \kappa |H|^2}{\lambda_S}} + \mathrm{O}(\partial^2)$$

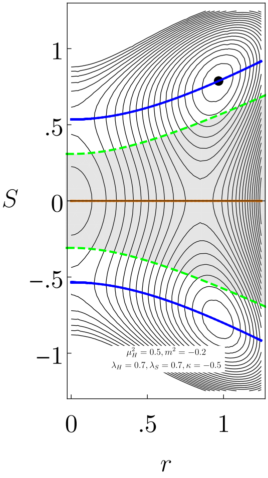
Choose the non-trivial branch of S

$$\mathcal{L}_{\rm EFT} = |\partial H|^2 - \frac{\kappa^2 \left(\partial_\mu |H|^2\right)^2}{4\lambda_S \left(m^2 + \kappa |H|^2\right)} - \left(-\mu_H^2 |H|^2 + \lambda_H |H|^4 - \frac{\left(m^2 + \kappa |H|^2\right)^2}{4\lambda_S}\right)$$

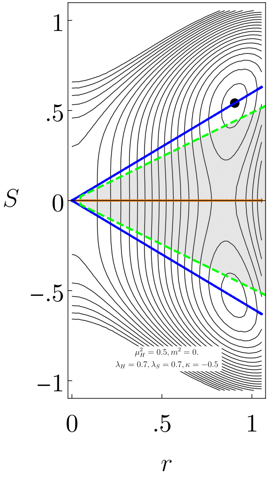
$$R\,(H = 0) = -\frac{3\kappa^2}{2\,m^2\,\lambda_S}$$

$\mathbb{Z}_2$ singlet example: in pictures ($r = \sqrt{2H^\dagger H}$)

$$\mathcal{L}_{\text{UV}} = |\partial H|^2 + \frac{1}{2}(\partial S)^2 - \left(-\mu_H^2|H|^2 + \lambda_H|H|^4 + \frac{1}{2}\left(m^2 + \kappa|H|^2\right)S^2 + \frac{1}{4}\lambda_S S^4\right)$$



✓SMEFT



✗SMEFT

Triplet example

$$\mathcal{L}_{\text{UV}} = |\partial H|^2 + \frac{1}{2}\left(\partial\Phi\right)^2$$
$$- \left(-\mu_H^2 |H|^2 + \lambda_H |H|^4 + \frac{1}{2} m^2 \Phi^2 - \frac{1}{2}\mu H^\dagger \sigma^a H \Phi_a + \kappa |H|^2 \Phi^2 + \frac{1}{4}\lambda_\Phi \Phi^4 \right)$$

Reparameterise as

$$\Phi_a = \frac{2f}{r^2} \exp \begin{pmatrix} 0 & 0 & \beta_1 \\ 0 & 0 & \beta_2 \\ -\beta_1 & -\beta_2 & 0 \end{pmatrix} \begin{pmatrix} H^\dagger \sigma^1 H \\ H^\dagger \sigma^2 H \\ H^\dagger \sigma^3 H \end{pmatrix} \quad \text{with} \quad \begin{matrix} f \in \mathbb{R} \\ \beta_i \in [0, 2\pi) \end{matrix} ,$$

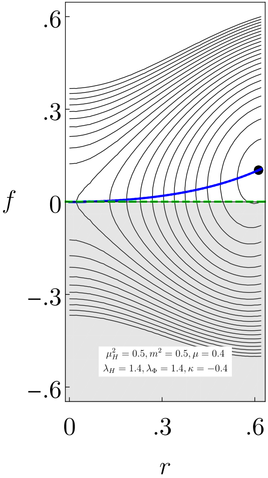
where $r \equiv \sqrt{2 H^\dagger H}$ and $f^2 = \Phi^2$, giving potential

$$V = -\frac{1}{2}\mu_H^2 r^2 + \frac{1}{2} m^2 f^2 + \frac{1}{4}\lambda_H r^4 + \frac{1}{2}\kappa r^2 f^2 + \frac{1}{4}\lambda_\Phi f^4$$
$$- \frac{1}{4}\mu f r^2 + \frac{\mu f}{r^2}(1 - \cos\beta)\left[\frac{\left(H^\dagger \sigma^i H \beta^i\right)^2}{\beta^2} + \left(H^\dagger \sigma^3 H\right)^2 \beta^2 \right]$$

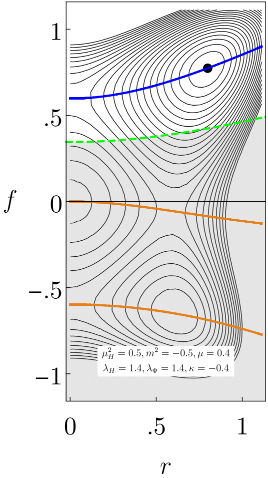
$$\frac{\partial V}{\partial \beta^i} = 0 \implies \beta^i = 0; \left.\frac{\partial V}{\partial f}\right|_{\beta^i = 0} = -\frac{1}{4}\mu r^2 + \left(m^2 + \kappa r^2\right) f + \lambda_\Phi f^3 = 0$$

Triplet example: in pictures

$$\mathcal{L}_{\rm EFT} = \left[1+\frac{4f^2}{r^2}\right]|\partial H|^2+\frac{1}{2}\left[\frac{(f')^2}{r^2}+\frac{4f^2}{r^4}\right]\left(\partial(H^\dagger H)\right)^2+\left[\frac{2f^2}{r^4}\right]\left(H^\dagger\overleftrightarrow{\partial}H\right)^2-V\,,$$



✓SMEFT



✗SMEFT

$\mathbb{Z}_2$ singlet example: loop level

$$\mathcal{L}_{\text{UV}} = |\partial H|^2 + \frac{1}{2}(\partial S)^2 - \left(-\mu_H^2 |H|^2 + \lambda_H |H|^4 + \frac{1}{2}\left(m^2 + \kappa |H|^2\right) S^2 + \frac{1}{4}\lambda_S S^4\right)$$

$$\exp(iS_{\text{EFT}}[\phi_{\text{SM}}]) = \int \mathcal{D}\Phi_{\text{BSM}} \exp(iS_{\text{UV}}[\phi_{\text{SM}}, \Phi_{\text{BSM}}]) = \exp\left(iS_{\text{UV}}[\phi_{\text{SM}}, \Phi^{\text{c}}_{\text{BSM}}] - \frac{1}{2}\mathrm{Tr}\log\left(\frac{\delta S_{\text{UV}}}{\delta\Phi\delta\Phi}\Big|_{\Phi=\Phi^{\text{c}}}\right)\right)$$

Choose $m^2, \kappa > 0$ such that we're on the trivial $S^{\text{c}} = 0$ branch

$$\mathcal{L}_{\text{EFT}} = |\partial H|^2 + \frac{1}{384\pi^2}\frac{\kappa^2}{m^2 + \kappa|H|^2}\left(\partial|H|^2\right)^2 + \mu_H^2|H|^2 - \lambda_H|H|^4 + \frac{1}{64\pi^2}\left(m^2 + \kappa|H|^2\right)^2\left(\ln\frac{\mu^2}{m^2 + \kappa|H|^2} + \frac{3}{2}\right)$$

$$R(H=0) = \frac{1}{32\pi^2}\frac{\kappa^2}{m^2}$$

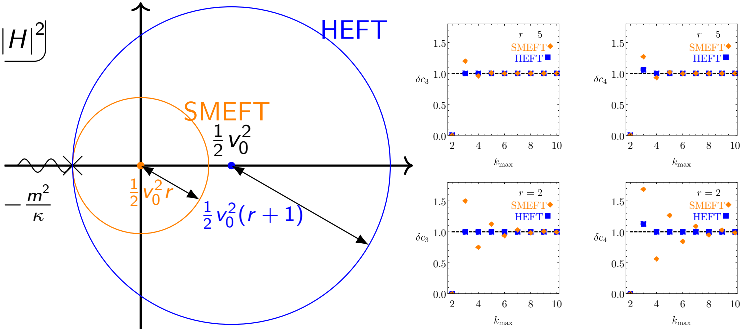
EFT convergence

Expand $\Delta V = -\frac{1}{64\pi^2}\left(m^2 + \kappa |H|^2\right)^2 \left(\ln \frac{\mu^2}{m^2+\kappa |H|^2} + \frac{3}{2}\right)$ in powers of

$$X_{\text{SMEFT}} = \frac{\kappa |H|^2}{m^2} = \frac{\kappa v_0^2}{2\, m^2}\left(1 + \frac{h}{v_0}\right)^2$$

$$X_{\text{HEFT}} = \frac{\kappa \left(|H|^2 - \frac{1}{2} v_0^2\right)}{m^2 + \frac{1}{2}\kappa v_0^2} = \frac{\kappa v_0^2}{2\, m^2 + \kappa v_0^2}\left[2\,\frac{h}{v_0} + \left(\frac{h}{v_0}\right)^2\right]$$

and consider radius of convergence in terms of $r \equiv \frac{m^2}{\frac{1}{2}\kappa v_0^2}$.

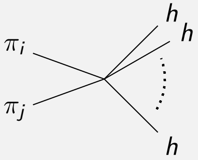


Amplitudes are covariant quantities

$$\mathcal{L} = \frac{1}{2}\left(\sum \frac{1}{n!} g_{ij,k_1\ldots k_n}\phi^{k_1}\ldots\phi^{k_n}\right)\partial\phi^i\partial\phi^j - \left(\sum \frac{1}{n!} V_{,k_1\ldots k_n}\phi^{k_1}\ldots\phi^{k_n}\right)$$

The partial derivatives $g_{ij,(k_1\ldots k_n)}$ and $V_{,k_1\ldots k_n}$ must combine in amplitudes to form coordinate invariant objects: **covariant derivatives** of the Riemann curvature tensor and potential

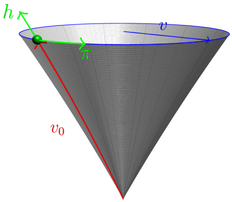
$$M(\pi_i\pi_j \to h^n) =$$



$$= sR_{\pi_i hh\pi_j;h\ldots h} + V_{;(\pi_i\pi_j h\ldots h)}$$

Singularity causes factorial growth of amplitudes

If R diverges at fixed point, so does Taylor expansion about vacuum



$$\sum_n R_{;\underbrace{hh\ldots h}_{n}} \frac{(-v_0)^n}{n!} \to \infty \implies |R_{;\underbrace{hh\ldots h}_{n}}| \to \frac{n!}{v_0^n}$$

As $R = -2g^{hh}g^{\pi_i\pi_j}R_{\pi_i hh\pi_j} + g^{\pi_i\pi_j}g^{\pi_k\pi_l}R_{\pi_i\pi_k\pi_j\pi_l}$, then

$$|R_{\pi_i hh\pi_j;h\ldots h}| \to \frac{n!}{v_0^n} g_{\pi_i\pi_j} = \frac{n! v^2}{v_0^n}\delta_{ij}, \text{ and/or}$$

$$|R_{\pi_i\pi_k\pi_l\pi_j;h\ldots h}| \to \frac{n!}{v_0^n}(g_{\pi_i\pi_l}g_{\pi_k\pi_j} - g_{\pi_i\pi_j}g_{\pi_k\pi_l}) = \frac{n! v^4}{v_0^n}(\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl})$$

at our vacuum in the large n limit.

(Similarly if $\nabla^2 V = g^{hh}V_{;hh} + g^{\pi_i\pi_j}V_{;\pi_i\pi_j}$ diverges at fixed point

$$|V_{;\pi_i\pi_j h\ldots h}| \to \frac{n!}{v_0^n} g_{\pi_i\pi_j} = \frac{n! v^2}{v_0^n}\delta_{ij}$$

in the large n limit.)

Perturbative unitarity limits

Following (Chang and Luty 2019), (Abu-Ajamieh, Chang, Chen, and Luty 2020)

Normalise multiparticle states:

$$\langle P', \alpha | P, \beta \rangle = (2\pi)^4 \delta^{(4)}(P' - P)\delta_{\alpha\beta}$$

such that $\langle P', \alpha | S | P, \beta \rangle = (2\pi)^4 \delta^{(4)}(P' - P)(\delta_{\alpha\beta} + i\hat{M}_{\alpha\beta})$

Then unitarity ($SS^\dagger = 1$) implies

$$\sum_\beta (\delta_{\alpha\beta} + i\hat{M}_{\alpha\beta})(\delta_{\alpha\beta} - i\hat{M}^*_{\alpha\beta}) = \delta_{\alpha\alpha} = 1$$

$$\implies 2\Im \hat{M}_{\alpha\alpha} = \sum_\beta |\hat{M}_{\alpha\beta}|^2$$

$$\implies \sum_{\beta \neq \alpha} |\hat{M}_{\alpha\beta}|^2 = 1 - (1 - \Im \hat{M}_{\alpha\alpha})^2 - (\Re \hat{M}_{\alpha\alpha})^2$$

$$\leq 1$$

Singularity causes growth in the inelastic cross section

(Chang and Luty 2019), (Abu-Ajamieh, Chang, Chen, and Luty 2020), (Falkowski and Rattazzi 2019)

$$M(\pi_i\pi_j \to h^n) = E_{\text{CoM}}^2 R_{\pi_i hh\pi_j;h\ldots h} + V_{;(\pi_i\pi_j h\ldots h)}$$

$$|R_{\pi_i hh\pi_j;h\ldots h}|, |V_{;\pi_i\pi_j h\ldots h}| \to \frac{n!v^2}{v_0^n}\delta_{ij}$$

$$1 \geq \sum_\beta |\hat{M}_{\alpha\beta}|^2 \geq \sum_n |M(\pi_1\pi_1 \to h^n)|^2 \frac{\mathrm{Vol}_2}{2!}\frac{\mathrm{Vol}_n}{n!}$$

$$\geq \sum_n |M(\pi_1\pi_1 \to h^n)|^2 \frac{1}{8(4\pi)^2}\frac{1}{n!(n-1)!(n-2)!}\left(\frac{E_{\text{CoM}}}{4\pi}\right)^{2n-4}$$

$$\geq \sum_n \left|\frac{R_{\pi_1 hh\pi_1;h\ldots h} v_0^{n-2} v^2}{(n-2)!}\right|^2 \frac{2\pi^2 v_0^4}{v^4}\frac{1}{n!(n-1)}\left(\frac{E_{\text{CoM}}}{4\pi v_0}\right)^{2n}$$

leading to perturbative unitarity violation at $E_{\text{CoM}} \sim 4\pi v_0$.

(Ultimately unitarised by the particle whose mass comes entirely from EWSB. E.g. $m^2 = \lambda|H|^2 \lesssim (4\pi v_0)^2$.)

Summary (1)

A geometric picture makes manifest the connection between scalar field theory and its amplitudes, its connection to UV completions, and the physical significance of non-analyticities.

There is no SMEFT expansion for a manifold that looks like a

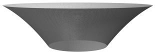
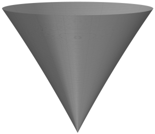
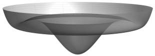
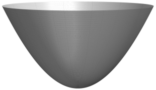
- **funnel** (extra sources of EWSB);
- **cone** (particles that get all their mass from EWSB).

Even if there is a SMEFT expansion, it will not converge at our vacuum unless (roughly) all BSM particles get the minority of their mass from EWSB ($v < \Lambda \sim \frac{M}{g_*}$).

Cones make

- $\frac{n!}{v_0^n}$ growth in amplitudes with n Higgses (v_0 is geodesic distance to singularity from our vacuum);
- perturbative unitarity violations $E_{\text{CoM}} \sim (4\pi v_0)$.

Summary (2)

	✗SMEFT	Singularity infinite distance away. From integrating out extra sources of EWSB.
	✗SMEFT	Singularity finite distance away, HEFT power counting and unitarity violation $E_{\text{CoM}} \sim 4\pi v_0$. From integrating out massless particles.
	$\sim$ SMEFT	Valid SMEFT expansion about fixed point, does not converge at our vacuum. Impractical.
	✓SMEFT	Valid SMEFT expansion converges at fixed point and our vacuum. All BSM particles get minority of mass from EWSB.

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