Looking at QED through the glasses of Very Special Relativity

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Pontificia Universidad Católica de Chile

Talk in Cambridge

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-“Is there any other point to which you would wish to draw my attention?”

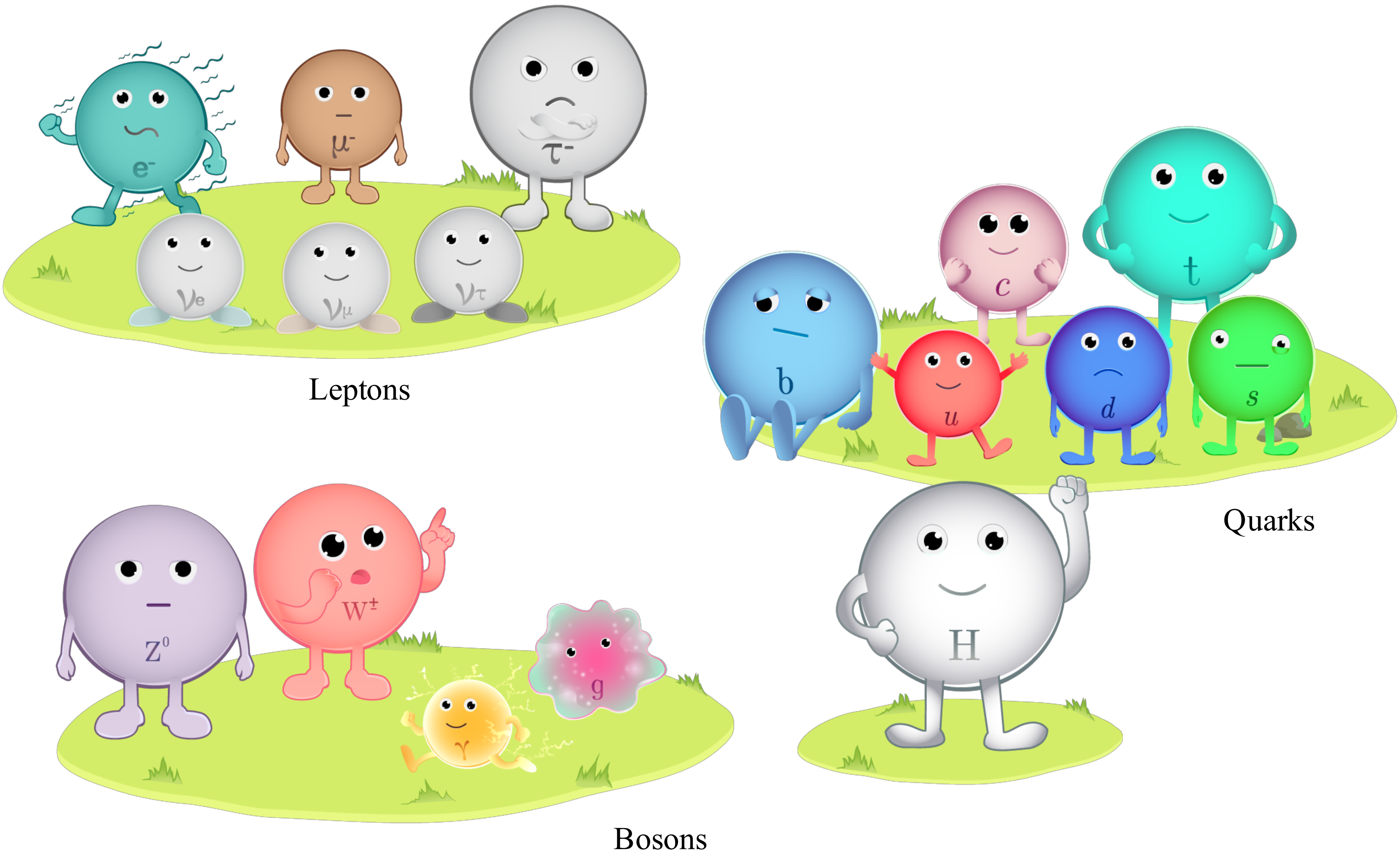
-“To the curious incident of the dog in the night-time.”

-“The dog did nothing in the night-time.”

-“That was the curious incident.”

The adventure of Silver Blaze

Arthur Conan Doyle



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Bosons


e⁻
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νe
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ντ
Leptons
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s
Quarks
H
Z⁰
W±
γ
g
Bosons

How do I get mass?

Plan of the talk

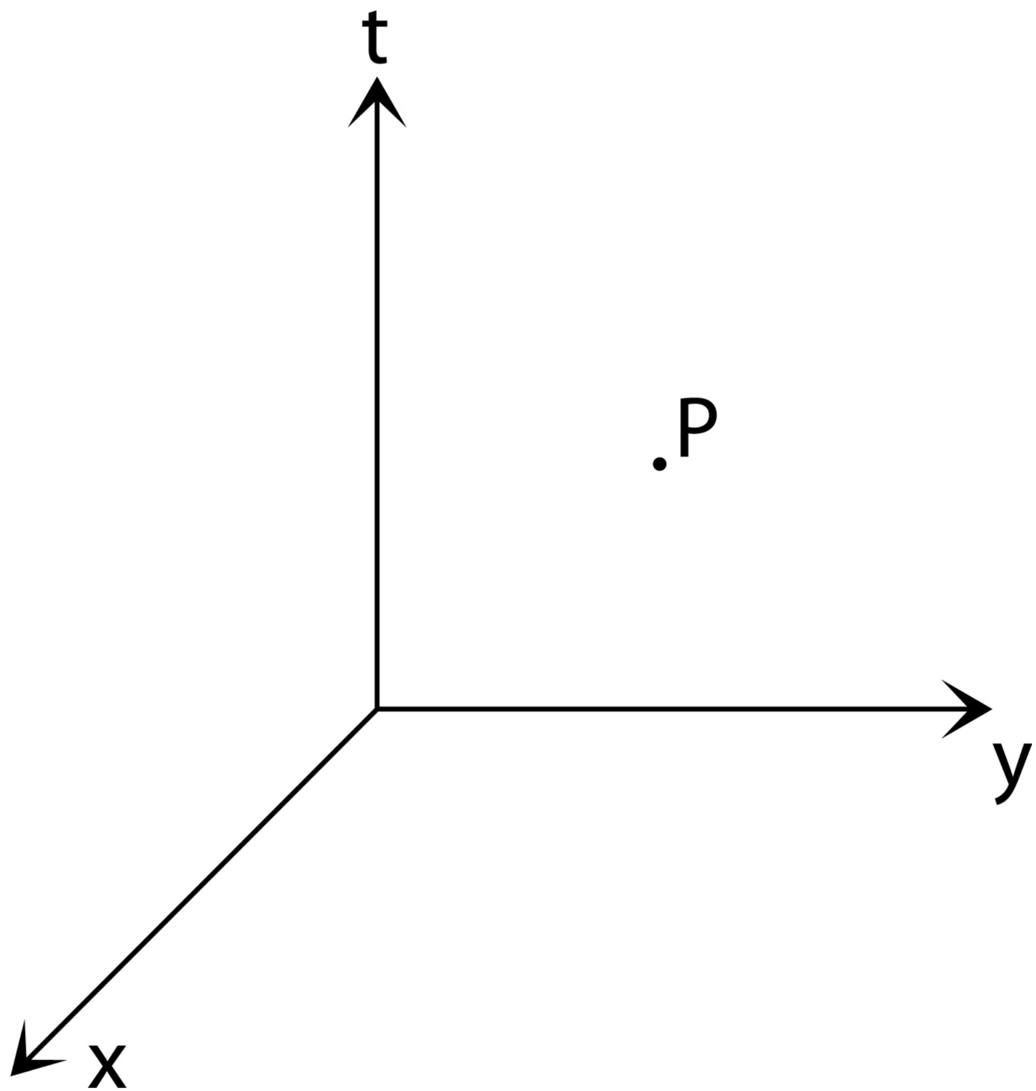
- VSR Framework
- QED in 1+1 dimensions
- QED in 2+1 dimensions
- QED in 3+1 dimensions
- Summary

VSR Framework

Very Special Relativity

Andrew G. Cohen* and Sheldon L. Glashow†
Physics Department, Boston University
Boston, MA 02215, USA
(Dated: Jan 26, 2006)

Phys.Rev.Lett. 97 (2006) 021601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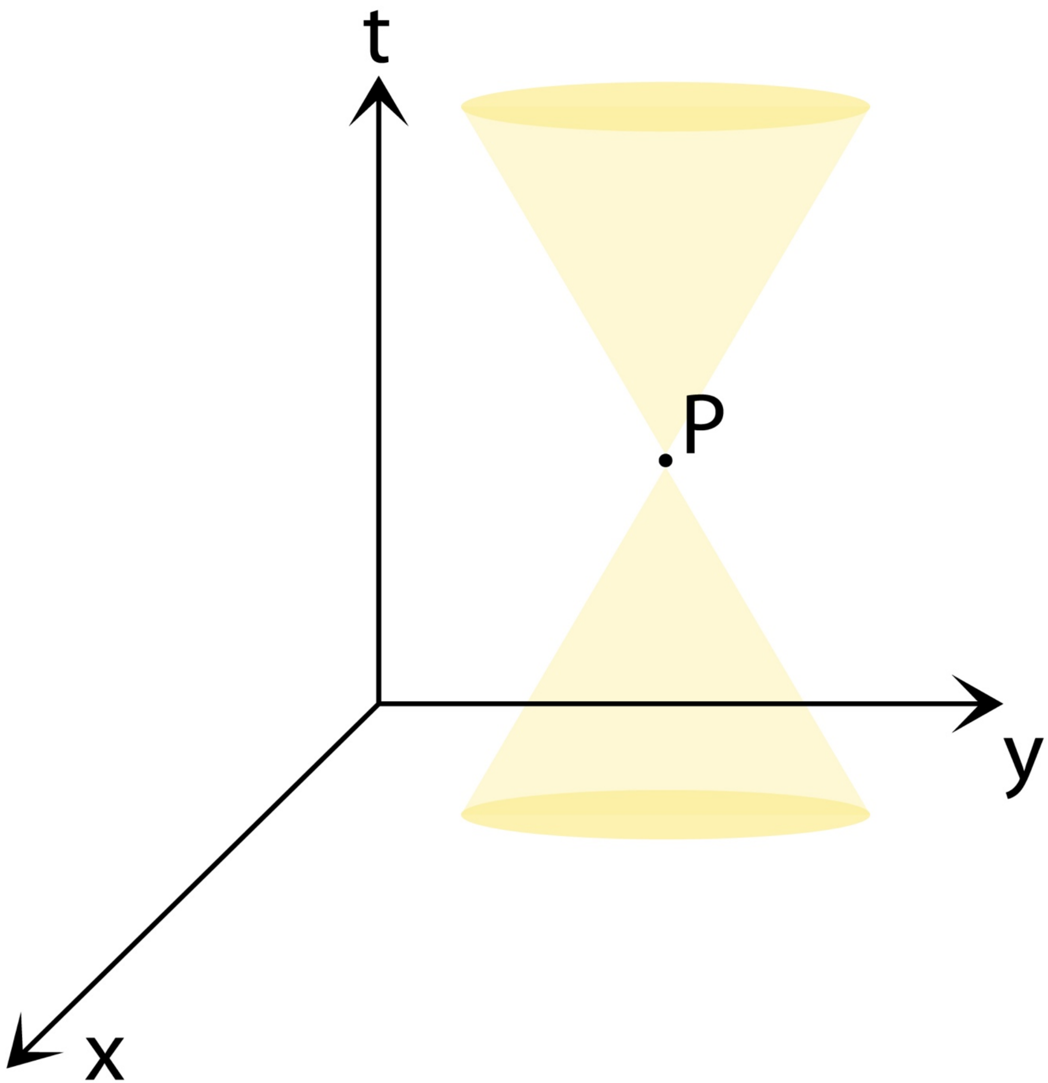
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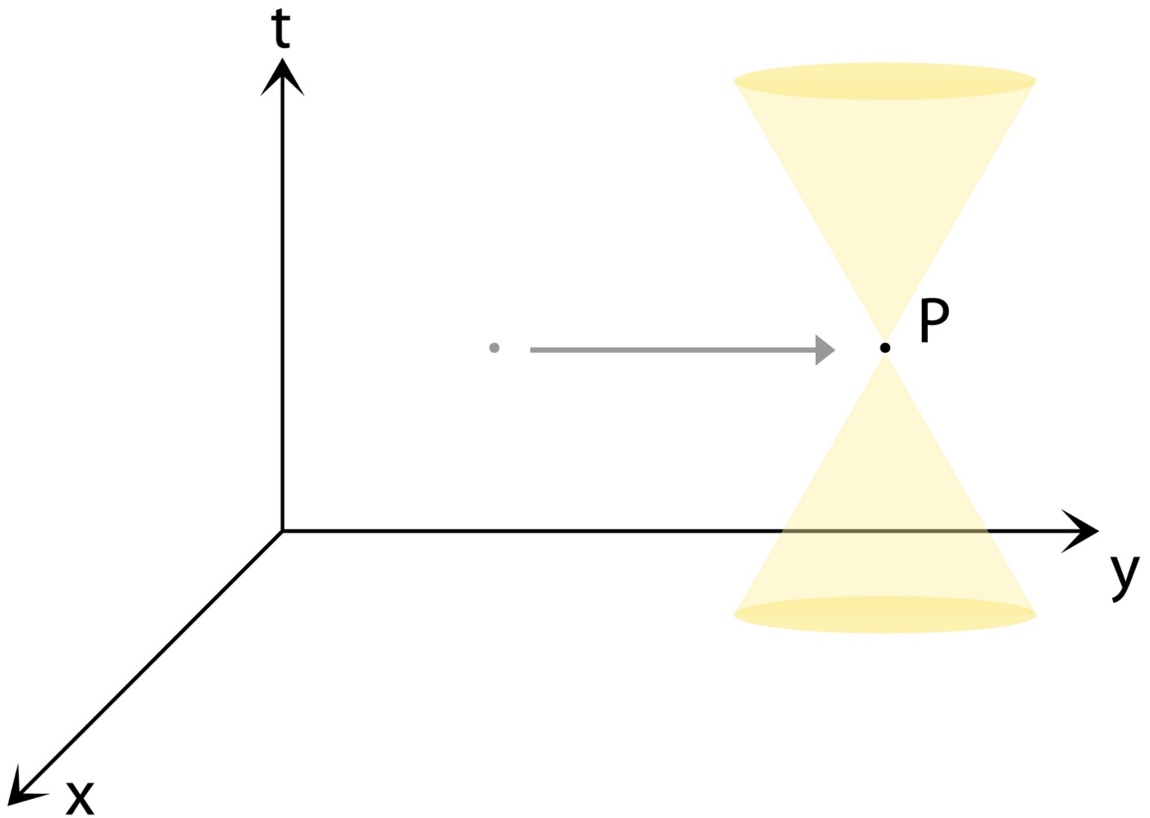
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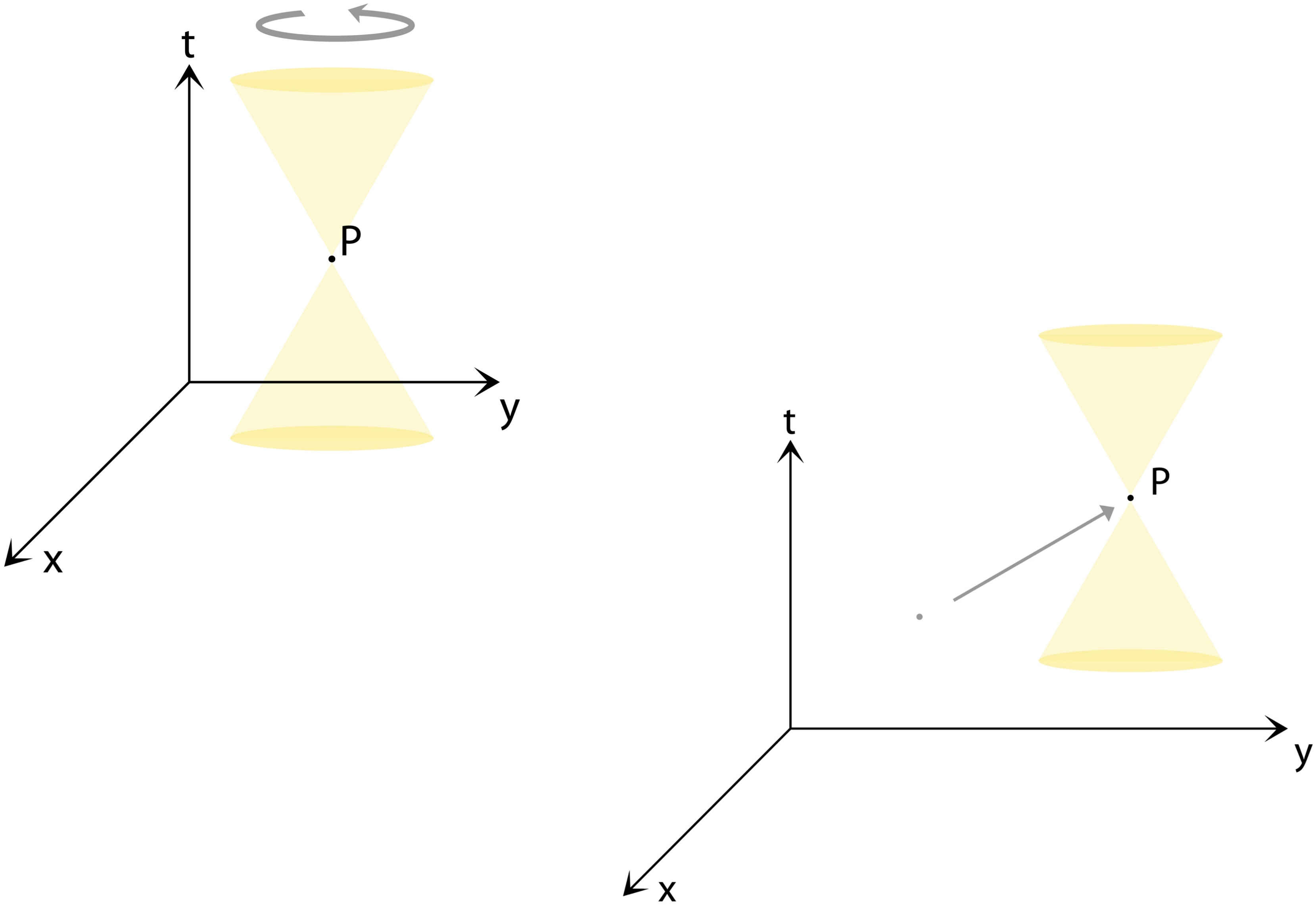
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The group of all transformations that left invariant the light cone (without translations) is the Lorentz Group. It is a 6 parameter group.

$$J_1, J_2, J_3, K_1, K_2, K_3$$

In a different basis

$$T_1 = K_1 + J_2$$

$$T_2 = K_2 - J_1$$

$$Y_1 = -K_1 + J_2$$

$$Y_2 = -K_2 - J_1$$

$$J_3$$

$$K_3$$

We see the effect of parity or time reversion in the boosts and rotations

$$PJ_iP^{-1} = J_i$$
$$PK_iP^{-1} = -K_i$$
$$TJ_iT^{-1} = J_i$$
$$TK_iT^{-1} = -K_i$$

Therefore

$$PT_1P^{-1} = Y_1$$
$$PT_2P^{-1} = Y_2$$

We need only a four parameter group.

$$T_1, T_2, J_3, K_3 \longrightarrow \text{SIM(2)}$$

An important feature of SIM(2) is the following null vector

$$n \rightarrow (1,0,0,1)$$

It transforms as

$$n \rightarrow e^{\phi} n$$

This allows us introduce new terms as

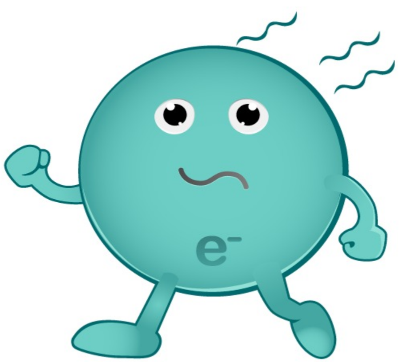
$$\frac{n \cdot p_1}{n \cdot p_2}$$

It is not Lorentz invariant but VSR

A privileged direction is part of the theory

VSR Equation for Neutrino: $$\left(\not{p} - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)\nu = 0$$

Dispersion relation: $$p^2 = m^2$$



VSR Equation for electron: $$\left(\not{p} - M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)\psi = 0$$

Dispersion relation: $$p^2 = M_e^2 \qquad M_e^2 = M^2 + m^2$$

The QED lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{GF}}$$

with

$$\mathcal{L}_{\text{fermion}} = \bar{\psi}\left[i\left(\not{D} + \frac{1}{2}m^2\frac{\not{n}}{n\cdot D}\right) - M\right]\psi$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m_\gamma^2}{2}(n^\alpha F_{\mu\alpha})\frac{1}{(n\cdot\partial)^2}(n_\beta F^{\mu\beta})$$

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi}(\partial^\mu A_\mu)^2$$

and

$$D_\mu = \partial_\mu + ieA_\mu$$

The QED lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{GF}}$$

with

$$\mathcal{L}_{\text{fermion}} = \bar{\psi}\left[i\left(\not{D} + \frac{1}{2}m^2\frac{\not{n}}{n \cdot D}\right) - M\right]\psi$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m_\gamma^2}{2}(n^\alpha F_{\mu\alpha})\frac{1}{(n \cdot \partial)^2}(n_\beta F^{\mu\beta})$$

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi}(\partial^\mu A_\mu)^2$$

and

$$D_\mu = \partial_\mu + ieA_\mu$$

Here, a photon mass is allowed!
It doesn't break gauge invariance!

In VSR we cannot include P or T, or composed transformations as CP or CT.

But including C in the free fermion part

$$\mathcal{L}^{c}_{\text{free fermion}} = \bar{\psi}^{c}\Big(i\partial\!\!\!/ + i\frac{m^2}{2}\frac{n\!\!\!/}{n\cdot\partial}\Big)\psi^{c}$$

$$\psi^c = \eta_\psi C\bar{\psi}^T \text{ and } \bar{\psi}^c = -\eta^*_\psi \psi^T C^{-1} \qquad C^{-1}\gamma^\mu C = -(\gamma^\mu)^T$$

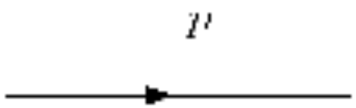
$$\bar{\psi}^c\gamma^\mu\left(\frac{1}{n.\partial}\psi^c\right) = -\Big(\frac{1}{n.\partial}\bar{\psi}\Big)\gamma^\mu\psi.$$

$$\frac{1}{n.\partial} = \int_0^\infty ds\ e^{-s(n.\partial)}$$

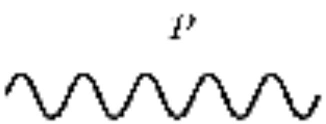
$$\bar{\psi}^c\gamma^\mu\left(\frac{1}{n.\partial}\psi^c\right) = -\int d^3x\Big(\frac{1}{n.\partial}\bar{\psi}\Big)\gamma^\mu\psi = \int d^3x\ \bar{\psi}\gamma^\mu\left(\frac{1}{n.\partial}\psi\right)$$

Is C invariant

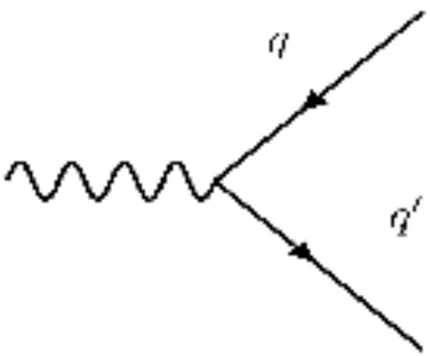
Feynman Rules



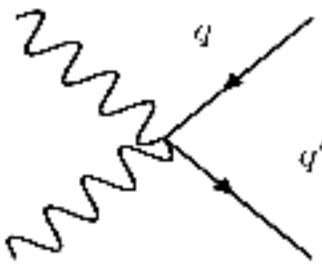
$$i\frac{\not p + M - \frac{m^2}{2}\frac{\not n}{n\cdot p}}{p^2 - M^2 - m^2 + i\varepsilon}$$



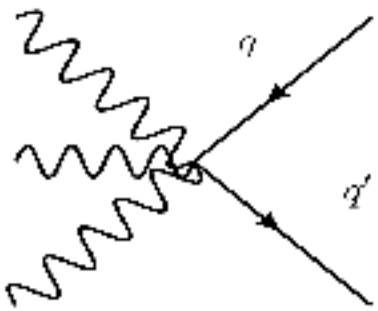
$$-\frac{i}{p^2-m_\gamma^2}\left[g_{\mu\nu} + \frac{m_\gamma^2}{(n\cdot p)^2}n_\mu n_\nu - \frac{m_\gamma^2}{p^2 n\cdot p}(p_\mu n_\nu + p_\nu n_\mu)\right]$$



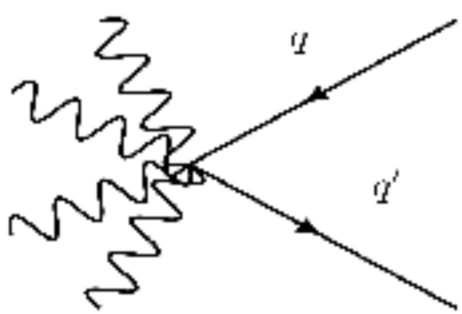
$$-ie\left(\gamma_\mu + \frac{1}{2}m^2\frac{\not n n_\mu}{n\cdot q\, n\cdot q'}\right)$$



$$-ie^2\frac{1}{2}\not{n}m^2\frac{n_\mu n_\nu}{n\cdot q\, n\cdot q'}\left(\frac{1}{n\cdot(p+q)}+\frac{1}{n\cdot(p'+q)}\right)$$



$$-ie^3\frac{1}{2}\not{n}m^2\frac{n_\mu n_\nu n_\rho}{n\cdot q\, n\cdot q'}\left[\left(\frac{1}{n\cdot(p_3+q)}+\frac{1}{n\cdot(p_2+p_3+q)}\right)+\text{perm.}\right]$$



$$-ie^4\frac{1}{2}\not{n}m^2\frac{n_\mu n_\nu n_\rho n_\sigma}{n\cdot q\, n\cdot q'}\left[\left(\frac{1}{n\cdot(p_4+q)}+\frac{1}{n\cdot(p_3+p_4+q)}+\frac{1}{n\cdot(p_2+p_3+p_4+q)}\right)+\text{perm.}\right]$$

QED in 1+1 dimensions

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Schwinger model à la Very Special Relativity

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Instituto de Física, Pontificia Universidad de Católica de Chile, Av. Vicuña Mackenna 4860, Santiago, Chile

Phys.Lett. B797 (2019) 134923

The Lorentz Group is only one-parameter group.
The most general transformation is

$$\Lambda = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$$

However, for the null vector $n = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ transforms as $\Lambda n = e^{\theta} n$.

Lorentz Group admits VSR terms

The free VSR fermion lagrangian

$$\mathcal{L} = \bar{\psi}\Big(i\not{\partial} + \frac{i}{2}m^2\frac{\not{n}}{n\cdot\partial}\Big)\psi$$

The vector current is

$$j^\mu = \bar{\psi}\gamma^\mu\psi + \frac{1}{2}m^2\Big(\frac{1}{n\cdot\partial}\bar{\psi}\Big)\not{n}n^\mu\Big(\frac{1}{n\cdot\partial}\psi\Big)$$

The axial current is

$$j^{\mu 5} = \bar{\psi}\gamma^\mu\gamma^5\psi + \frac{1}{2}m^2\Big(\frac{1}{n\cdot\partial}\bar{\psi}\Big)\not{n}\gamma^5 n^\mu\Big(\frac{1}{n\cdot\partial}\psi\Big)$$

They are conserved classically

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j^{\mu 5} = 0$$

and

$$j^{\mu 5} = -\epsilon^{\mu\nu} j_\nu$$

Now, we couple the fermion with an external electromagnetic field

$$\mathcal{L}=\bar{\psi}\left(i\not{D}+\frac{i}{2}m^2\frac{\not{n}}{n\cdot D}\right)\psi$$

With $D_\mu=\partial_\mu+ieA_\mu$

We expand at first order $\frac{1}{n\cdot D}$

Now, the vector current is

$$j^\mu=\bar{\psi}\gamma^\mu\psi+\frac{1}{2}m^2\left(\frac{1}{n\cdot\partial}\bar{\psi}\right)\not{n}n^\mu\left(\frac{1}{n\cdot\partial}\psi\right)$$

$$+\frac{1}{2}iem^2\left(\frac{1}{n\cdot\partial}n\cdot A\frac{1}{n\cdot\partial}\bar{\psi}\right)\not{n}n^\mu\left(\frac{1}{n\cdot\partial}\psi\right)-\frac{1}{2}iem^2\left(\frac{1}{n\cdot\partial}\bar{\psi}\right)\not{n}n^\mu\left(\frac{1}{n\cdot\partial}n\cdot A\frac{1}{n\cdot\partial}\psi\right)$$

The axial current is

$$j^{\mu 5} = \bar{\psi}\gamma^{\mu}\gamma^{5}\psi + \frac{1}{2}m^{2}\left(\frac{1}{n\cdot\partial}\bar{\psi}\right)\not{n}n^{\mu}\gamma^{5}\left(\frac{1}{n\cdot\partial}\psi\right)$$

$$+\frac{1}{2}iem^{2}\left(\frac{1}{n\cdot\partial}n\cdot A\frac{1}{n\cdot\partial}\bar{\psi}\right)\not{n}n^{\mu}\gamma^{5}\left(\frac{1}{n\cdot\partial}\psi\right) - \frac{1}{2}iem^{2}\left(\frac{1}{n\cdot\partial}\bar{\psi}\right)\not{n}n^{\mu}\gamma^{5}\left(\frac{1}{n\cdot\partial}n\cdot A\frac{1}{n\cdot\partial}\psi\right)$$

There is a modification

Despite the modification

$$\partial_{\mu}j^{\mu} = 0$$

$$\partial_{\mu}j^{\mu 5} = 0$$

They are conserved classically

$$j^{\mu 5} = -\epsilon^{\mu\nu}j_{\nu}$$

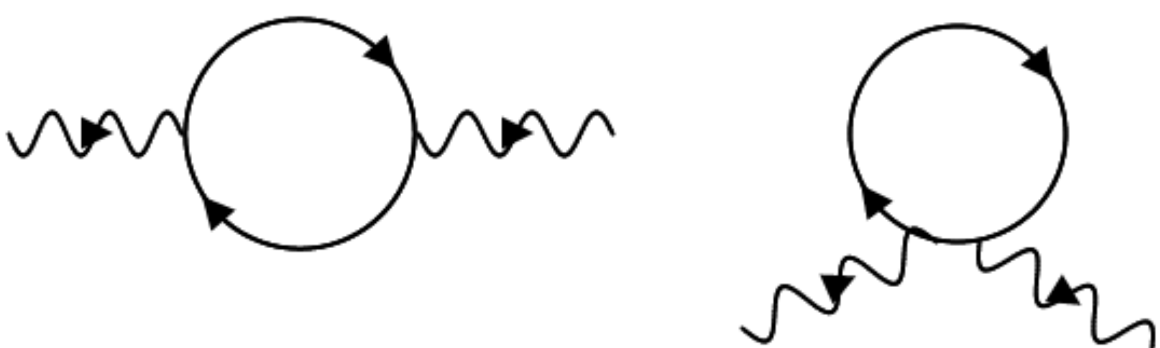
In the quantum level, we will use

$$\langle j^{\mu 5}\rangle = -\epsilon^{\mu\nu}\langle j_\nu\rangle$$

We move on to the path integrals to compute the expectation value

$$\langle j^\mu(x)\rangle = \frac{1}{Z}\int D\bar{\psi}D\psi\, j^\mu \exp[i\int d^2x\,\mathcal{L}_0]\Big(1 - ie\int d^2x\,\bar{\psi}\Big(\not{A} - \frac{m^2}{2}\frac{\not{n}}{n\cdot\partial}\, n\cdot A\frac{1}{n\cdot\partial}\Big)\psi\Big)$$

We arrive to $\langle j^\mu(q)\rangle = \frac{i}{e}\Pi^{\mu\nu}A_\nu(q)$



To compute the integrals with $\frac{1}{n \cdot p}$

Which are of this form:

$$\int \frac{d^{2\omega}p}{(2\pi)^{2\omega}} \frac{1}{p^2 + 2p\cdot q - \Lambda^2} \frac{1}{n\cdot p}$$

We compute using the Mandelstam-Leibbrandt prescription.
See Alfaro, Phys. Rev D93 (2016), 065033 and Alfaro, Phys. Lett. B772 (2017)

We get

$$i\Pi_{\mu\nu} = \alpha(q^2)\{q^2 g_{\mu\nu} - q_\mu q_\nu\} + \beta(q^2)\left\{\left(\frac{n_\mu q_\nu + n_\nu q_\mu}{n\cdot q}\right) - g_{\mu\nu} - \frac{n_\mu n_\nu}{(n\cdot q)^2} q^2\right\}$$

$$\alpha(q^2) = -\frac{i}{\pi} e^2 \int dx \left(\frac{x\,(1-x)}{m^2 - x(1-x)q^2 - i\varepsilon}\right)$$

$$\beta(q^2) = \frac{i}{\pi} e^2 \frac{m^2}{2} \int dx \int_0^1 dt\, \frac{x\,q^2}{[m^2 - x\,q^2 + x^2 q^2 (1-t) - i\varepsilon]^2}$$

$$q_\mu \Pi^{\mu\nu} = 0$$

Since

$$\langle j^{\mu}(q)\rangle = \frac{i}{e}\Pi^{\mu\nu}A_{\nu}(q)$$

In the quantum level vector current stills conserved

$$q_{\mu}\langle j^{\mu}\rangle = 0$$

But, as

$$\langle j^{\mu 5}\rangle = -\epsilon^{\mu\nu}\langle j_{\nu}\rangle$$

For the axial current

$$q_{\mu}\langle j^{\mu 5}\rangle = \left[\frac{e}{2\pi} + \frac{em^2}{\pi q^2\sqrt{1-\frac{4m^2-i\varepsilon}{q^2}}}\log\left(\frac{1+\sqrt{\frac{q^2-4m^2+i\varepsilon}{q^2}}}{-1+\sqrt{\frac{q^2-4m^2+i\varepsilon}{q^2}}}\right)\right]\varepsilon^{\mu\nu}F_{\mu\nu}(q)$$

QED in 2+1 dimensions

Induced Maxwell-Chern-Simons Effective Action in Very Special Relativity

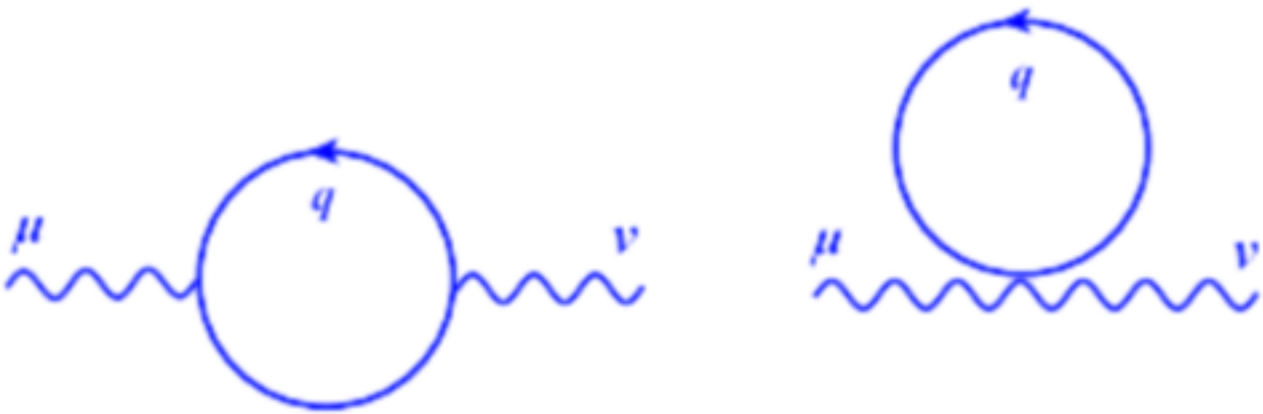
R. Bufalo*[1], M. Ghasemkhani†[2], Z. Haghgouyan‡[2] and A. Soto§[3]

ArXiv:2004.02176

Chern-Simons terms can be induced by radiative quantum corrections.

$$\frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho$$

$$i\Gamma[A] = \int \frac{d^3p}{(2\pi)^3}\int d^3x_1 d^3x_2 e^{ip.(x_1-x_2)}A_\mu(x_1)A_\nu(x_2)\Pi^{\mu\nu}(p)\Big|_{p^2\ll m_e^2}.$$



$$\mathrm{Tr}\left(\gamma^\sigma\gamma^\rho\right) = 2\eta^{\sigma\rho}, \quad \mathrm{Tr}\left(\gamma^\alpha\gamma^\sigma\gamma^\rho\right) = -2i\epsilon^{\alpha\sigma\rho}, \quad \mathrm{Tr}\left(\gamma^\alpha\gamma^\sigma\gamma^\lambda\gamma^\rho\right) = 2\left(\eta^{\alpha\sigma}\eta^{\lambda\rho} - \eta^{\alpha\lambda}\eta^{\sigma\rho} + \eta^{\alpha\rho}\eta^{\lambda\sigma}\right)$$

In the limit $p^2 \ll M^2$

$$-i\Pi^{\mu\nu}(p)|_{p^2\ll M^2} = \frac{e^2}{12\pi M}(p^\mu p^\nu - g^{\mu\nu}p^2)\left(1+\frac{1}{10}\frac{p^2}{M^2}\right) - \frac{i\,e^2}{4\pi}\epsilon^{\mu\nu\alpha}p_\alpha\left(1+\frac{1}{12}\frac{p^2}{M^2}\right)$$

$$+\frac{e^2}{16\pi M}\left(\frac{m^2}{M^2}\right)\left[\frac{n^\mu p^\nu + n^\nu p^\mu}{n{\cdot}p} - g^{\mu\nu} - \frac{n^\mu n^\nu}{(n{\cdot}p)^2}p^2\right]p^2$$

$$-\frac{i\,e^2}{16\pi}\left(\frac{m^2}{M^2}\right)\left[\epsilon^{\mu\nu\alpha}\frac{n_\alpha}{n{\cdot}p} + \epsilon^{\alpha\sigma\nu}p_\alpha\frac{n^\mu n_\sigma}{(n{\cdot}p)^2} + \epsilon^{\alpha\mu\sigma}p_\alpha\frac{n^\nu n_\sigma}{(n{\cdot}p)^2}\right]p^2$$

Comparing with the classical VSR 2+1. We can start from:

$$\mathscr{L}_{2+1} = -\frac{1}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu} + \frac{m_e}{4}\varepsilon^{\mu\nu\lambda}\tilde{F}_{\mu\nu}A_\lambda$$

$$\tilde{\partial}_\mu = \partial_\mu + \frac{m^2}{2}\frac{n_\mu}{(n.\partial)}$$

$$\tilde{F}_{\mu\nu} = \tilde{\partial}_\mu A_\nu - \tilde{\partial}_\nu A_\mu = F_{\mu\nu} + \frac{m^2}{2}n^\alpha\left(\frac{n_\nu}{(n.\partial)^2}F_{\mu\alpha} - \frac{n_\mu}{(n.\partial)^2}F_{\nu\alpha}\right)$$

$$\begin{aligned}\mathcal{L}_{2+1} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{m^2}{2}(n_\nu F^{\mu\nu})\frac{1}{(n.\partial)^2}(n^\alpha F_{\mu\alpha}) \\ &+ \frac{m_e}{4}\epsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda + \frac{m^2 m_e}{4}\epsilon^{\mu\nu\lambda}\frac{n_\nu n^\alpha}{(n.\partial)^2}F_{\mu\alpha}A_\lambda + \frac{m^2 m_e}{8}\epsilon^{\mu\alpha\nu}\frac{n_\nu n^\lambda}{(n.\partial)^2}F_{\mu\alpha}A_\lambda.\end{aligned}$$

$$\mathcal{L}_{2+1} = \frac{1}{2}A_\lambda \mathcal{O}^{\lambda\alpha}A_\alpha$$

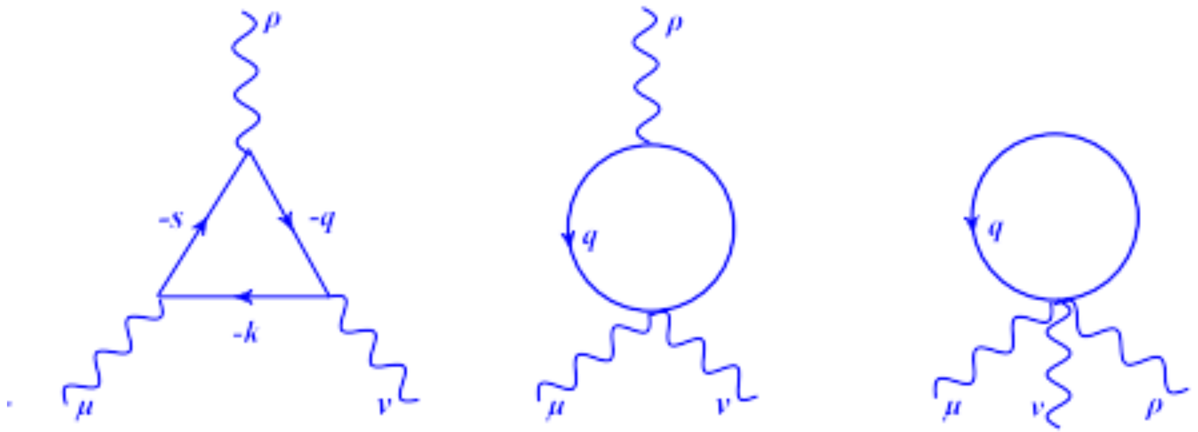
$$
\begin{aligned}
\widetilde{\mathcal{O}}^{\lambda\alpha} &= p^\lambda p^\alpha - \eta^{\lambda\alpha} p^2 + i m_e \epsilon^{\mu\alpha\lambda} p_\mu \\
&\quad + m^2 \left(\frac{n^\lambda n^\alpha p^2}{(n.p)^2} - \frac{n^\lambda p^\alpha + n^\alpha p^\lambda}{(n.p)} + \eta^{\alpha\lambda} \right) \\
&\quad + i m_e \frac{m^2}{2} \left[\frac{\epsilon^{\alpha\nu\lambda} n_\nu}{(n.p)} - \frac{\epsilon^{\mu\nu\lambda} p_\mu n_\nu n^\alpha}{(n.p)^2} + \frac{\epsilon^{\mu\nu\alpha} p_\mu n_\nu n^\lambda}{(n.p)^2} \right]
\end{aligned}
$$

$$
\begin{aligned}
&-i\Pi^{\mu\nu}(p)|_{p^2 \ll M^2} = \frac{e^2}{12\pi M}(p^\mu p^\nu - g^{\mu\nu} p^2)\left(1 + \frac{1}{10}\frac{p^2}{M^2}\right) - \frac{i e^2}{4\pi} \epsilon^{\mu\nu\alpha} p_\alpha \left(1 + \frac{1}{12}\frac{p^2}{M^2}\right) \\
&+ \frac{e^2}{16\pi M}\left(\frac{m^2}{M^2}\right)\left[\frac{n^\mu p^\nu + n^\nu p^\mu}{n\cdot p} - g^{\mu\nu} - \frac{n^\mu n^\nu}{(n\cdot p)^2} p^2\right] p^2 \\
&- \frac{i e^2}{16\pi}\left(\frac{m^2}{M^2}\right)\left[\epsilon^{\mu\nu\alpha}\frac{n_\alpha}{n\cdot p} + \epsilon^{\alpha\sigma\nu} p_\alpha \frac{n^\mu n_\sigma}{(n\cdot p)^2} + \epsilon^{\alpha\mu\sigma} p_\alpha \frac{n^\nu n_\sigma}{(n\cdot p)^2}\right] p^2
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{\mathcal{O}}^{\lambda\alpha} &= p^\lambda p^\alpha - \eta^{\lambda\alpha}p^2 + i m_e \epsilon^{\mu\alpha\lambda}p_\mu \\
&+ m^2\left(\frac{n^\lambda n^\alpha p^2}{(n.p)^2} - \frac{n^\lambda p^\alpha + n^\alpha p^\lambda}{(n.p)} + \eta^{\alpha\lambda}\right) \\
&+ i m_e \frac{m^2}{2}\left[\frac{\epsilon^{\alpha\nu\lambda}n_\nu}{(n.p)} - \frac{\epsilon^{\mu\nu\lambda}p_\mu n_\nu n^\alpha}{(n.p)^2} + \frac{\epsilon^{\mu\nu\alpha}p_\mu n_\nu n^\lambda}{(n.p)^2}\right]
\end{aligned}
$$

$$
\begin{aligned}
-i\Pi^{\mu\nu}(p)\big|_{p^2\ll M^2} =& \frac{e^2}{12\pi M}(p^\mu p^\nu - g^{\mu\nu}p^2)\left(1+\frac{1}{10}\frac{p^2}{M^2}\right) - \frac{i e^2}{4\pi}\epsilon^{\mu\nu\alpha}p_\alpha\left(1+\frac{1}{12}\frac{p^2}{M^2}\right) \\
&+\frac{e^2}{16\pi M}\left(\frac{m^2}{M^2}\right)\left[\frac{n^\mu p^\nu + n^\nu p^\mu}{n\cdot p} - g^{\mu\nu} - \frac{n^\mu n^\nu}{(n\cdot p)^2}p^2\right]p^2 \\
&-\frac{i e^2}{16\pi}\left(\frac{m^2}{M^2}\right)\left[\epsilon^{\mu\nu\alpha}\frac{n_\alpha}{n\cdot p} + \epsilon^{\alpha\sigma\nu}p_\alpha\frac{n^\mu n_\sigma}{(n\cdot p)^2} + \epsilon^{\alpha\mu\sigma}p_\alpha\frac{n^\nu n_\sigma}{(n\cdot p)^2}\right]p^2
\end{aligned}
$$

Higher derivative terms



$$\Xi = 0$$

Here the Furry Theorem is satisfied.

QED in 3+1 dimensions
(with Photon mass)

Photon mass in very special relativity

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(Received 28 March 2019; published 18 September 2019)

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The QED lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{GF}}$$

with

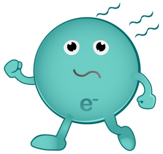
$$\mathcal{L}_{\text{fermion}} = \bar{\psi}\left[i\left(\not{D} + \frac{1}{2}m^2\frac{\not{n}}{n\cdot D}\right) - M\right]\psi$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m_\gamma^2}{2}(n^\alpha F_{\mu\alpha})\frac{1}{(n\cdot\partial)^2}(n_\beta F^{\mu\beta})$$

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi}(\partial^\mu A_\mu)^2$$

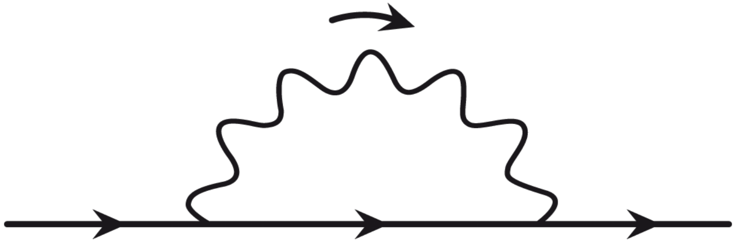
and

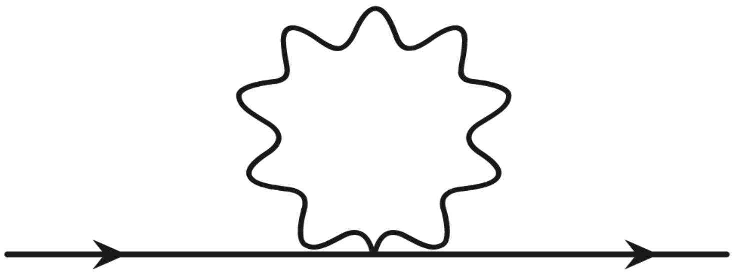
$$D_\mu = \partial_\mu + ieA_\mu$$



Self energy of electron

In VSR we have two diagrams





$$-i\Sigma(q) = (-ie)^2 \left[C \frac{\not{n}}{n \cdot q} + D \not{q} + E \right]$$

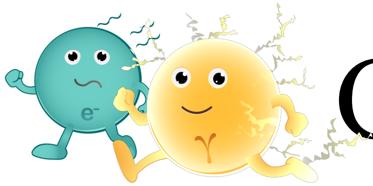
$$C = (-ie)^2 m^2 \Bigg[-\frac{i}{16\pi^2} \int_0^1 dx \frac{1}{x} \log\left(1 + \frac{x^2 q^2}{(1-x)M_e^2 - xq^2 + xm_\gamma^2 - i\varepsilon}\right)$$

$$+ \frac{2i}{(4\pi)^\omega} \int_0^1 dx \frac{\Gamma(2-\omega)}{[(1-x)M_e^2 - x(1-x)q^2 + xm_\gamma^2 - i\varepsilon]^{2-\omega}} + \frac{i}{8\pi^2} \int_0^1 dx \log\left(1 + \frac{m_\gamma^2(1-x)}{xM_e^2 - x(1-x)q^2}\right) \Bigg]$$

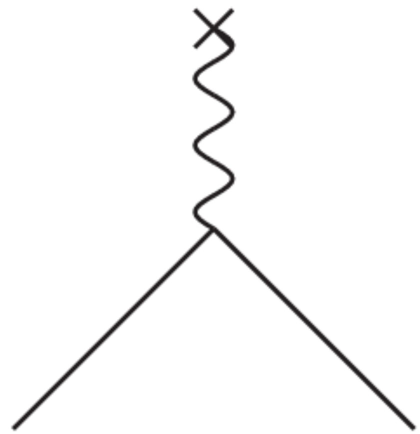
$$D = -2(-ie)^2(\omega - 1) \frac{i}{(4\pi)^\omega} \int_0^1 dx \frac{x\Gamma(2-\omega)}{[(1-x)M_e^2 - x(1-x)q^2 + xm_\gamma^2 - i\varepsilon]^{2-\omega}} + \frac{i}{8\pi^2} \int_0^1 dx \log\left(1 + \frac{m_\gamma^2(1-x)}{xM_e^2 - x(1-x)q^2}\right)$$

$$E = (-ie)^2 2\omega M \frac{i}{(4\pi)^\omega} \int_0^1 dx \frac{\Gamma(2-\omega)}{[(1-x)M_e^2 - x(1-x)q^2 + xm_\gamma^2 - i\varepsilon]^{2-\omega}}$$

$$+ M \frac{i}{8\pi^2} \int_0^1 dx \log\left(1 + \frac{m_\gamma^2(1-x)}{xM_e^2 - x(1-x)q^2}\right).$$



Coulomb Scattering

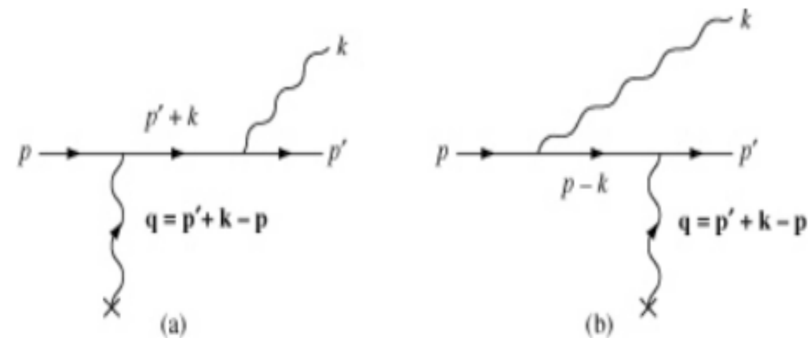


Mott Formula in VSR:

$$\frac{d\sigma}{d\Omega} = \frac{Z\alpha^2}{4|\vec{p}|^2\beta^2\sin^4\left(\frac{\theta}{2}\right)}\Bigg[1-\beta^2\sin^2\left(\frac{\theta}{2}\right)-\frac{m^2}{2M_e^2}+\frac{m^2}{4M_e^2}\left(\frac{1-\beta\sin\eta\sin\phi}{1-\beta\sin\eta\sin(\phi-\theta)}+\frac{1-\beta\sin\eta\sin(\phi-\theta)}{1-\beta\sin\eta\sin\phi}\right)$$
$$-\frac{m^2}{2M_e^2}\beta^2\sin^2\left(\frac{\theta}{2}\right)\frac{1}{(1-\beta\sin\eta\sin\phi)(1-\beta\sin\eta\sin(\phi-\theta))}\Bigg].$$

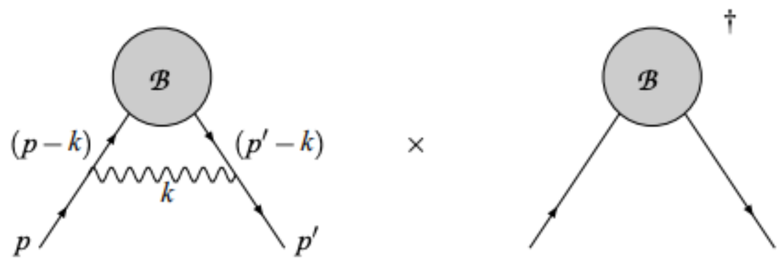
We compute radiative corrections.
Bremsstrahlung part:

$$|\mathcal{M}_{brem}|^2 \;=\; e^2 \int \frac{d^3\vec{k}}{(2\pi)^2 2\sqrt{|\vec{k}|^2+m_\gamma^2}} \left[\frac{2}{p\cdot k p'\cdot k}\left(p\cdot p' + m_\gamma^2 \frac{n\cdot p n\cdot p'}{(n\cdot k)^2}\right) - \frac{1}{(p'\cdot k)^2}\left(M_e^2 + m_\gamma^2 \frac{(n\cdot p')^2}{(n\cdot k)^2}\right) - \frac{1}{(p\cdot k)^2}\left(M_e^2 + m_\gamma^2 \frac{(n\cdot p)^2}{(n\cdot k)^2}\right)\right] |\mathcal{M}|^2.$$



Vertex correction to first order:

$$|\mathcal{M}_{mix}|^2 = -2e^2 \int \frac{d^3\vec{k}}{(2\pi)^2 2\sqrt{|\vec{k}|^2+m_\gamma^2}} \frac{1}{p\cdot k p'\cdot k}\left(p\cdot p' + m_\gamma^2 \frac{n\cdot p n\cdot p'}{(n\cdot k)^2}\right) |\mathcal{M}|^2.$$



Summary

- Very Special Relativity can solve the neutrino mas problem without introduce any new particle. The main feature is a privileged direction and a small Lorentz violation. There is a window to explore signals of that direction.
- We have shown some VSR-QED results in 1+1, 2+1 and 3+1 dimensions. There is a lot of new possible things to do.

Thanks!