AMPLITUDES, RESONANCES & THE ULTRAVIOLET COMPLETION OF GRAVITY

Rodrigo Alonso — IPPP

Durham University

HEP seminar, DAMPT April 30th

[R.A. & A. Urbano 1906.11687]

OUTLINE

1. ON SHELL AMPLITUDES

2. UNITARITY

3. RESONANCES FOR GRAVITY

4. ANALYSIS.

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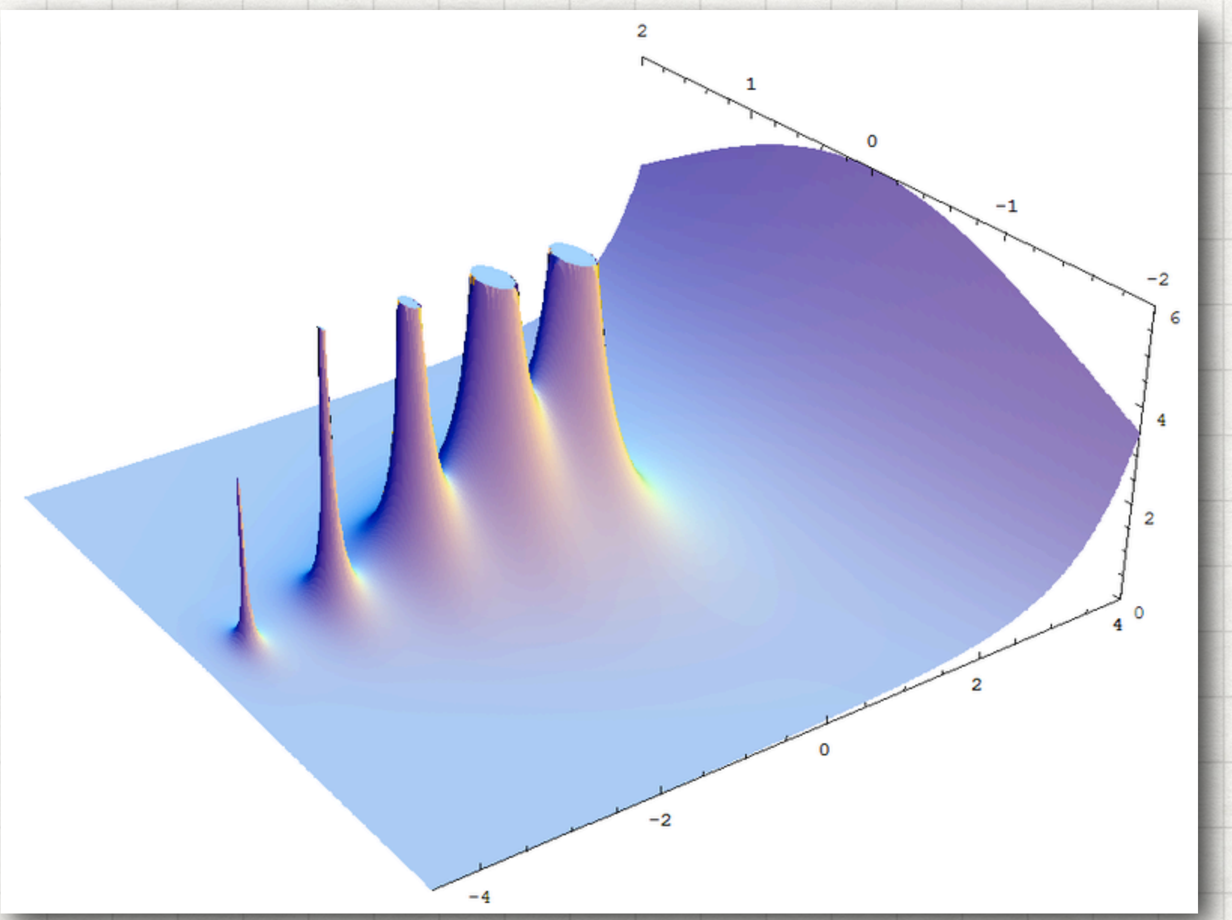
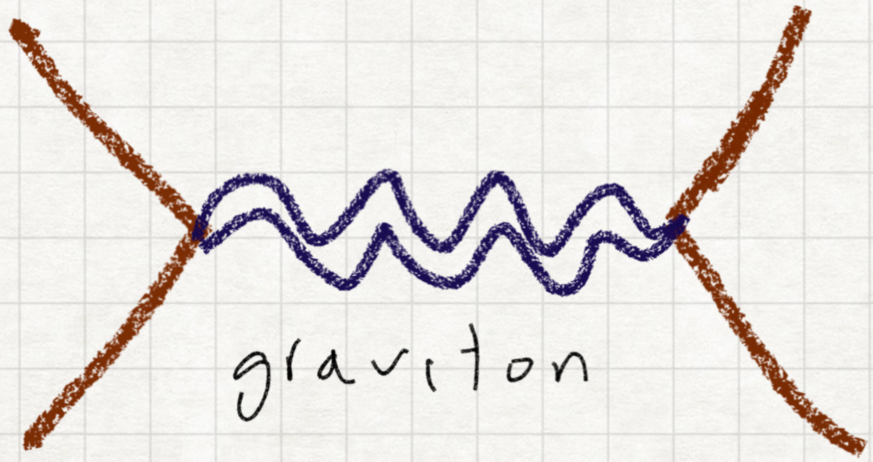
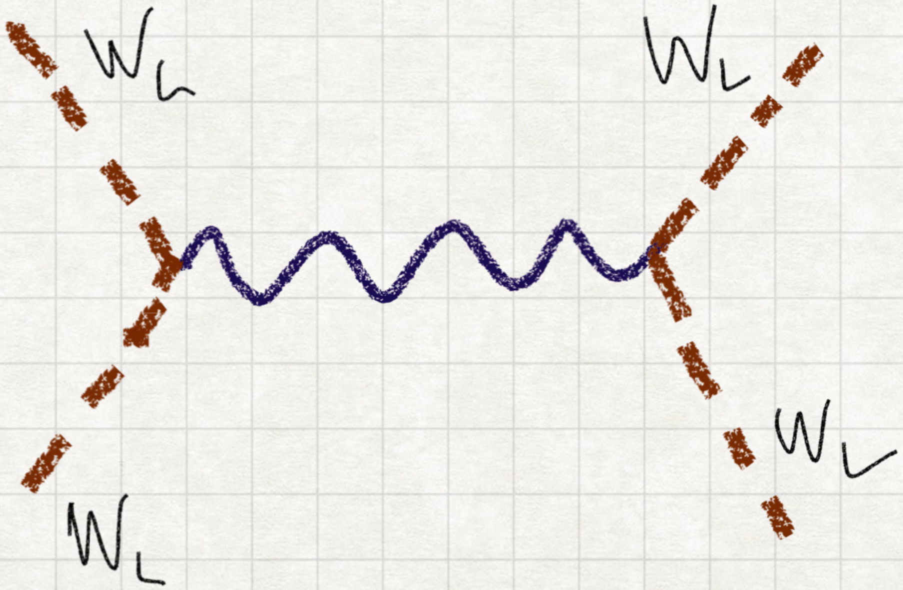


This talk in a nutshell



$$\mathscr{A} \sim \frac{s}{M_{\rm pl}^2}$$

Let's add Resonances to fix this



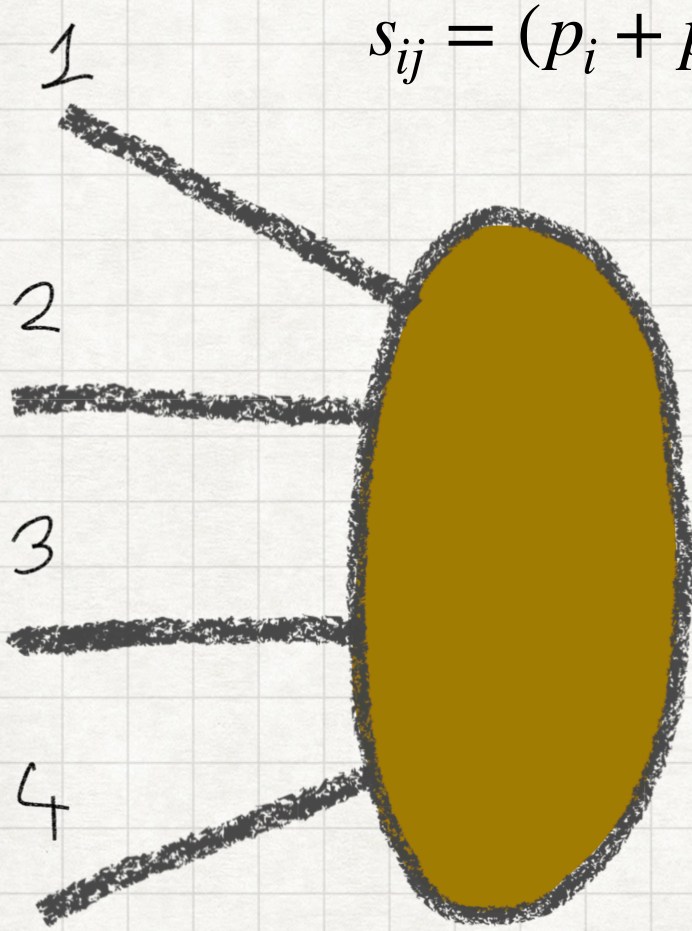
$$\mathscr{A} \sim \frac{s}{v_{\rm ew}^2}$$

The Higgs

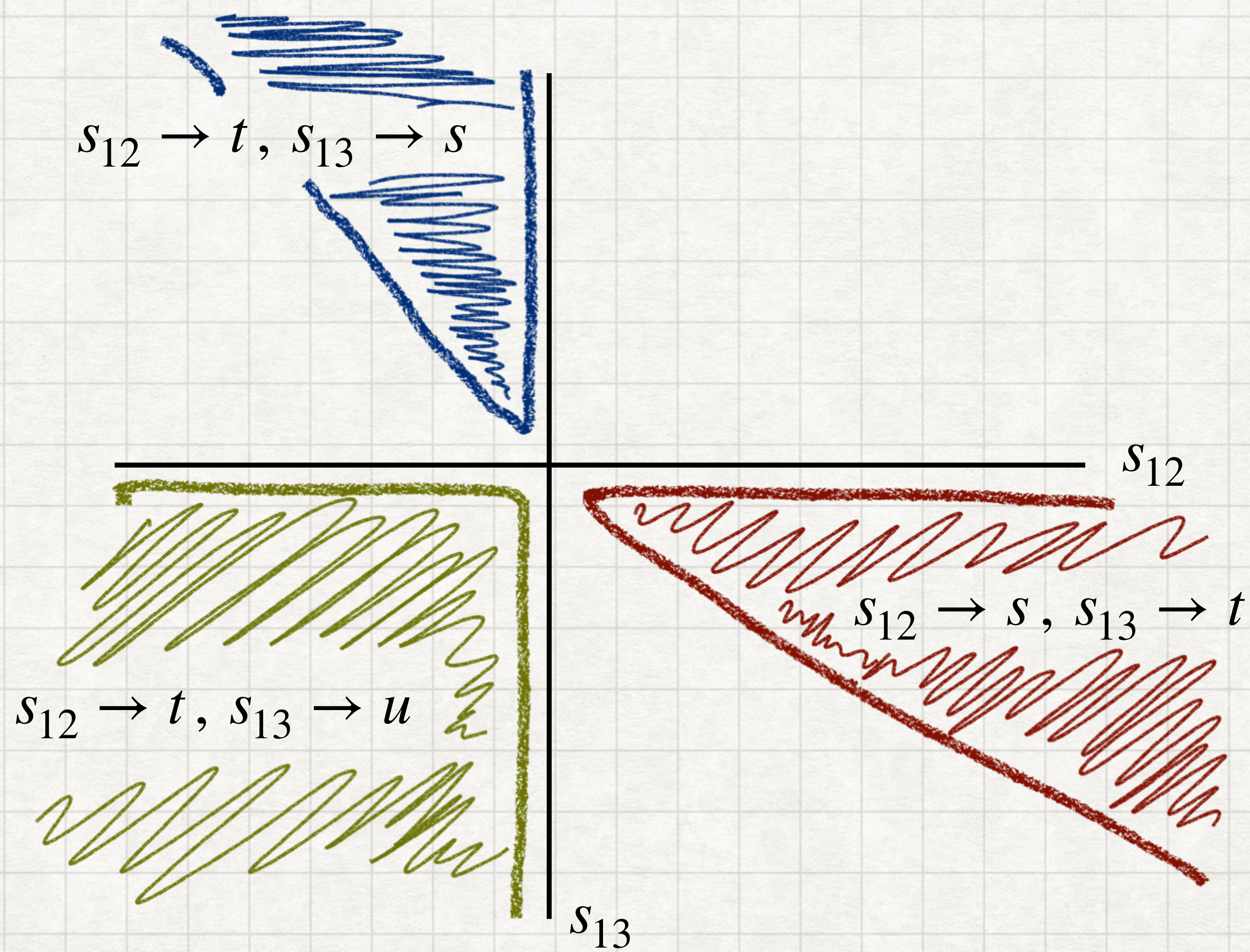
I. On-shell amplitudes

Setting the stage

$s_{ij} = (p_i + p_j)^2$



$\mathscr{A}(s_{12}, s_{13})$



$p_{2,4} \to -p_{2,4}$

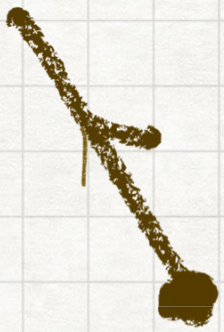
$p_{3,4} \to -p_{3,4}$

$p_{2,3} \to -p_{2,3}$

\+ Helicity Flip

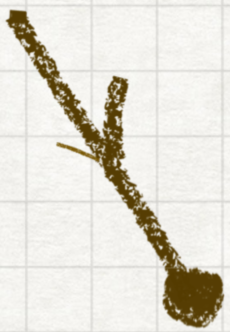
$$s_{12} + s_{13} + s_{14} = \sum m_i^2$$

Helicity scaling or Little group Reps.



$_\alpha \left|p\right\rangle \qquad SO(1,3); \quad \left|p\right\rangle \to i\Lambda^{\mu\nu}\sigma_{\mu\nu}\left|p\right\rangle \qquad U(1)_{LG}; \quad \left|p\right\rangle \to \left|p\right\rangle e^{-i\phi/2}$

(Just Weyl spinors)



$^{\dot\alpha} \left|p\right] \qquad SO(1,3); \quad \left|p\right] \to i\Lambda^{\mu\nu}\bar\sigma_{\mu\nu}\left|p\right] \qquad U(1)_{LG}; \quad \left|p\right] \to \left|p\right] e^{i\phi/2}$

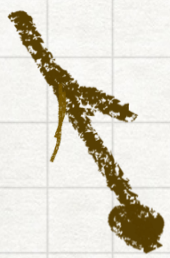
$$\varepsilon_+^\mu = \frac{\langle\xi\right|\sigma^\mu\left|p\right]}{\sqrt{2}\,\langle\xi\, p\rangle} \qquad U(1)_{LG}; \quad \frac{\langle\xi\right|\sigma^\mu\left|p\right]e^{i\phi/2}}{\sqrt{2}\,\langle\xi\, p\rangle\, e^{-i\phi/2}}$$

In this talk all external lines -> massless

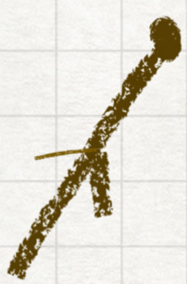
Learn more

[Dixon 9601359, Elvang & Huang 1308.1697]

Helicity scaling or Little group Reps. Massive



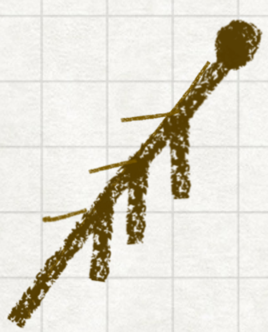
$$_\alpha \left|p_I\right\rangle \qquad SO(1,3); \quad \left|p_I\right\rangle \to i\Lambda^{\mu\nu}\sigma_{\mu\nu}\left|p_I\right\rangle \qquad SU(2)_{LG}; \quad \left|p\right\rangle \to \left|p_J\right]\, i[T_a]_{IJ}\phi_a$$



$$^{\dot\alpha} \left|p_I\right] \qquad SO(1,3); \quad \left|p_I\right] \to i\Lambda^{\mu\nu}\bar\sigma_{\mu\nu}\left|p_I\right] \qquad SU(2)_{LG}; \quad \left|p\right] \to \left|p_J\right]\, i[T_a]_{IJ}\phi_a$$

$$u(p) = \begin{pmatrix} \left|p_I\right\rangle \\ \left|p_I\right] \end{pmatrix}$$

Just half of
Dirac
Spinors



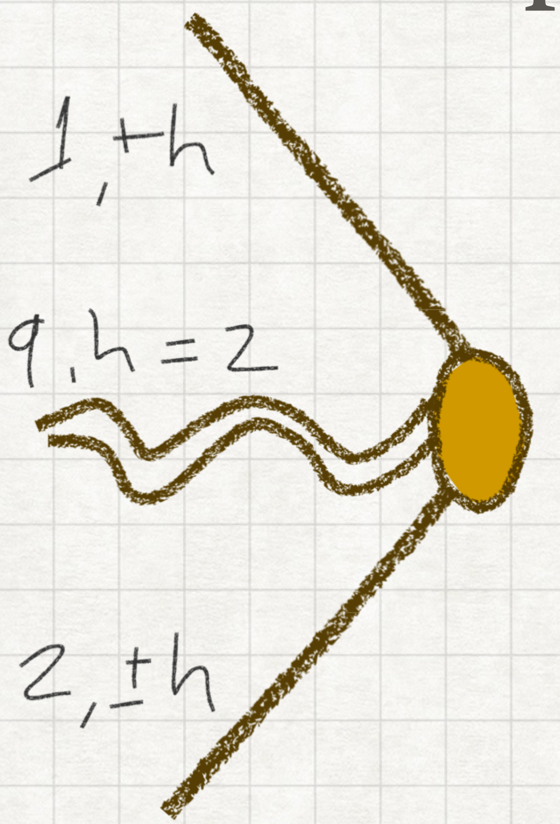
Spin J
Massive
State

$$\otimes^{2J}_{\text{sym}} \left|p\right] = \left|p_{I_1}\right] \times \left|p_{I_2}\right] \times \ldots \left|p_{I_{2J}}\right] \quad (2J+1 \text{ elements})$$

Learn more

[Arkani-Hamed &Huang(x2), 1709.04891]

3 point amplitude for gravity



$$\mathscr{A}_3^{\mathrm{GR}} = C\,|1]^{2h} \times |2]^{\pm 2h} \times |q]^4 = C[12]^a\,[1q]^b\,[2q]^c$$

h, h

h, −h

$$[12]^{2h-2}[1q]^2[2q]^2\,, \qquad [12]^{-2}[1q]^{2+2h}[2q]^{2-2h}\,.$$

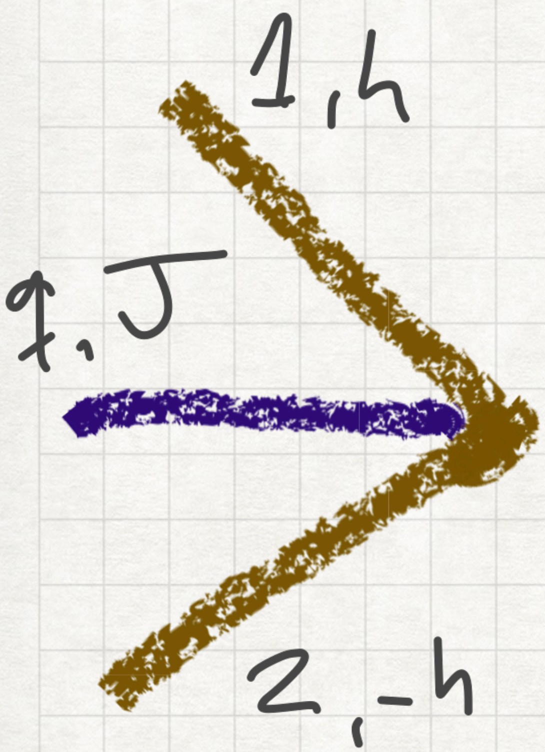
Mass dimension of C

$$\dim(\mathscr{A}_n) = 4 - n \quad \Rightarrow \quad [C] = \begin{pmatrix} 1 - 2h - 2 \\ 1 - 2 \end{pmatrix} \qquad C = \frac{\sqrt{8\pi}}{M_{\mathrm{pl}}}$$

Only opposite helicity case in GR

$$\frac{\sqrt{8\pi}}{M_{\mathrm{pl}}}\frac{[1q]^{2+2h}[2q]^{2-2h}}{[12]^2}$$

3 point amplitude massive spin J

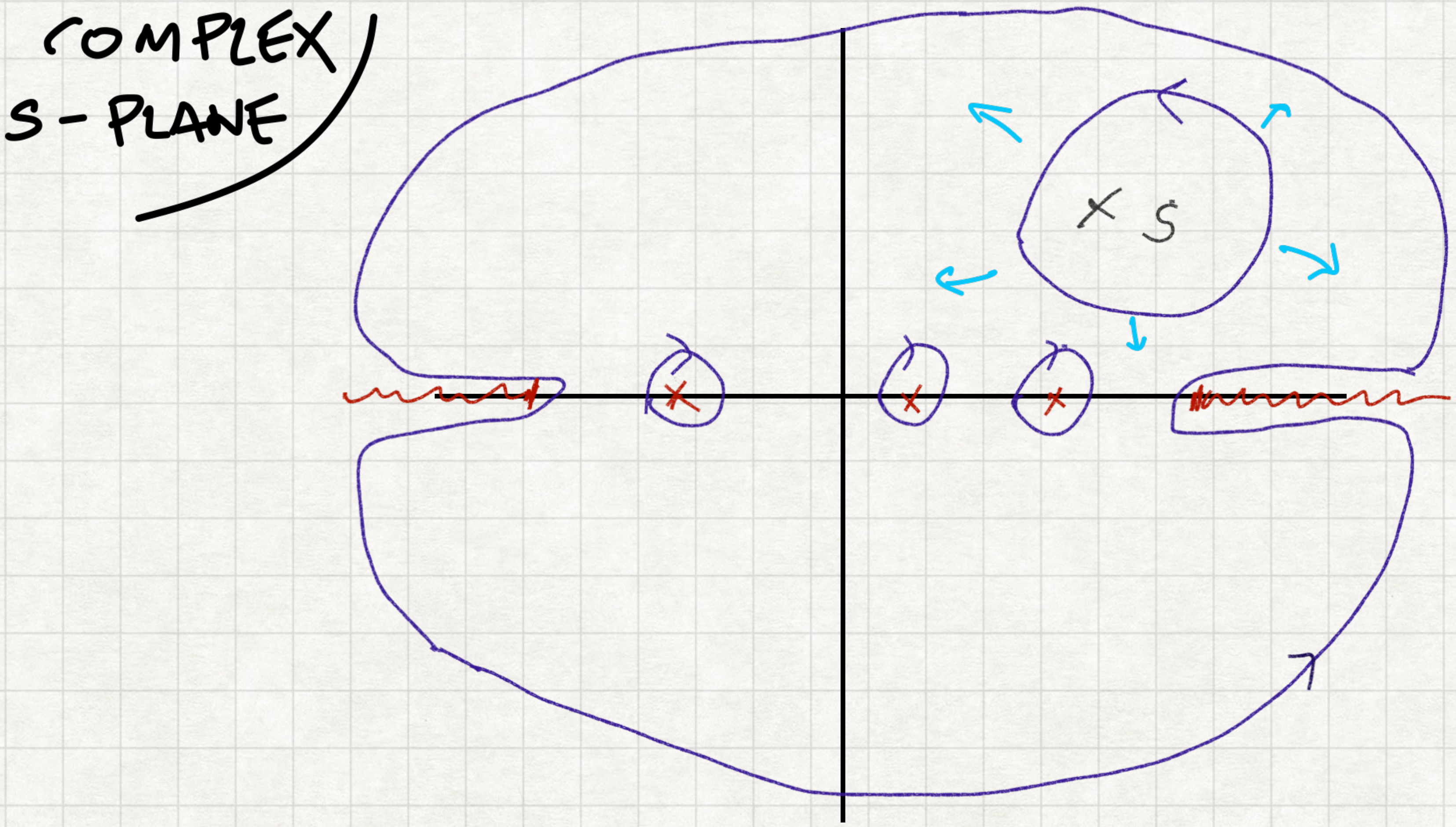


$$gM\frac{(\langle q_I \hat{P}_{12}\, q_I])^{J-2h}([1 q_I]\langle 2 q_I\rangle)^{2h}}{M^{2J}}$$

$$\hat{P} = \sigma_\mu P^\mu \qquad P_{ij} = p_i - p_j$$

On shell methods @ tree level

$$2\pi i \mathscr{A}(s,t) = \oint \frac{\mathscr{A}(s',t)}{s'-s} ds'$$

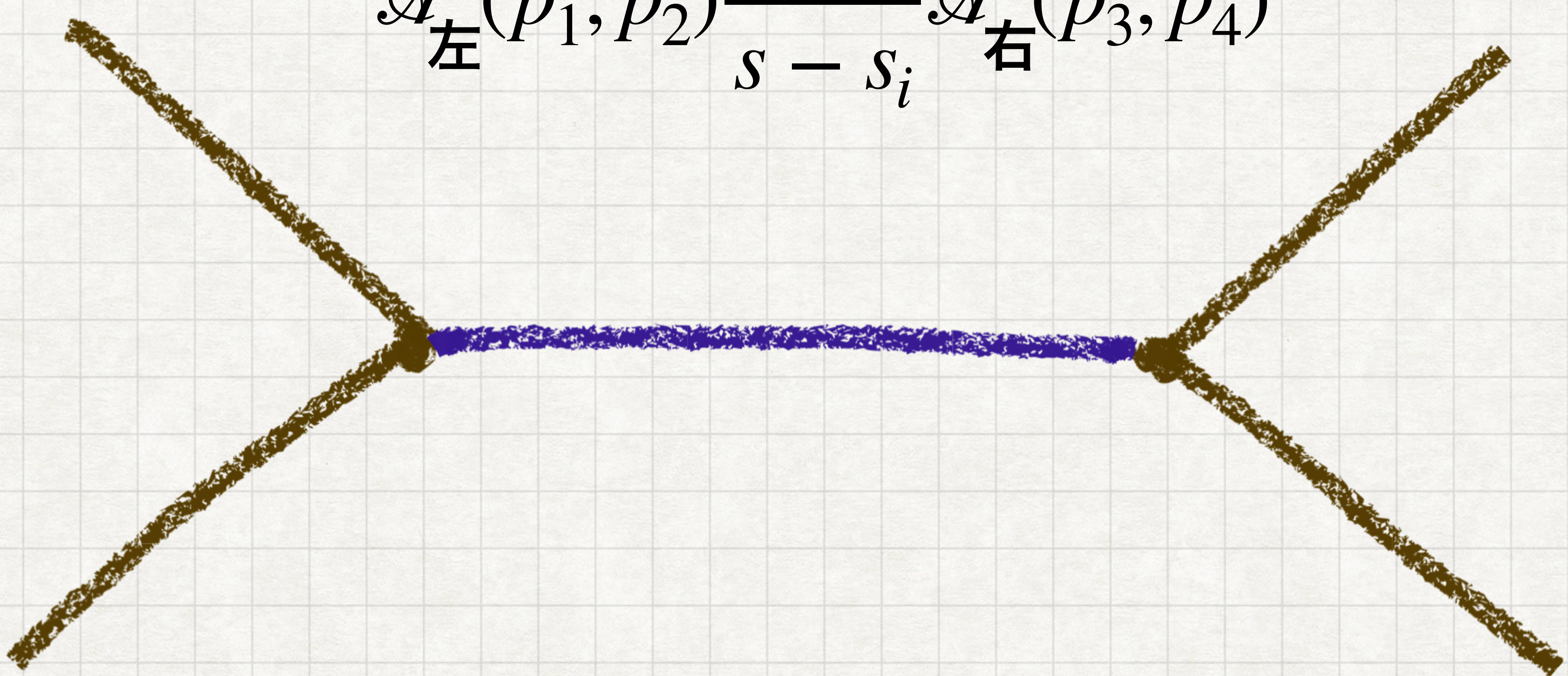


$$2\pi i \mathscr{A}(s,t) = -\oint_{\text{Poles}} \frac{\mathscr{A}(s',t)}{s'-s} ds' + \oint_{\infty} \frac{\mathscr{A}(s',t)}{s'-s} ds'$$

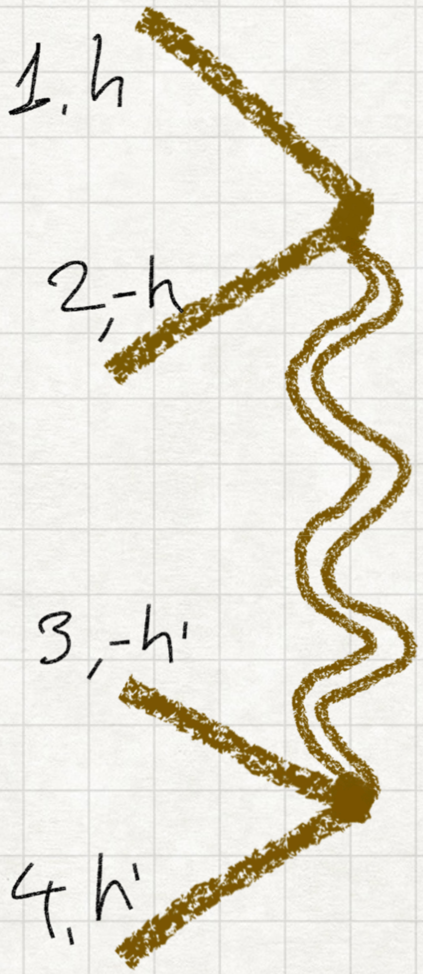
On shell part of the amplitude

$$2\pi i \mathscr{A}(s,t) = -\oint_{\text{Poles}} \frac{\mathscr{A}(s',t)}{s'-s} ds' + B = -\frac{\text{Res}(\mathscr{A})_{s=s_i}}{s_i - s} + B$$

$$\mathscr{A}_{\text{左}}(p_1, p_2) \frac{1}{s - s_i} \mathscr{A}_{\text{右}}(p_3, p_4)$$



4 pt amplitude from 3 pt in Gravity



$$\frac{\sqrt{8\pi}}{M_{\rm pl}}\frac{[1q]^{2+2h}[2q]^{2-2h}}{[12]^2}$$

$$\frac{1}{s_{12}}$$

$$\frac{\sqrt{8\pi}}{M_{\rm pl}}\frac{\langle 3q\rangle^{2+2h'}\langle 4q\rangle^{2-2h'}}{\langle 34\rangle^2}$$

Using on-shell relations like

$$|q\rangle[q| = \bar{\sigma}_\mu q^\mu = -\bar{\sigma}_\mu p_1^\mu - \bar{\sigma}_\mu p_2^\mu$$

$$\hat{P} = \sigma_\mu P^\mu$$

We obtain:

$$= \frac{8\pi}{M_{\rm pl}^2} c_r \left(\frac{s_{14}}{s_{13}}\right)^r \frac{(s_{13}s_{14})^{1-h'}}{s_{12}} ([14]\langle 23\rangle)^{2h}(\langle 3\hat{P}_{12}4])^{2h'-2h}$$

$$h-1 < r < 1-h \qquad h'-1 < r < 1-h'$$

Ambiguity in fermions and scalars

-> Scalar

$$\frac{8\pi}{M_{\mathrm{pl}}^2}\left(\frac{s_{13}s_{14}}{s_{12}} - as_{12}\right)$$

Contact terms

-> Fermion

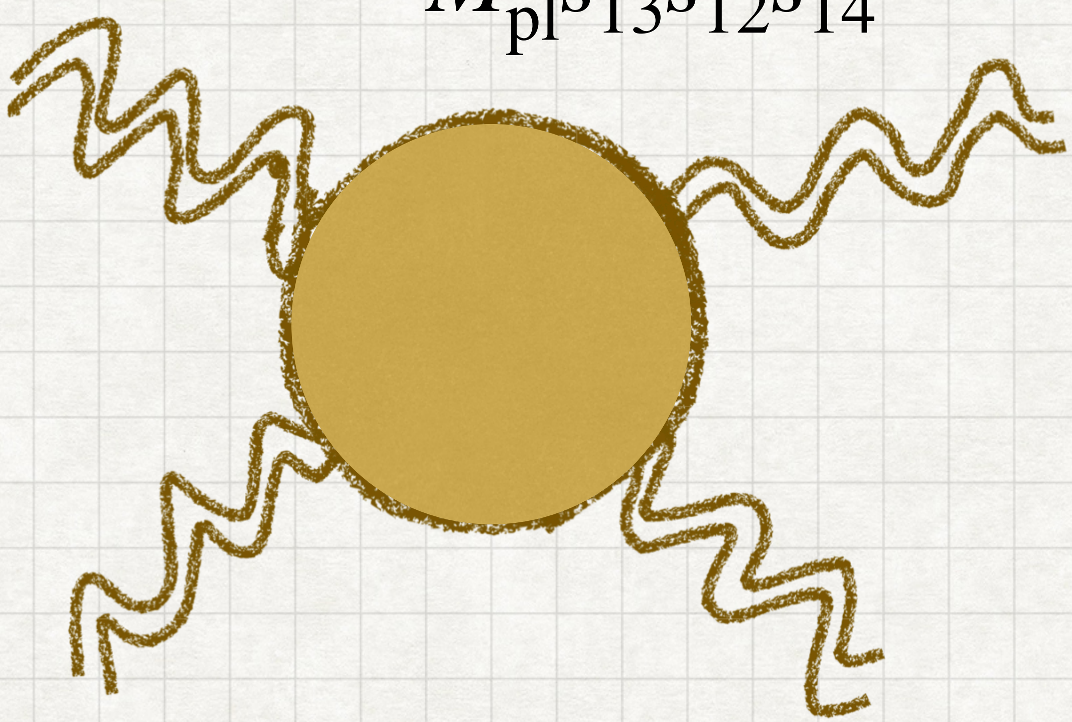
$$\frac{8\pi[14]\langle 23\rangle}{M_{\mathrm{pl}}^2}\left(\frac{s_{13}}{s_{12}} + \frac{b}{2}\right)$$

We have computed more than we know!

$$\mathscr{A} = \frac{8\pi}{M_{\rm pl}^2}\frac{(s_{13}s_{14})^{1-h'}}{s_{12}}([14]\langle 23\rangle)^{2h}(\langle 3\hat{P}_{12}4])^{2h'-2h}$$

$$h = h' = 2$$

$$\frac{8\pi[14]^4\langle 23\rangle^4}{M_{\rm pl}^2 s_{13}s_{12}s_{14}}$$



The Feynman rule computation has ~1000 terms!

[B. DeWitt, Phys. Rev. 162 1239, 1967]

These are all of them

$\mathcal{A}^{\mathrm{GR}}_{1^h 2^{-h} 3^{-h'} 4^{h'}}$	**Scalar**	**Fermion**	**Vector**	**Graviton**
Scalar	$\frac{8\pi}{M_{\mathrm{Pl}}^2}\left(\frac{s_{13}s_{14}}{s_{12}} - a s_{12}\right)$ $\frac{8\pi}{M_{\mathrm{Pl}}^2}\left(\frac{s_{13}s_{14}}{s_{12}} + \frac{s_{12}s_{14}}{s_{13}} + \frac{s_{13}s_{12}}{s_{14}}\right)$	$\frac{8\pi\left(\langle 3\vert\hat{P}_{12}\vert 4]\right)}{M_{\mathrm{Pl}}^2}\left(\frac{s_{13}-s_{14}}{2s_{12}}\right)$	$-\frac{8\pi\left(\langle 3\vert\hat{P}_{12}\vert 4]\right)^2}{M_{\mathrm{Pl}}^2 s_{12}}$	$\frac{8\pi\left(\langle 3\vert\hat{P}_{12}\vert 4]\right)^4}{M_{\mathrm{Pl}}^2 s_{12}s_{13}s_{14}}$
Fermion		$-\frac{8\pi\langle 2\,3\rangle[1\,4]}{M_{\mathrm{Pl}}^2}\left(\frac{s_{13}}{s_{12}} + \frac{b}{2}\right)$ $-\frac{8\pi\langle 2\,3\rangle[1\,4]}{M_{\mathrm{Pl}}^2}\left(\frac{s_{13}}{s_{12}} + \frac{s_{12}}{s_{13}} + b\right)$	$\frac{8\pi\langle 2\,3\rangle[1\,4]\left(\langle 3\vert\hat{P}_{12}\vert 4]\right)}{M_{\mathrm{Pl}}^2 s_{12}}$	$-\frac{8\pi\langle 2\,3\rangle[1\,4]\left(\langle 3\vert\hat{P}_{12}\vert 4]\right)^3}{M_{\mathrm{Pl}}^2 s_{12}s_{13}s_{14}}$
Vector			$-\frac{8\pi\langle 2\,3\rangle^2[1\,4]^2}{M_{\mathrm{Pl}}^2 s_{12}}$ $-\frac{8\pi\langle 2\,3\rangle^2[1\,4]^2}{M_{\mathrm{Pl}}^2}\left(\frac{1}{s_{12}} + \frac{1}{s_{13}}\right)$	$\frac{8\pi\langle 2\,3\rangle^2[1\,4]^2\left(\langle 3\vert\hat{P}_{12}\vert 4]\right)^2}{M_{\mathrm{Pl}}^2 s_{12}s_{13}s_{14}}$
Graviton				$\frac{8\pi\langle 2\,3\rangle^4[1\,4]^4}{M_{\mathrm{Pl}}^2 s_{12}s_{13}s_{14}}$

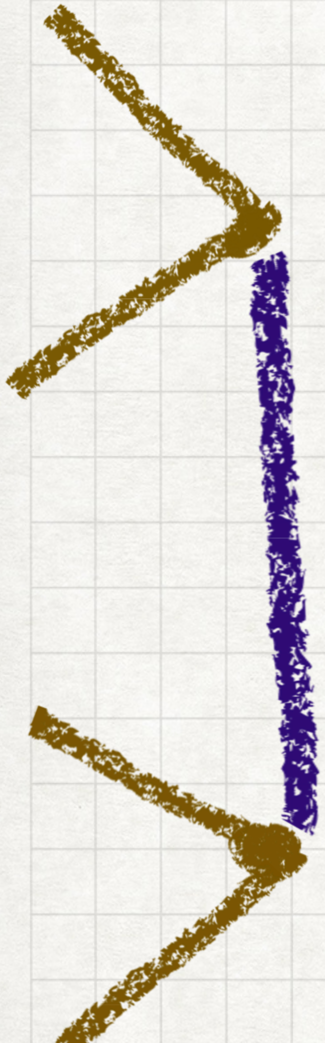
Maximal Helicity (non) Violation

$$\sum h_i = 0$$

If we exchange a massive spin J? Legendre

$$|q^I\rangle[q_I| = \bar{\sigma}_\mu q^\mu = -\bar{\sigma}_\mu p_1^\mu - \bar{\sigma}_\mu p_2^\mu$$

USING THIS



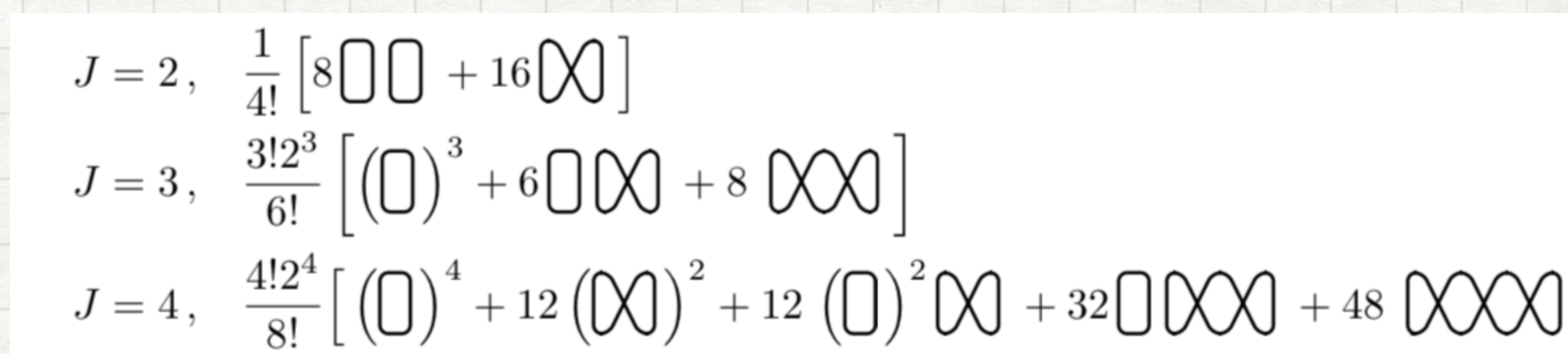
$$gM\left(\frac{\langle q_I \hat{P}_{12}\, q_I]}{M^2}\right)^J$$

$$\frac{1}{q^2 - M^2} =$$

$$gM\left(\frac{\langle q_I \hat{P}_{34}\, q_I]}{M^2}\right)^J$$

$$\langle \mathbf{q}_{\mathbf{I}_1} | \hat{P}_{12} | \mathbf{q}_{\mathbf{I}_2}] \times \langle \mathbf{q}_{\mathbf{I}_3} | \hat{P}_{12} | \mathbf{q}_{\mathbf{I}_4}] \times \langle \mathbf{q}_{\mathbf{I}_5} | \hat{P}_{12} | \mathbf{q}_{\mathbf{I}_6}] \times \ldots$$

$$\langle \mathbf{q}^{\mathbf{I}_1} | \hat{P}_{34} | \mathbf{q}^{\mathbf{I}_2}] \times \langle \mathbf{q}^{\mathbf{I}_3} | \hat{P}_{34} | \mathbf{q}^{\mathbf{I}_5}] \times \langle \mathbf{q}^{\mathbf{I}_4} | \hat{P}_{34} | \mathbf{q}^{\mathbf{I}_6}] \times \ldots$$



$$\mathrm{tr}(\bar{\sigma}^\mu p_\mu \sigma^\mu k_\mu)^n = 2^n \sum_m \binom{n}{2m} (p \cdot k)^{n-2m} \left[(p \cdot k)^2 - p^2 k^2\right]^m .$$

If we exchange a massive spin J? Legendre

$$|q^I\rangle[q_I| = \bar{\sigma}_\mu q^\mu = -\bar{\sigma}_\mu p_1^\mu - \bar{\sigma}_\mu p_2^\mu$$

USING THIS

$$gM\left(\frac{\langle q_I \hat{P}_{12}\, q_I]}{M^2}\right)^J$$

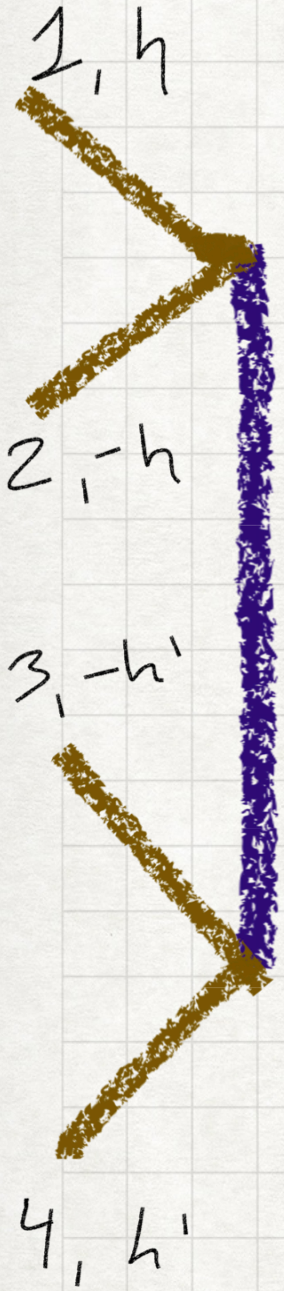
$$\frac{1}{q^2 - M^2}$$

$$gM\left(\frac{\langle q_I \hat{P}_{34}\, q_I]}{M^2}\right)^J$$

$$= \frac{g^2(2J)!!}{(2J-1)!!}\frac{M^2}{s_{12}-M^2} \times \frac{(2J-2m)!(-P_{12}^2P_{34}^2)^m(P_{12}\cdot P_{34})^{J-2m}}{m!(J-m)!(J-2m)2^J M^{2J}}$$

$$= \frac{P_J(x)}{4^J}, \qquad x = 1 + \frac{2s_{13}}{M^2}.$$

If we exchange a massive spin J? Jacobi



$$gM\,\frac{(\langle q_I \hat{P}_{12}\, q_I])^{J-2h}([1q_I]\langle 2q_I\rangle)^{2h}}{M^{2J}}$$

$$\frac{1}{q^2-M^2}$$

$$gM\,\frac{(\langle q_I \hat{P}_{12}\, q_I])^{J-2h'}([4q_I]\langle 3q_I\rangle)^{2h'}}{M^{2J}}$$

$$= \frac{g^2(2J)!!(J-2h')!(J+2h')!}{(2J-1)!!J!^2}\left(\frac{[14]\langle 32\rangle}{M^2}\right)^{2h}\left(\frac{\langle 3\hat{P}_{12}4]}{M^2}\right)^{2h'-2h} P_{J-2h'}^{(2h'-2h,2h'+2h)}(x)\,\frac{M^2}{s_{12}-M^2}$$

JACOBI POLYNOMIALS

$$P_n^{(a,b)}(x) \equiv \sum \binom{n+a}{n-k}\binom{n+b}{k}\left(\frac{x-1}{2}\right)^k\left(\frac{x+1}{2}\right)^{n-k}$$

$$x = 1 + \frac{2s_{13}}{M^2}.$$

II. Unitarity

Angular Analysis and unitarity? Wigner

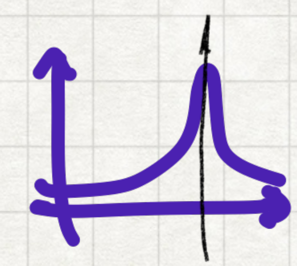
$$s_{12} \to s\,, \quad s_{13} \to -\,s\sin^2(\theta/2)\,, \quad \langle 13\rangle \to \sqrt{s}\,\sin(\theta/2)\,, \quad \ldots$$

$$\mathscr{A} = \left(\frac{[14]\langle 32\rangle}{M^2}\right)^{2h} \left(\frac{\langle 3\hat{P}_{12}4]}{M^2}\right)^{2h-2h} P_{J-2h'}^{(2h'-2h,2h'+2h)}(x)$$

$$\left(\frac{s\cos^2(\theta/2)}{M^2}\right)^{2h} \left(\frac{s\sin(\theta/2)\cos(\theta/2)}{M^2}\right)^{2h-2h}$$

$$P_{J-2h'}^{(2h'-2h,2h'+2h)}(1-(1-\cos\theta)s/M^2)$$

On-shell — $s = M^2$ —



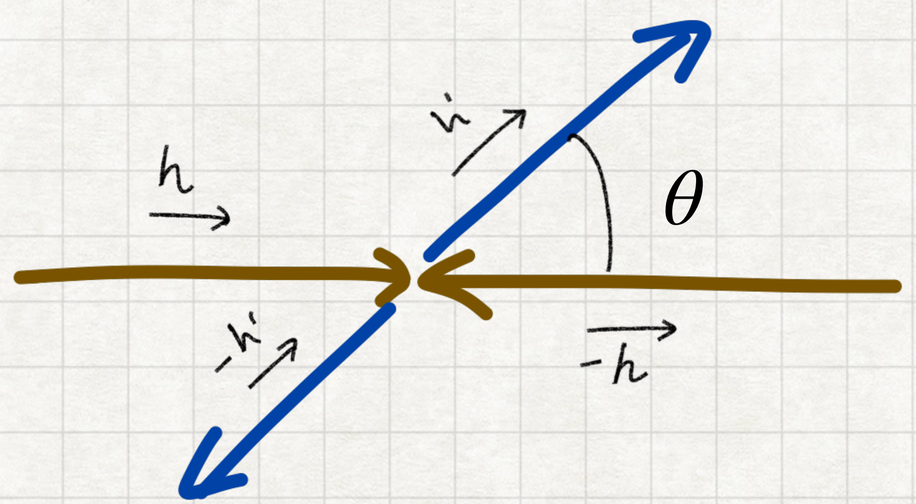
$$P_{J-2h'}^{(2h'-2h,2h'+2h)}(\cos\theta)$$

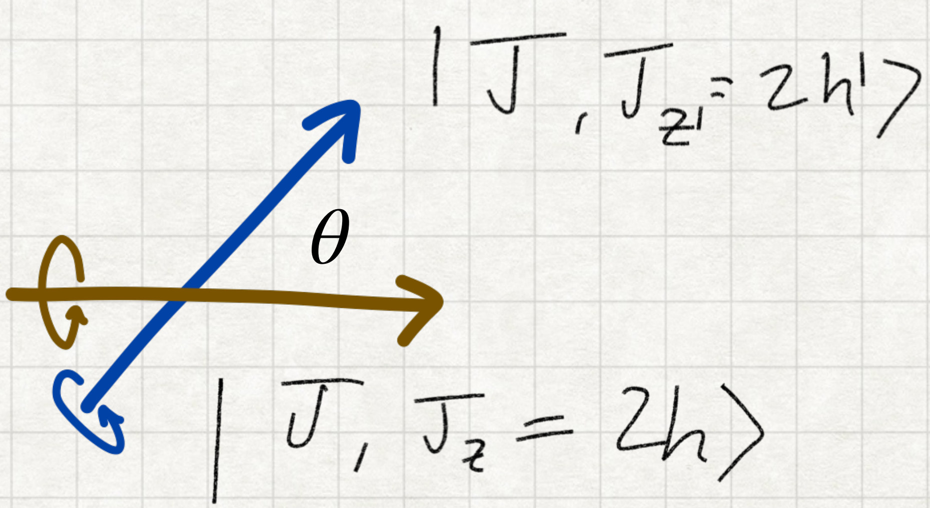
Wigner d-function and angular analysis

$$\sin^{2h'-2h}(\theta/2)\ \cos^{2h+2h'}(\theta/2)\, P_{J-2h'}^{(2h'-2h,2h'+2h)}(\cos\theta)$$

$$\propto d^J_{2h,2h'}(\theta)$$

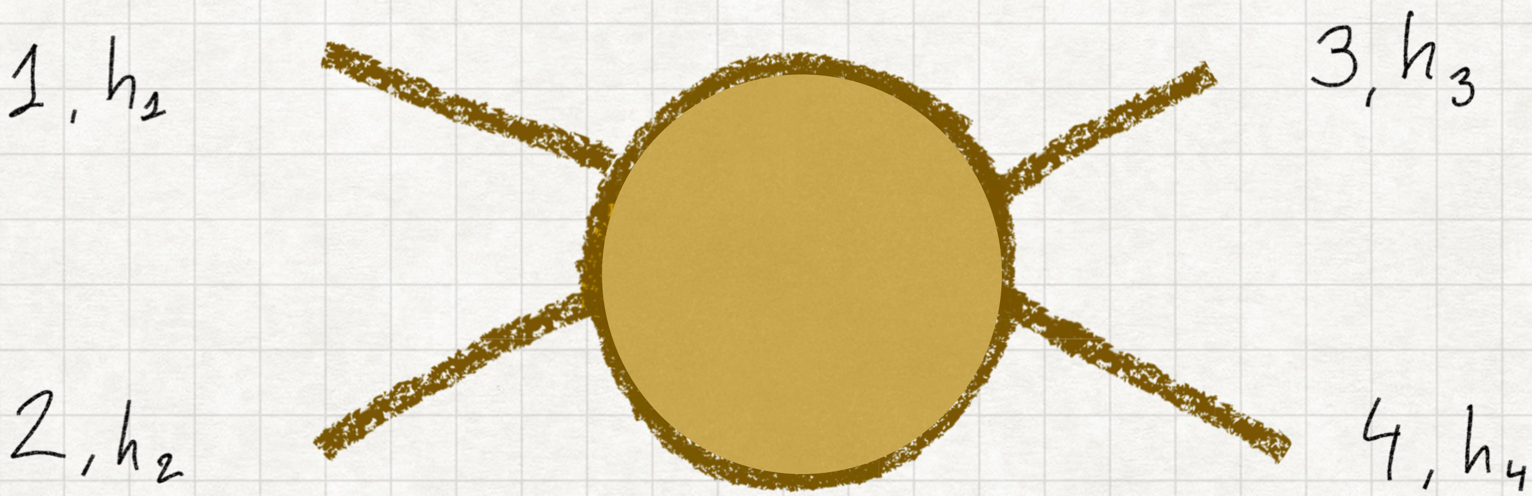
$$d^J_{m',m} = \langle J, m' \,|\, R(\theta) \,|\, J, m\rangle$$





Wigner d-function and angular analysis

$$\mathscr{A} = \sum 16\pi(2J+1)a^J(s)d^J_{h_1-h_2,h_3-h_4}(\theta)$$



Partial wave expansion

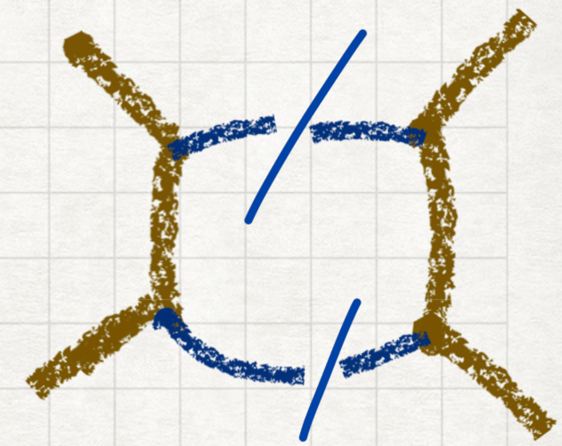
a's bounded to be less than one to conserve probability

$$a^J = \frac{1}{32\pi}\int d\cos(\theta)\, d^J_{h_1-h_2,h_3-h_4}(\theta)\mathscr{A}(s,\theta)$$

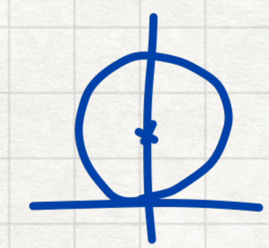
Unitarity

$$(1 + i\mathscr{A})(1 + i\mathscr{A})^\dagger = 1$$

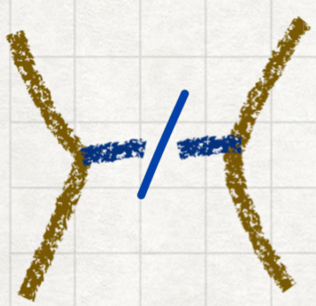
$$2\,\mathrm{Im}(\mathscr{A}_{12\to 34}) = \int \prod_i \frac{dp_i^3}{(2E_i)(2\pi)^3} \mathscr{A}^*_{34\to n} \mathscr{A}_{12\to n}$$



If 12=34 and we do angular dec. $\mathrm{Im}(a^J) \leq 1,\ \mathrm{Re}(a^J) \leq 1/2\,.$



For a single particle intermediate state n



$$2\,\mathrm{Im}(\mathscr{A}_{12\to 34}) = \pi \mathscr{A}^*_{34\to J}(\theta)\mathscr{A}_{12\to J}(\theta)\,\delta(s - M^2)$$

$$\frac{1}{s - M^2 + i\epsilon} = \mathscr{PV}\left[\frac{1}{s - M^2}\right] - i\pi\delta(s - M^2)$$

Unitarity

$$(1 + i\mathscr{A})(1 + i\mathscr{A})^\dagger = 1$$

$$2\,\mathrm{Im}(\mathscr{A}_{12\to 34}) = \pi \mathscr{A}^*_{34\to J}(\theta)\mathscr{A}_{12\to J}(\theta)\,\delta(s - M^2)$$

If 12=34

$$2\,\mathrm{Im}(\mathscr{A}_{12\to 12}) = \pi\,|\,\mathscr{A}_{12\to J}(\theta)\,|^2\,\delta(s - M^2)$$

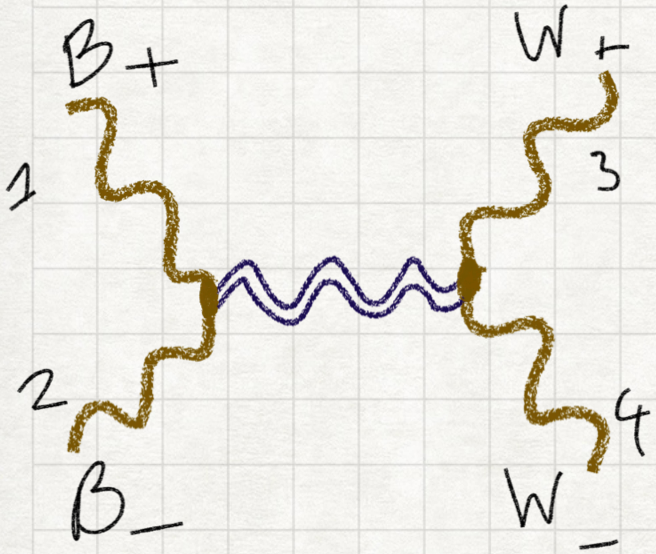
Positivity

If in the forward direction

$$2\,\mathrm{Im}(\mathscr{A}_{12\to 34})(\theta = 0) = 16\pi(2J + 1)M\Gamma_{J\to 12}\,\delta(s - M^2)$$

III. Gravity in the ultraviolet

Gravity in the ultraviolet

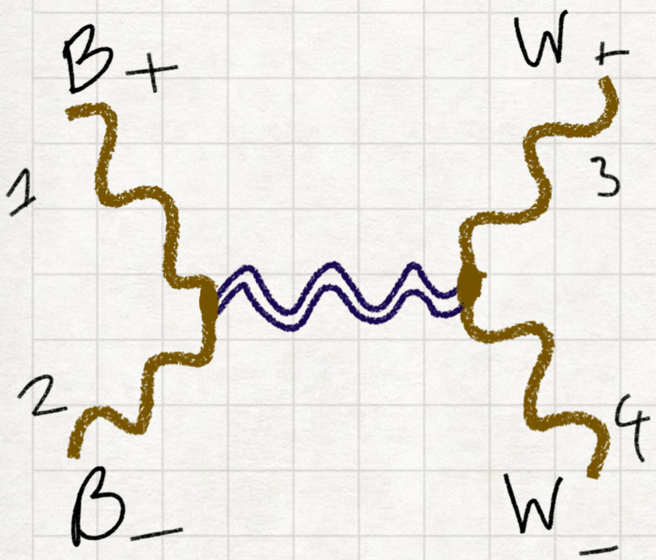


$$\mathscr{A}_{B,W} = -\frac{8\pi}{M_{\rm pl}^2}\frac{\langle 32\rangle^2[41]^2}{s_{12}}$$

Second partial wave coeff.

$$a_{\rm GR}^2 = \frac{s}{10M_{\rm pl}^2}$$

Gravity in the ultraviolet



$$\mathcal{A}_{B,W} = -\frac{8\pi}{M_{\rm pl}^2}\frac{\langle 32\rangle^2[41]^2}{s_{12}}$$

Second partial wave coeff.

$$a^2_{\rm GR} = \frac{s}{10M_{\rm pl}^2}$$

$= 1$

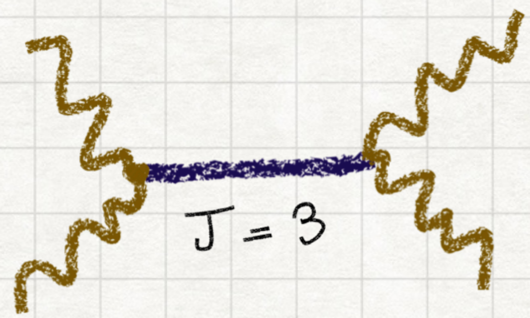
+

$$\mathcal{A}^{\rm GR}_{B,W} + \mathcal{A}^{J=2}_{B,W} = \langle 23\rangle^2[14]^2\left(\frac{8\pi}{M_{\rm pl}^2 s} + \frac{g_B g_W P_0^{(0,4)}(x)}{M_2^2(s-M_2^2)}\right)$$

$$g_B g_W = -8\pi\frac{M_2^2}{M_{\rm pl}^2}$$

But opposite sign does not extend to a third especies, e.g. gluon

Gravity in the ultraviolet

+ 

$1 + 6t/M_3^2$

$$\mathscr{A}^{\rm GR}_{B,W} + \mathscr{A}^{J=3}_{B,W} = \langle 23\rangle^2[14]^2\left(\frac{8\pi}{sM_{\rm pl}^2} + \frac{g_B g_W P_1^{(0,4)}(x)}{M_3^2(s-M_3^2)}\right)$$

$$a^2_{\rm GR} = \frac{s}{10M_{\rm pl}^2} - \frac{g_B g_W s^2}{80M_3^4}$$

Gravity in the ultraviolet

$$\mathscr{A}^{\rm GR}_{B,W} + \sum_J \mathscr{A}^J_{B,W} = \langle 23\rangle^2[14]^2\left(\frac{8\pi}{sM_{\rm pl}^2} + \sum_J \frac{C_J P_J^{(0,4)}(1+2t/M_J^2)}{s-M_J^2}\right)$$

$$\equiv \frac{\langle 23\rangle^2[14]^2}{sM_{\rm pl}^2}\frac{\prod_n (t-f_n(s))}{\prod_i (s-M_i^2)}$$

Solving for fn is not straightforward so we will make a number of assumptions

Gravity in the ultraviolet

$$\frac{\langle 23\rangle^2[14]^2}{sM_{\rm pl}^2}\frac{\prod(t-f_n(s))}{\prod(s-M_i^2)}\Bigg|_{s=M_i} \propto \prod^{\infty}(t-f_n(M_i^2))$$

✱ fn analytic around M

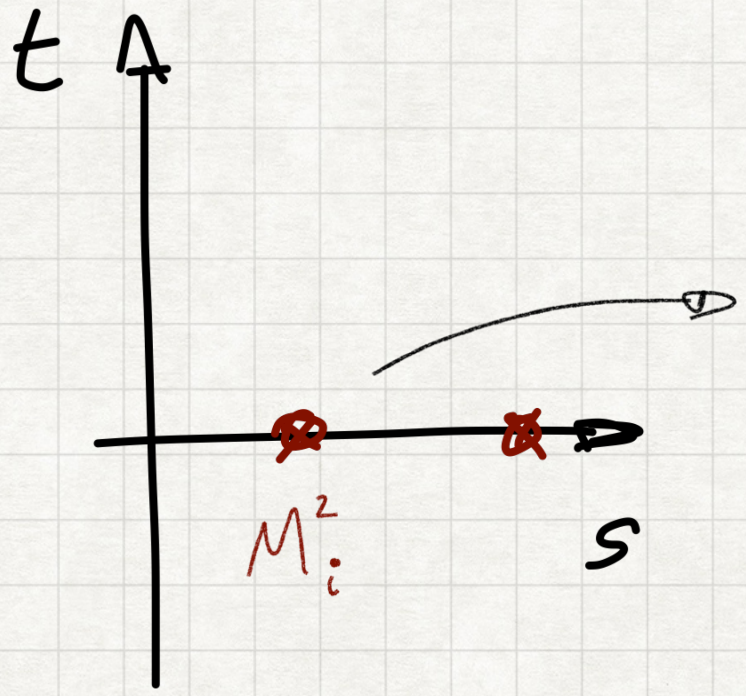
We don't want an infinite degree (infinite spin)

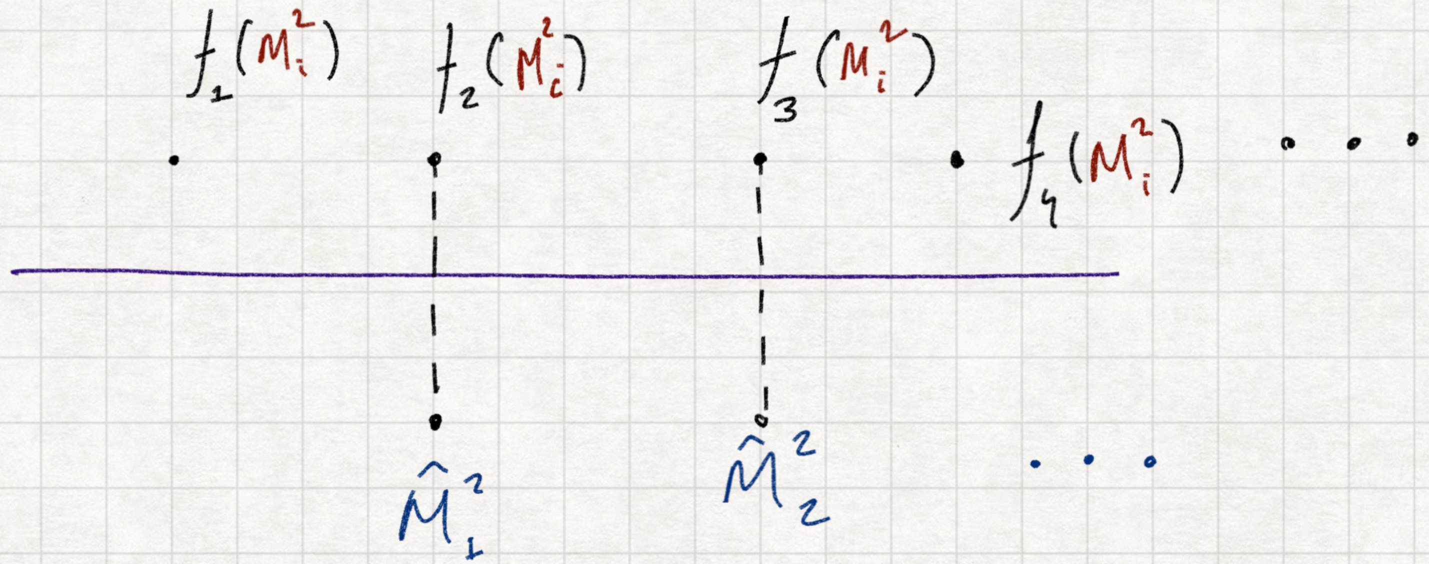
So we introduce inverse powers of t

$$\frac{\langle 23\rangle^2[14]^2}{sM_{\rm pl}^2}\frac{\prod(t-f_n(s))}{\prod(s-M_i^2)\prod(t-\hat{M}_j^2)}$$

Gravity in the ultraviolet

$$\frac{\langle 23\rangle^2[14]^2}{sM_{\rm pl}^2}\frac{\prod(t-f_n(s))}{\prod(s-M_i^2)\prod(t-\hat{M}_j^2)}\Bigg|_{s=M_i^2} \propto \frac{\prod(t-f_n(M_i^2))}{\prod(t-\hat{M}_i^2)}$$



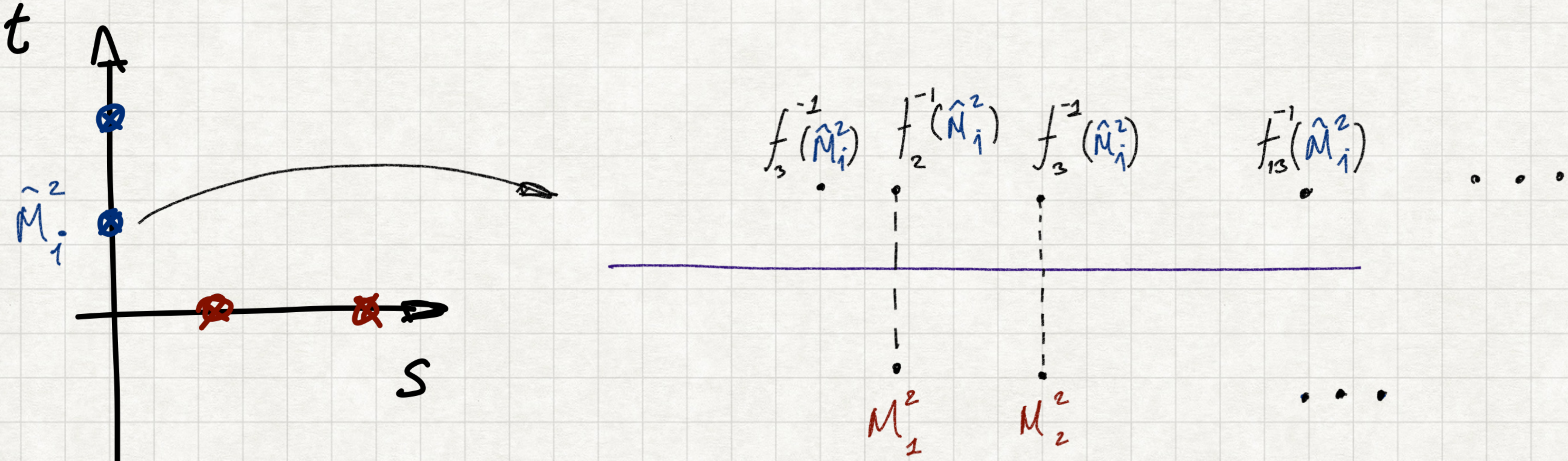


$$\{f_n(s_i)\} \supset \{\hat{M}_j^2\}\,, \quad \forall i\,.$$

The zeroes contain the poles

Gravity in the ultraviolet

$$\frac{\langle 23\rangle^2[14]^2}{sM_{\rm pl}^2}\frac{\prod(t-f_n(s))}{\prod(s-M_i^2)\prod(t-\hat{M}_j^2)}\Bigg|_{t=\hat{M}_j^2} \propto \frac{\prod(\hat{M}_j^2-f_n(s))}{\prod(s-M_i^2)}$$

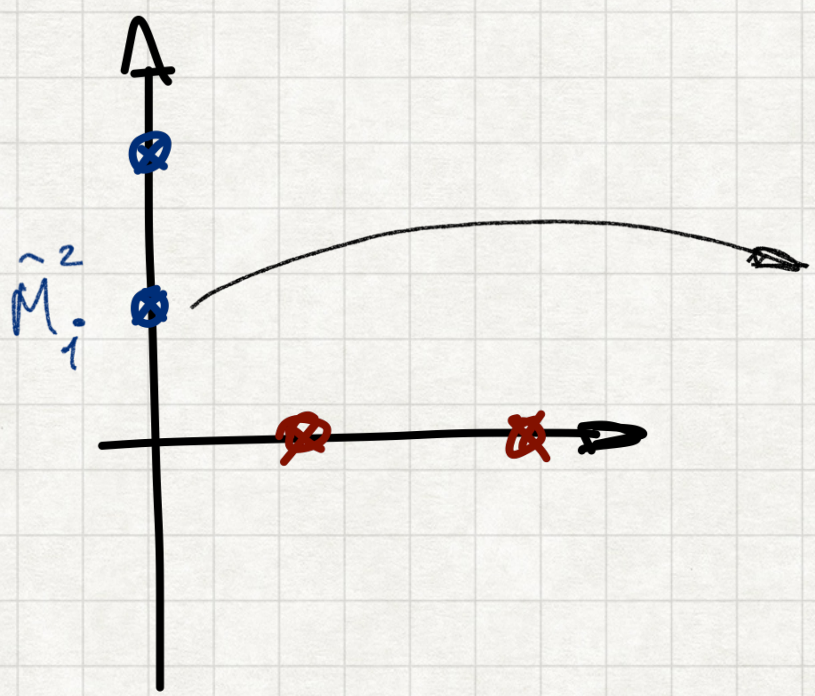


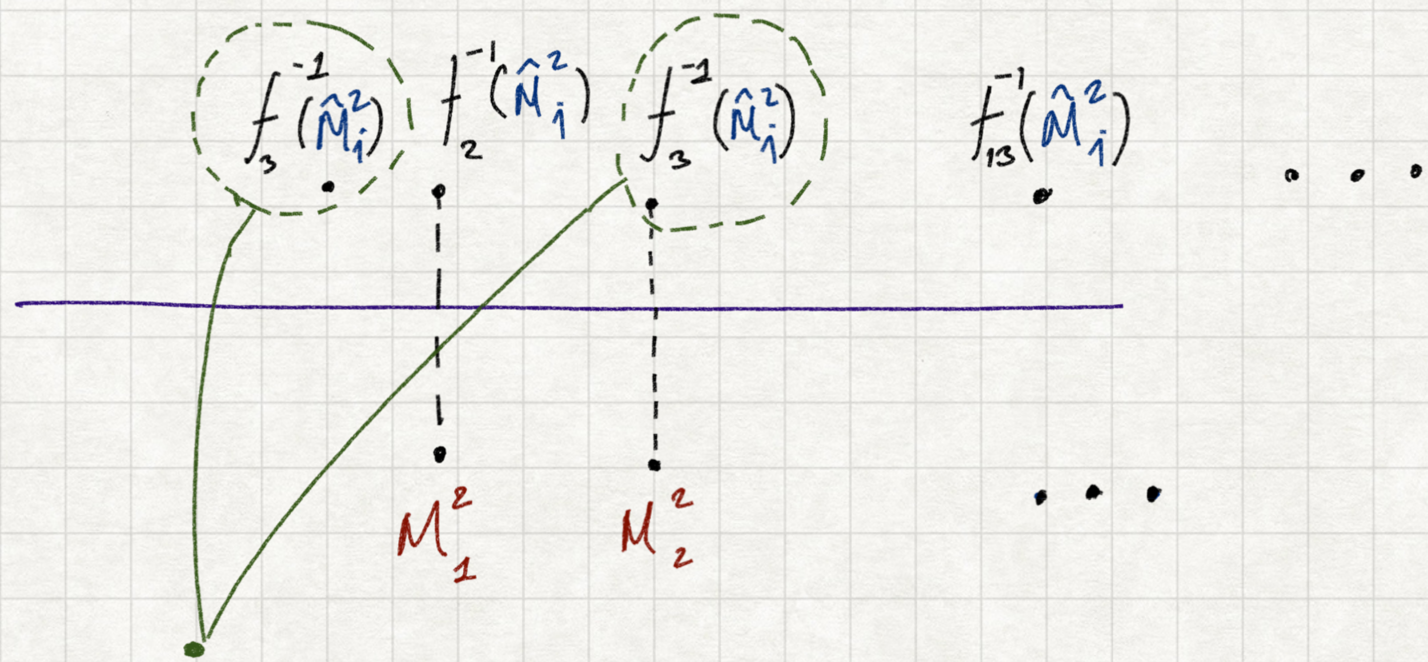
$$\{f_n^{-1}(t_j)\} \supset \{M_i^2\}\,, \quad \forall j\,.$$

The zeroes contain the poles (also in s)

Gravity in the ultraviolet

$$\frac{\langle 23\rangle^2[14]^2}{sM_{\rm pl}^2}\frac{\prod(t-f_n(s))}{\prod(s-M_i^2)\prod(t-\hat{M}_j^2)}\Bigg|_{t=\hat{M}_j^2} \propto \frac{\prod(\hat{M}_j^2-f_n(s))}{\prod(s-M_i^2)}$$

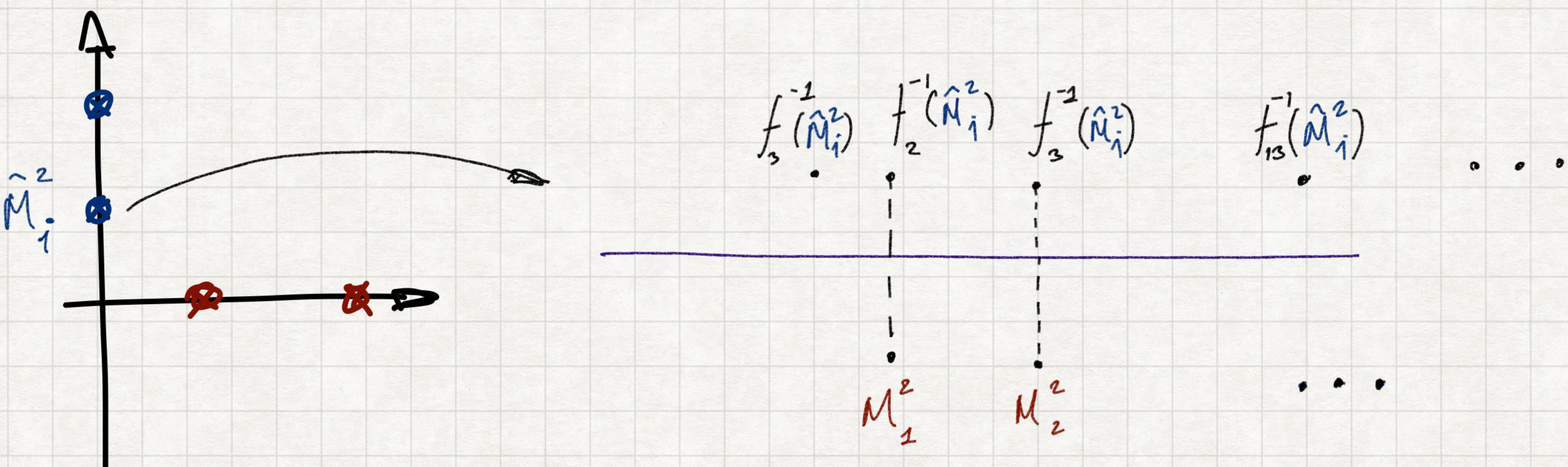




But how many zeroes for the inverse of fn?

Gravity in the ultraviolet

$$\frac{\langle 23\rangle^2[14]^2}{sM_{\rm pl}^2}\frac{\prod(t-f_n(s))}{\prod(s-M_i^2)\prod(t-\hat{M}_j^2)}\Bigg|_{t=\hat{M}_j^2}\propto\frac{\prod(\hat{M}_j^2-f_n(s))}{\prod(s-M_i^2)}$$



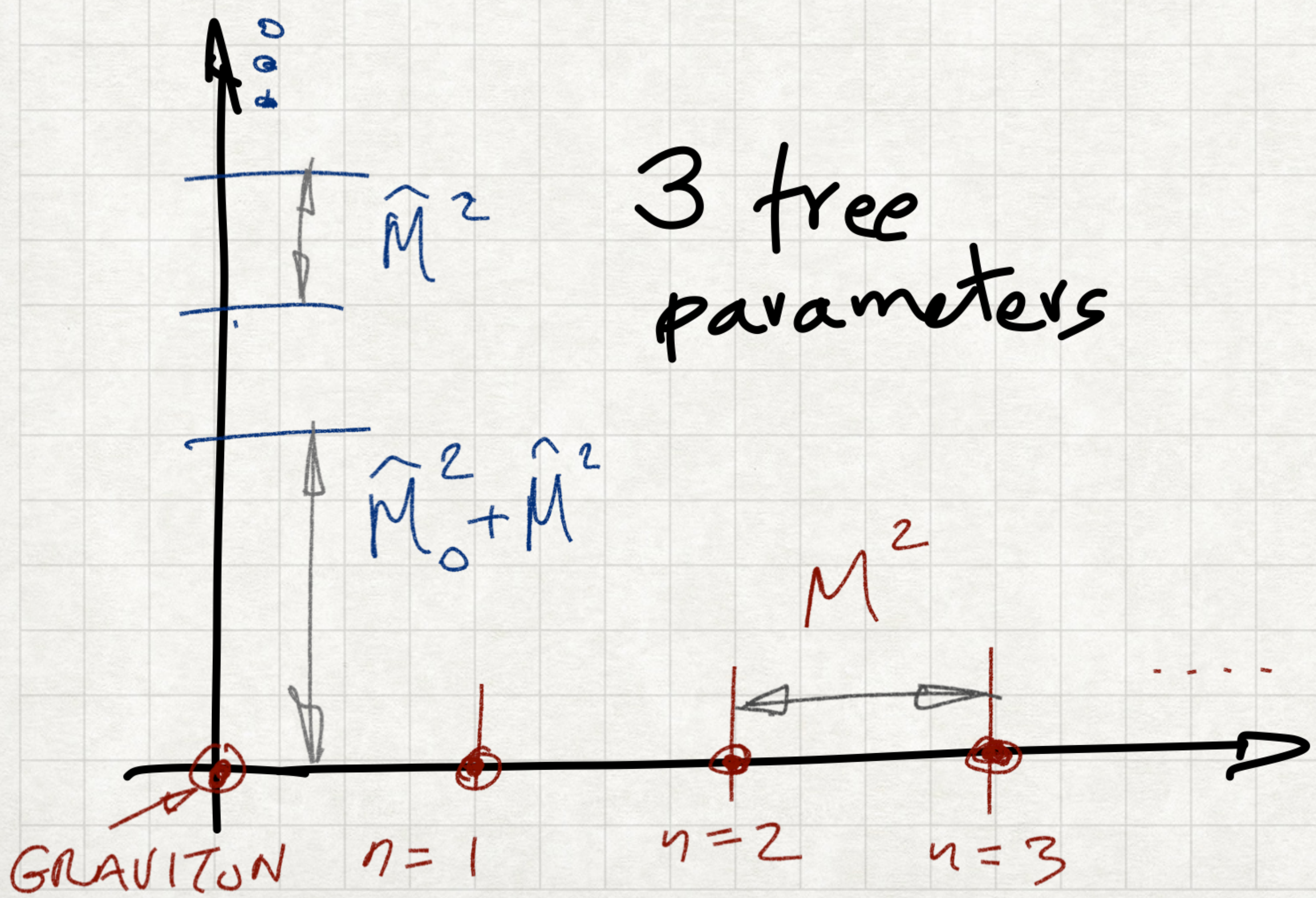
★ Assume f_n^{-1} are singled valued

→ linear $f_n = f_n' s + f_{n,0}$

Gravity in the ultraviolet

Finally we assume the spin (degree of polynomial in t) of resonances increases with n

$$\frac{\langle 23\rangle^2[14]^2}{sM_{\rm pl}^2}\prod_n \frac{M^2 t+\hat{M}^2 s-M^2(n\hat{M}^2+\hat{M}_0^2)}{(s-nM^2)(t-\hat{M}_0^2-n\hat{M}^2)}, \qquad n\in\mathbb{N}\,.$$



Recalling the following definition of the Gamma function

$$\Gamma(z)=\frac{1}{z}\prod\frac{(1+1/n)^z}{(1+z/n)}$$

Gravity in the ultraviolet

$$\mathcal{A} = \frac{8\pi\langle 23\rangle^2[14]^2}{M_{\rm pl}^2 s}\frac{\Gamma(1-\tilde{s})\Gamma(1-\hat{t})}{\Gamma(1-\tilde{s}-\hat{t})}$$

$$\tilde{s} = s/M^2\,, \quad \hat{t} = (t-\hat{M}_0^2)/\hat{M}^2$$

This looks familiar but… did we solve the problem we set out to?

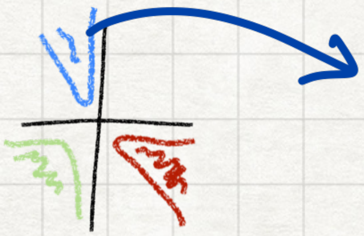
$$\mathcal{A} \to s e^{R\tilde{s}} \qquad R = \log\left((1-\eta s_{\theta/2}^2)^{\eta s_{\theta/2}^2}(\eta s_{\theta/2}^2)^{\eta s_{\theta/2}^2}\right)$$

$\eta \equiv M^2/\hat{M}^2$ for η less (or =) than one it decays exponentially!

IV. Analysis

Positivity from unitarity

The t resonances we found are accessible in the crossed process which furthermore is subject to positivity



$s_{12} \to t\,, \quad [14] \to \sqrt{s}\cos(\theta/2) \quad$ etc

$$\mathcal{A} = \frac{8\pi\langle 23\rangle^2[14]^2}{s_{12}M_{\rm pl}^2}\frac{\Gamma(1-\eta\tilde{s}_{13})\Gamma(1-\tilde{s}_{12})}{\Gamma(1-\tilde{s}_{12}-\eta\tilde{s}_{13})}$$

Make (blue) substitutions & evaluate @ poles

$\eta\,\tilde{s}_{13} = n$

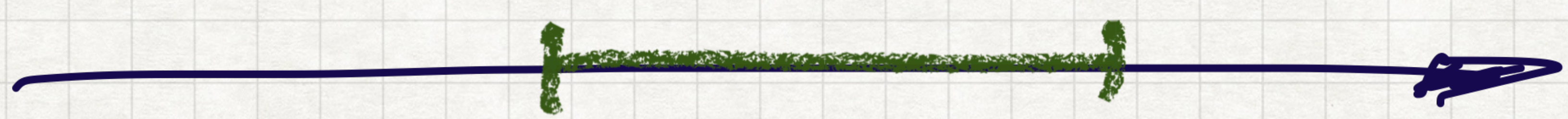
$$\alpha^J_{4,4} = \frac{1}{32\pi}\int dc_\theta d^J_{4,4}(\theta)\mathrm{Res}(\mathcal{A}(s,\theta))_{s=M_n^2}$$

$$\alpha^J \geq 0 \quad \Rightarrow \quad (\eta^{-1}-1) \leq \frac{3}{2n} \quad n = 1,2,\ldots \quad \Rightarrow M = \hat{M} \;!$$

Fermions are special

$$\frac{8\pi\langle 23\rangle[14]}{M_{\rm pl}^2 s_{12}}\left(\frac{s_{13}}{s_{12}}+\frac{s_{12}}{s_{13}}+b\right)\frac{\Gamma(1-\tilde{s}_{12})\Gamma(1-\tilde{s}_{13})}{\Gamma(1-\tilde{s}_{12}-\tilde{s}_{13})}$$

$$\frac{2}{3}\leq b\leq\frac{22}{5}$$

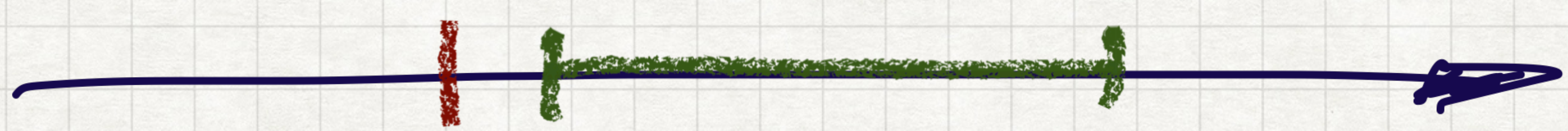


But b we can compute from Feynman rules of GR!

Fermions are special

$$\frac{8\pi\langle 23\rangle[14]}{M_{\rm pl}^2 s_{12}}\left(\frac{s_{13}}{s_{12}}+\frac{s_{12}}{s_{13}}+b\right)\frac{\Gamma(1-\tilde{s}_{12})\Gamma(1-\tilde{s}_{13})}{\Gamma(1-\tilde{s}_{12}-\tilde{s}_{13})}$$

$$\frac{2}{3}\leq b\leq\frac{22}{5}$$



But b we can compute from Feynman rules of GR!

$$b = 1/2$$

Modify the low energy content of gravity

We can fix this introducing a 3-form H

$$-\frac{g}{M}\epsilon^{\mu\nu\rho\sigma}H_{\mu\nu\rho}\psi^\dagger\sigma_\mu\psi + (H^2)$$

Which nonetheless is not dynamical but it integrates out to

$$\frac{g^2}{M^2}\left(\psi^\dagger\sigma_\mu\psi\right)^2$$

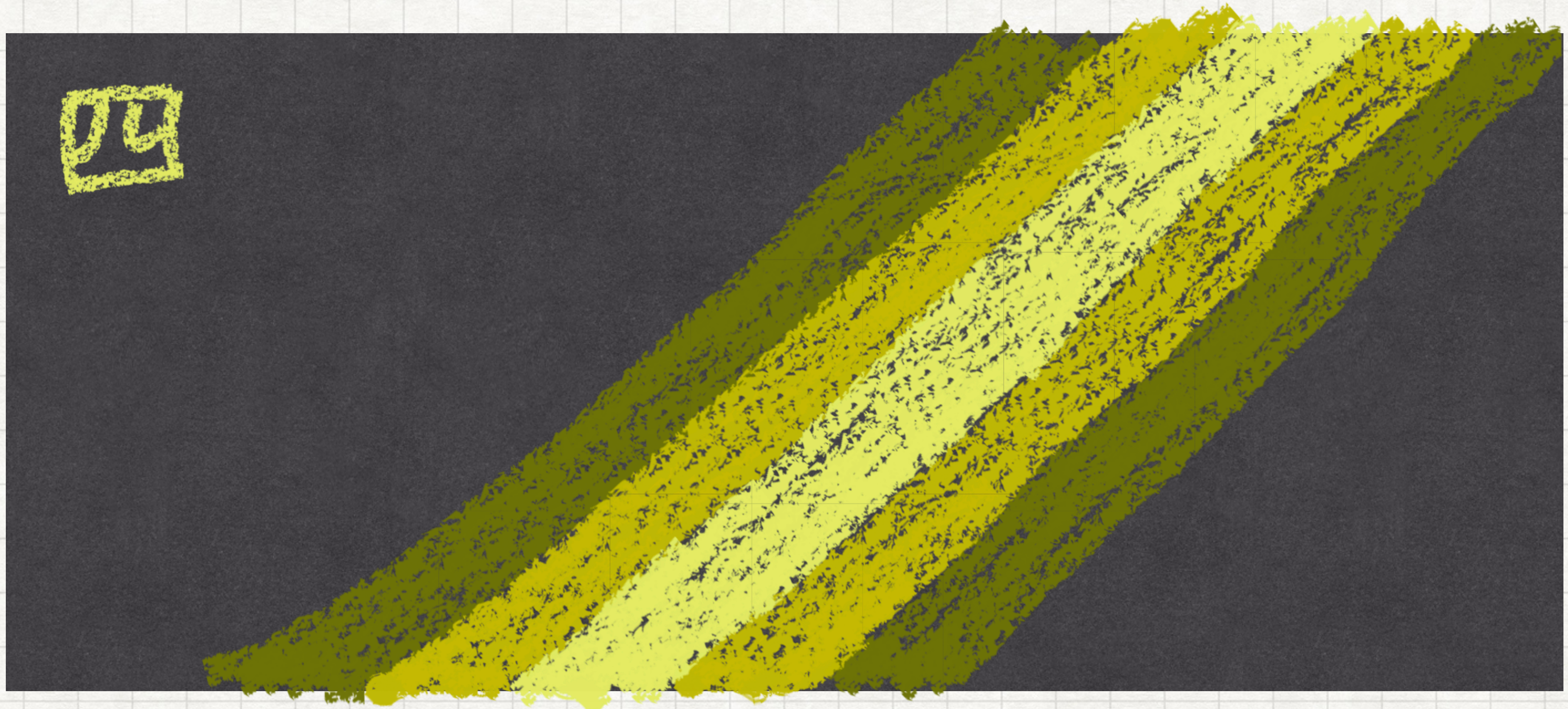
and the amplitude we obtain is reconciled with the low energy EFT for the range

$$\frac{1}{108} \leq \frac{g^2 M_{\rm pl}^2}{\pi M^2} \leq \frac{13}{60}$$

Summary

A sample of how amplitude methods
Provide a new angle to approach gravity

... and quite remarkably give predictions ^-^



Analysis of results