# **Quantum Computing for Quantum Field** *Theory*

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w/ Chancellor and Spannowsky,

#### *Overview*

- Quantum annealers background
- Simple examples of Ising encodings
- Field theory problem: tunnelling in QFT
- Ising encoding of QFT
- Results for thin wall limit
- Thick wall limit and solving PDEs
- Multiple fields and dimensions

#### *Quantum annealers background*

Quantum computing has a long and distinguished history but is only now becoming practicable. (Feynman '81, Zalka '96, Jordan, Lee, Preskill ... see Preskill 1811.10085 for review). Two types of Quantum Computer:



imated from its classical action:  $\uparrow$  10)



respectively. And the calculate, but for our purposes time-dependent transverse field:  $\frac{1}{2}$ 

*,* (3)

piboth hybes operate on the bloch sphere: basically measuring  $\epsilon = \epsilon_{0}$ <sub>w</sub>About types operate on the Bloch sphere: basically measuring  $\sigma_i^2 = \left( \begin{array}{c} 0 \ 0 \end{array} \right)$ ne minima is represents a single qubit To limits the bees this take wall limits  $\epsilon = \epsilon_{0}$  above which the barrier  $\alpha$  the false vacuum. This critical suring  $\sigma^Z_i =$  $(1 \quad 0$  $0 -1$ ◆ it acts upon, and *<sup>X</sup>* is its friend pointing in the *<sup>X</sup>*-direction. The gradual decrease of (*t*) ! <sup>0</sup> from a large value *i j i i* where *<sup>Z</sup> <sup>i</sup>* =  $\epsilon_{0}$ <sub>1</sub>  $\lambda$  $_{\rm{alse}}$ ے<br>N  $(\sigma^{\mathcal{Y}}|0) = |0\rangle$ ,  $\sigma^{\mathcal{Z}}|1\rangle = -|1\rangle$  are the possible eigenvector eqns  $\sigma^{\mathcal{Y}}$ Let us suppose the central problem, which is the continuous state and  $\sqrt{2}$  are the possible eigenvector equantum on  $\sqrt{2}$ a minima is  $\epsilon$ 

IIIHIIA <sup>18</sup> represents a single qubit  $\mathbf{f}$  are system into the ground state of the time-independent part of the Hamiltonian, and  $\bullet$  Latin represents a single qubit

 $\bullet$  A discrete quantum gate system is good for looking at things like entanglement, Bell's inequality etc. Also discrete problems, cryptographical problems, 3hor's, Grover's algorithms, etc.  $\bullet$   $\Lambda$  discrete quantum gate system is good for looking at things like value of its interest of its solution of its solution of its solution, i.e. that is controlled with its solution, i.e. the interest of the int

 $\blacklozenge$ A & Ammetric solutions good for looking at network problems but *I*, igaditally allhedict to good for identity at heritors problems bate from our perspective it is also a more natural tool for thinking about field theory. It is based on the general transverse field Ising model (Kadowaki, Nishimori):  $\mathrm{d}\ O(3)$  symmetric solutions 634 for looking at network problems but about field theory. It is based on the general transverse field ising<br>model (Kadowaki Nishimori) could use to adjust the potential of a system in the potential  $\mathcal{L}$  system in the  $\mathcal{L}$  system in the  $\mathcal{L}$  $\mathsf{n}$ inking it can be encoded. We will further split the Hamiltonian into the Hamiltonian into the  $\mathsf{n}$ about field theory. It is based on the general transverse field Ising However our first task is to encode continuous field values over a continuous domain, with only the discrete Ising *t*!0  $\mu$ <sub>D</sub><br>F<sub>D</sub><br>*F*  $\mu$ <sub>D</sub>  $\mu$ <sub></sub> about field theory. It is based on the general transverse field Ising

$$
\mathcal{H}_{\text{QA}}(t) = \sum_{j}^{5} \sum_{j} J_{ij} \sigma_i^Z \sigma_j^Z + \sum_{i} h_i \sigma_i^Z + \Delta(t) \sum_{i} \sigma_i^X
$$
 the actions can be expressed in  $j$ 

 $\bullet$  What does the "anneal" mean?  $t_{\rm{t}}$  transverse field: • What does the "anneal" mean?  $\epsilon$  ) of the state and the *"conneal"* resear. a mathematical problem, e.g. the calculation of (⇢) for Eq. 7, we have to find a mapping such that the expectation  $\omega$ 

 $\Delta(t)$ 

$$
\mathcal{H}_{\mathrm{QA}}(t) = \sum_{i} \sum_{j} J_{ij} \sigma_{i}^{Z} \sigma_{j}^{Z} + \sum_{i} h_{i} \sigma_{i}^{Z} + \Delta(t) \sum_{i} \sigma_{i}^{X}
$$
  

$$
\Delta(t) \text{ induces bit-hopping in the Hamming/Hilbert space}
$$
 (001)

 $T$ The idea is to dial this parameter to land in the global minimum (i.e. the solution) of some "probler The idea is lution The idea is to dial this parameter to land in the global minimum (i.e. the<br>solution) of some "problem space" described by *Lb*: it acts upon, and *<sup>X</sup>* is its friend pointing in the *<sup>X</sup>*-direction. The gradual decrease of (*t*) ! <sup>0</sup> from a large value it acts upon, and *X* is its friend point the point  $\alpha$  is the *x*-direction. The gradual decrease of  $\alpha$  is  $\beta$  of  $\alpha$ . The gradual decrease of  $\beta$  of  $\alpha$  $\frac{1}{20}$ and not als to that this parameter to land in the globa<br>
solution) of some "problem space" described by *J,h*:  $j,h$ : lem to be solved (as bitstring energies) more on next slides (as bitstring energies) more on  $\mathcal{L}$ 



#### Thermal (classical) and Quantum Annealing are complementary: *Thermal (classical) and Quantum Annealing are complementary:*

- Thermal tunnelling is fast over broad shallow potentials (Quantum "tunnelling" is exponentially slow) problems people care about?
- Quantum Tunnelling is fast through tall thin potentials (Thermal "tunnelling" is exponentially slow  $-$  Boltzmann suppression) o Quantum Tunnolling is fast through tall thin notantials (Thormal "tunnelling" is exponentially slow - Boltzmann suppression)



 $H = H \cdot H \cdot \mathcal{L} = H \cdot \math$ problems approach to Quantum rumbaning can be aboy.<br>plution landscape: Hence hybrid approach to Quantum Annealing can be useful depending on the solution landscape:



More specifically: thermal annealing uses Metropolis algorithm: accept  $\mathsf{random}\quad \sigma_i^Z\; \textit{ flips with probability}$  $\int e^{-x}$  $P =$  $\int 1 \quad \Delta H \leq 0$  $e^{-\Delta H/KT}$   $\Delta H > 0$ **y** - $\overline{1}$   $\overline{$  $P=\begin{cases} 1 & \Delta H \leq 0 \ & P=\begin{cases} 1 & \Delta H < 0 \ & 0 \end{cases}$ More specifically: thermal annealing uses Metropolis algorithm: accept

Quantum tunnelling in QFT happens with probability  $\;\;P\sim e^{-w\sqrt{2m\Delta H/\hbar}}$ *so by contrast it can be operative for tall barriers if they are made thin*<br> Quantum tunnelling in QFT happens with probability  $\quad P \sim e^{-w\sqrt{2m\Delta H}/\hbar}$ 



## **Simple examples of Ising encodings**

#### *Encoding network problems* **Encoding network problems in a general Ising model**

• Example 1: how many vertices on a graph can we colour so that none touch? NP-hard problem (from N.Chancellor). ! σ!N+<sup>i</sup> etouche ive-nare  $E$ vomplo 1.



- ad vortic  $\iota$ • Let non-coloured vertices have  $\sigma_i^Z = -1$  and coloured ones have  $\sigma_i^Z = +1$ .
- Add a reward for every coloured vertex, and for each link between vertices *i,j* we add 2 are two +1 eigenvalues: and the regularization of a physics is a physical project in the notation of a physical project in the set of a physics in the set of a physics of a physics in the set of a physics in the set of a physics in the set of a p a penalty if there are two +1 eigenvalues: = Λ ! !m=1 ij=1  $\overline{\phantom{a}}$

$$
\mathcal{H} = -\Lambda \sum_{i} \sigma_i^Z + \sum_{\text{linked pairs } \{i,j\}} \left[ \sigma_i^Z + \sigma_j^Z + \sigma_i^Z \sigma_j^Z \right]
$$

- Example 2: N^2 students are to sit an exam in a square room with NxN desks 1.5m apart. half the students (A) have a virus while half of them (B) do not. How can they be arranged to minimise the number of ill students that are less than 2m from healthy students?
- Call the eigenvalue of A == +1 and that of B == -1. That is if I measure  $\sigma^Z$  at a point to Can the eigenvalue of  $A = -1$  and that of  $D = -1$ . That is if inteasure  $U$  at a point<br>have value +1 then I conclude that I should put an ill person there, and vice-versa.  $\mathbf{r} \cdot \mathbf{r}$  $\int \sigma^Z$ at a point to  $\bullet$  Call the eigenvalue of  $\Lambda$  --  $\pm 1$  and that of R --  $\pm 1$  That is then reductude that i should put an in person there, and y
	- There are N^2 spins  $\sigma_{\ell N+j}^2$  arranged in rows and columns. I do not care if A>=<A or B>=<B, but if A>=<B then I put a penalty of +2 on the Hamiltonian (ferromagnetic coupling). So ... it acts upon, and *<sup>X</sup>* is its friend pointing in the *<sup>X</sup>*-direction. The gradual decrease of (*t*) ! <sup>0</sup> from a large value • There are N<sup>^2</sup> sp inc<br>...  $\sigma_{\ell,N+j}^Z$  arranged in rows and columns. I do not care if A>=<A or

$$
\mathcal{H} = \sum_{\ell m=1}^{N} \sum_{ij=1}^{N} \left( \delta_{\ell m} (\delta_{(i+1)j} + \delta_{(i-1)j}) + \delta_{ij} (\delta_{(\ell+1)m} + \delta_{(\ell-1)m}) \right) \left[ 1 - \sigma_{\ell N + i}^{Z} \sigma_{m N + j}^{Z} \right]
$$

• Finally I need to apply the constraint that #A = #B:  $\mathcal{H}^{(\text{constr})} = \Lambda \left(\#A-\#B\right)^2$  $\mu$ <sub>ij</sub>  $\mu$   $\mu$   $\sigma$ *i*<sup>*i*</sup> countring and  $\mu$   $\mu$   $\mu$ <sup>2</sup> could use the potential  $\mathcal{H}^{(\mathcal{L}^{\text{unif}})} = \Lambda \left(\#A - \#B\right)$  $\frac{1}{\sqrt{N}}$  in the divisor of the constraint that  $\frac{1}{\sqrt{N}}$  and  $\frac{1}{\sqrt{N}}$  and  $\frac{1}{\sqrt{N}}$  and  $\frac{1}{\sqrt{N}}$  and  $\frac{1}{\sqrt{N}}$ 

$$
= \Lambda \left(\sum_{\ell, i}^N \sigma_{\ell N + i}^Z\right)^2 \newline = \Lambda \sum_{\ell m = 1}^N \sum_{ij = 1}^N \sigma_{\ell N + i}^Z \sigma_{m N + j}^Z
$$

• Example 2 done with classical thermal annealing using the Metropolis algorithm. Note this represents a search over  $\;\;_{100}C_{50}\sim 2^{100}\;$  configurations:  $\frac{1}{\sqrt{2}}$ ırch over  $_{100}C_{50} \sim 2^{100}$  configuratio  $\Gamma$ lermai  $\frac{100}{100}$  $a_0 \sim 2^{100}$ 



• Importantly the constraint hamiltonian cannot be too big otherwise the hills are too high and it freezes too early. This makes the process require a (polynomial sized) bit of "thermal tuning".

- In principle this could be done more easily on a quantum annealer as the constraints could be high and it would still work.
- To do this we would simply fill h and J and call the quantum annealer from python as follows:  $response = sampler.sumple\_ising(h, J, seed=1234+i, num\_reads=3000000, num_sweeps=1)$
- "response" is a list of  $[+1,-1,+1,+1,$ ....] spins ordered by energy
- However the architecture (connectivity of J,h) is limited. (Later)

## *A field theory problem: Tunnelling in QFT*

- We think of the general Ising model as a "universal QFT computer"
- Simple problem to demonstrate encoding  $QFT$  quantum tunnelling in a scalar theory
- Advantage 1: easy to prepare the initial state (this non-perturbative process is much easier than preparing scattering states).
- Advantage 2: we could in principle observe genuine tunnelling in the annealer rather than just simulate it.

$$
U(\phi) = V(\phi) - V(\phi_{+})
$$
\n
$$
U(\phi) = \frac{\lambda}{8}(\phi^{2} - v^{2})^{2} + \frac{\epsilon}{2v}(\phi - v)
$$
\n7 thick wall situation

\n

 $\bullet$  A system tranned in the false vacuum will decay by forming bubbles • A system trapped in the false vacuum will decay by forming bubbles ...  $\alpha$  custom tranned in the folcours

respectively. The quantum determinant prefactors *A*4*, A*<sup>3</sup> are notoriously difficult to calculate, but for our purposes



- The analytic result for the tunnelling rate was worked out in several famous papers by Callan, Coleman, de Luccia and Linde = *U*<sup>0</sup> *,* (2) several famous papers by Calian, Coleman, de Luccia and Linde າ, de Lucc
- Decay rate per unit volume is given by the Euclidean actions of the  $O(4)$  or  $O(3)$  symmetric "bounce" solution (for instanton or thermal resp): tunnelling *O*(4) symmetric instanton, respectively. The required "bounce" is subject to the boundary condition that  $\bullet$  Decay rate per unit volume is given by the Euclidean actions of  $\bullet$  . Decay rate per unit volume is given by the Euclidean actions or the  $\Omega(4)$  or  $\Omega(2)$  symmetric "bounce" solution (for instanton or  $\mathfrak{m}$ e O(4) or O(3) symmetric  $\mathfrak{m}$ ounce $\mathfrak{m}$  solution ( $\mathfrak{m}$  instanton or  $\mathbf{r}$   $\mathbf{r}$   $\mathbf{r}$   $\mathbf{r}$   $\mathbf{r}$ line Lucilue all  $\cdot$   $\cdot$   $\cdot$   $\cdot$  $\epsilon$



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• Normally solution found by solving Euler-Lagrange equations with boundary conditions: where the value of the value 2 or 3 for 3 for 3 for 3 for a finite temperature *or 3* for a purely symmetric bubble • Normally solution found by solving Euler-Lagrange equations with boundary conditions: where  $\alpha$  is the value 2 or 3  $\alpha$  symmetric bubble, or a  $\alpha$  $\frac{1}{100}$ : tound by solving Fuler-L e−∆H/KT ∆H > 0

$$
\frac{d^2\phi}{d\rho^2} + \frac{c}{\rho}\frac{d\phi}{d\rho} = U' \ , \quad d\phi/d\rho = 0 \quad \text{as} \quad \rho \to 0, \infty
$$



 $\mathsf{v}$  and  $\mathsf{v}$   $\$ • "Escape point" found with overshoot/undershoot method. value corresponds to ✏<sup>0</sup> = 2*a*<sup>4</sup>*/*3 . Thick-wall approximation: rescaling arguments give answer in terms of "standard action"  $\frac{1}{2}$  argum ime  $\frac{1}{2}$  is give answer in terms of "standard action"

the bounce action can be active action can be active around the value  $\mathcal{L}=\mathcal{L}^2$ 

$$
S_4 = \frac{3\xi}{\lambda} S_4^0 \; ; \quad S_4^0 = 91
$$
\n
$$
S_3 = \frac{3v\xi^{3/2}}{\lambda^{1/2}} S_3^0 \; ; \quad S_3^0 = 19.4
$$
\nwhere

\n
$$
\epsilon_0 = 2\lambda v^4 / 3\sqrt{3}
$$

Then following the rescaling procedure of [45], the tunnelling actions for the *O*(4) and *O*(3) symmetric solutions can  $\bullet$  Thin-wall approximation: action written in terms of c=0 action (Z2 domain wall)

$$
S_4 = \frac{27\pi^2 S_1^4}{2\epsilon^3} \; ; \; S_3 = \frac{16\pi^3 S_1^3}{3\epsilon^2} \; .
$$

ve can encode this field theory on a quantum annealer, we would be able to vary the parameters and perform a tunnelling experiment. As a first step, *S*<sup>4</sup> = we will determine S1: finding the extremum of the action is a quasi-convex problem <sup>2</sup>✏<sup>3</sup> ; *<sup>S</sup>*<sup>3</sup> <sup>=</sup> all by the action is a quasi convex problem In principle if we can encode this field theory on a quantum appealer, we would be *(convex in a finite box).* In principle if we can encode this field theory on a quantum annealer, we would be

This means for the  $\;c\,=\,0\;$  action we will attempt to minimise the Euclidean action holding the endpoints fixed at  $+/- v$ :

$$
S_1 \, = \, 2 \pi^2 \int_0^\infty d\rho \; \frac{1}{2} \dot{\phi}^2 + U(\phi)
$$

## **Ising chain encoding of scalar QFT**

Consider encoding a continuous field value  $\phi(\rho)$  at some point, and discretise into N  $\alpha$ 

$$
\phi(\rho) = \phi_0 + \alpha_l \xi = \phi_0 + \xi \dots \phi_0 + N \xi
$$

Wish to represent it as a point on a spin chain == domain wall encoding (Chancellor):

$$
[-1]-1]-1]-1]-1]-1]-1]-1]+1|+1|+1|+1|+1|+1
$$

We translate this to a field value using

$$
\phi = \phi_0 + \frac{\xi}{2} \sum_{i=1}^{N} (1 - \sigma_i^Z)
$$

For this to work as a consistent encoding we have to avoid e.g.

$$
-1 - 1 - 1 + 1 - 1
$$

This is the domain-wall encoding. Begin in the Ising model with a ferromagnetic interaction that favours as few flips as possible, but frustrate at least one by having the<br>endpoints pinned at -1 ... +1. (Note this is a 1D version of the exam-room example). endpoints pinned at -1 ... +1. (Note this is a 1D version of the exam-room example).  $\mathbf{d}$ ole, but mustre<br>1D version of t  $\overline{a}$ 

$$
\mathcal{H}^{(\text{chain})} = \Lambda \left( \sigma_1^Z - \sigma_N^Z - \sum_i^{N-1} \sigma_i^Z \sigma_{i+1}^Z \right)
$$

we have  $\alpha$ rk as a consistent encoding we have to avoid e ground subspace of the Hamiltonian, where exactly one spin  $\alpha$  $\frac{1}{\sqrt{2}}$ For this to work as a consistent encoding we have to avoid e.g.



r flips as poss<br>Note this is a  $\mathbf{d}$ ole, but mustre<br>1D version of t  $\overline{a}$ che ising moder with a refromagnetic<br>sible, but frustrate at least one by having th an ain-wall encoding. Begin in the Ising model with a ferromagnetic said and the meaning t of t  *he*  $\nu$ ersion of t ււ ս<br>1e e  $\alpha$  this is a 1D version of the exam-room example). international extent of the the term of the term district the term of the term district the term of the term d<br>The term of the term of the term district the term district to the term district to the term district to the t<br> י<br>פא ion or the exam<br> endpoints pinned at -1 ... +1. (Note this is a 1D version of the exam-room example).  $\blacksquare$  is the domain-wa *I* encoding. Begin in the Ising model with a ferromagnetic<br> *n* as few flips as possible, but frustrate at least one by having the  $\frac{1}{2}$ *j*=1 ins is the domain wan encoding. Begin in the ising moder with a refromagnetic<br>interaction that favours as few flips as possible, but frustrate at least one by having the This is the domain-wall encoding. Begin in the Ising model with a ferromagnetic<br>interaction that favours as few flips as possible, but frustrate at least one by havi



To add a potential we can add a contribution to the linear *h* couplings  $\mathop{\sf tion}$  to the linear



Next add the discretised radial spacetime coordinate:  $\rho_{\ell} = \ell \nu = \nu ... M \nu$  $\phi$ Next add the discretised radial spacetime coordinate:  $\rho_{\ell}$ <br> $\phi$  $U(\phi) = \frac{1}{2}$  $-\phi$  $\phi$ @  $\frac{1}{2}$ `*N*+*j*+1 *<sup>Z</sup>*



*j*=1

Everything done so far is then trivially extended in the *l* spacetime index: ig done so far is then trivially extended in the r spacetime index: Everything done so far is then trivially extended in the *l* spacetime index:

$$
h_{\ell N+j}^{(\text{chain})} = \Lambda \left( \delta_{j1} - \delta_{jN} \right) \qquad J_{\ell N+i,mN+j}^{(\text{chain})} = -\frac{\Lambda}{2} \delta_{\ell m} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \end{pmatrix}_{ij}
$$

$$
h_{N\ell+j}^{(\text{QFT})} = \begin{cases} -\frac{\nu \xi}{2} U'(\phi_0 + j\xi) \; ; & j < N \\ \frac{\nu}{2} U(\phi_0 + (N-1)\xi) \; ; & j = N \end{cases}
$$

hen kinetic terms are as follov , the potential is somewhat easier to deal with the kinetic terms, because it can be can  $ms\ x$ Then kinetic terms are as follows: `*N*+*j*+1 *<sup>Z</sup>* Than kinatic tarms are as follows: **bilinear interactions are discretised in a set of the set of**  $\alpha$  **are discretised in**  $\alpha$  $\overline{z}$  $\overline{a}$ w<br>W S:

$$
S_{KE} \equiv \int_0^{\Delta \rho} d\rho \frac{1}{2} \dot{\phi}^2 = \lim_{M \to \infty} \sum_{\ell=1}^{M-1} \frac{1}{2\nu} \left( \phi(\rho_{\ell+1}) - \phi(\rho_{\ell}) \right)^2
$$

$$
= \sum_{\ell=1}^{M-1} \sum_{ij}^{N} \frac{\xi^2}{8\nu} \left[ \sigma_{(\ell+1)N+i}^Z - \sigma_{\ell N+i}^Z \right] \times
$$

$$
\left[ \sigma_{(\ell+1)N+j}^Z - \sigma_{\ell N+j}^Z \right]
$$

ig done so far is then trivially extended in the r spacetime index: Everything done so far is then trivially extended in the *l* spacetime index: discrete representation of the field values as well using Eq.(15), we find Everything done so far is then trivially extended in the *l* spacetime index:

$$
h_{\ell N+j}^{(\text{chain})} = \Lambda (\delta_{j1} - \delta_{jN}) \qquad J_{\ell N+i,mN+j}^{(\text{chain})} = -\frac{\Lambda}{2} \delta_{\ell m} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \end{pmatrix}_{ij}
$$

$$
h_{N\ell+j}^{(\text{QFT})} = \begin{cases} -\frac{\nu \xi}{2} U'(\phi_0 + j\xi) ; & j < N \\ \frac{\nu}{2} U(\phi_0 + (N-1)\xi) ; j = N \end{cases}
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$$
J_{\ell N + i, mN + j}^{(\text{QFT})} = \frac{\xi^2}{8\nu} \left( 2\delta_{\ell m} - \delta_{\ell (m+1)} - \delta_{(\ell+1)m} \right)
$$

Next we need to impose the physical boundary condition with: wall states we have <sup>h</sup>*<sup>Z</sup> n*<sub>t</sub> we need to impose the priyoned `*N*+*<sup>i</sup>* + 1i for *j>i*. As bilinear terms may be hard to engineer on real Next we need to impose the physical boundary condition with:<br> $Z^2$  only indicates

$$
\mathcal{H}^{(BC)}=\tfrac{\Lambda'}{2}(\phi(0)+v)^2+\tfrac{\Lambda'}{2}(\phi(\rho_M)-v)^2
$$

link of these as just boundary mass-term pc<br>
———————————————————— We can think of these as just boundary mass-term potentials in *U*: large parameter. This is simply an extra contribution to *h* which follows directly from Eq.(20), of the form

$$
h_{N\ell+j}^{(\text{BC})} = \begin{cases} -\Lambda'(\phi_0 + j\xi + v) \; ; \; \ell = 1, \forall j \\ -\Lambda'(\phi_0 + j\xi - v) \; ; \; \ell = M - 1, \forall j \end{cases}
$$

Finally add everything together with  $\mathbf{r}$ Finally add everything together!<br> **Example 10** of a system in the potential *ultimate* in the postem in the po

$$
\mathcal{H} = \mathcal{H}^{(\text{chain})} + \mathcal{H}^{(\text{QFT})} + \mathcal{H}^{(\text{BC})}.
$$

# *Results for thin wall limit*

Can solve classical simulated annealing with the Metropolis algorithm. Again have to

be careful how we set the temperatures and parameters:

Too hot



Too cold



Just right (two stage annealing process)



Same result on Dwave using hybrid quantum/classical Kerberos annealer (It finds best samples of parallelised tabu search + simulated annealing + D-Wave subproblem sampling) rest samples of paraliensed tabu search + simulated annealing + D-Wave subproblem<br>-<br>- ^^^^ling respectively). And the sampling  $\bm{p}$ 



Figure 2: Solutions for this computer is solutions for the thin-wall solution computed using the thing the thin $t_{\text{t}}$  as discussed is overlaid on the right panel. Notably the Kerberos sampler is much more robust than pure simulated annealing.

**Why not pure Quantum annealer?** The connectivity is not general enough for this problem (in particular encoding the kinetic terms): it has a Chimera structure ...



But the principle has been proven: we can encode a pure field theory potential on the chimera structure, so we can experiment with QFT tunnelling (c.f. Johnson 2011)

#### *Thick wall limit: solving PDEs*

To find the  $c = 3$  solution shown here is less easy because just using the action tends resorted to a hybrid asynchronous decomposition sampler (the Kerberos solver [51]), which can solve problems of o find the C =5 solution shown here is less easy because just using the action tends we calculate the solution ( $\frac{1}{2}$  for  $\frac{1}{2}$  for  $\frac{1}{2}$   $\frac{1}{2$ 

to give the black line: simulated annealing and D-Wave subproblem sampling on variables that have high-energy impact. Using this method



This is because the critical point of  $S_{c+1} = 2\pi^2 \int d\rho \rho^c \left( \frac{1}{2} \dot{\phi}^2 + U(\phi) \right)$  is a saddle. Instead the correct bubble profile is found by solving the E-L PDE by minimising  $\int^{\infty}$ 0  $d\rho \rho^c$  $(1)$ 2  $\dot{\phi}^2 + U(\phi)$ ◆ is a saddle. However the correct babble prome is foally by solving the E ET DE by minimining This is because the critical point of  $\,S_{c+1}\,=\,2\pi^2\, \int\,d\rho\,\rho^c\, \big(\,\,\frac{}{\,2\,}\phi^2+U(\phi)\,\big)\,$  . Is a saddle,  $\begin{array}{c} 0 \ V^+ \ \end{array}$ inside the correct bubble pr

$$
\tilde{S}_{c+1} \,=\, \int_0^\infty d\rho \, \left( \frac{d^2\phi}{d\rho^2} + \frac{c}{\rho}\frac{d\phi}{d\rho} - U' \right)^2
$$

It is squared in derivatives, so it can be written mostly as adjustments in J ... boundary conditions (in the ` = 1*,* 2*,M,M* 1 case). stly a *d*⇢ ¨ + ⇢ *.* (30)

$$
\frac{4\nu^3}{\xi^2} J_{\ell N + i, mN + j}^{(\text{QFT})} = \frac{c^2}{\ell m} \left( 2\delta_{\ell m} - \delta_{\ell (m+1)} - \delta_{(\ell+1)m} \right) + \left( 6\delta_{\ell m} - 4\delta_{\ell (m+1)} - 4\delta_{(\ell+1)m} + \delta_{\ell (m+2)} + \delta_{(\ell+2)m} \right) \n+ \frac{c}{m} \left( 3\delta_{(\ell+1)m} + \delta_{\ell (m+1)} - \delta_{(\ell+2)m} - 3\delta_{\ell m} \right) + \frac{c}{\ell} \left( 3\delta_{(m+1)\ell} + \delta_{m(\ell+1)} - \delta_{(m+2)\ell} - 3\delta_{\ell m} \right) \n- \nu^2 U''(\phi_0 + i\xi) \left( \delta_{\ell (m+1)} - \delta_{\ell (m+2)} + \left( 1 - \frac{c}{m} \right) \left( \delta_{\ell (m+1)} - \delta_{\ell m} \right) \right) \n- \nu^2 U''(\phi_0 + j\xi) \left( \delta_{m(\ell+1)} - \delta_{m(\ell+2)} + \left( 1 - \frac{c}{\ell} \right) \left( \delta_{m(\ell+1)} - \delta_{m\ell} \right) \right) + \nu^4 U''(\phi_0 + i\xi) U''(\phi_0 + j\xi) \delta_{\ell m} ,
$$

together with ... *v* together with …

$$
h_{N\ell+j}^{(QFT)}=\frac{\epsilon}{v}\frac{\xi c}{2\nu}\left(\frac{1}{\ell-1}-\frac{1}{\ell}\right)
$$

and boundary condition terms for 
$$
\dot{\phi}(0) = \dot{\phi}(\infty) = \ddot{\phi}(0) = \ddot{\phi}(\infty) = 0,
$$
  
 $\phi(\infty) = \phi_+$ ,

**Multiple fields and dimensions:** the U(1) string

Consider 2D system with 2 fields: the *U*(1) topological string in two dimensions. Consider the following energy integral Extension to the case of multiple fields and multiple dimensions is straightforward. As an example will consider with  $\alpha$ *<sup>U</sup>*(*a*) = ration of the *Physical Consider* 2 Development of the *Phialder* 

$$
H_{U(1)} = \int_0^\infty d^2x \, \frac{1}{2} \nabla \phi_a \cdot \nabla \phi^a + U(\phi_a)
$$
  

$$
U(\phi_a) = \frac{\lambda}{8} (\phi_0^2 + \phi_1^2 - v^2)^2
$$

O(1) vortex is again a convex problem: can be discretised as before, and it has topologically stable statement. U(1) vortex is again a convex problem: can be discretised as before, *S*O(1) voltex is again a convex problem. Can be discretised as before,  $U(1)$  vortex is again a convex problem: can be discretised as before, Obviously this two-dimensional theory has a *U*(1) ⇠ *SO*(2) rotational symmetry, and it has a topologically stable U(1) vortex is again a convex problem: can be discretised as before,  $f(1)$  vortex is again a convex problem: can be discre  $U(1)$  vortex is again a convex problem: can be discretised U(1) vortex is again a convex problem: can be discretised as before. finding the solution for a string in a finite box, which is also not rotationally symmetric).

finding the solution for a string in a finite box, which is also not rotationally symmetric). finding the solution for a string in a finite box, which is also not rotationally symmetric). We first need to go back to the beginning and decide how we are going to assign the dimensions and fields to Ising spins. The simplest is to vectorise the indices ` ! `*µ*=0*...d*<sup>1</sup> and *i* ! *ia*=0*...n*<sup>1</sup>, where *d* is the number of space

Flatten the Ising model indices as:  $\begin{equation} \mathsf{F}}[\mathsf{a}]\mathsf{f}\mathsf{f}[\mathsf{a}] \mathsf{f}[\mathsf{b}] \mathsf{f$ 

$$
\{i_a\} \equiv (n\ell_\mu M^{\mu-1} + a)N + i_a \; ; \quad a = 0 \dots n-1
$$

 $\blacksquare$  Ising model is  $\hspace{0.1em} nN$  $\blacksquare$  Ising model is  $\; nM^dN \times nM^dN$ 



#### **Future directions**

- We have seen how the general Ising model can be used to encode QFT
- Genuine tunnelling of metastable nontrivial (d=0 system)?
- Deduce quantum prefactors as well as classical actions
- GPU encoding for finite temperature (simulated annealling) (c.f. Parisi et al)
- Soliton dynamics?