Time-dependent wave-packet approach to ^{12}C + ^{12}C sub-barrier fusion using microscopic potentials

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Introduction

Understanding the dynamics of the 12C + 12C reaction is an important factor in determining the path of stellar evolution, one that at low astrophysical energies is not completely explained.

Fusion cross sections of the 12C + 12C reaction determine the nucleosynthesis of heavier ions in carbon burning for stars with M > $8M_{\odot}$.

The Gamow peak for the 12C + 12C is approximately 1.5 MeV. The Gamow window is the energy range for a reaction in a stellar burning phase where the Maxwell-Boltzmann distribution of velocities overlaps with the low energy tail of the reaction cross section.

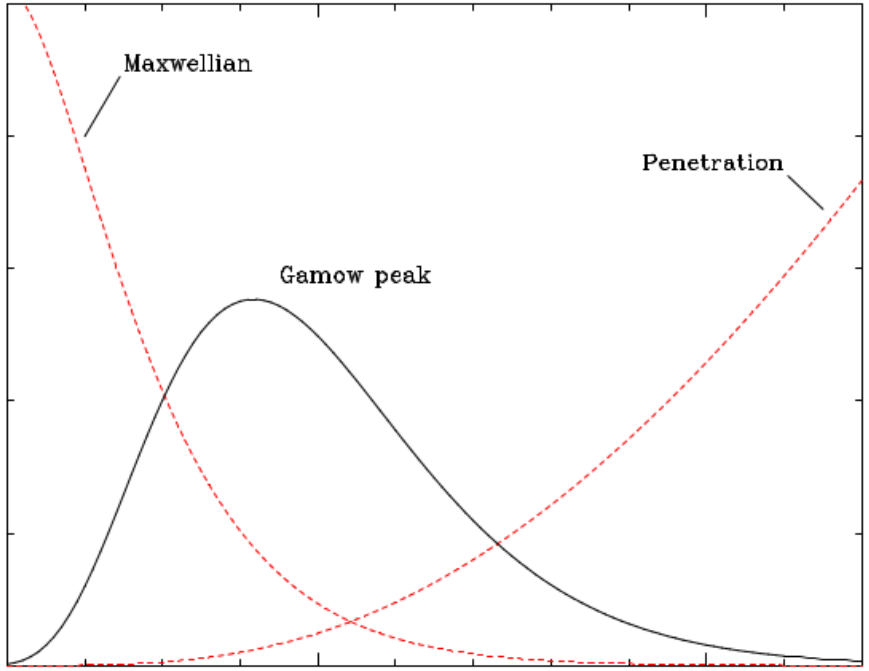


Figure from [1]

Nuclear Molecule States

- Direct measurements in the fusion cross section have given insights down to the center-of-mass energy, $E_{c.m}$ = 2.1 MeV, but struggle to probe further due to prominent Coulomb effects.
- Resonant structures in the fusion cross-section have been observed in the sub-barrier energy region, $E_{c.m}$ < 6 MeV,
- These structure could be related to collective excitation modes when at small separation distances.

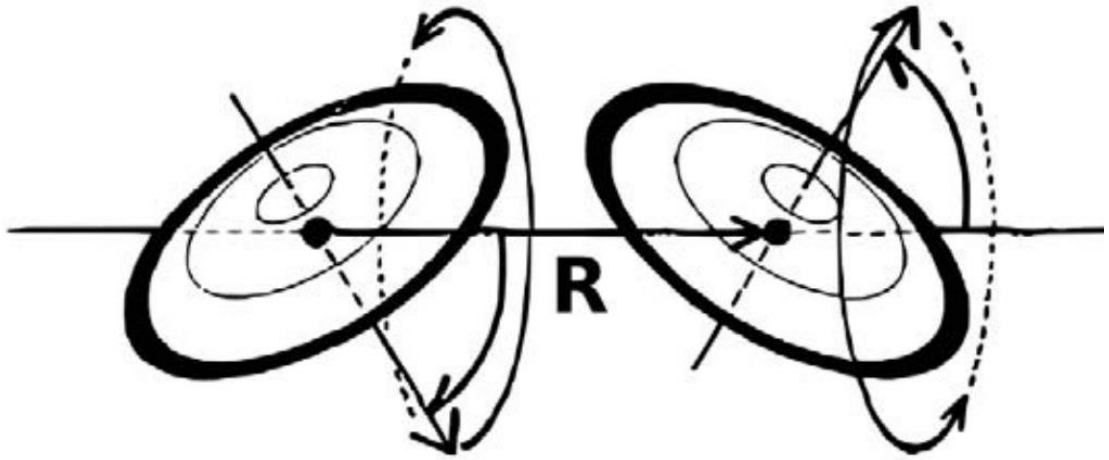


Figure from [2]

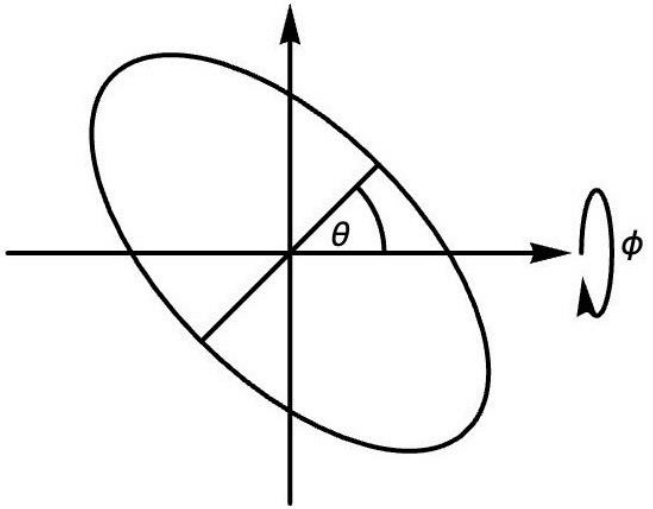
Time-dependent wave-packet (TDWP) method

1). Definition of the initial wave-packet, $\Psi(0)$. Sufficiently far away in their ground state ($j^{\pi} = 0^{+}$).

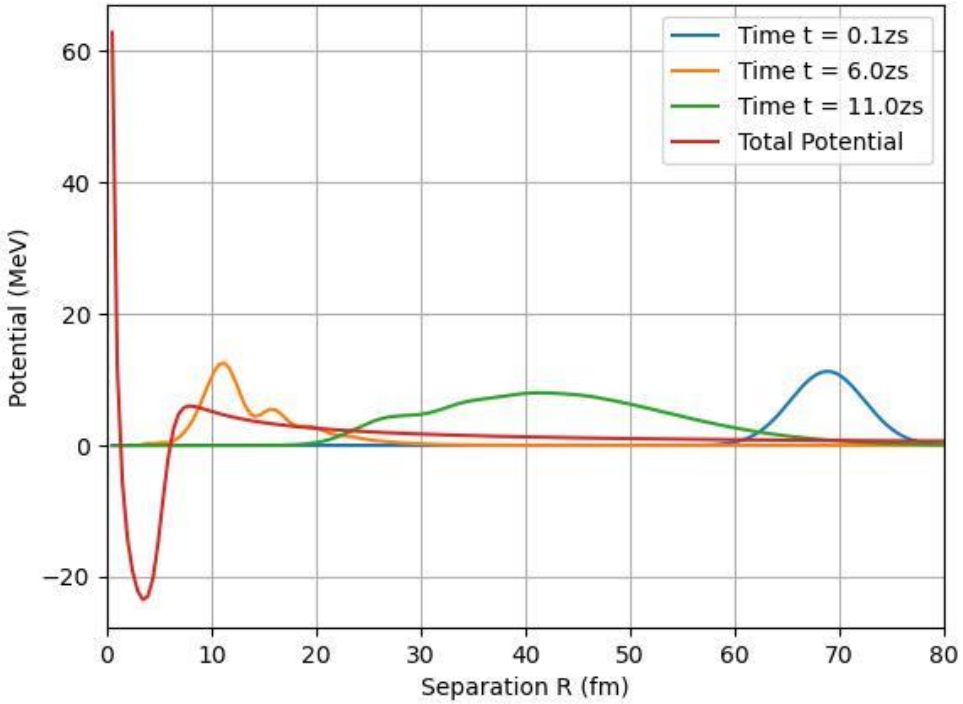
2). Evolving the wave-packet in time by solving the time-dependent Schrödinger equation.

3). After a sufficient amount of time, for the wave-packet to have finished interacting with the potential wells, fusion probabilities and cross-sections are calculated from the wave function.

$$\Psi(R, \theta_1, k_1, \theta_2, k_2) = \chi(R)\psi(\theta_1, k_1, \theta_2, k_2)$$



Example Simulation



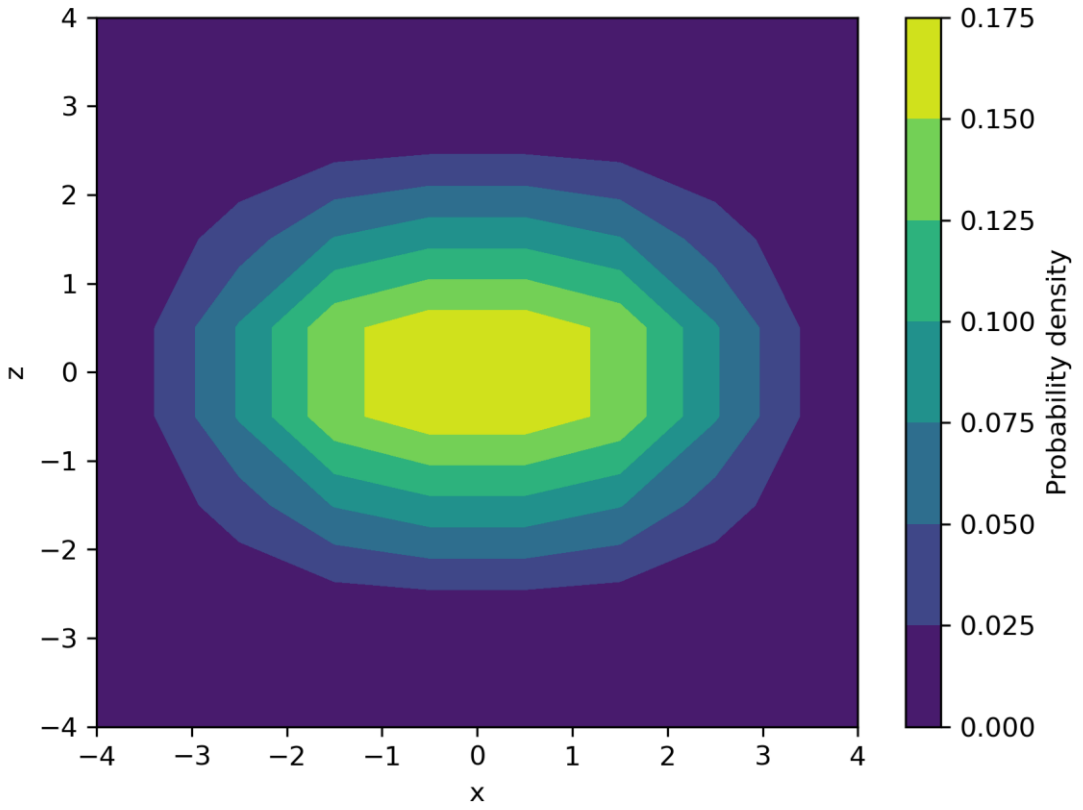
Time-dependent Hartree-Fock (TDHF) method

Sky3D - 3 dimensional TDHF code.

Skyrme energy density functional calculation.

Binding energy per nucleon:
6.38 MeV, Experimental: 7.68 MeV

Quadrupole Deformation:
β_2 = -0.22, Experimental: β_2 = -0.57



Sky3D publication [3]

Density-constrained time-dependent Hartree-Fock (DC-TDHF) Method

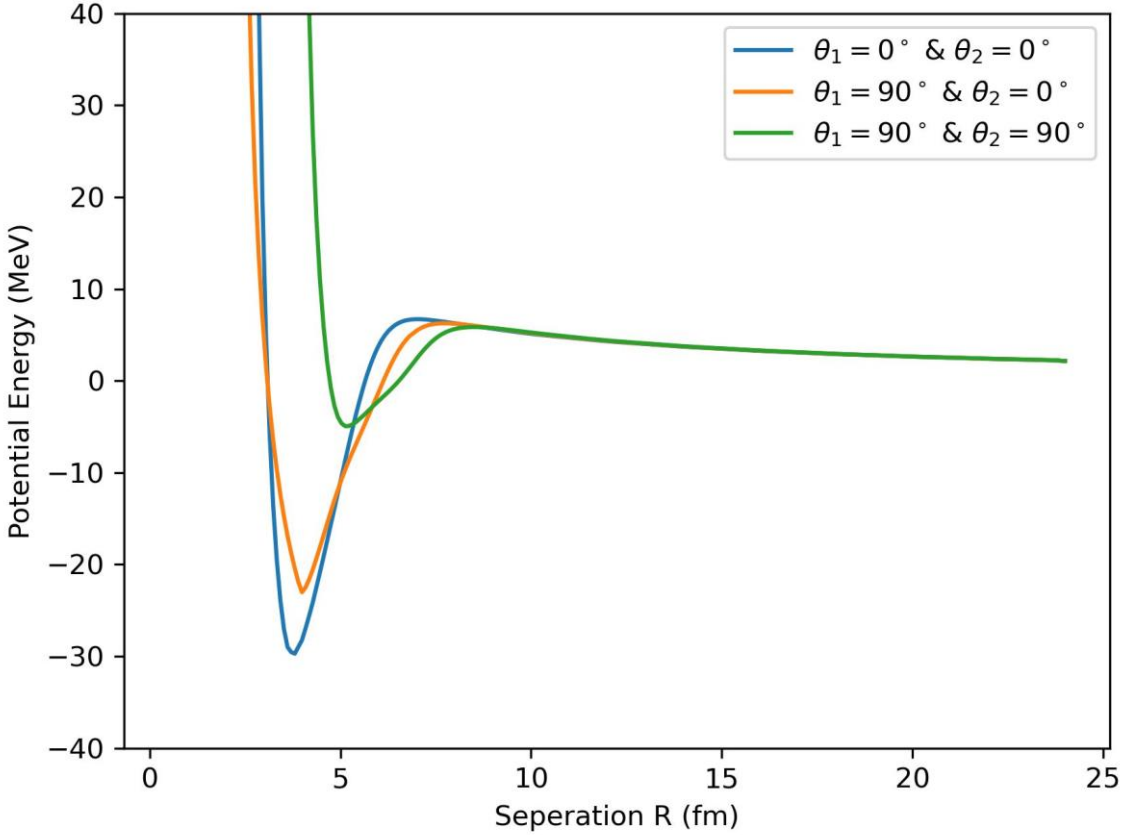
1). Start a TDHF collision.

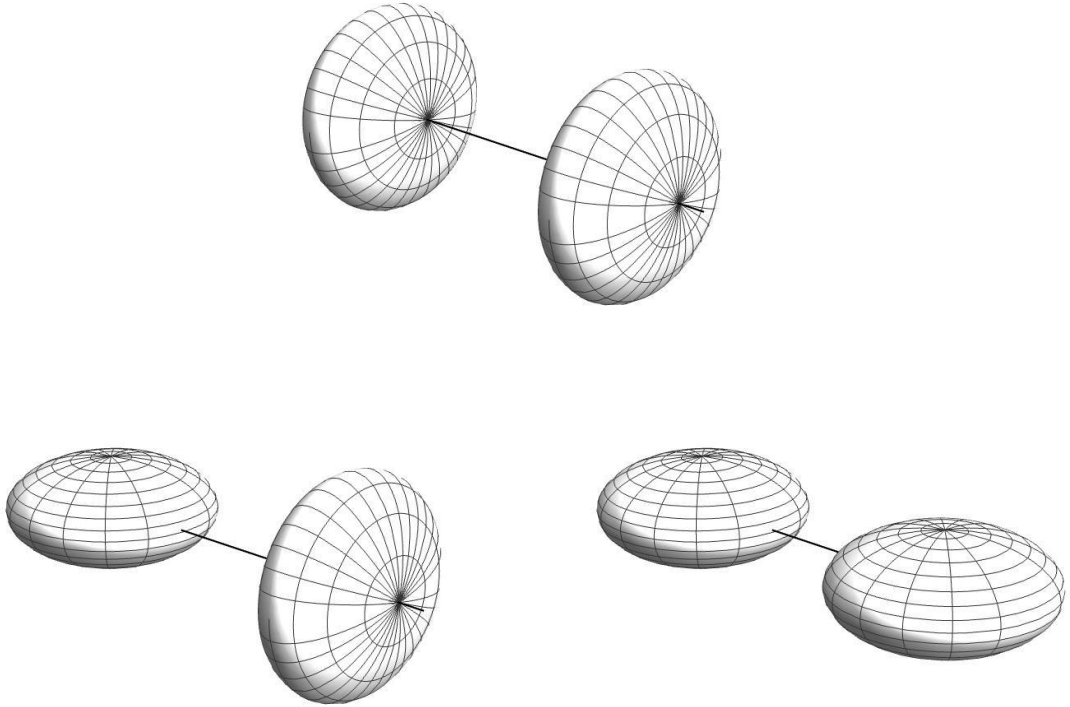
2). After a specified number of time steps, the reaction is paused to start a static HF calculation of the dinuclear system. The density operators of the nucleons are constrained to the values of the density operators during the TDHF procedure.

$$\delta\left\langle \hat{H} - \sum_{q=p,n} \int d\boldsymbol{r}\lambda_q(\boldsymbol{r})(\rho_q(\boldsymbol{r}) - \rho_q^{TDHF}(\boldsymbol{r}))\right\rangle = 0$$

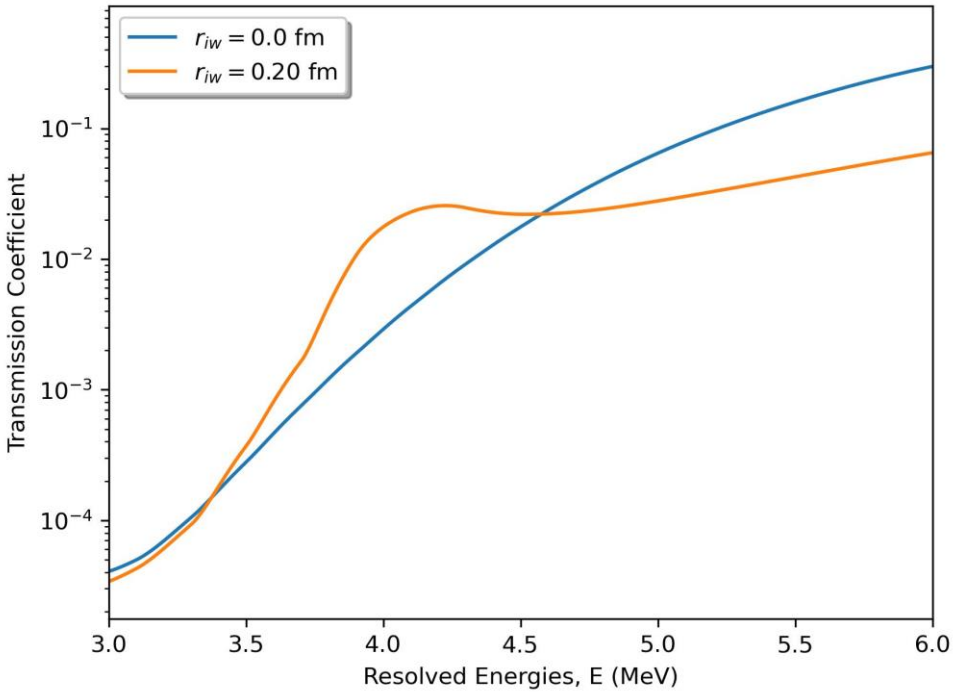
3). Calculate interaction potential by $V_{DC}(R) = E_{DC}(R) - E_1 - E_2$.

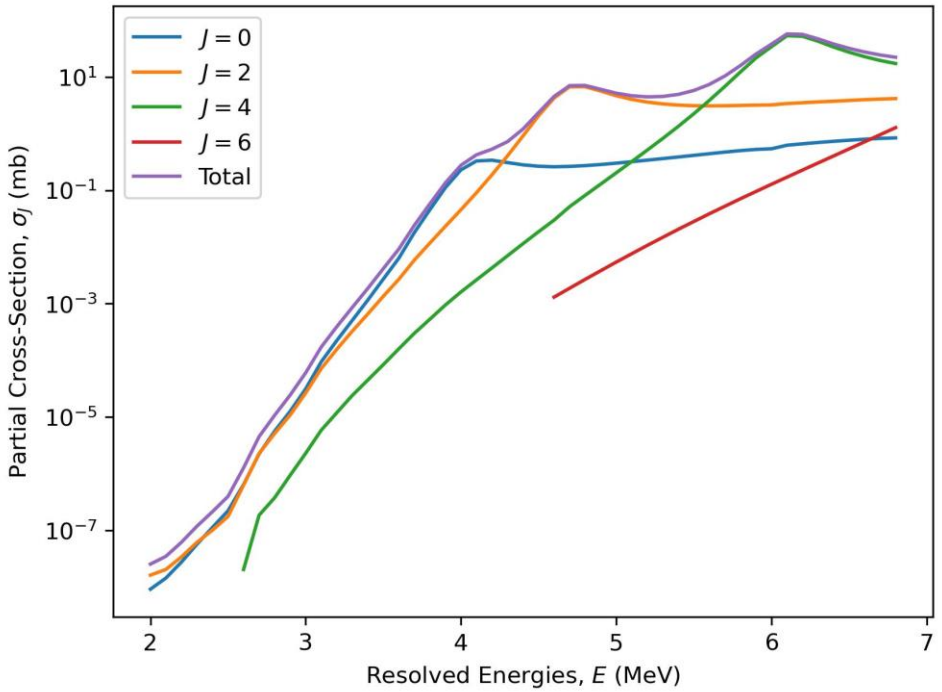
Interaction Potentials





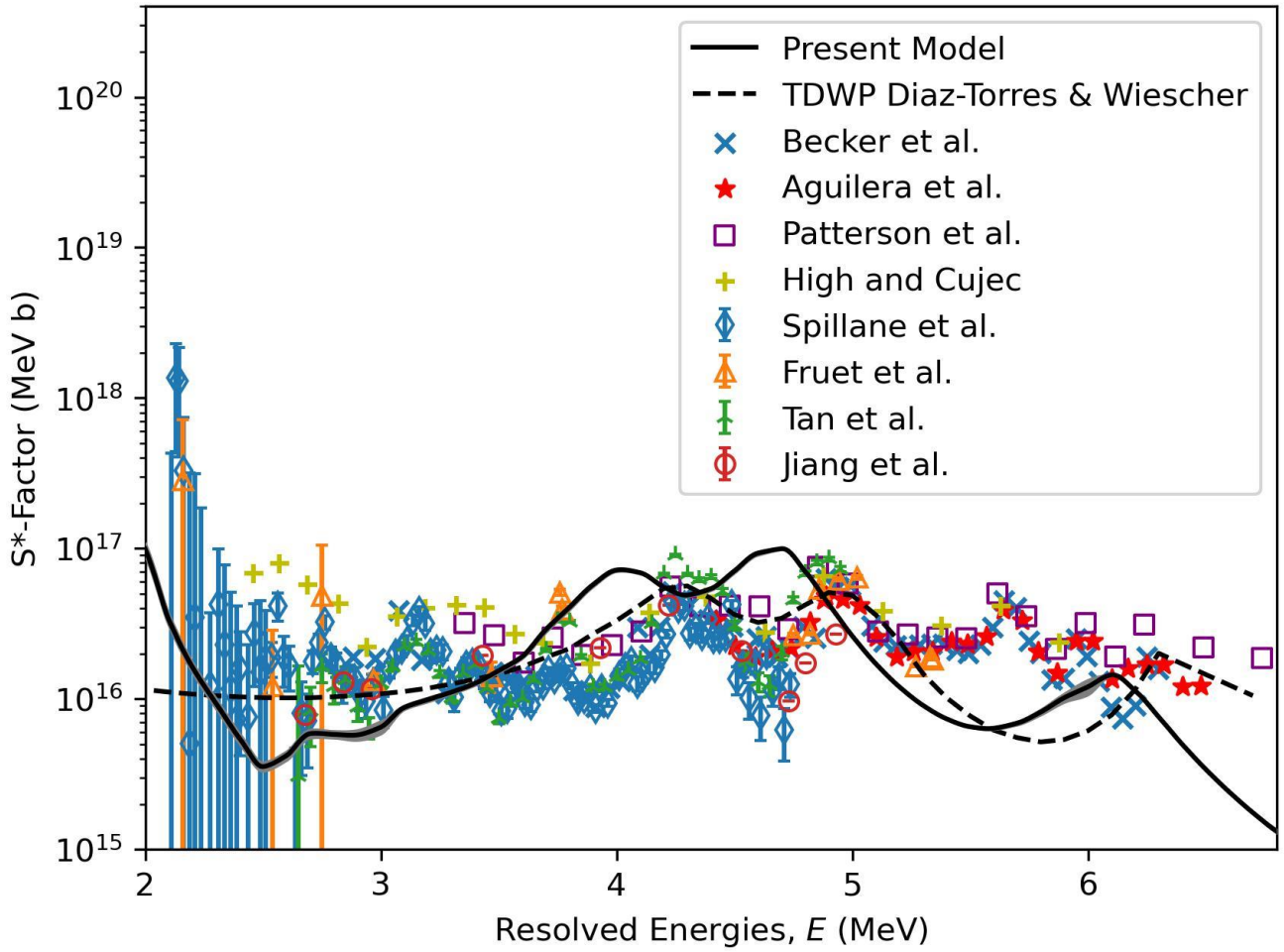
Resonance Peak and Cross-section





$$T(E) = 1 - \frac{\langle \Psi_f | \hat{\Delta}(E) | \Psi_f \rangle}{\langle \Psi_i | \hat{\Delta}(E) | \Psi_i \rangle}$$

Modified S*-factor



Any Questions?

References:

[1]. https://andromedageek.wordpress.com/2015/02/01/the-gamow-window/
[2]. A.Diaz-Torres, M.Wiescher, Quantum partner-dance in the 12C + 12C system yields sub-Coulomb fusion resonances. J. Phys. Conf. Ser. 492 (2014) 012006
[3]. J. Maruhn, P.-G. Reinhard, P. Stevenson, A. Umar, The TDHF code Sky3D, Comput. Phys. Commun. 185 (7) (2014) 2195–2216

Internal Wavefunction

$$\Psi(R, \theta_1, k_1, \theta_2, k_2) = \chi(R)\psi(\theta_1, k_1, \theta_2, k_2)$$

$$\psi(\theta_1, k_1, \theta_2, k_2) = [\zeta_{j1,m1}(\theta_1, k_1)\zeta_{j2,m2}(\theta_2, k_2) + (-1)^J \zeta_{j2,-m2}(\theta_1, k_1)\zeta_{j1,-m1}(\theta_2, k_2)] / \sqrt{2 + 2\delta_{j1,j2}\delta_{m1,-m2}}$$

Kinetic Energy Operator

$$\frac{2\hat{T}}{\hbar^2} = -\frac{1}{\mu}\frac{\partial^2}{\partial R^2} + \left(\frac{1}{I_1} + \frac{1}{\mu R^2}\right)\hat{j}_1^2 + \left(\frac{1}{I_2} + \frac{1}{\mu R^2}\right)\hat{j}_2^2$$
$$+ \frac{1}{\mu R^2}\left(\hat{j}_{1-}\hat{j}_{2+} + \hat{j}_{1-}\hat{j}_{2+} + J(J+1) - 2k_1^2 - 2k_1k_2 - 2k_2^2\right)$$
$$- \frac{C_+(J,K)}{\mu R^2}\left(\hat{j}_{1+} + \hat{j}_{2+}\right) - \frac{C_-(J,K)}{\mu R^2}\left(\hat{j}_{1-} + \hat{j}_{1-}\right)$$

Controls the translational and rotational movement of the nuclei during the reaction. As well as the reorientation and Coriolis effects.

Energy Projection

$$\widehat{\Delta}(E) = \frac{\zeta^{2^n}}{(\widehat{H} - E)^{2^n} + \zeta^{2^n}}$$