The MUonE Experiment:

Pioneering Measurement of the Hadronic Contribution to Muon $g-2$

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Oxford Seminar

UNIVERSITY OF LIVERPOOL

LEVERHULME TRUST

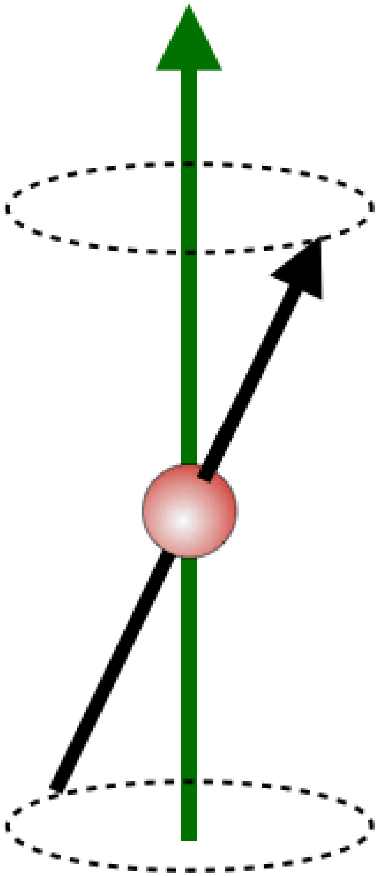
The MUonE Experiment:

Pioneering Measurement of the Hadronic Contribution to Muon $g-2$

- Muon $g-2$ Puzzle
- The MUonE Experiment
  - Principle
  - Setups — tracker, ECAL, beam, ...
  - Test Runs
- Outlook & Summary

Muon $g-2$

- The anomalous magnetic moment of the muon is connected to spin $\vec{s}$ via gyromagnetic ratio g: $\vec{\mu} = g\frac{e}{2m}\vec{S}$
  - g - factor quantifies interaction strength
  - Dirac predicted $\boldsymbol{g = 2}$ for spin-1/2 fermions



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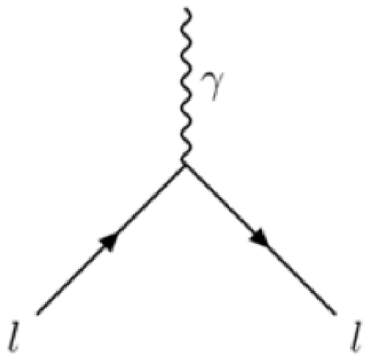

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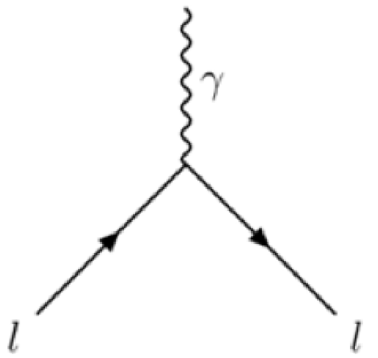
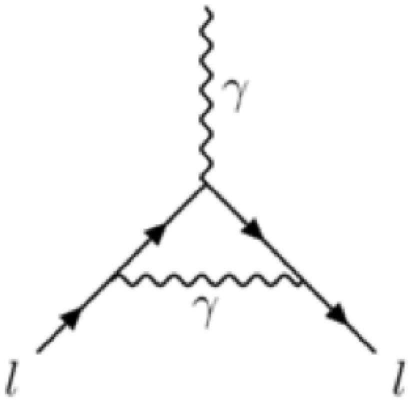


- Interactions with virtual particles cause g to deviate from 2 ($g > 2$). **Muon magnetic anomaly** is defined as:

$$a_\mu = \frac{g_\mu - 2}{2}$$

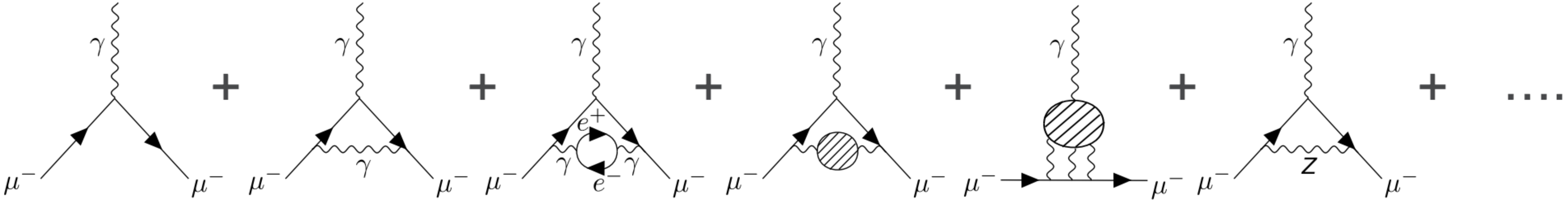
$$a_l = \frac{g_l - 2}{2} = \frac{\alpha}{2\pi} + \mathrm{O}(\alpha^2)$$

$$= 0.001\,161\,40\,.$$

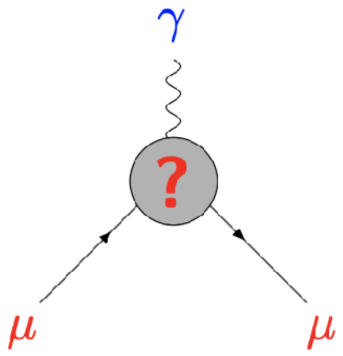


Muon $g-2$

- a_μ receives contribution from QED, EW and QCD effects in the SM



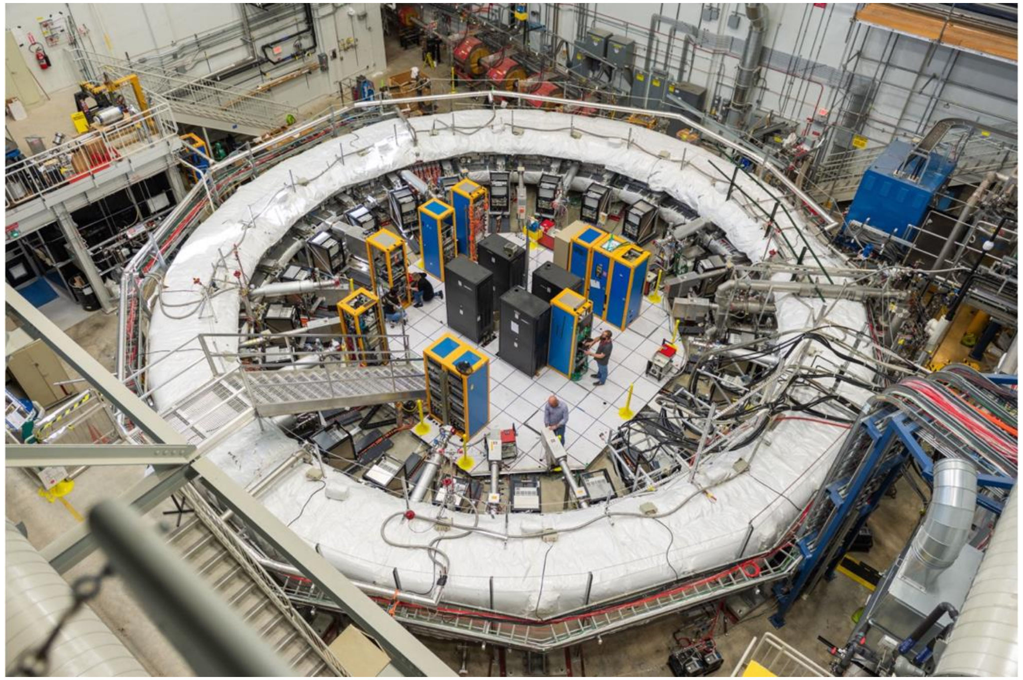
- A probe to new physics beyond SM:
  - For possible new physics $a_\mu^{NP} \propto (\frac{m_l}{\Lambda_{NP}})^2$
  - Muon is more sensitive by a factor of $(\frac{m_\mu}{m_e})^2 \approx 4.3 \times 10^4$

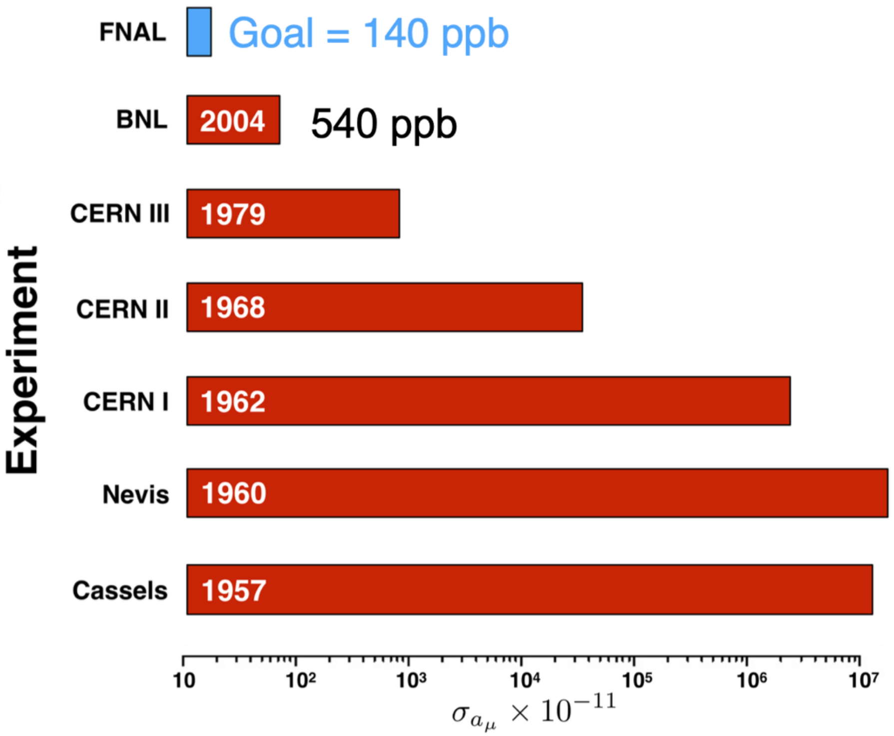


Muon $g - 2$

Experiment at Fermilab

- Fermilab utilizes an approach similar to BNL with the same storage ring





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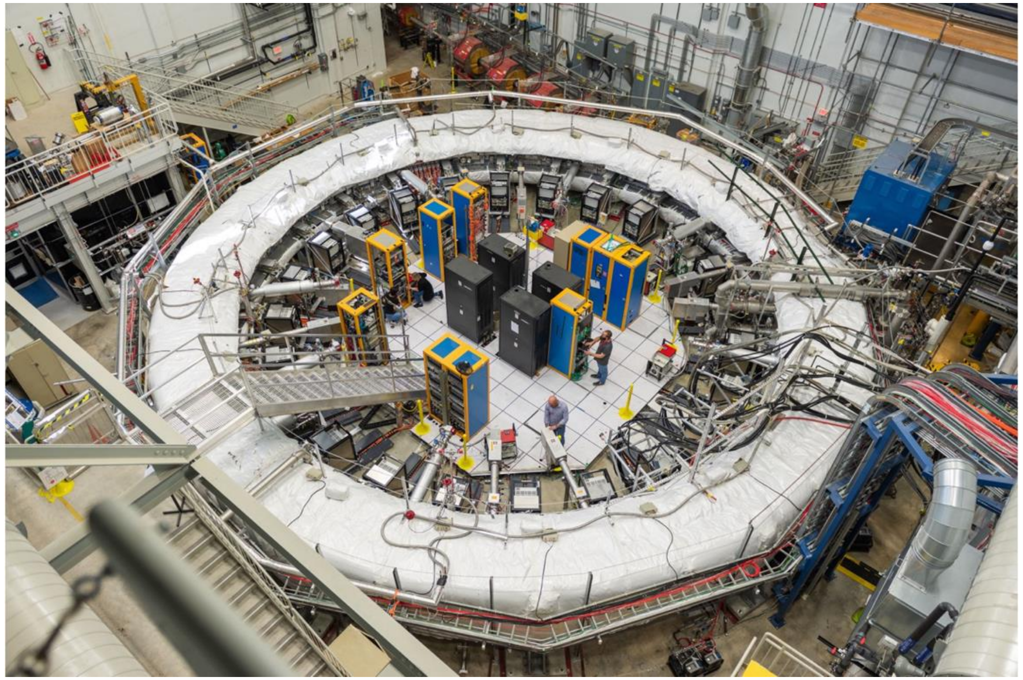

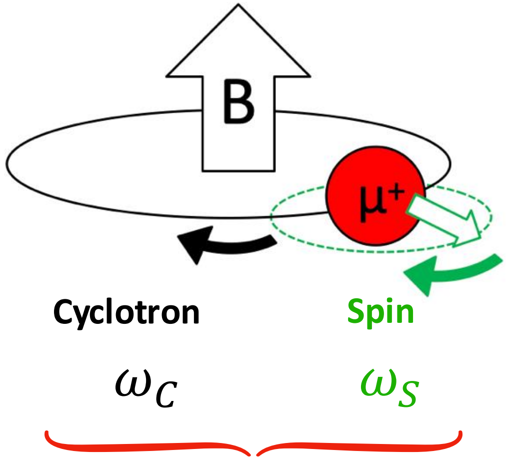

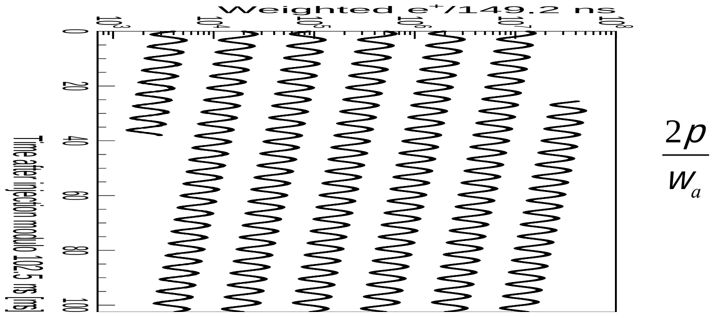

$$\boldsymbol{\omega_a} = \omega_S - \omega_C = -\frac{q}{m_\mu}\boldsymbol{a_\mu}B$$

Muon $g-2$

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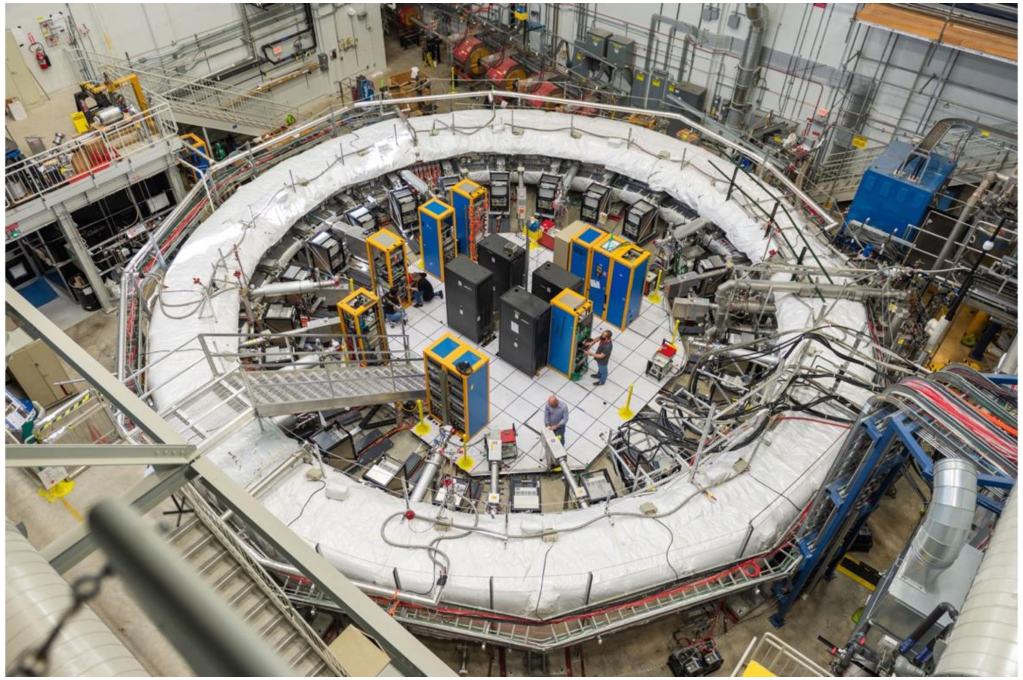
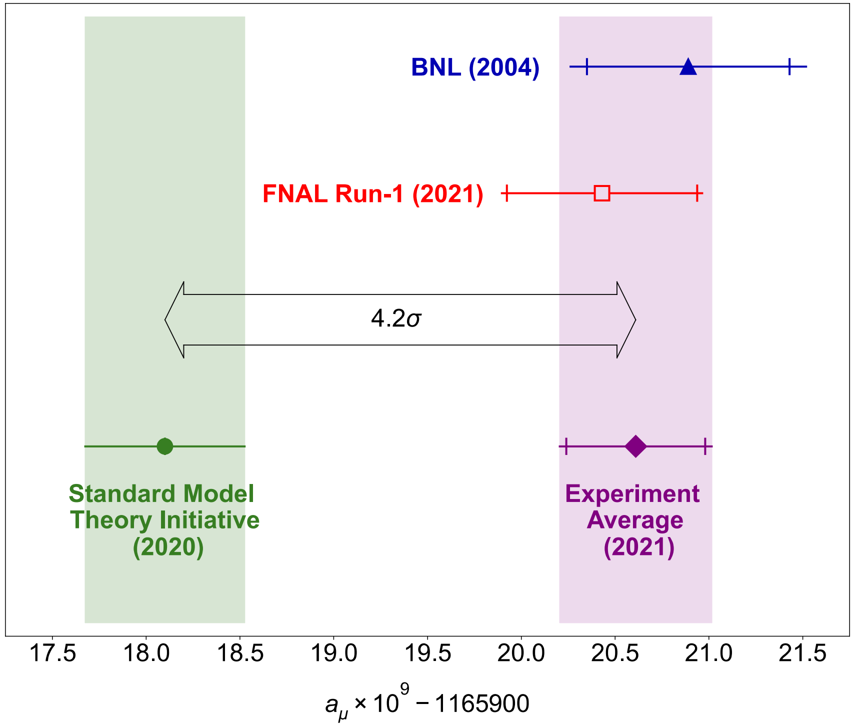
- Run-1 results were released in 2021, with Run-2/3 results following in 2023





- FNAL Run-1 (2021) confirmed BNL (Brookhaven, 2004) measurement
- FNAL (2021) + BNL average in tension with Theory Initiative White Paper (2020) at $4.2\ \sigma$

Muon $g-2$

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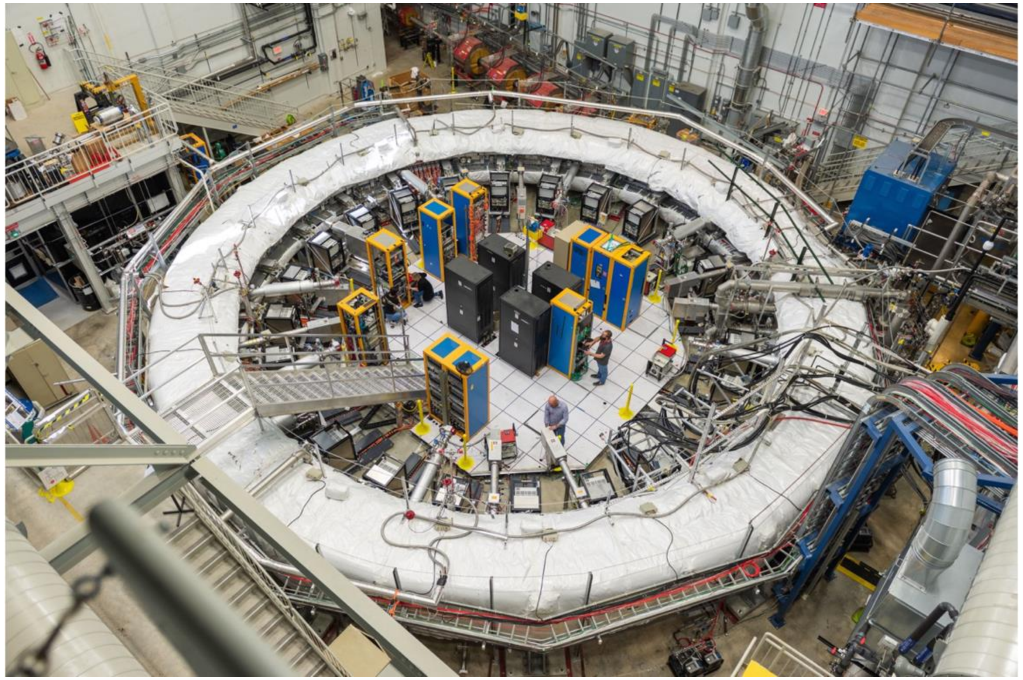
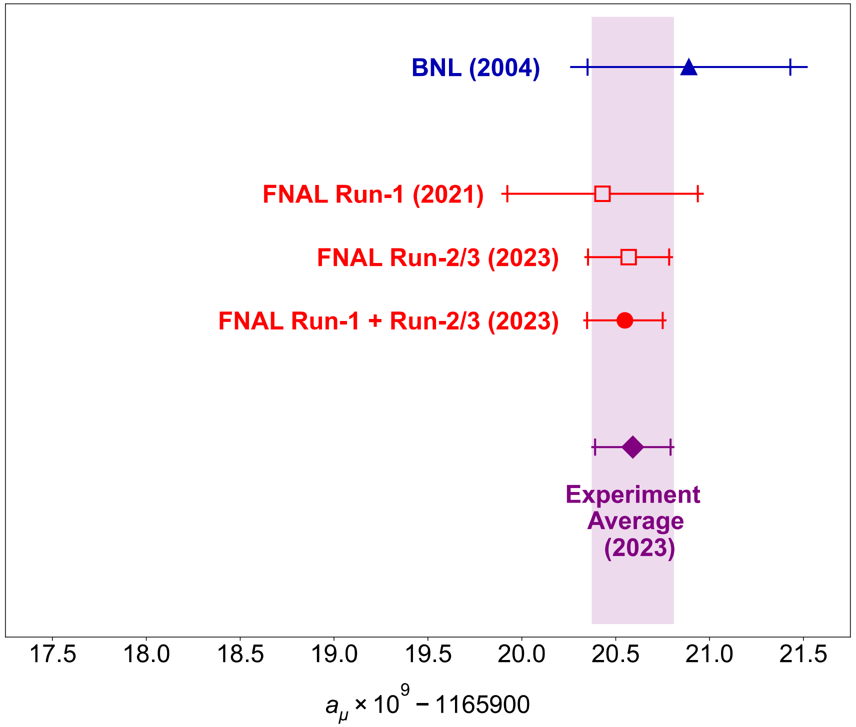
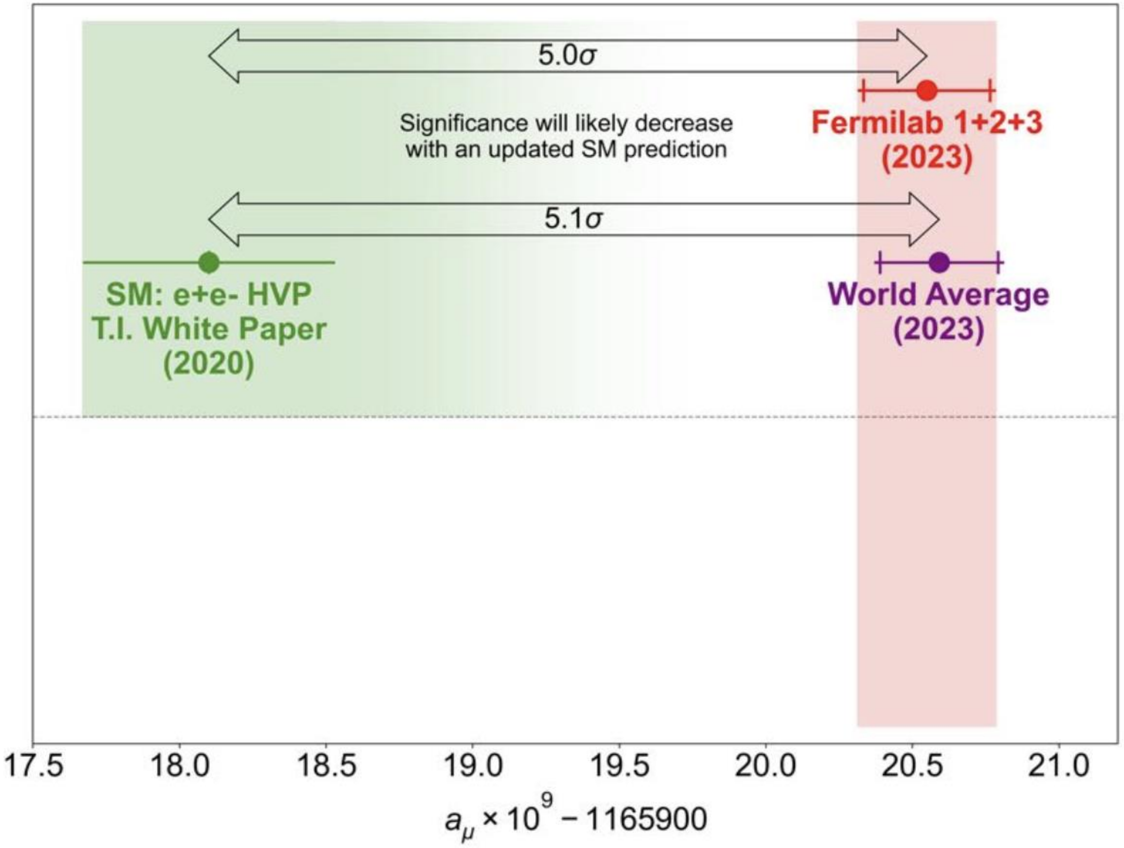




- Combined FNAL result uncertainty: **203 ppb**
- Combined world average uncertainty is **190 ppb**
- Average is **dominated by FNAL** value

Muon $g-2$ Puzzle

Discrepancy between experiments & theories



- New experimental average with SM prediction (WP-2020) gives > 5σ

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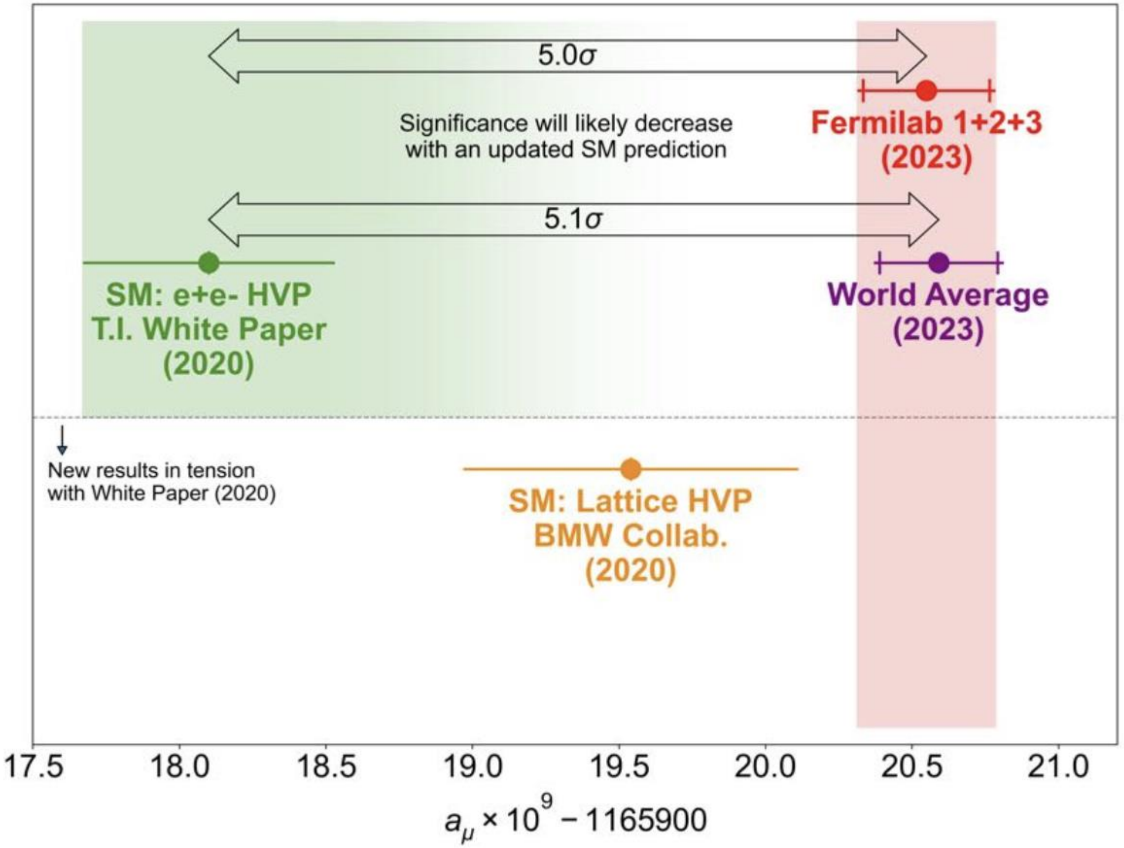

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  - Lattice QCD from the BMW (2020)

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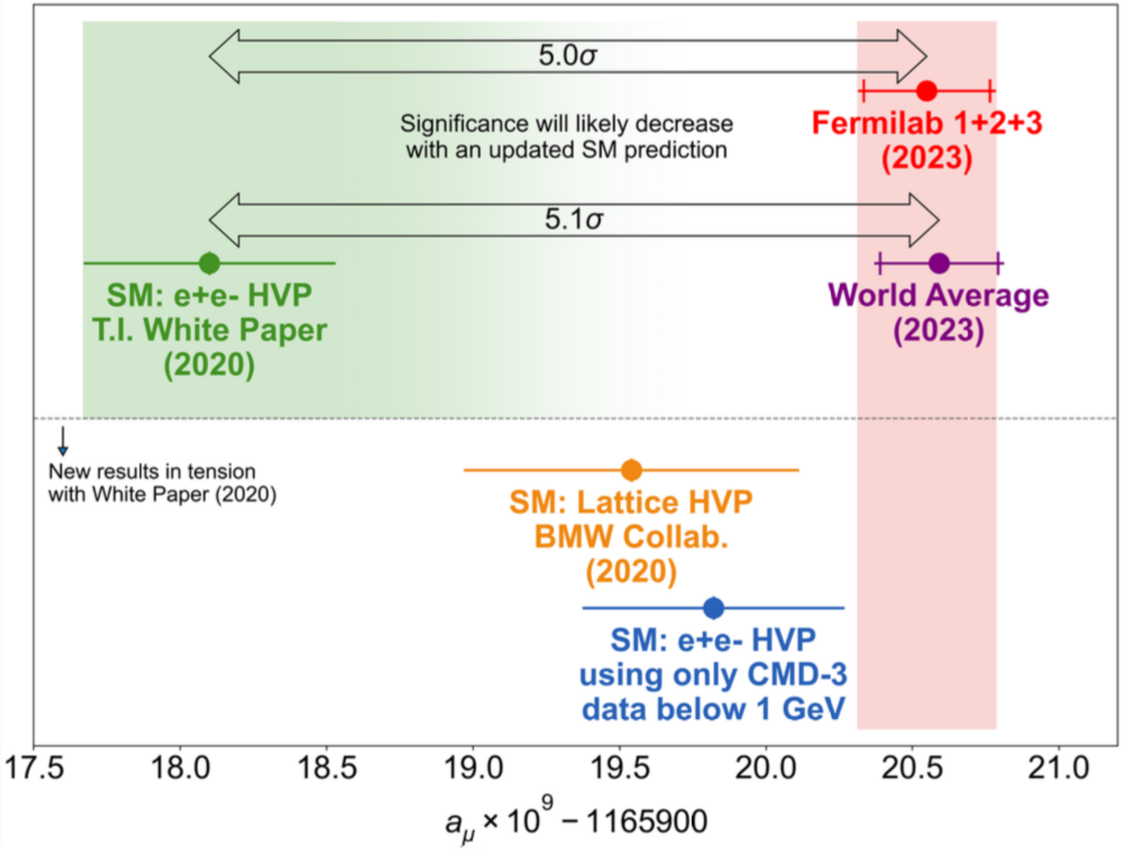

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- Since then, two important developments on SM prediction:
  - Lattice QCD from the BMW (2020)
  - New $e^+e^- \to \pi^+\pi^-$ cross section from CMD-3 (2023)

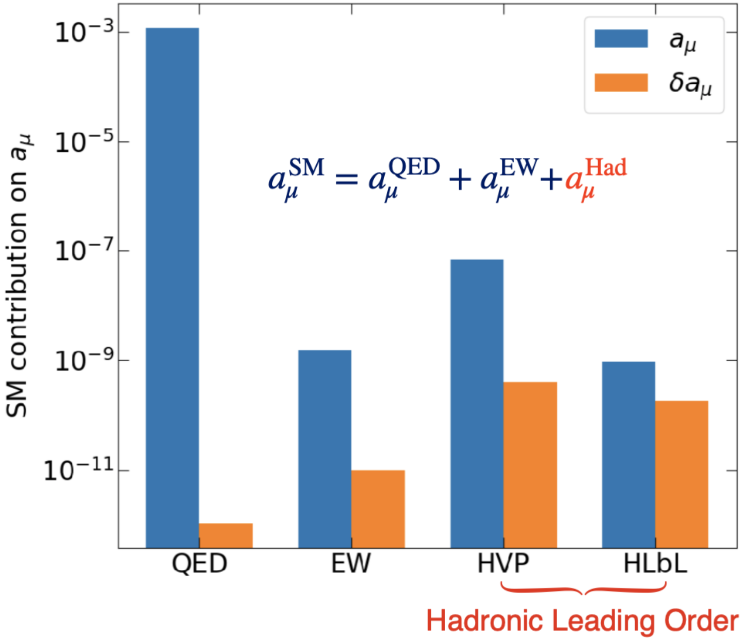
> ➢ **Disclaimer:**
> The CMD-3 point is a visual exercise. It is not a fully updated SM prediction!

- TI White Paper result has been substituted by CMD-3 only for 0.33 → 1.0 GeV.
- The NLO HVP has not been updated.
- It is purely for demonstration purposes → should not be taken as final!

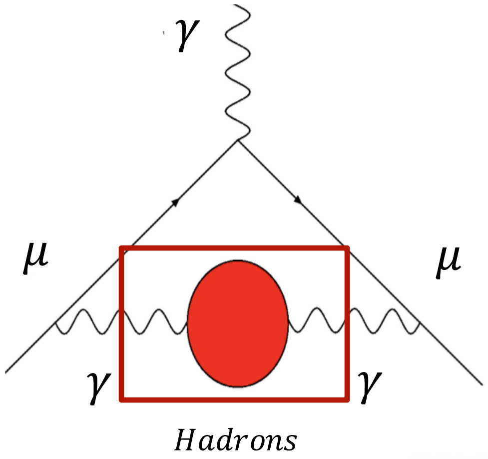
Muon $g-2$ **Puzzle**

Standard Model (SM) predictions

- The uncertainty in the SM prediction of a_μ is **entirely limited** by our knowledge of the hadronic leading order contribution $\boldsymbol{a_\mu^{\mathrm{HLO}}}$ $\boldsymbol{(a_\mu^{\mathrm{HVP},LO})}$



- **HVP**: **hadronic vacuum polarization →**
- **HLbL**: hardronic light-by-light



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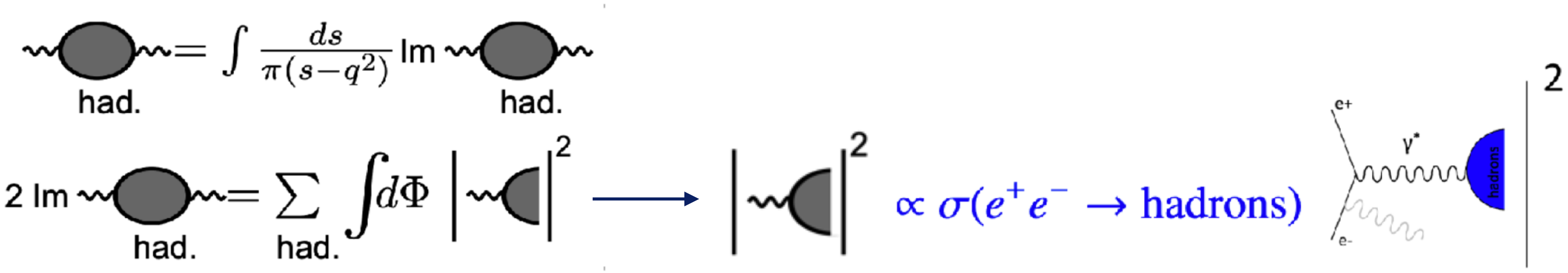
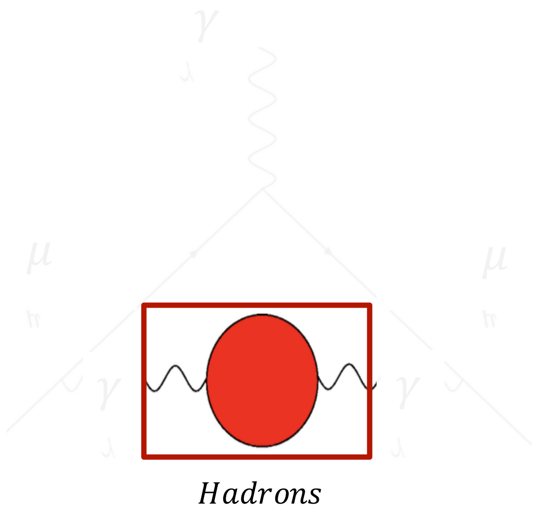
- Approaches (at low-E):
  1) Lattice QCD Method: Ab-initio calculation on lattice
  2) Dispersive Method: using σ(e^+e^-→ hadrons) data





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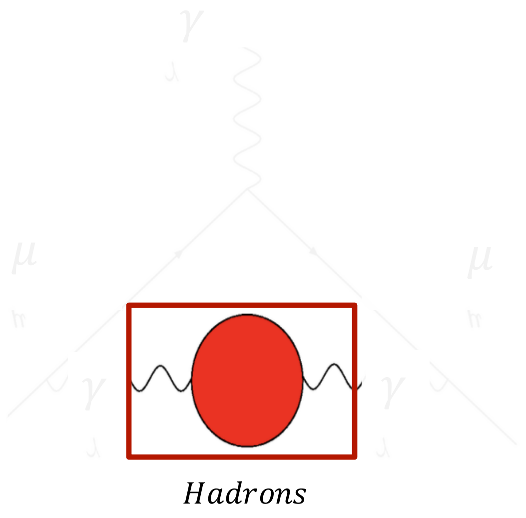
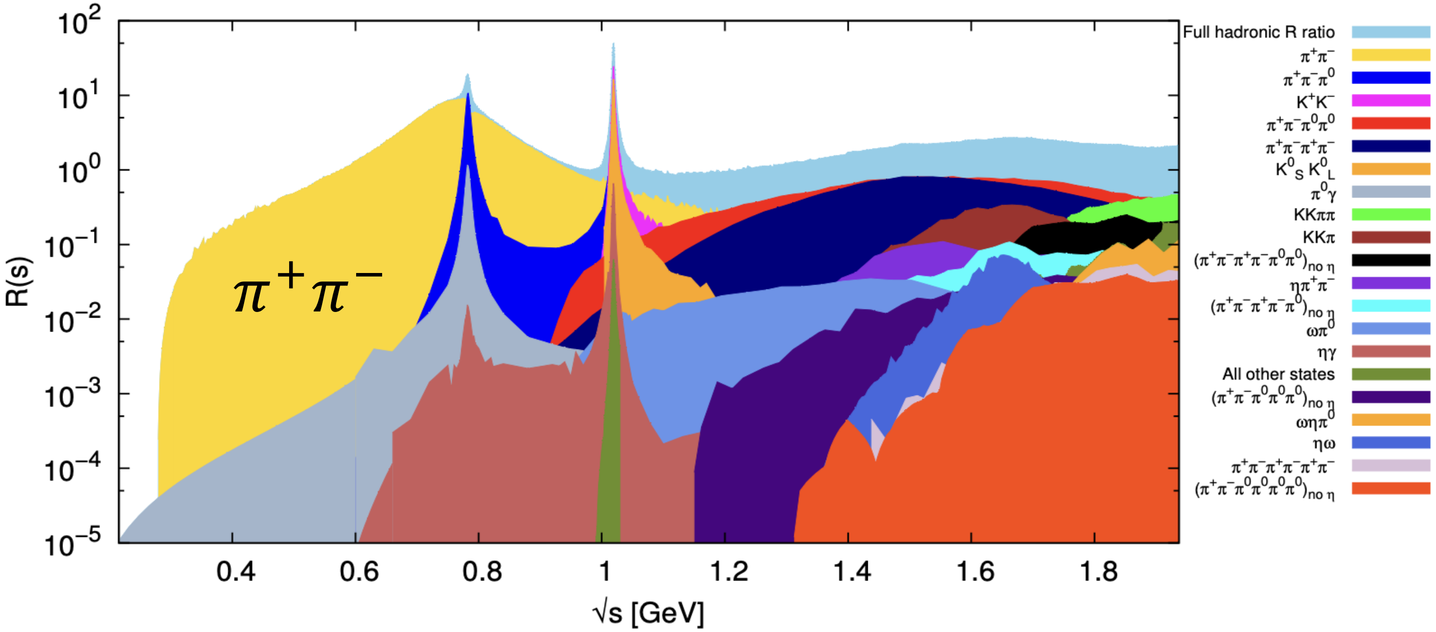
$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds \, \frac{\boldsymbol{R_{\mathrm{had}}(s)}\, K(s)}{s^2}, \quad R_{\mathrm{had}}(s) = \sigma(e^+e^- \to \mathrm{hadrons}) \Big/ \frac{4\pi\,\alpha(s)}{(3s)}$$

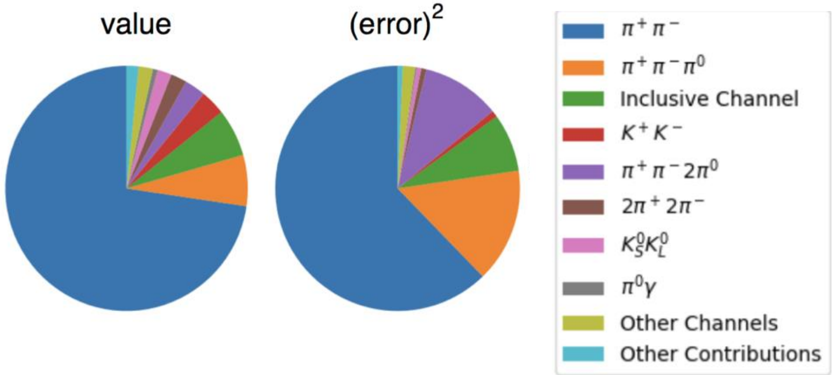


Muon $g-2$ Puzzle

Chaotics in $e^+e^- \to \pi^+\pi^-$ for a_μ^{HVP}

- $e^+e^- \to \pi^+\pi^-$ **channel** is the major source of uncertainty in a_μ^{HVP}





Contribution of $\sigma(e^+e^- \to \mathrm{hadrons})$ across different hadronic channels

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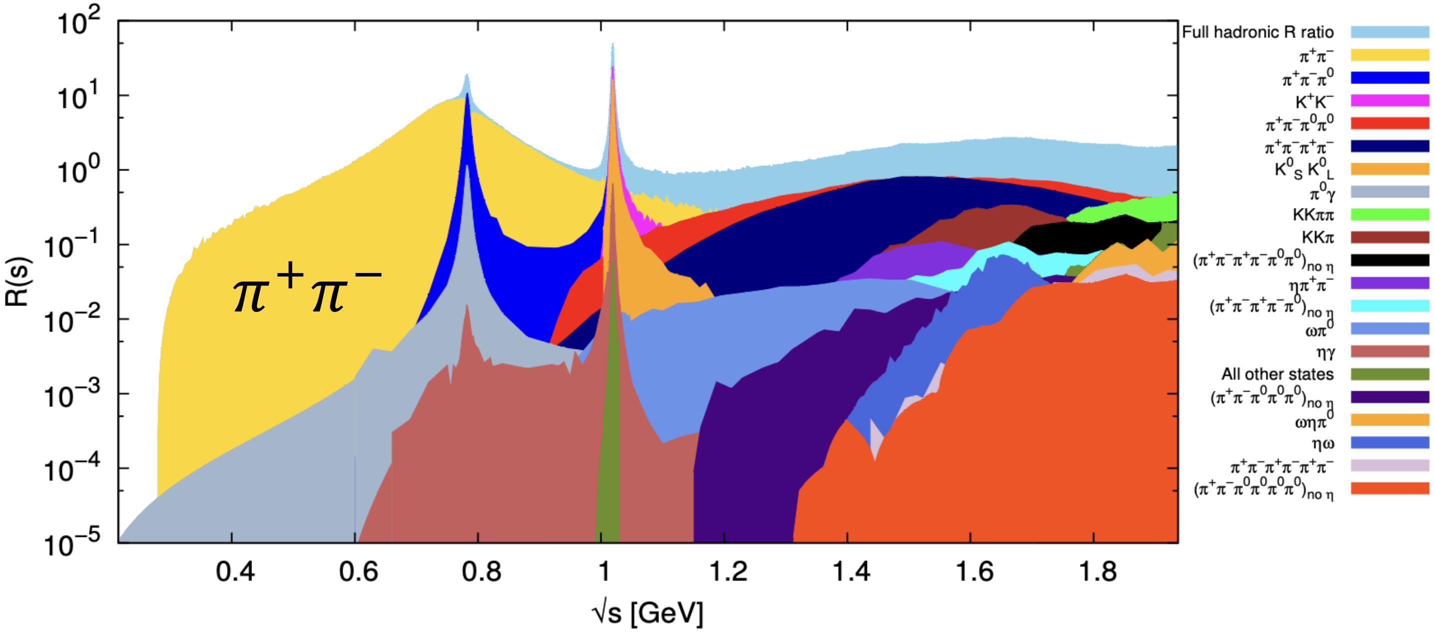
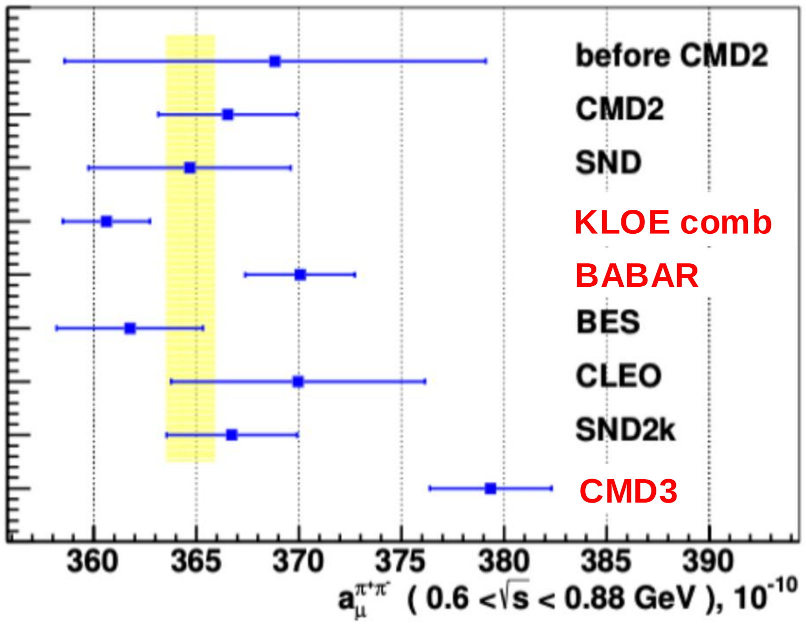
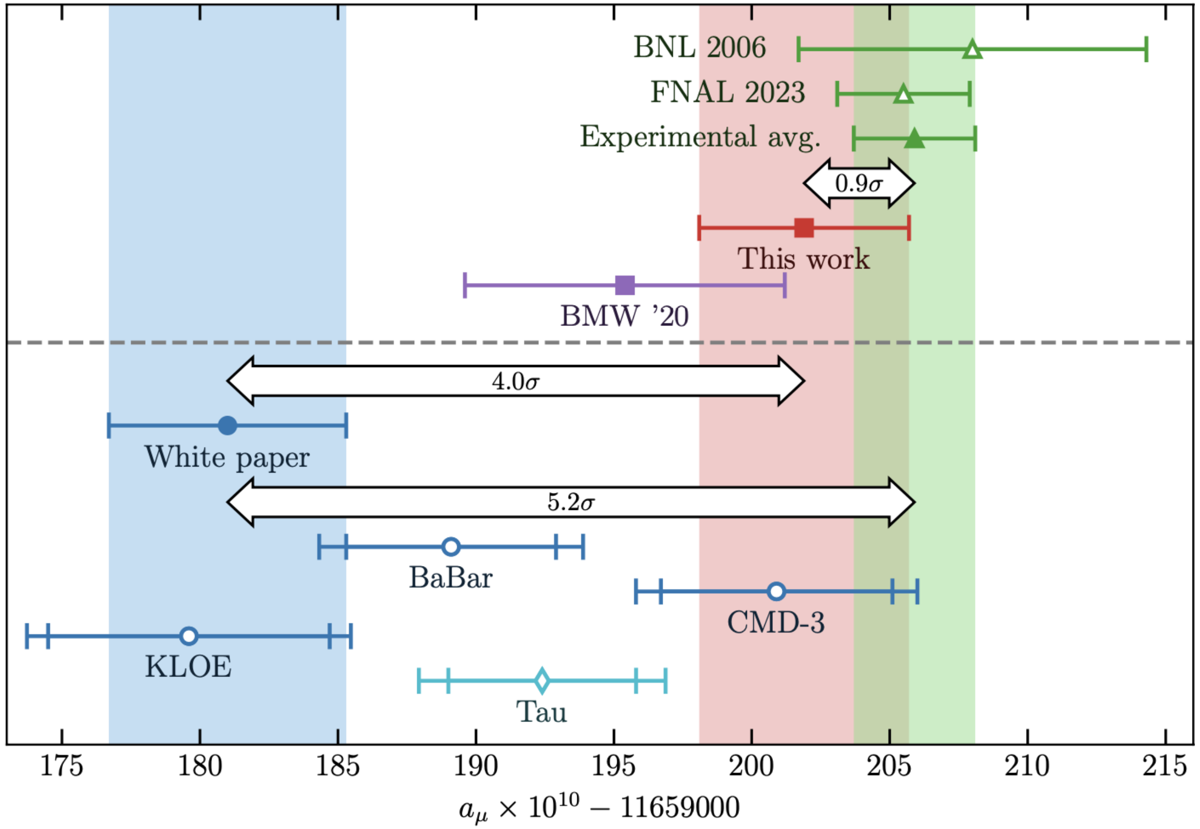




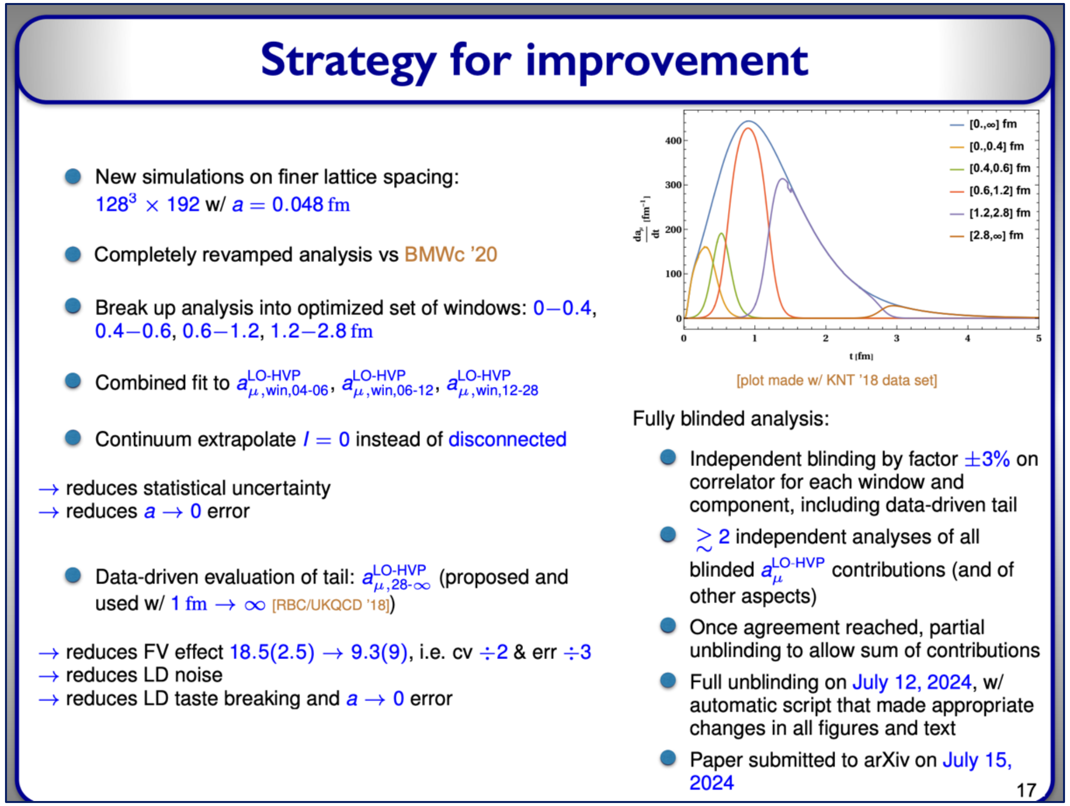
- Another puzzle: measurement by CMD-3 (2023) was significantly higher!

Muon $g-2$ **Puzzle**

More chaos from recent Lattice QCD updates



BMW/DMZ24 (2407.10913)



Strategy for improvement

- New simulations on finer lattice spacing: $128^3 \times 192$ w/ $a = 0.048\,\mathrm{fm}$
- Completely revamped analysis vs BMWc '20
- Break up analysis into optimized set of windows: $0-0.4$, $0.4-0.6$, $0.6-1.2$, $1.2-2.8\,\mathrm{fm}$
- Combined fit to $a^{\text{LO-HVP}}_{\mu,\text{win,04-06}}$, $a^{\text{LO-HVP}}_{\mu,\text{win,06-12}}$, $a^{\text{LO-HVP}}_{\mu,\text{win,12-28}}$
- Continuum extrapolate $I = 0$ instead of disconnected

→ reduces statistical uncertainty
→ reduces $a \to 0$ error

- Data-driven evaluation of tail: $a^{\text{LO-HVP}}_{\mu,28\text{-}\infty}$ (proposed and used w/ $1\,\mathrm{fm} \to \infty$ [RBC/UKQCD '18])

→ reduces FV effect $18.5(2.5) \to 9.3(9)$, i.e. cv $\div 2$ & err $\div 3$
→ reduces LD noise
→ reduces LD taste breaking and $a \to 0$ error

[plot made w/ KNT '18 data set]

Fully blinded analysis:

- Independent blinding by factor $\pm 3\%$ on correlator for each window and component, including data-driven tail
- $\gtrsim 2$ independent analyses of all blinded $a^{\text{LO-HVP}}_{\mu}$ contributions (and of other aspects)
- Once agreement reached, partial unblinding to allow sum of contributions
- Full unblinding on July 12, 2024, w/ automatic script that made appropriate changes in all figures and text
- Paper submitted to arXiv on July 15, 2024

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Davide Giusti, MPP2024

Muon $g-2$ Puzzle

Putting all together

	Current Situation
Experiments	- Storage ring measurements are consistent (BNL + FNAL)

SM predictions (HVP contribution)	- Lattice ⇔ e^+e^- data-driven
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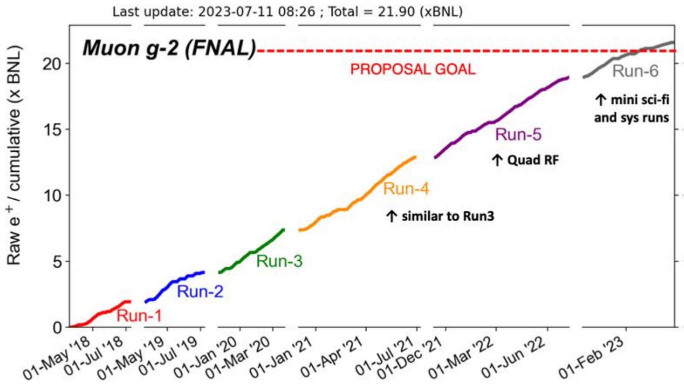

Dataset	Statistical Error [ppb]
Run 1	434
Run 2+3	201
Run 4+5+6	110 (est.)
All Run 1 to 6	90 (est.)

- For weighted e+ in the final ω_a fit, a factor 3.3 more data in Run-4/5/6 than Run-2/3 and 15.7 than Run-1.
- All using positive muons.

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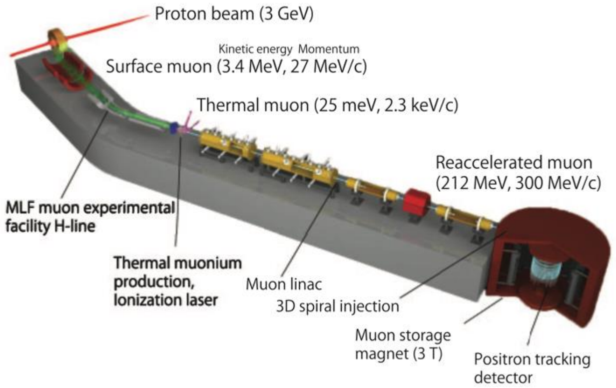

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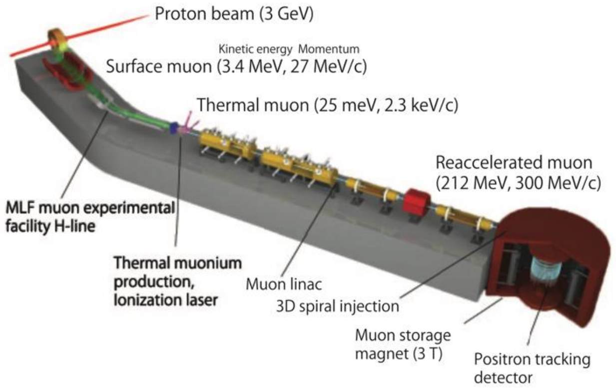

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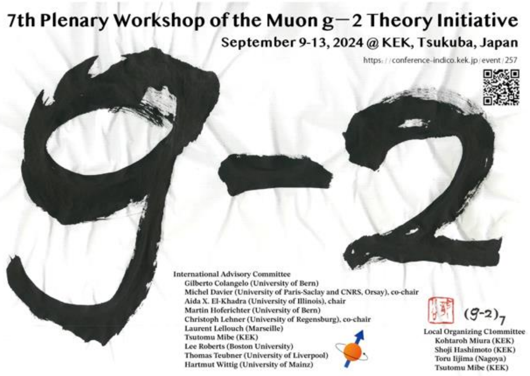

7th Plenary Workshop of the Muon g−2 Theory Initiative

September 9-13, 2024 @ KEK, Tsukuba, Japan

https://conference-indico.kek.jp/event/257

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Lee Roberts (Boston University)
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Hartmut Wittig (University of Mainz)

$(g-2)_7$

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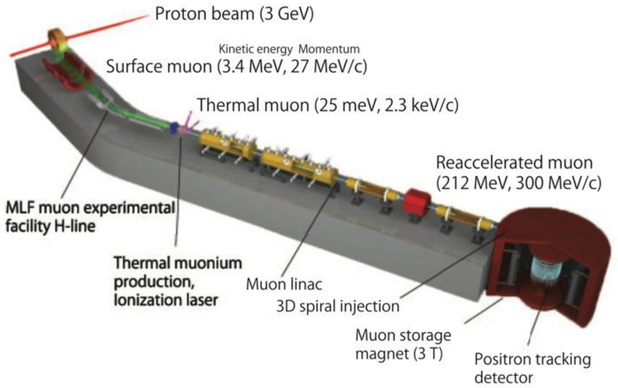

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The Status of MUonE Experiment:

Understanding Muon $g-2$ Puzzle via $\mu-e$ Scattering

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  - Principle
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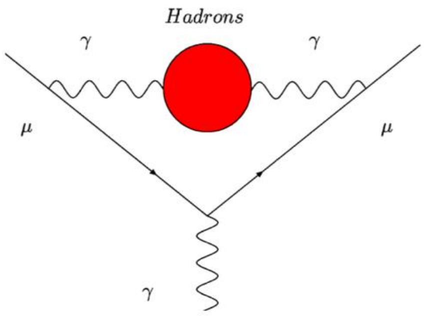
MUonE Experiment

A new approach measuring $\boldsymbol{a_\mu^{\mathrm{HVP}}}$ with running of $\Delta\boldsymbol{\alpha}_{\mathbf{had}}$

- The dispersive approach to compute $a_\mu^{\mathrm{HVP,LO}}$ is via the **time-like** formula:



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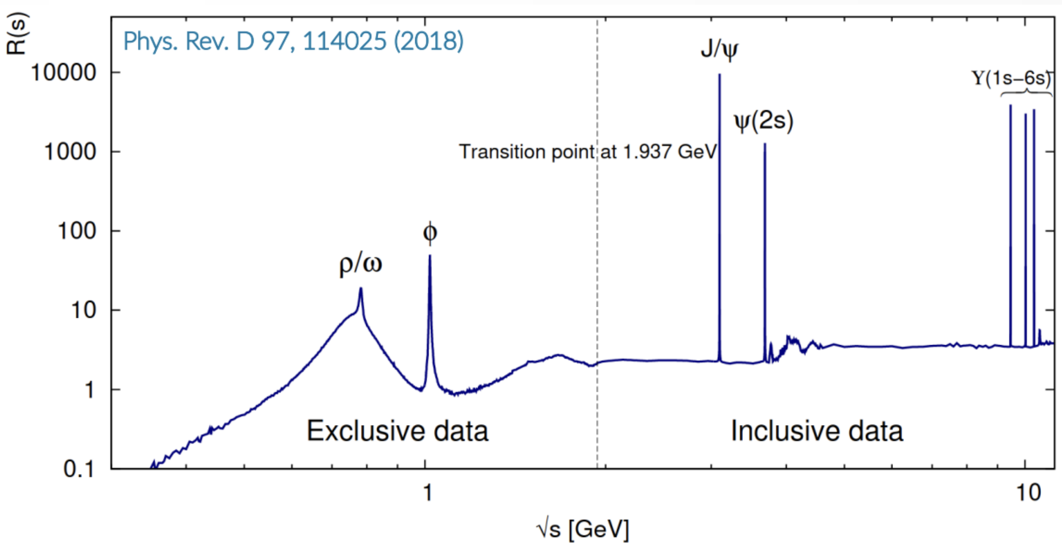
Phys. Rev. D 97, 114025 (2018)

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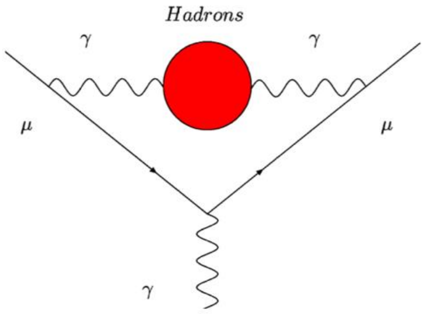
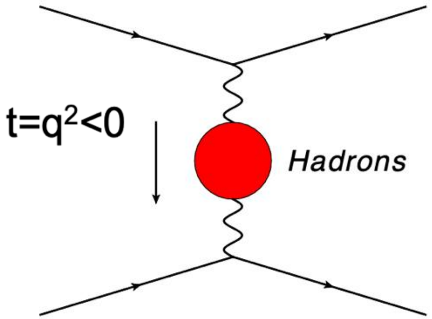
- The dispersive approach to compute $a_\mu^{\mathrm{HVP,LO}}$ is via the **time-like** formula:



$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\mathrm{had}}(s)\boldsymbol{K(s)}}{s^2}, \quad K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2+(1-x)(s/m_\mu^2)}$$

- Alternatively, exchanging the x and s integrations → **space-like** formula:



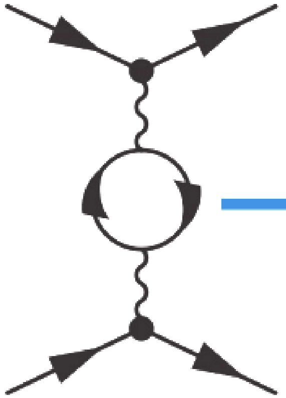
$$a_\mu^{\mathrm{HVP}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\boldsymbol{\Delta\alpha_{\mathrm{had}}[t(x)]}, \quad t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

- $\Delta\alpha_{\mathrm{had}}$ is the hadronic contribution to the **running α**

MUonE Experiment

Running of $\Delta\alpha_{\text{had}}$: ‘Time-like’ vs ‘Space-like’

- The electromagnetic coupling constant runs as a function of the momentum transfer, due to vacuum polarization effects



$$\Delta\alpha(q^2) = \Delta\alpha_{\text{lep}}(q^2) + \Delta\alpha_{\text{had}}(q^2) + \Delta\alpha_{\text{top}}(q^2)$$

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}$$

$\alpha(0) \sim 1/137$

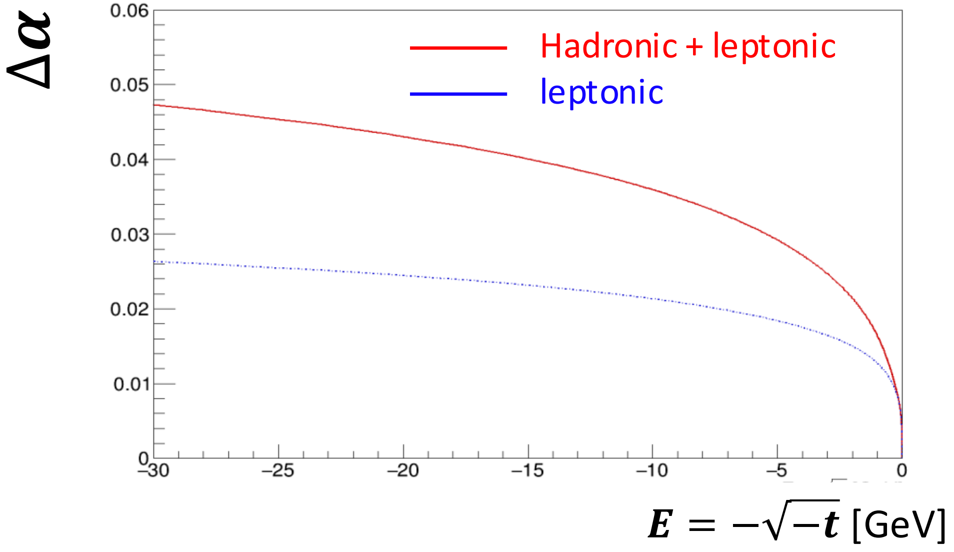
$\alpha(M^2_Z) \sim 1/127$

MUonE Experiment

Running of $\Delta\alpha_{\mathrm{had}}$: ‘Time-like’ vs ‘Space-like’

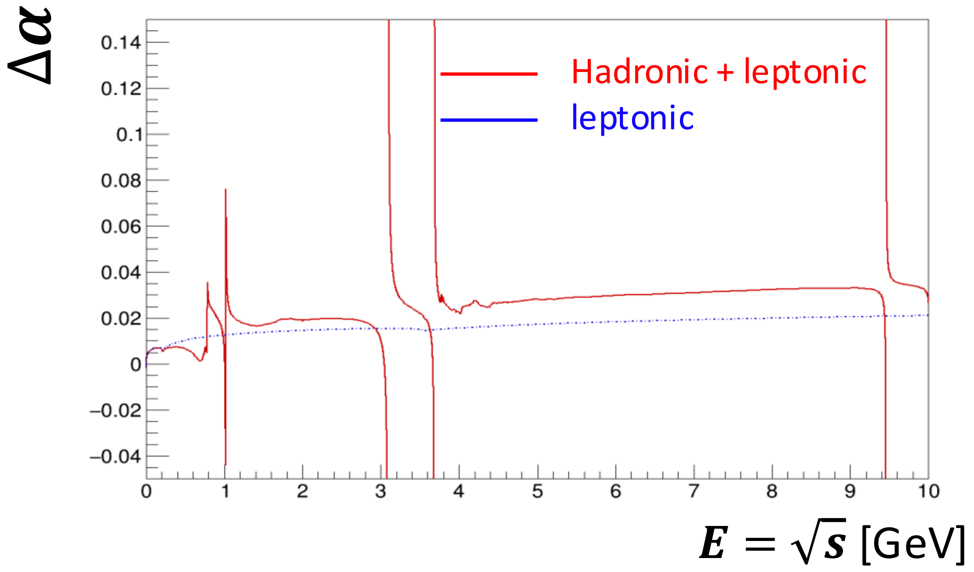
- The electromagnetic coupling constant runs as a function of the momentum transfer, due to vacuum polarization effects

- **Space-like**: a very smooth behavior
  - Inclusive measurement
  - Direct interplay with LQCD



$E = -\sqrt{-t}$ [GeV]

- **Time-like:** characterized by the opening of resonances

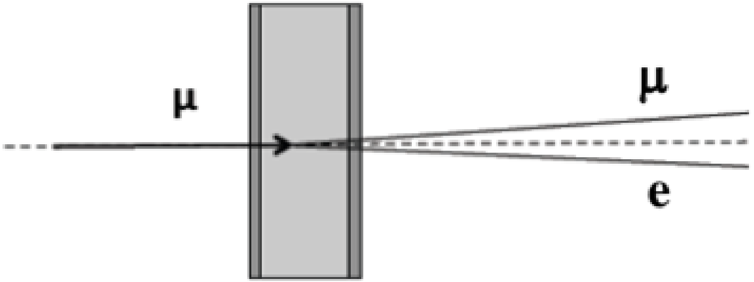


$E = \sqrt{s}$ [GeV]

MUonE Experiment

$\Delta\alpha_{\text{had}}$ via Muon-electron scattering

- $\boldsymbol{\Delta\alpha_{\text{had}}[t(x)]}$ can be extracted from the **shape** of the differential cross-section of **muon-electron scattering $\boldsymbol{\mu^+ e^- \to \mu^+ e^-}$**



$$R_{\text{had}} = \frac{d\sigma_{\text{data}}(\Delta\alpha_{\text{had}})}{d\sigma_{\text{MC}}(\Delta\alpha_{\text{had}} = 0)} \sim 1 + \underline{2\Delta\alpha_{\text{had}}(t)}$$

To be determined
in this experiment

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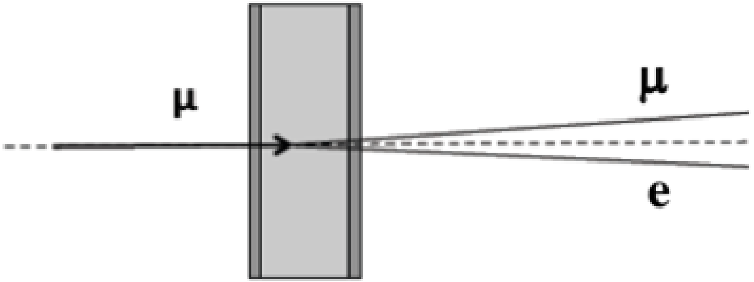

Shape measurement

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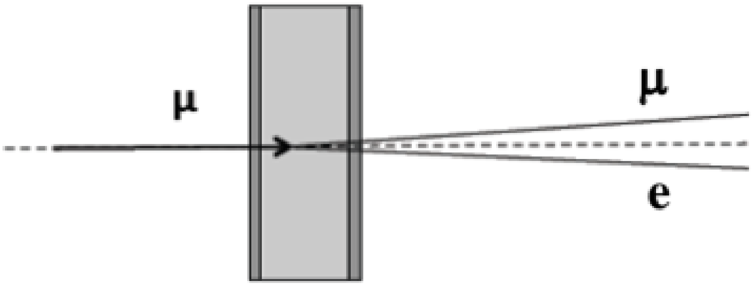

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The NNLO differential cross section from **theoretical calculation**

To be determined in this experiment

MUonE Experiment

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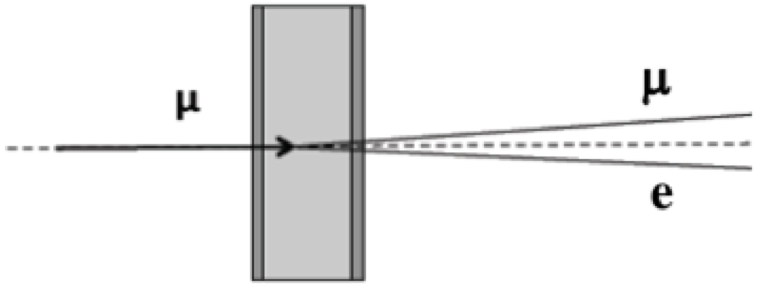

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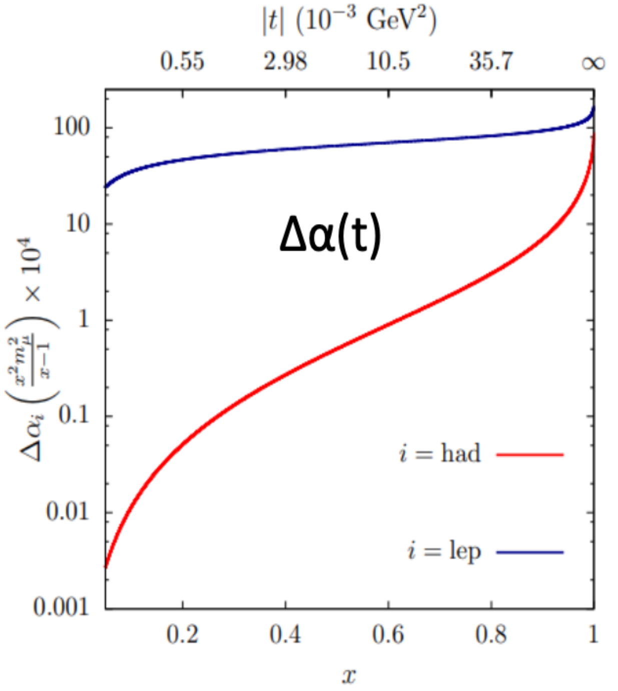

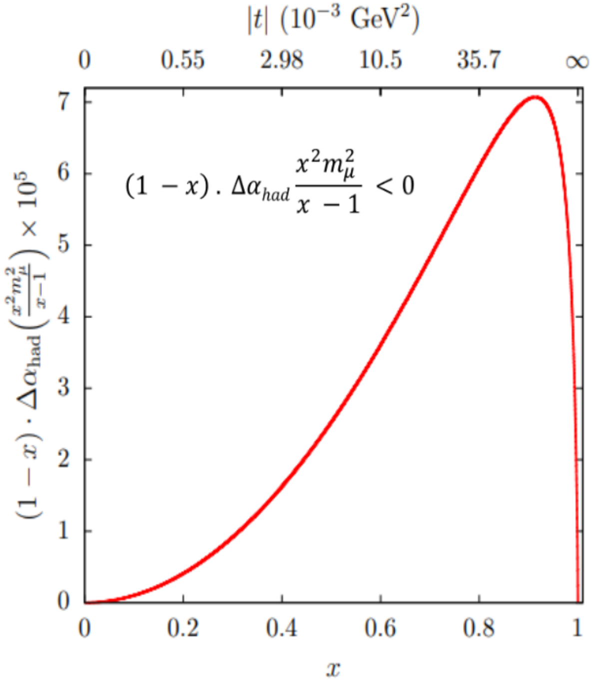

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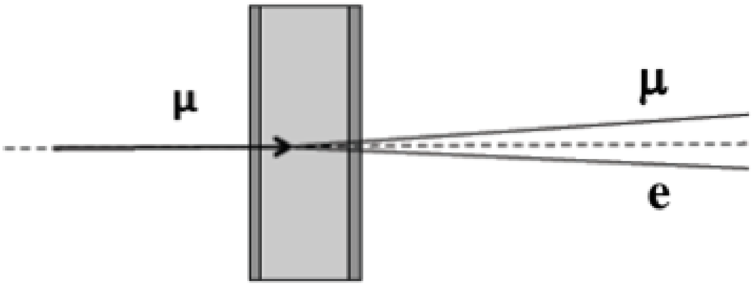

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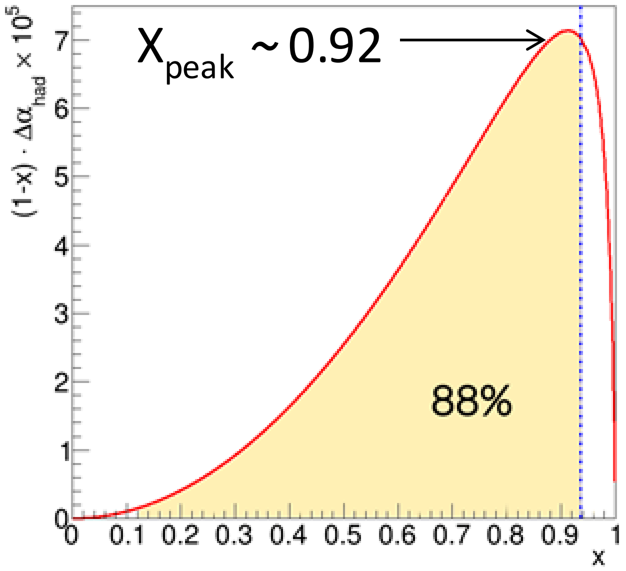
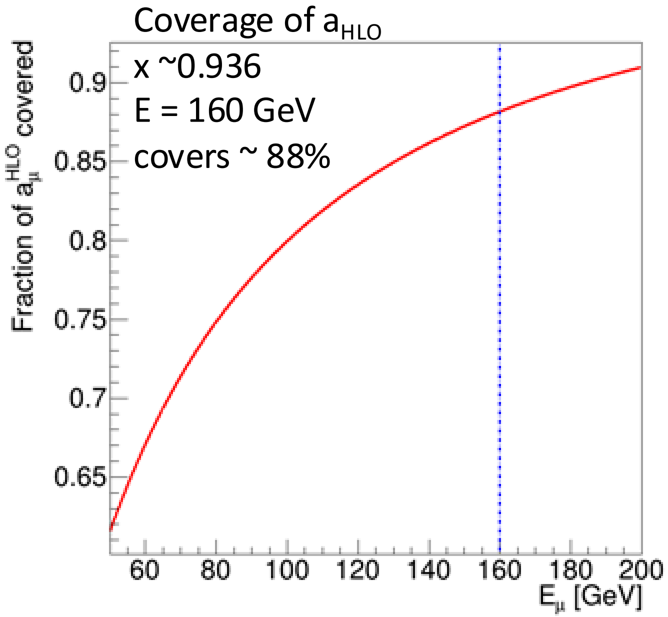


$X_{\text{peak}} \sim 0.92$

88%



Coverage of a_{HLO}
x ~0.936
E = 160 GeV
covers ~ 88%

MUonE Experiment

$\Delta\alpha_{\text{had}}$ via Muon-electron scattering

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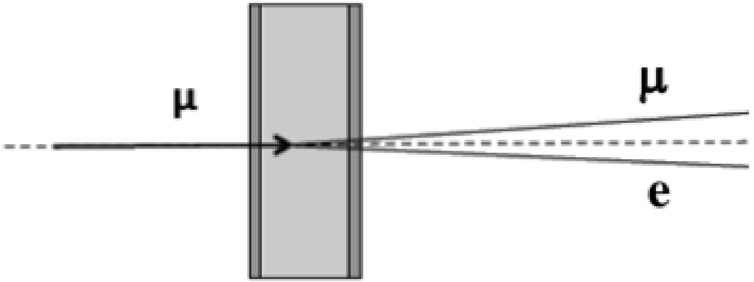

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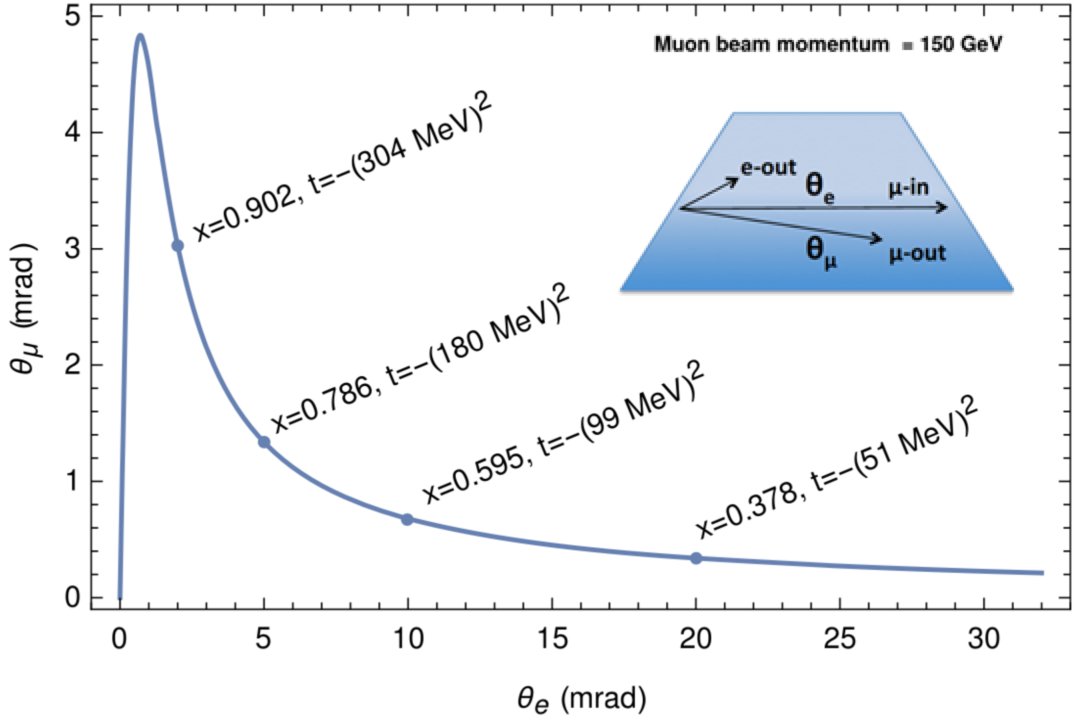
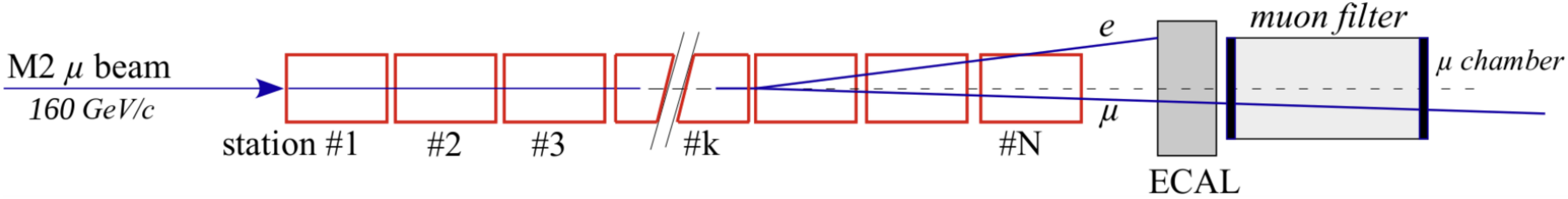


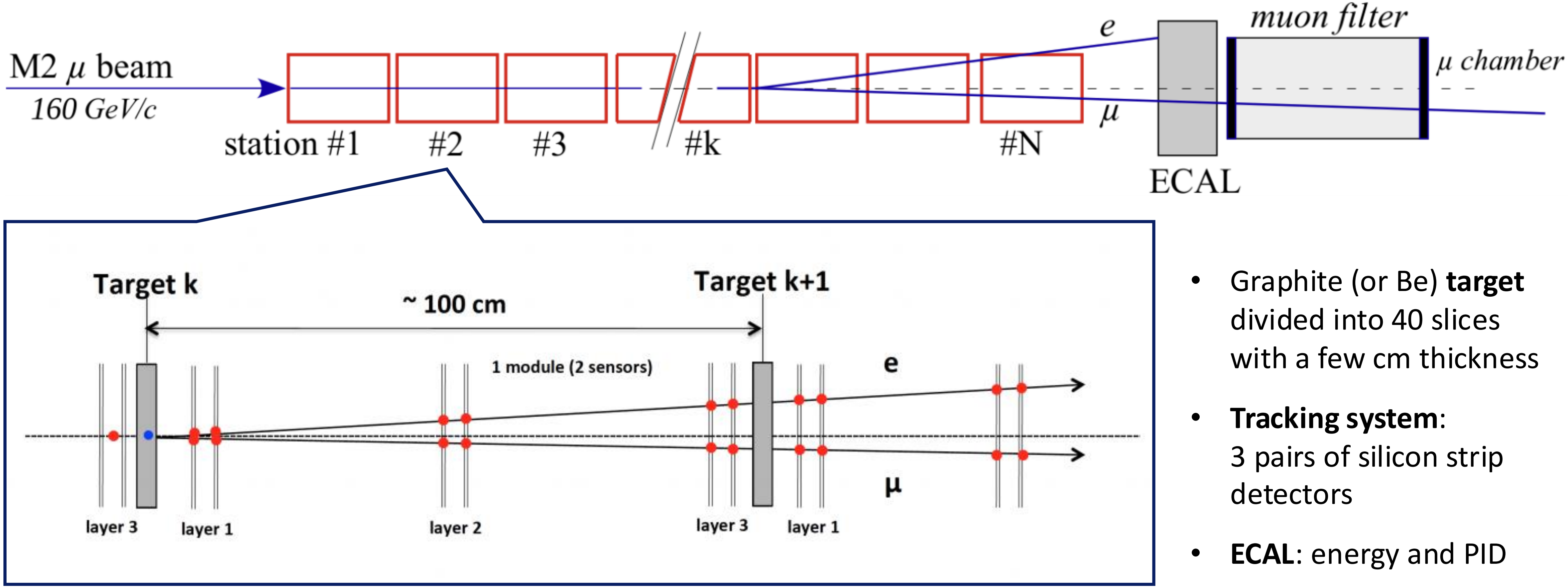
MUonE Experiment

Setup overview



MUonE Experiment

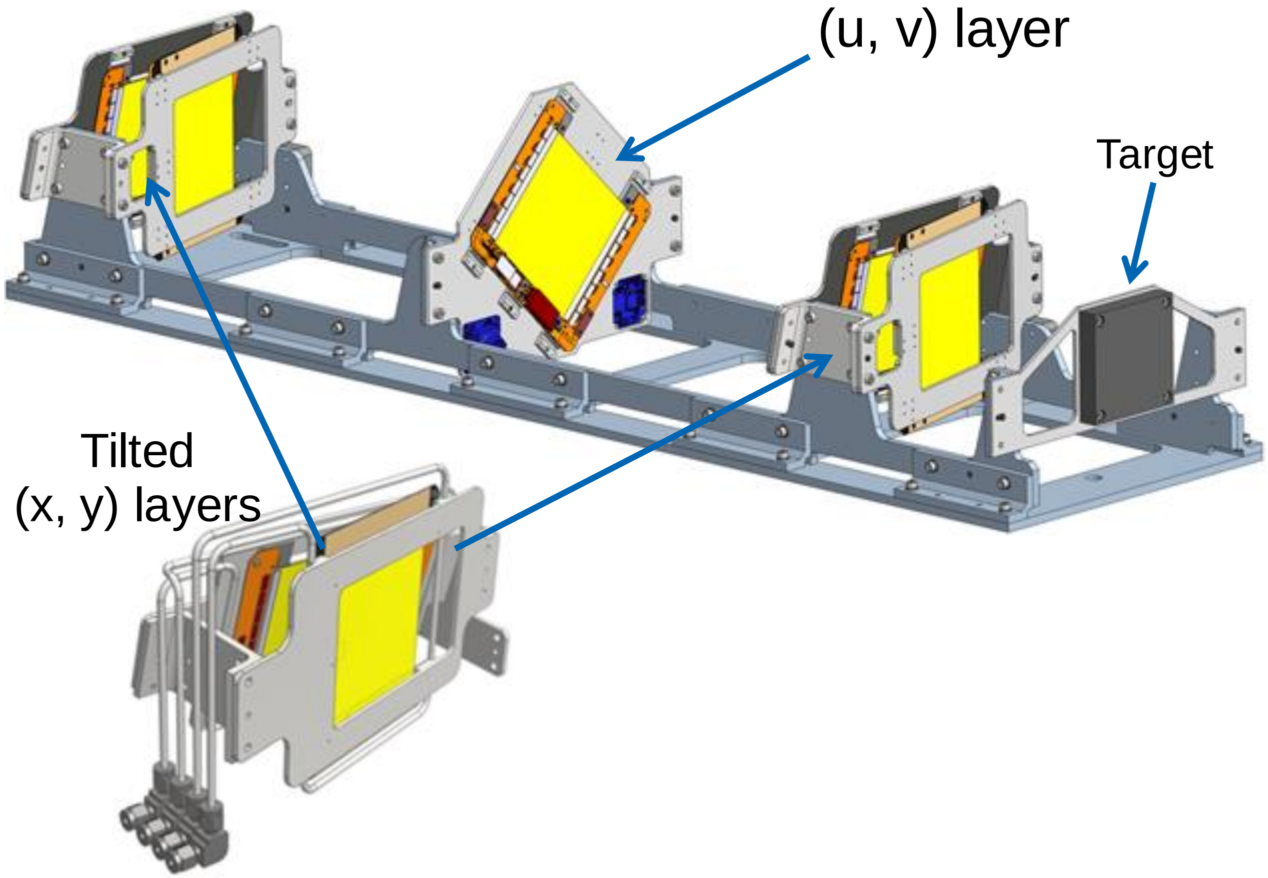
Setup overview



- Graphite (or Be) **target** divided into 40 slices with a few cm thickness
- **Tracking system**: 3 pairs of silicon strip detectors
- **ECAL**: energy and PID

Apparatus

The tracking station



- Two (x, y) layers and (u, v) layer
  - (x, y) layers tilted for better resolution
  - (u, v) layer rotated to solve reconstruction ambiguities.
- Relative position between stations must be stable at **10 μm** → **a super precise experiment!**
  - Low-CTE material (INVAR, carbon)
  - Well-controlled temperature
  - Laser system to monitor stability

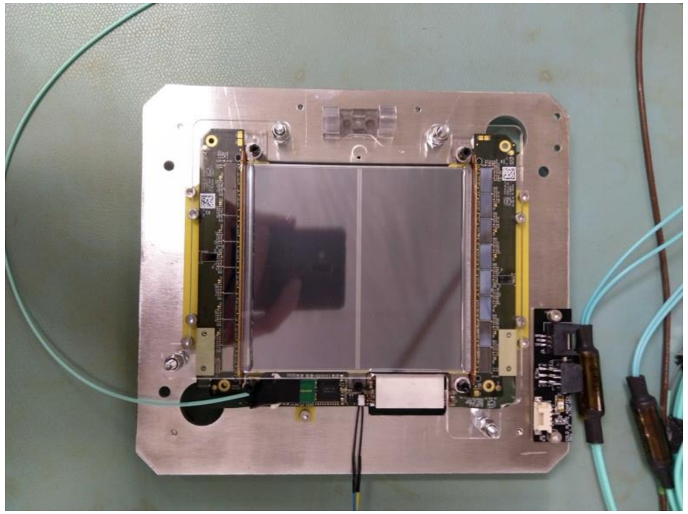
Apparatus

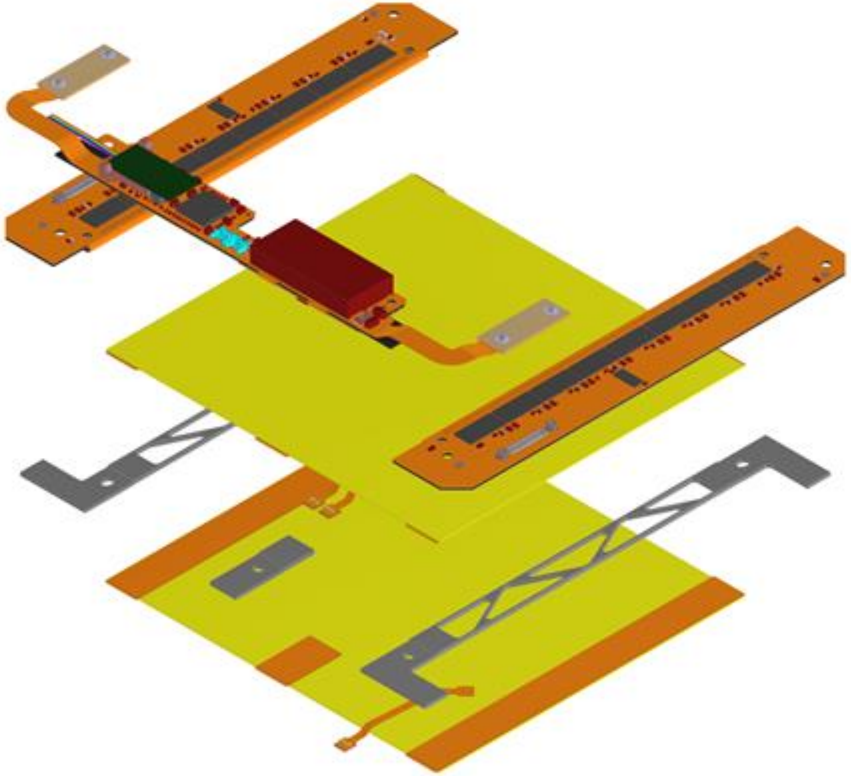
The tracker (CMS 2S Module)

- Silicon strip sensors currently in production for the CMS-Phase 2 upgrade (HL-LHC).
- Each module is divided in two independent halves.

A single half:

- 1016 strips
- 5 cm long
- Divided into 8 sectors
- Binary readout with a ~26 µm resolution

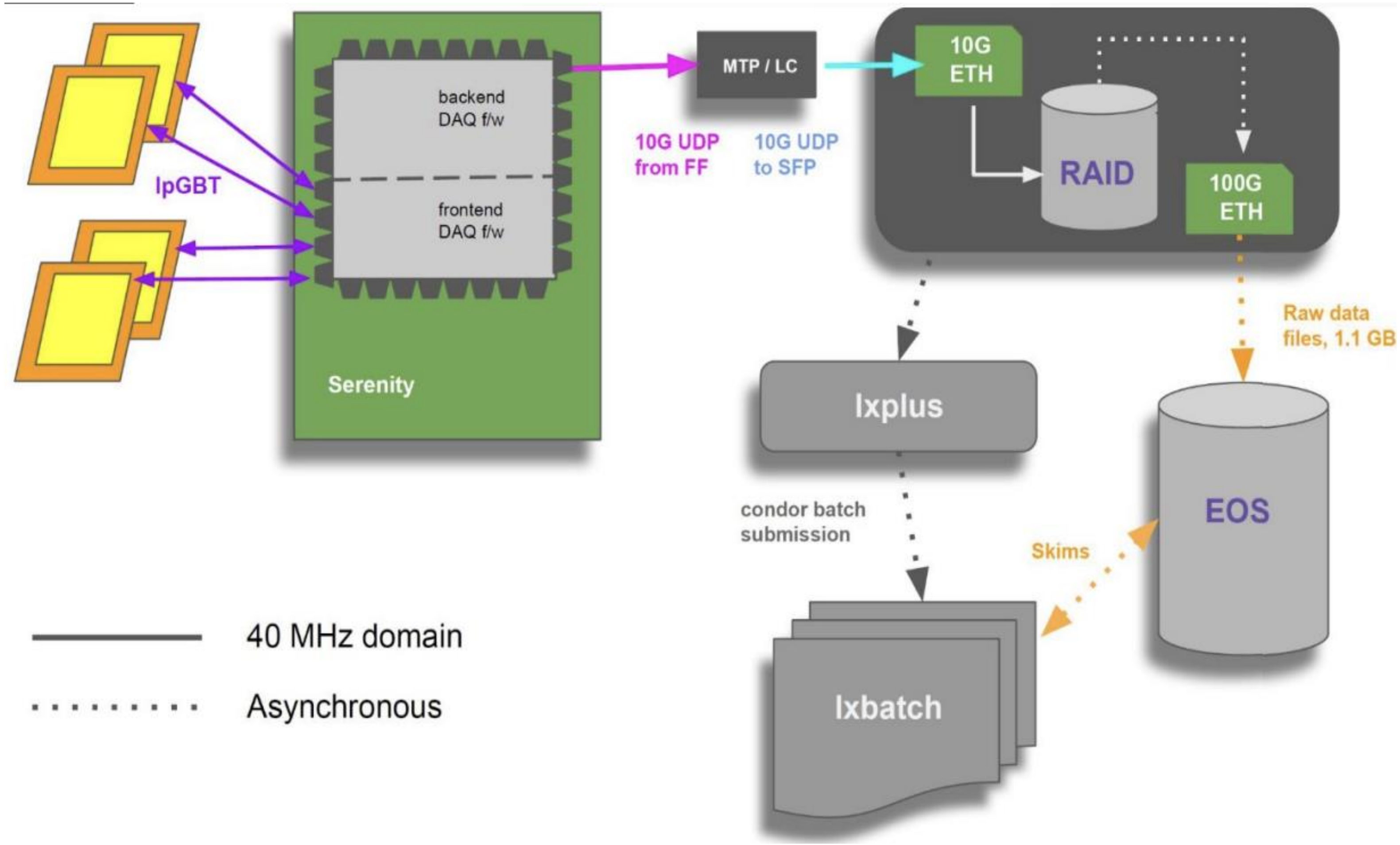


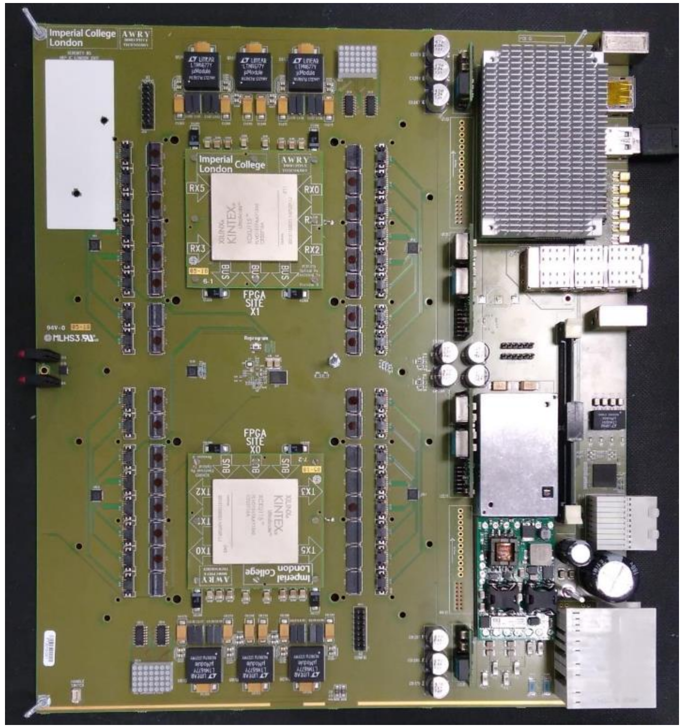


- TDR CMS Phase 2 Tracker Upgrade
- I. Zoi, POS 448 (VERTEX2023), 021

Apparatus

The tracker DAQ system

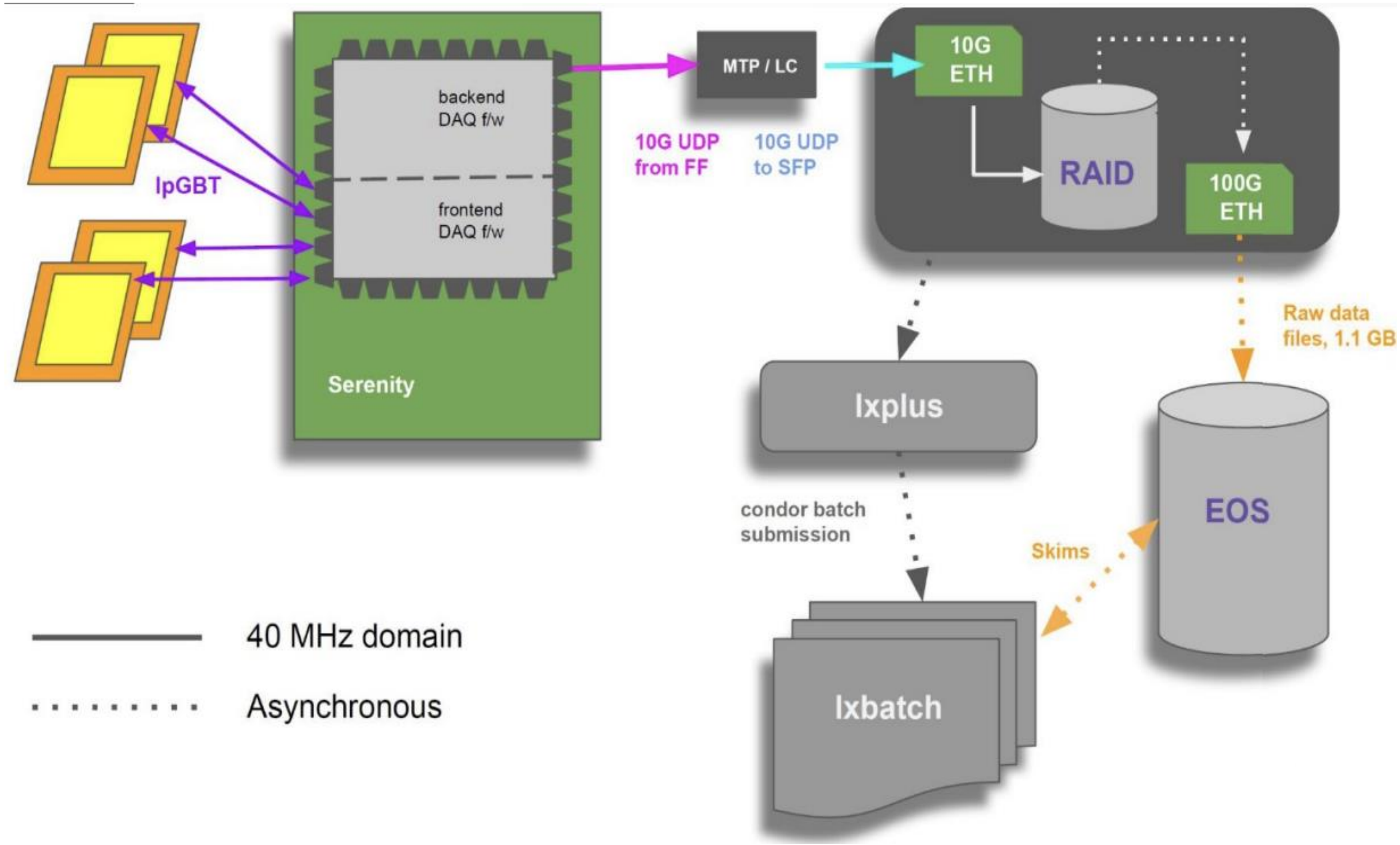




https://serenity.web.cern.ch/serenity/

Apparatus

The tracker DAQ system



- Triggerless readout @40MHz
- The CMS Serenity platform, with two FPGA daughtercards mounted
- Event aggregator on FPGA with online event filtering in 2025)

Apparatus

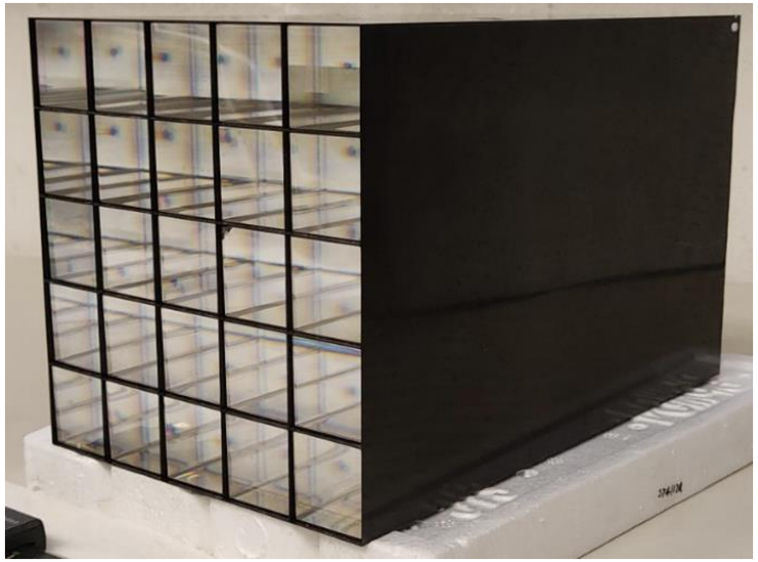
Calorimeter

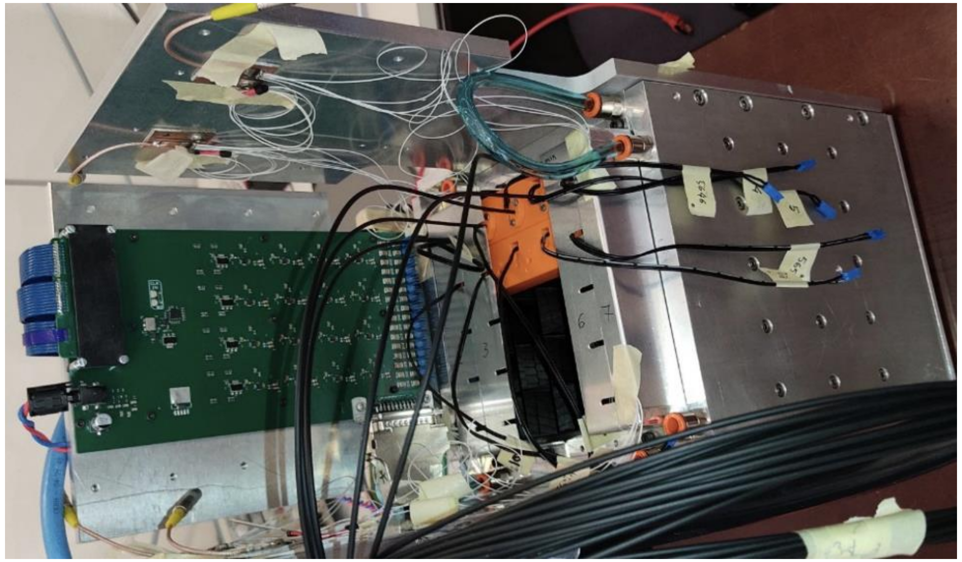
- A forward ECAL covering part of the total scattering acceptance
- Useful for **PID** & systematic study (an independent kinematic measurement)
- Considering recycling FNAL muon g-2 ECAL

5x5 PbWO4 crystals:

- Area: 2.85x2.85 cm^2
- Length: 22 cm (~25 X0).
- Total area: ~14x14 cm^2.
- Readout: APD sensors.

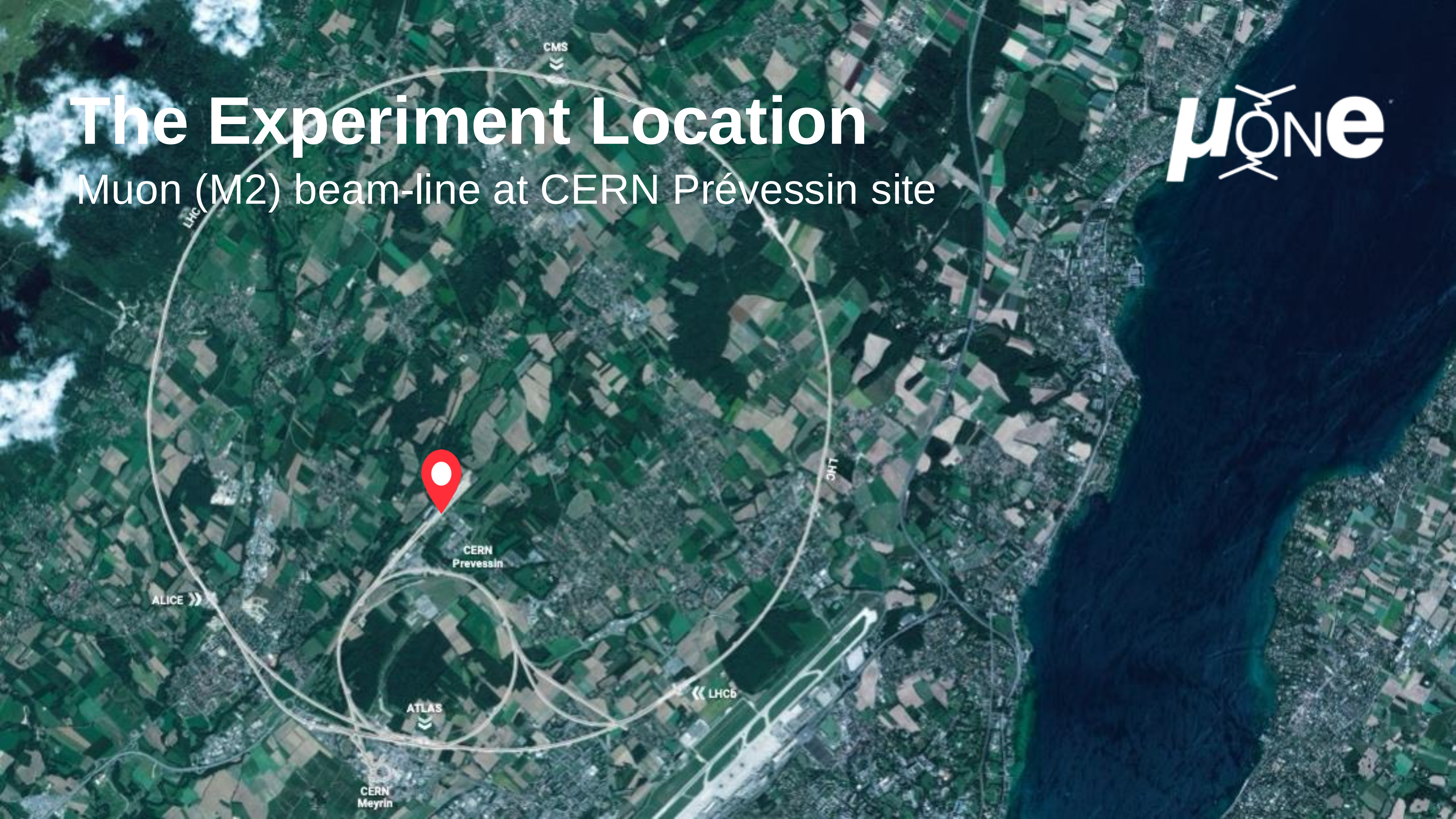
- Aram Hayrapetyan et al. Performance of the CMS Calo Crystals





The Experiment Location

Muon (M2) beam-line at CERN Prévessin site



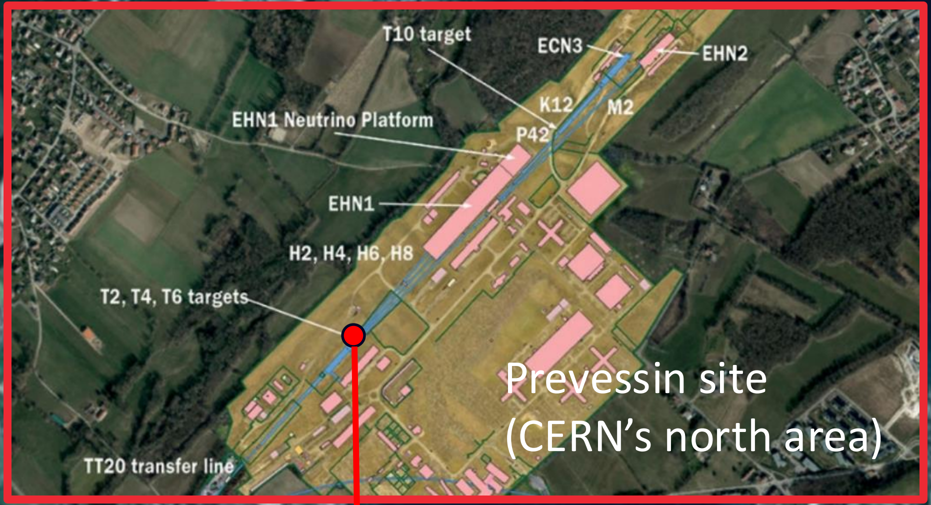
The Experiment Location

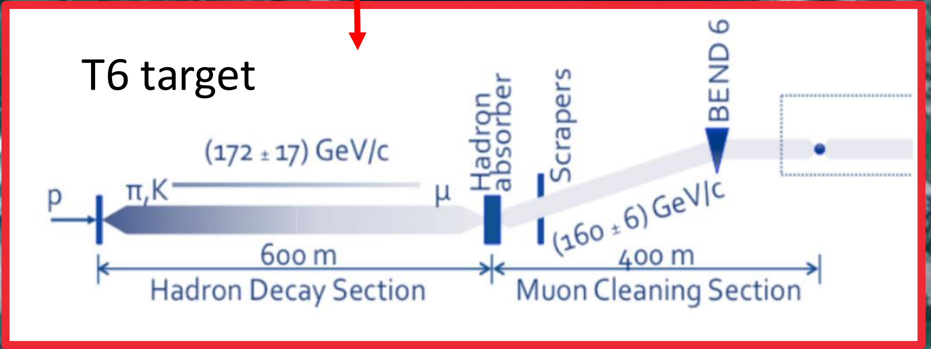
Muon (M2) beam-line at CERN Prévessin site

Prevessin site
(CERN's north area)

The Experiment Location

Muon (M2) beam-line at CERN Prévessin site





Test Runs

Joint test with CMS tracker group from 2021 to 2024

Oct – Nov 2021

- First test of the 2S module with tracker DAQ system
- Also confirmed thermal stability of the mechanical structure

July and Oct 2022

- 1 full station (6 modules) + ECAL in the proposed MUonE location
- Beam intensity and profile measured in the real beam conditions

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- 1 full station (6 modules) + ECAL in the proposed MUonE location
- Beam intensity and profile measured in the real beam conditions

Aug – Sep 2023

- **First physics data taking to for the $\Delta\alpha_{\text{lep}}$ measurement**

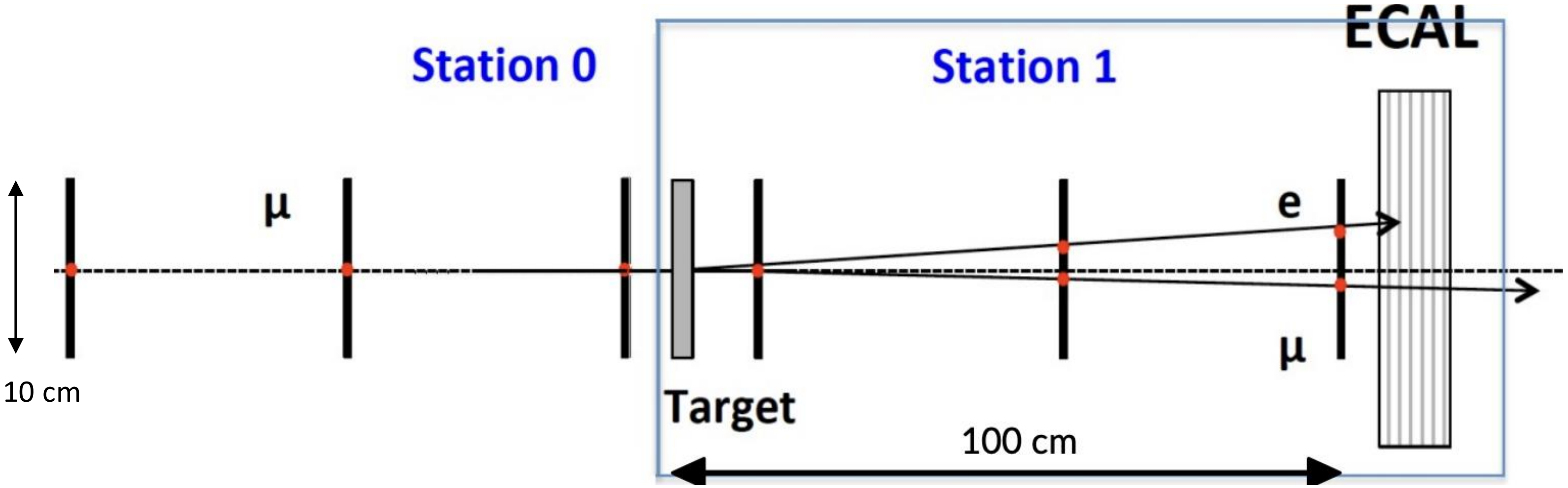
Sep – Oct 2024

- Dedicated ECAL test and its synchronization with tracker DAQ system

Test Run 2023

2 stations (beam telescope + target + tracker) + ECAL

- Expected luminosity: ~ $1pb^{-1}$
- $\sim 10^{12}$ μ accumulated on target with $\sim 2.5\times 10^{8}$ elastic events with $E_e > 1$ GeV
- Goal: demonstration measurement of $\Delta\alpha^{\mathrm{LEP}}$ with a few % precision



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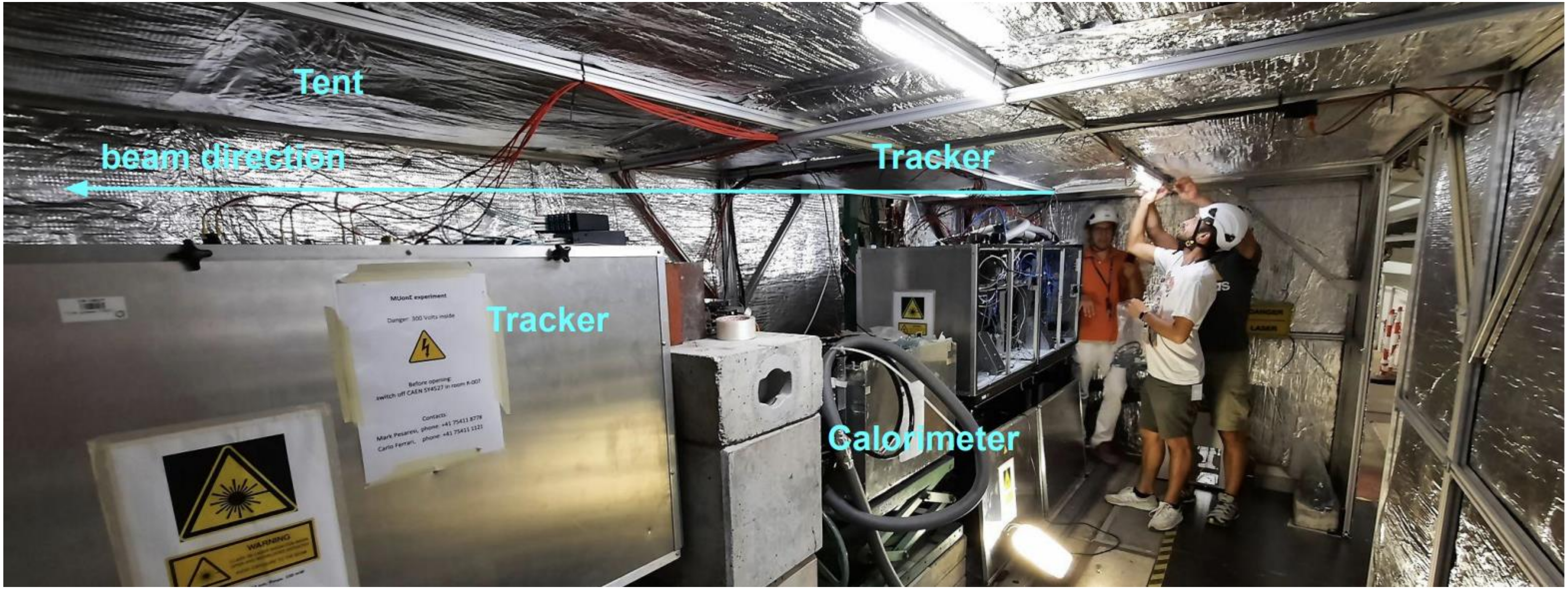

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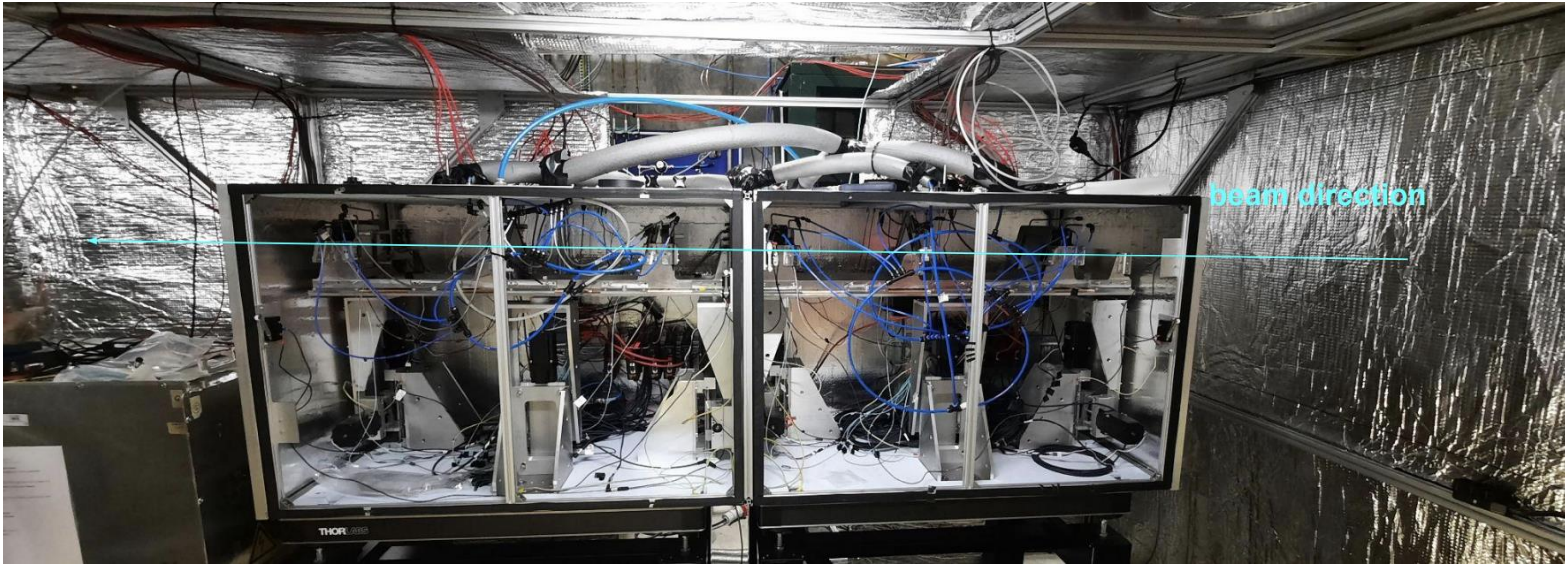
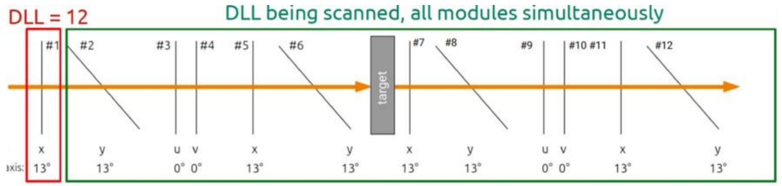
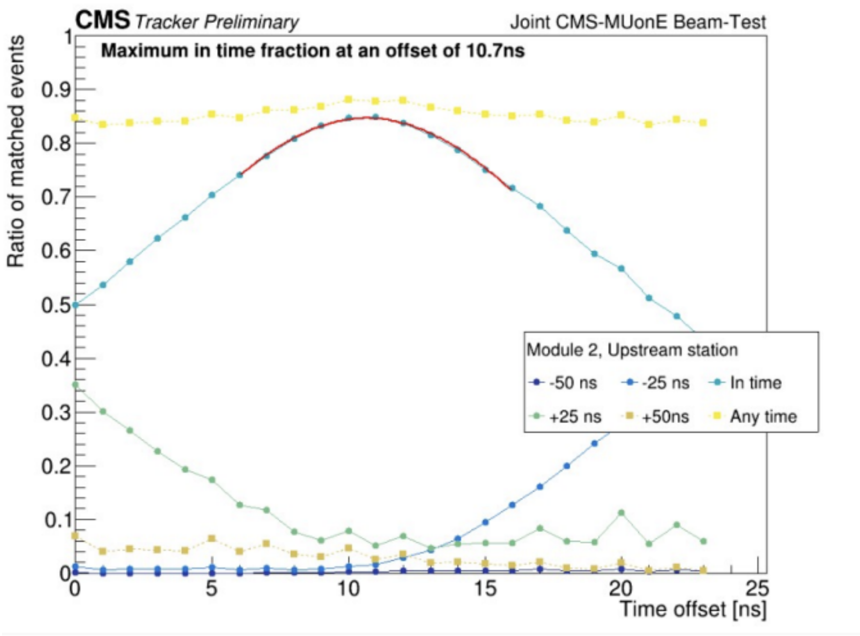


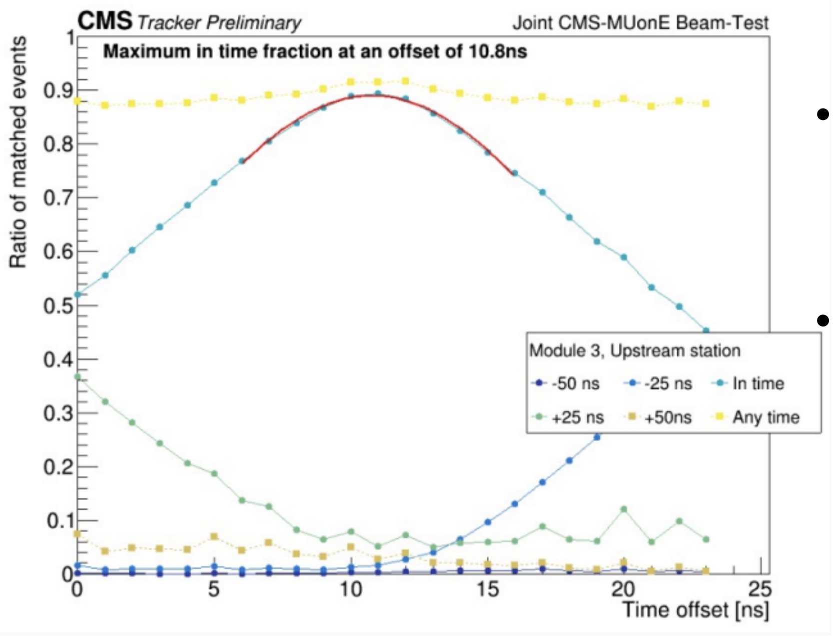
Test Run 2023

Tracking system performance

- **Module synchronization** were checked computing the fraction of events normalized to the first module, if a hit is found in the the module under test.





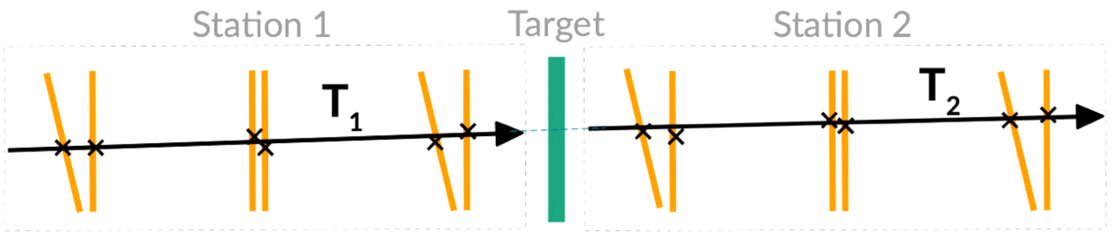


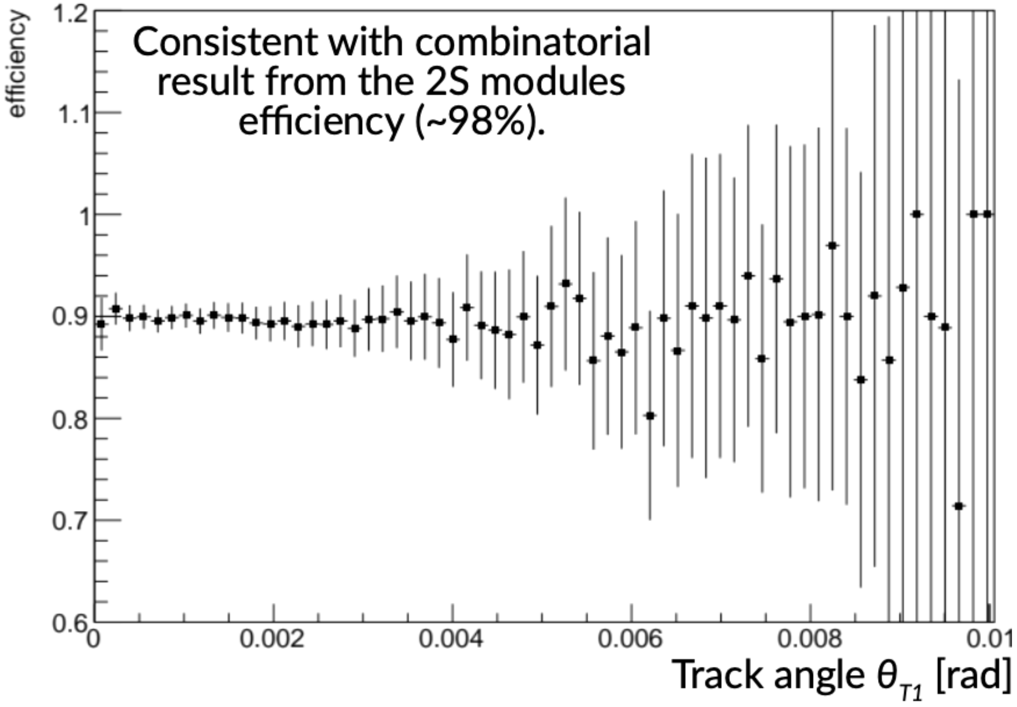
- Determine the relative timing of the modules (i.e. ~0.1 ns for #2 and #3).
- Best time offset ~10-15 ns

Test Run 2023

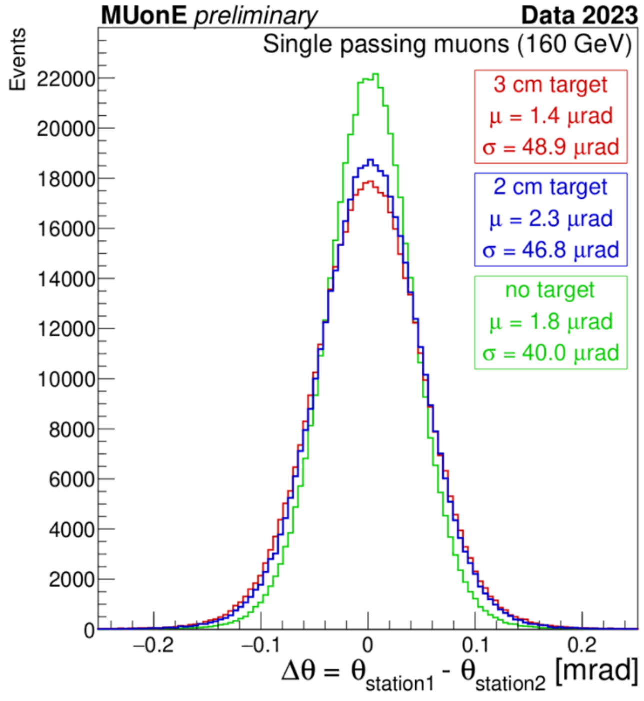
Tracking system performance

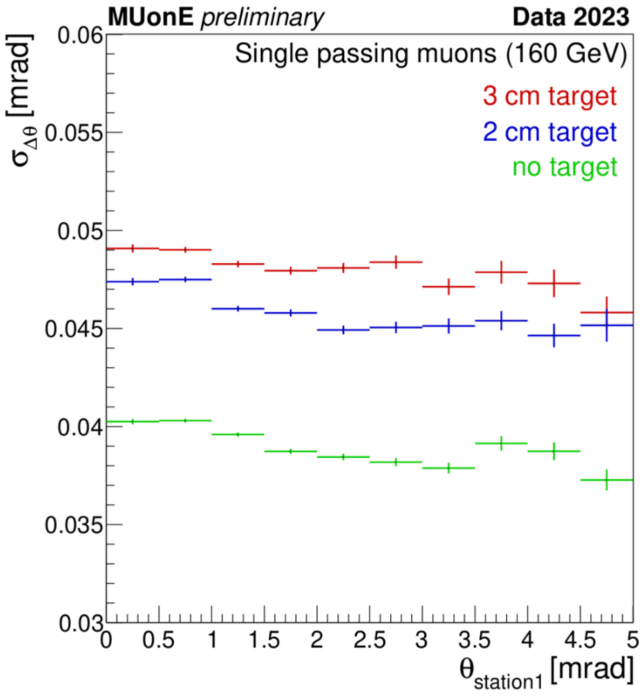
- **Module efficiency and angular resolution** were checked by selecting single-passing muon events for two stations separately.





Consistent with combinatorial result from the 2S modules efficiency (~98%).

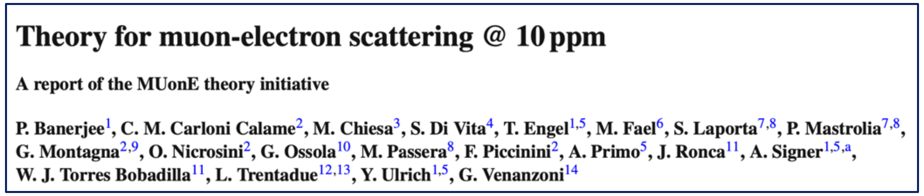




Analysis for Test Run 2023

Software framework

- **NNLO** Monte Carlo generator: **MESMER**
- ‘MuE’ fast simulation
  - (θ_e, θ_μ) up to the **NNLO** generator as well
  - Limited detector effects (multiple scattering) included
- ‘FairMUonE’ dedicatedly developed for this project
  - **Full detector effects** and **track reconstruction**
- Event selection & **weighted (θ_e, θ_μ)** for template fits
- ‘Combine’ tool for analysing systematic effects



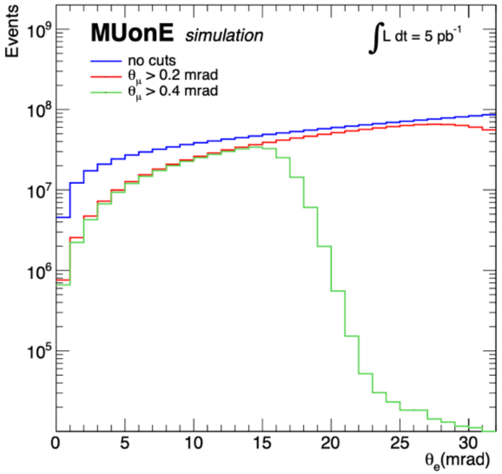
Theory for muon-electron scattering @ 10 ppm

A report of the MUonE theory initiative

P. Banerjee[1], C. M. Carloni Calame[2], M. Chiesa[3], S. Di Vita[4], T. Engel[1,5], M. Fael[6], S. Laporta[7,8], P. Mastrolia[7,8], G. Montagna[2,9], O. Nicrosini[2], G. Ossola[10], M. Passera[8], F. Piccinini[2], A. Primo[5], J. Ronca[11], A. Signer[1,5,a], W. J. Torres Bobadilla[11], L. Trentadue[12,13], Y. Ulrich[1,5], G. Venanzoni[14]

First NNLO prediction!

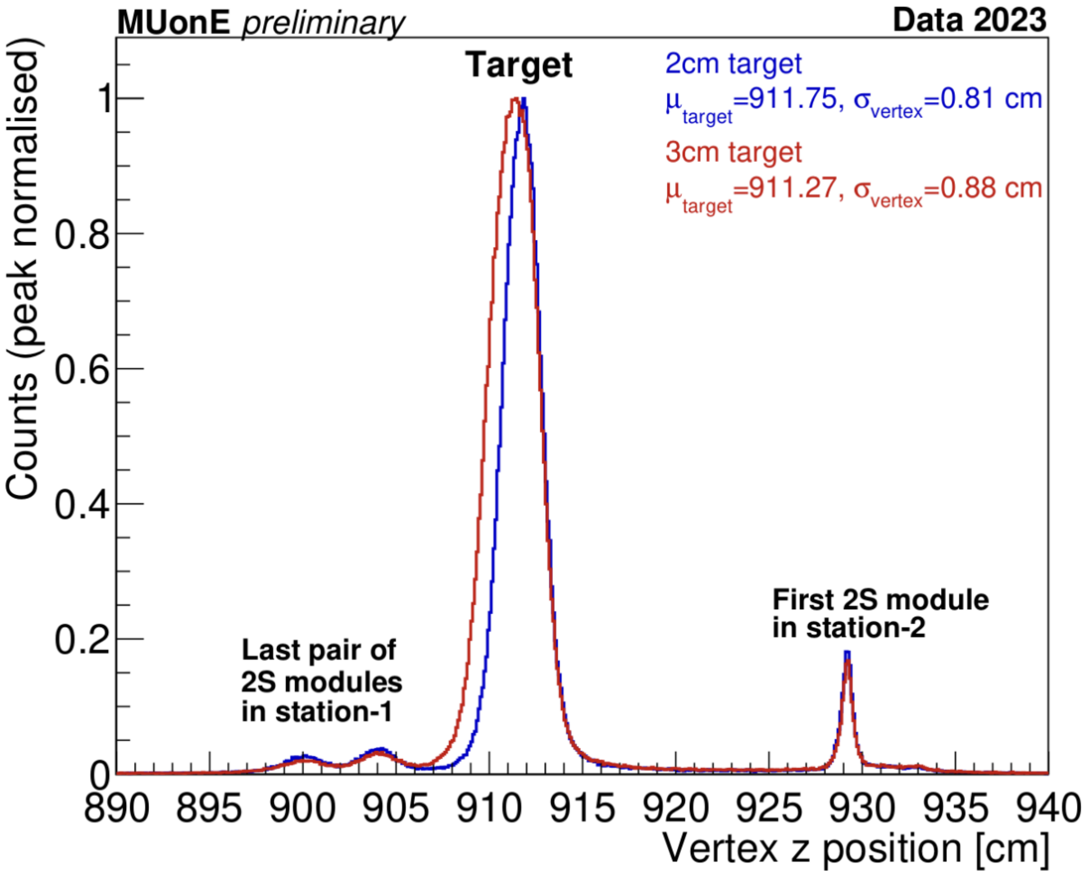
Eur. Phys. J. C (2020) 80:591



Analysis for Test Run 2023

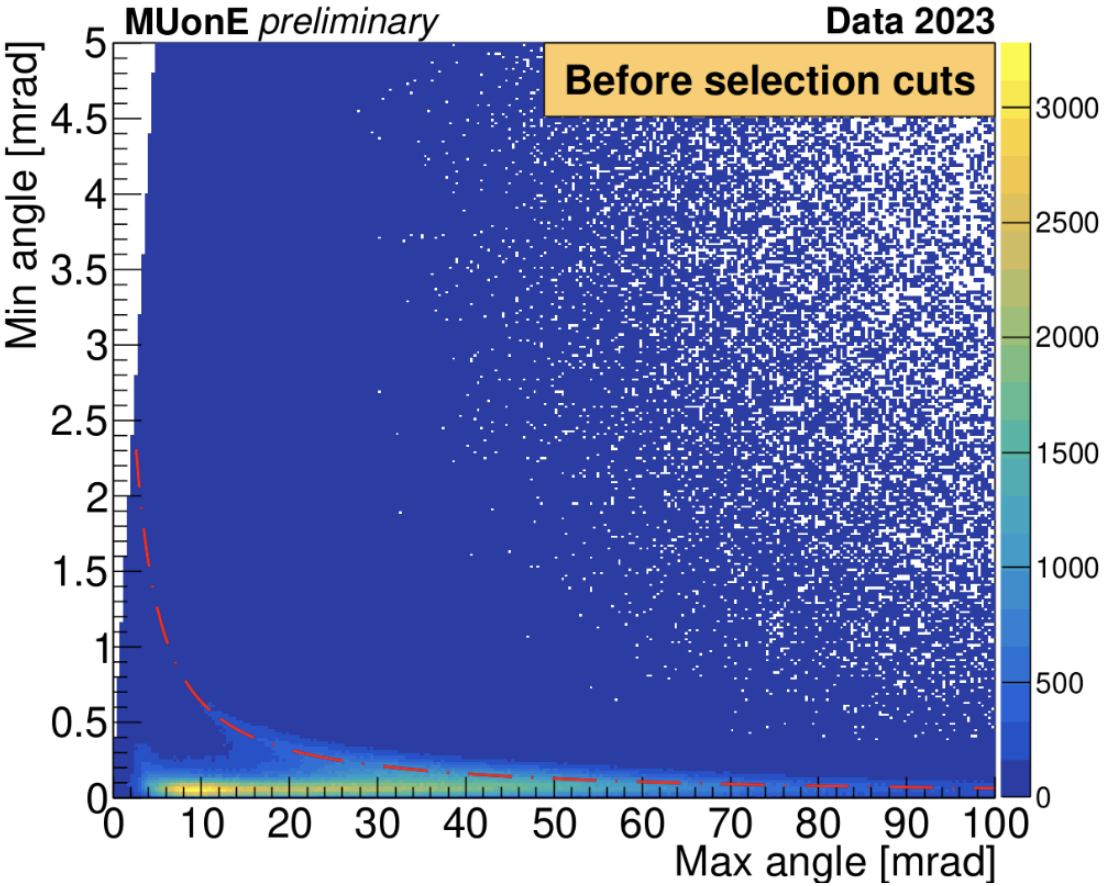
Event reconstruction & selections

- Some basic criteria
  - Vertex position in the target
  - Track vertex fit quality (χ^2)
  - Acoplanarity
- Boosted kinematics:
  - Single detector to cover full acceptance
  - $\theta_\mu < 5$ mrad, $\theta_e < 32$ mrad.



Analysis for Test Run 2023

First elastic scattering results

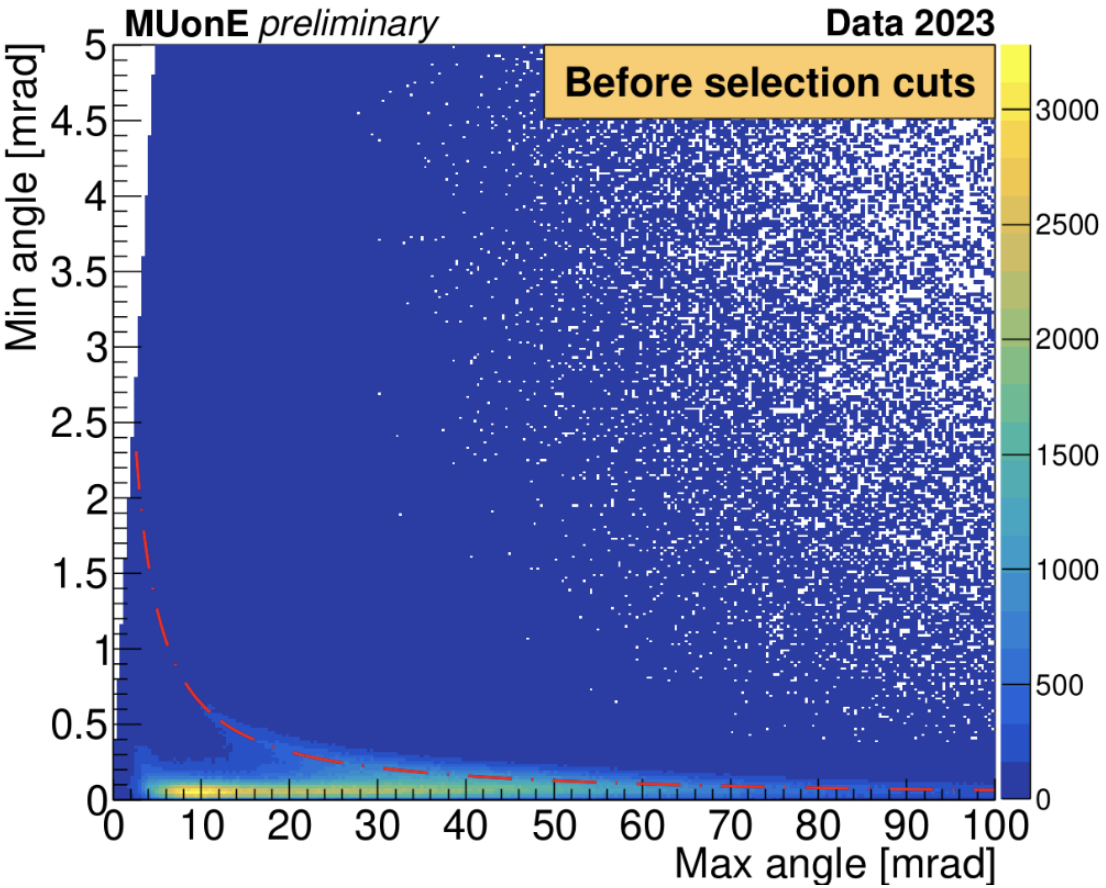


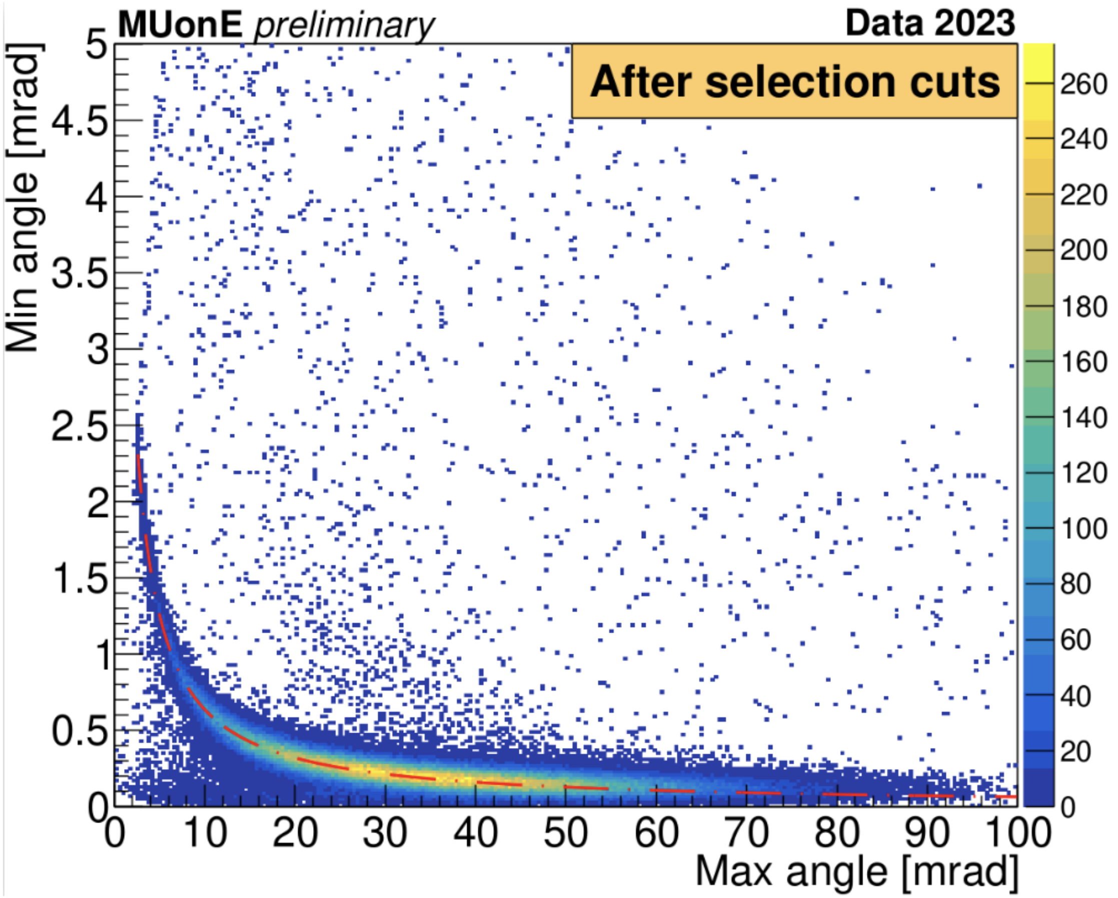
→ Making selections:

- Angles from the best vertex fit in FairMUonE
- 3 tracks (1 incoming & 2 out)
- Acoplanarity cut (<=1)
- Vertex reconstructed +/- 3 cm of the target mean position
- **Angles: θ_μ> 0.2 mrad; θ_e < 32 mrad**

Analysis for Test Run 2023

First elastic scattering results





Elastic Scattering Analysis

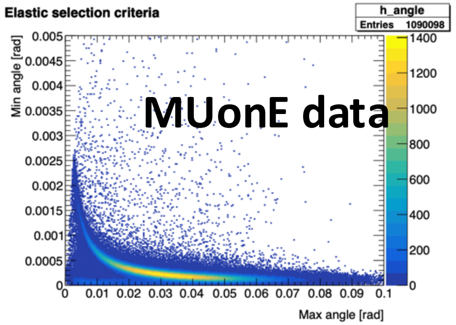
Extraction of $a_{\mu}^{\mathrm{HVP,LO}}$ from the template fit

- Extracting $\Delta\alpha_{had}(t)$ through **a template fit** to the (θ_e, θ_μ) distribution
- $\Delta\alpha_{had}$ parameterization **(K, M)**: $$\Delta\alpha_{had}(t) = KM\left\{-\frac{5}{9} - \frac{4}{3}\frac{M}{t} + \left(\frac{4}{3}\frac{M^2}{t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\ln\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$
  - ‘Lepton-like’ parameterization
    - K: related to $\boldsymbol{\alpha_0}$ and the electric charge of the lepton in the loop
    - M: related to the **squared mass of the particle** in the loop $(m_l^2, m_\mu^2, m_\tau^2)$
  - In the hadronic parameterization, K & M don’t really have physical meaning

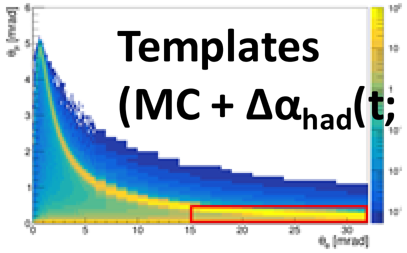
Elastic Scattering Analysis

Extraction of $a_\mu^{\mathrm{HVP,LO}}$ from the template fit

- Extracting $\Delta\alpha_{had}(t)$ through **a template fit** to the (θ_e, θ_μ) distribution
- $\Delta\alpha_{had}$ parameterization **(K, M)**: $\Delta\alpha_{had}(t) = KM\left\{-\frac{5}{9} - \frac{4}{3}\frac{M}{t} + \left(\frac{4}{3}\frac{M^2}{t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\ln\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$

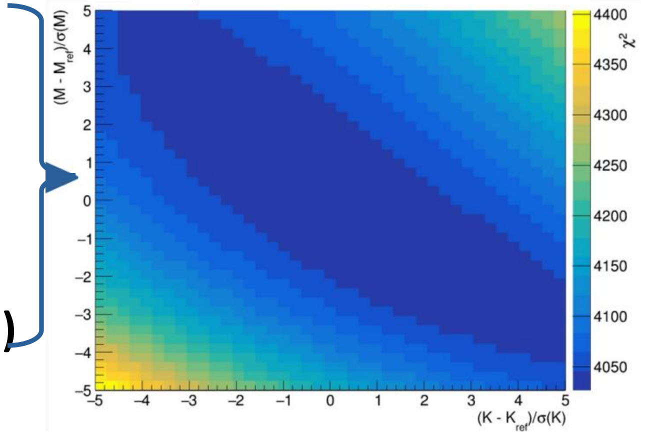


MUonE data



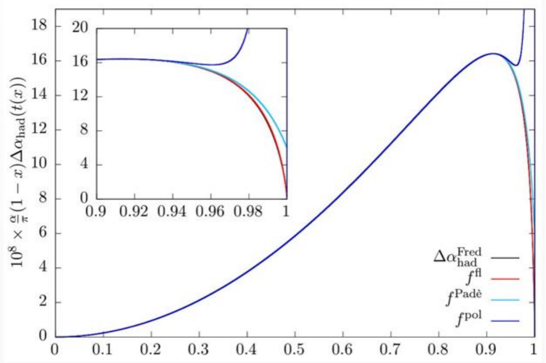
Templates
(MC + $\Delta\alpha_{had}(t; K_i, M_j)$)

$$\chi^2 = \sum_i^{\mathrm{bins}}\left(\frac{\mathrm{data}_i - \mathrm{templ}(K,M)_i}{\sigma_i^{\mathrm{data}}}\right)^2$$



→ **K_{best}, M_{best}**

$$a_\mu^{HLO} = \frac{\alpha_0}{\pi}\int_0^1 dx(1-x)\underline{\Delta\alpha_{had}[t(x)]}$$



- Dominant behaviour in the MUonE kinematic region (x<0.936)
- The lepton like function that allows us to extrapolate the remaining 12%
- An alternative approach ('derivative') covering 99%:
  - Phys. Lett. B 848 (2024) 138344

Systematic Effects

Achievable accuracy

- The main challenge of the MUonE is the control of systematic effects at the same level of the statistical precision
  - **40 stations + 3 years of data-taking** with full stations → 4E12 events
  - **~0.3%** statistical accuracy on $a_\mu^{\text{HVP,LO}}$
  - Competitive with dispersive data-driven method
  - Estimated **10 ppm** systematic uncertainty
    - Requires a **uniform detection efficiency** (modules, across all angular range)
    - Precise **alignment** (10 μm longitudinally)
    - ...

Systematic Effects

General considerations & strategy

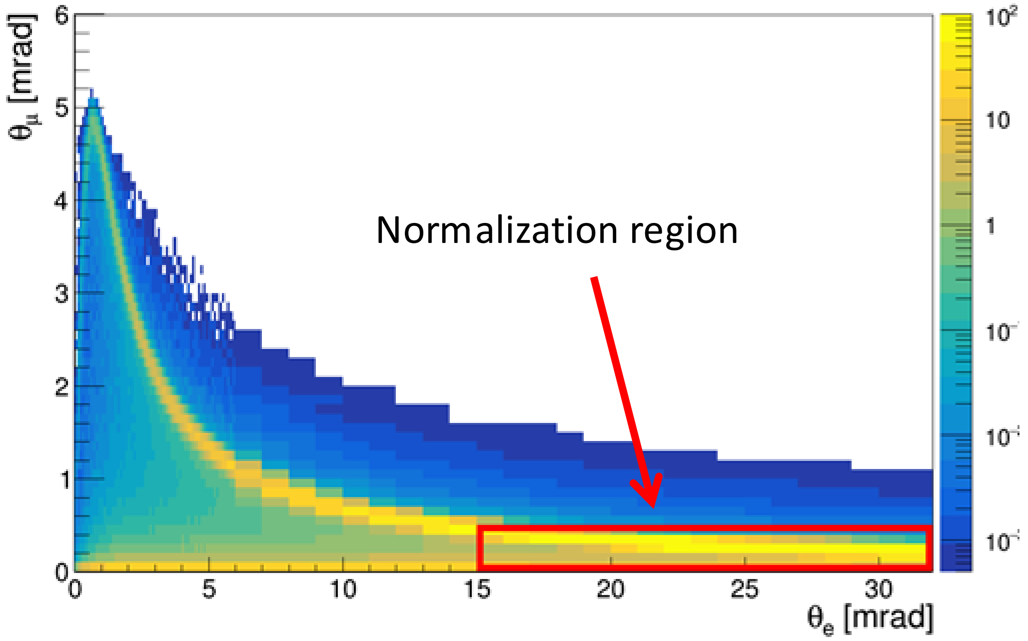
$$R_{\mathrm{had}} = \frac{d\sigma_{\mathrm{data}}(\Delta\alpha_{\mathrm{had}})}{\underline{d\sigma_{\mathrm{MC}}(\Delta\alpha_{\mathrm{had}} = 0)}} \sim 1 + 2\Delta\alpha_{\mathrm{had}}(t)$$

- Theory input: MC generator of radiative contributions at **NNLO** level
- Main experimental sources:
  - **Multiple-scattering** (accuracy of 1%)
  - Angular resolution (a few %)
  - Knowledge of the beam energy (a few MeV)
  - …

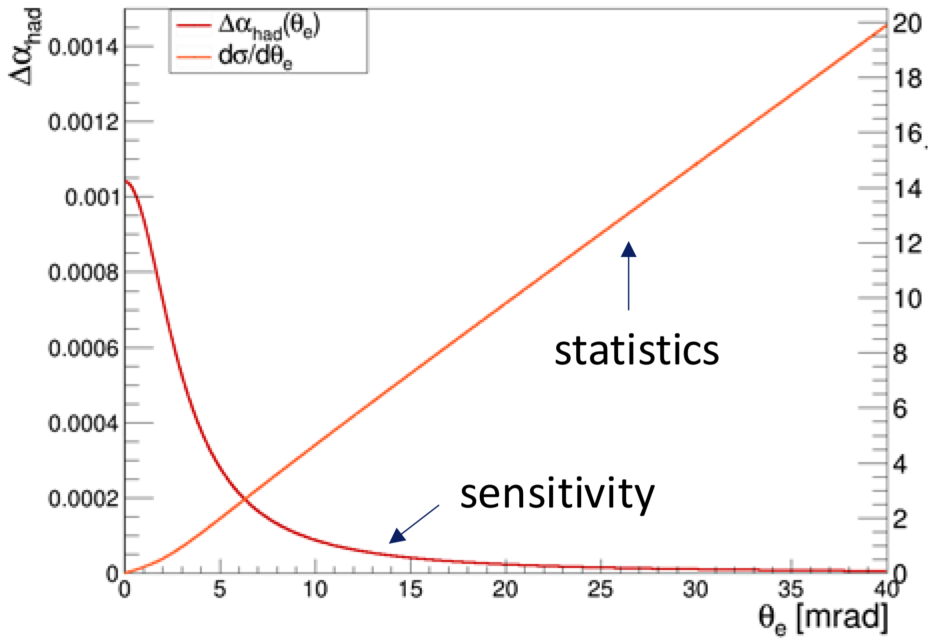
Systematic Effects

General considerations & strategy

- Main systematics have large effects in **the normalization region**.
- Large statistics but not sensitivity to Δa_{had}



Normalization region

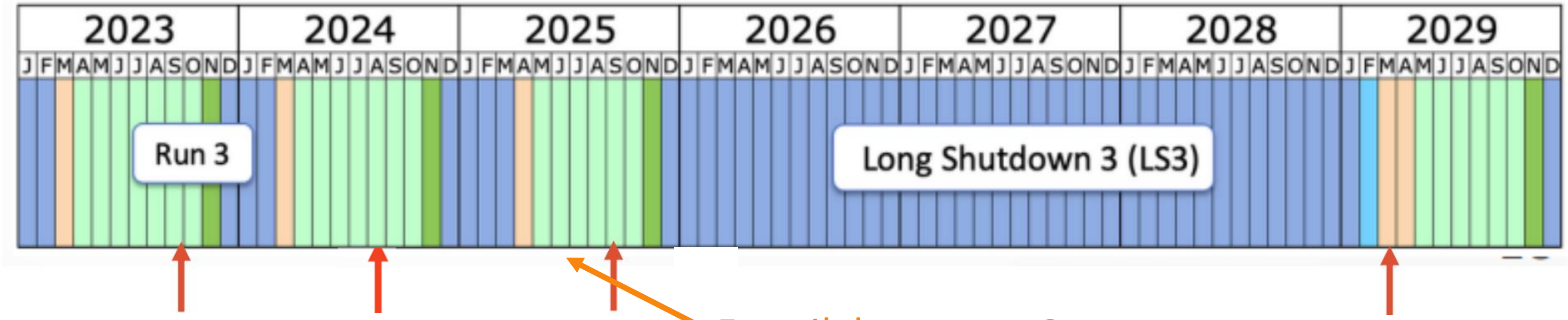


statistics

sensitivity

Timeline

- **An experimental proposal** submitted to SPSC on May 2024
- **The first physics run in 2025** before LHC Long Shutdown
  - **3** stations; 4 weeks data-taking; **about 20% precision of $\Delta\alpha_{\mathbf{had}}$**
- **Full run with 40 stations** after LS3 with final goal **~0.3%** stat and similar syst.



Test Run
2023

Test Run
2024

Physics
run
2025

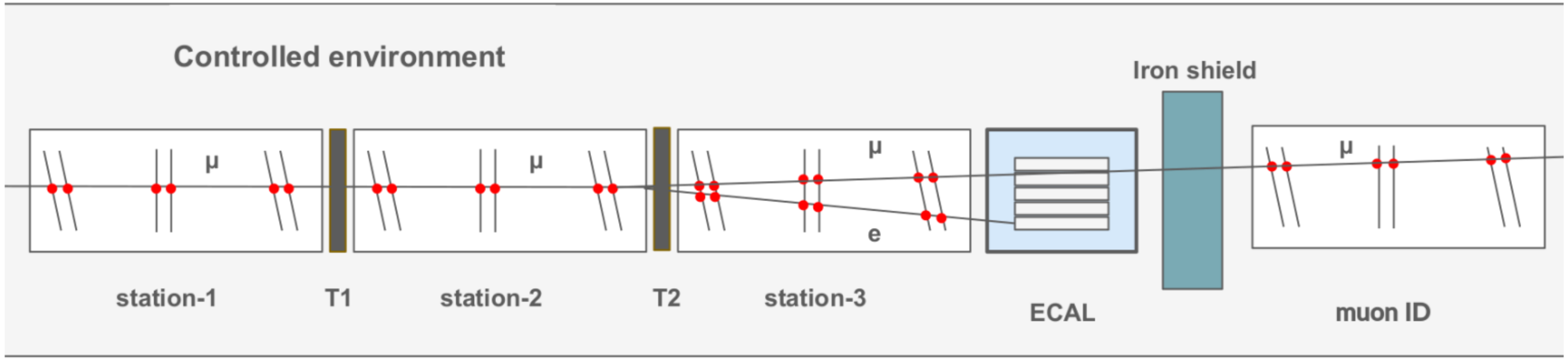
Fermilab muon g-2
final release

Full run
40 stations

Run 2025

BMS

....

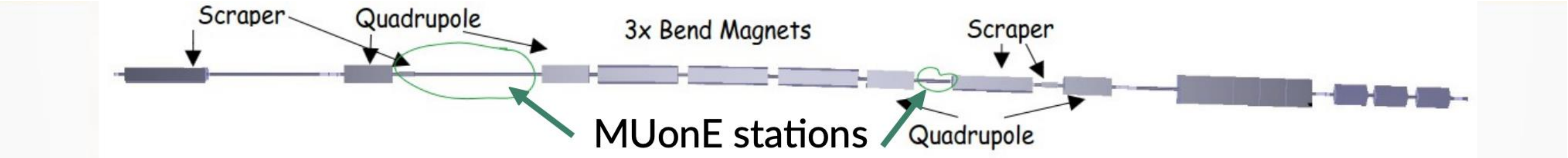


- Goals:

- **Systematic error studies:**
  - Exploit data from all the sub-detectors to study backgrounds and systematics.
  - Study uniformity of tracking efficiency, PID, backgrounds, detector modelization, beam control.
  - Demonstrate control of the systematic errors at *O*(500ppm).

- **Physics results:**
  - Preliminary measurement of $\Delta\alpha_{had}(t)$ with *O*(20%) statistical accuracy.
  - Measure $\Delta\alpha_{lep}(t)$ with a few percent precision, and compare with the measurement currently being performed with 2023 data.

BMS (Beam Momentum Spectrometer)



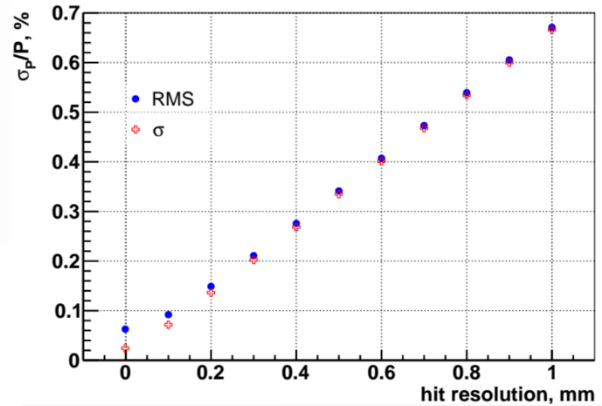
- Bending power:
  16 T*m (30 mrad @160 GeV).
- Determine the muon momentum event by event.
- Goal: < 0.5% momentum resolution.

UNIVERSITY OF LIVERPOOL

- Mechanics.
- Simulation and analysis.

Longer term tasks:

- Magnetic field measurement.



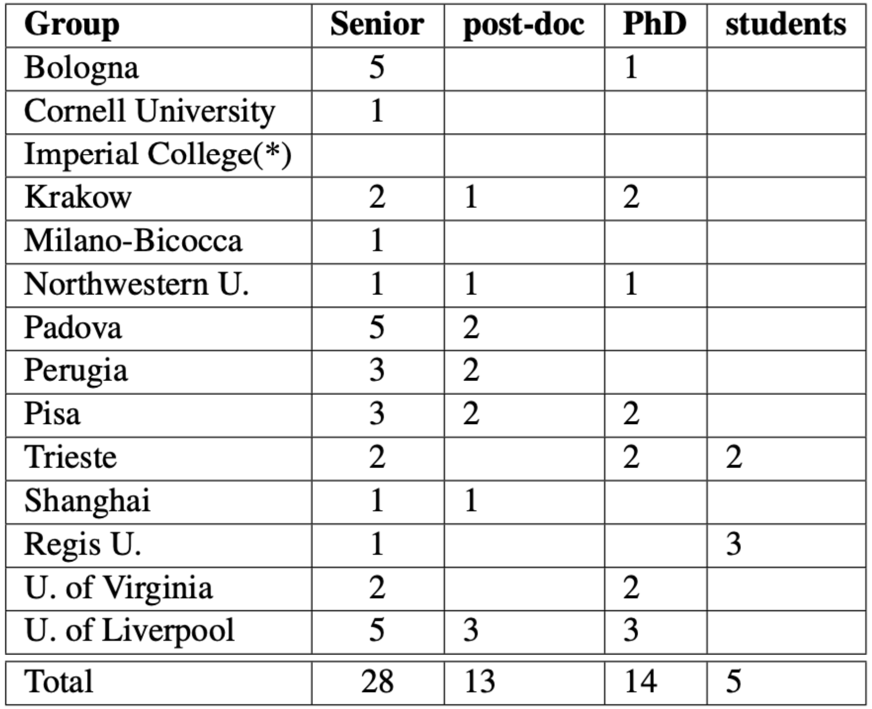
T. Bowcock
S. Charity
F. Ignatov
G. Venanzoni

Intrinsic resolution only
But also we have:
- Mag field uncert
- Alignment

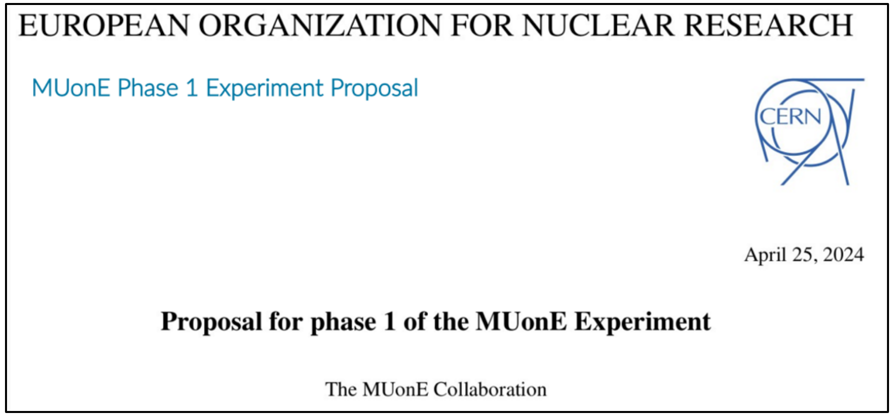
0.5% → 10ppm

The Collaboration

- 50+ people from over 9 countries
  - https://cds.cern.ch/record/2896293



Group	Senior	post-doc	PhD	students
Bologna	5		1	
Cornell University	1			
Imperial College(*)				
Krakow	2	1	2	
Milano-Bicocca	1			
Northwestern U.	1	1	1	
Padova	5	2		
Perugia	3	2		
Pisa	3	2	2	
Trieste	2		2	2
Shanghai	1	1		
Regis U.	1			3
U. of Virginia	2		2	
U. of Liverpool	5	3	3	
Total	28	13	14	5



EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

MUonE Phase 1 Experiment Proposal

April 25, 2024

Proposal for phase 1 of the MUonE Experiment

The MUonE Collaboration

In the UK:
- Imperial College (CMS tracker group): tracker module
- Liverpool: Analysis, BMS, theory, etc.

New collaborators are always welcome!

Conclusion

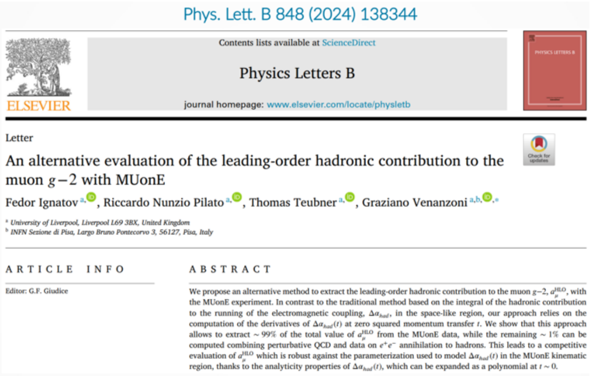
- Muon $g-2$ puzzles:
  - Conflict between **Muon $\boldsymbol{g-2}$ SM predictions** and experimental measurement;
  - $\boldsymbol{a_\mu^{\mathrm{HVP,LO}}}$ represents a major uncertainty in the e^+e^- data-driven method for SM prediction.
- MUonE: a new approach for $\boldsymbol{a_\mu^{\mathrm{HVP,LO}}}$ **via $\boldsymbol{\mu-e}$ scattering**
  - A **new, alternative, and independent way to measure $\Delta\alpha_{\mathrm{had}}$ for the first time;**
  - Main challenge: a very precise measurement on the shapes of differential distributions at the **10ppm level** of systematic uncertainty;
  - Planning a 4 weeks of the **data taking in 2025**, making a preliminary measurement of $\Delta\alpha_{\mathrm{had}}(t)$; A **full run for 2029+** after LS3.

Backup

Elastic Scattering Analysis

Extraction of $a_{\mu}^{\mathrm{HVP,LO}}$ from the template fit

- Another approach computing $\Delta\alpha_{had}(t)$ using MUonE data



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Fedor Ignatov [a], Riccardo Nunzio Pilato [a], Thomas Teubner [a], Graziano Venanzoni [a,b,*]

[a] University of Liverpool, Liverpool L69 3BX, United Kingdom
[b] INFN Sezione di Pisa, Largo Bruno Pontecorvo 3, 56127, Pisa, Italy

ARTICLE INFO

Editor: G.F. Giudice

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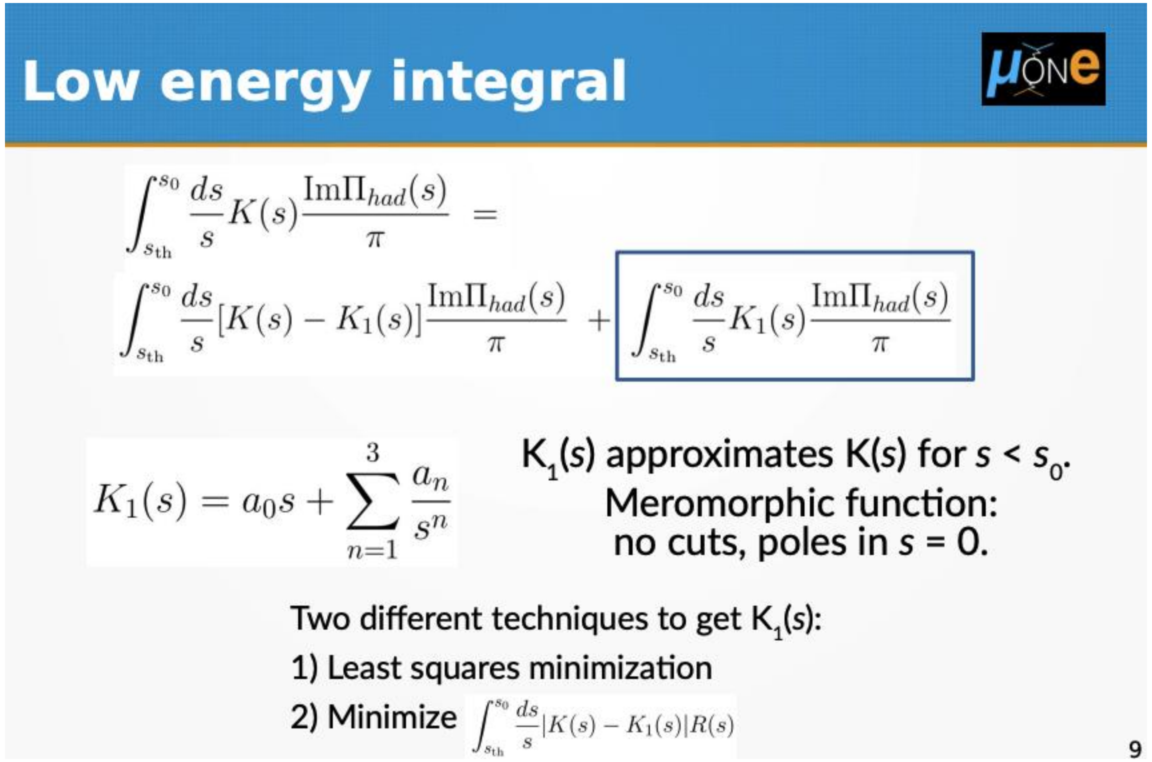


Low energy integral

$$\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}K(s)\frac{\mathrm{Im}\Pi_{had}(s)}{\pi} =$$

$$\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}[K(s)-K_1(s)]\frac{\mathrm{Im}\Pi_{had}(s)}{\pi} + \boxed{\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}K_1(s)\frac{\mathrm{Im}\Pi_{had}(s)}{\pi}}$$

$$K_1(s) = a_0 s + \sum_{n=1}^{3}\frac{a_n}{s^n}$$

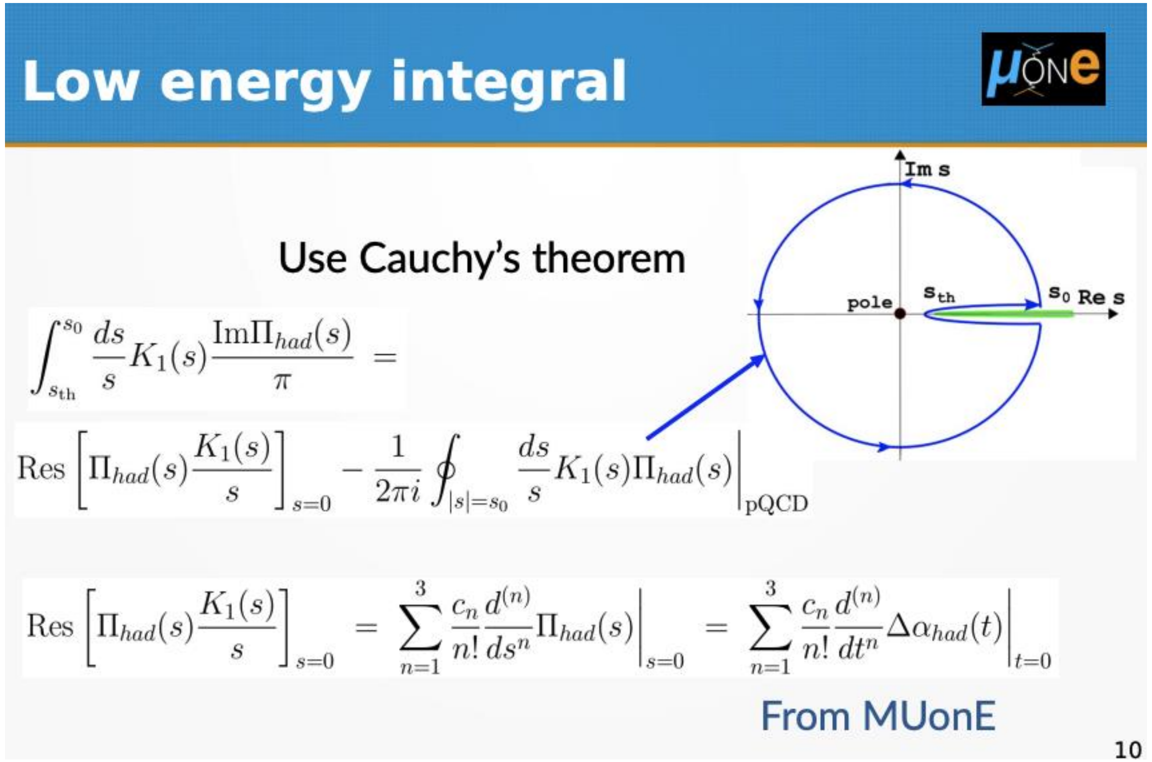
$K_1(s)$ approximates $K(s)$ for $s < s_0$.
Meromorphic function:
no cuts, poles in $s = 0$.

Two different techniques to get $K_1(s)$:

1) Least squares minimization

2) Minimize $\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}|K(s)-K_1(s)|R(s)$

9



Low energy integral

Use Cauchy's theorem

$$\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}K_1(s)\frac{\mathrm{Im}\Pi_{had}(s)}{\pi} =$$

$$\mathrm{Res}\left[\Pi_{had}(s)\frac{K_1(s)}{s}\right]_{s=0} - \frac{1}{2\pi i}\oint_{|s|=s_0}\frac{ds}{s}K_1(s)\Pi_{had}(s)\Big|_{\mathrm{pQCD}}$$

$$\mathrm{Res}\left[\Pi_{had}(s)\frac{K_1(s)}{s}\right]_{s=0} = \sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{ds^n}\Pi_{had}(s)\Big|_{s=0} = \sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

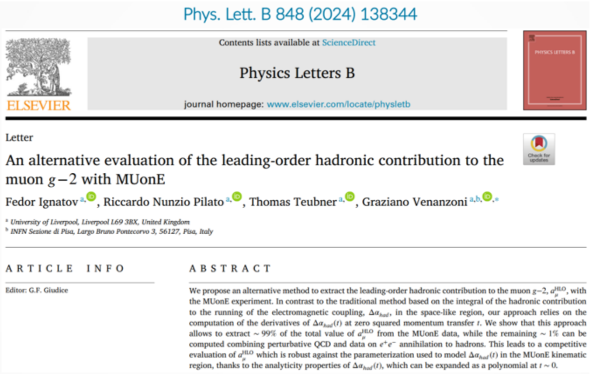
From MUonE

10

Elastic Scattering Analysis

Extraction of $a_\mu^{\mathrm{HVP,LO}}$ from the template fit

- Another approach computing $\Delta\alpha_{\mathrm{had}}(t)$ using MUonE data



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99%

$$a_\mu^{\mathrm{HLO\ (I)}} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

MUonE

1%

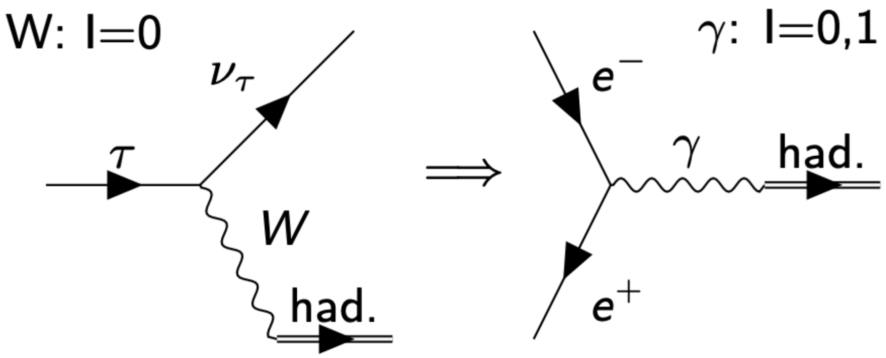
$$a_\mu^{\mathrm{HLO\ (II)}} = \frac{\alpha}{\pi}\frac{1}{2\pi i}\oint_{|s|=s_0}\frac{ds}{s}c_0 s\,\Pi_{had}(s)\Big|_{\mathrm{pQCD}}$$

$$a_\mu^{\mathrm{HLO\ (III)}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}[K(s)-K_1(s)]R(s)$$

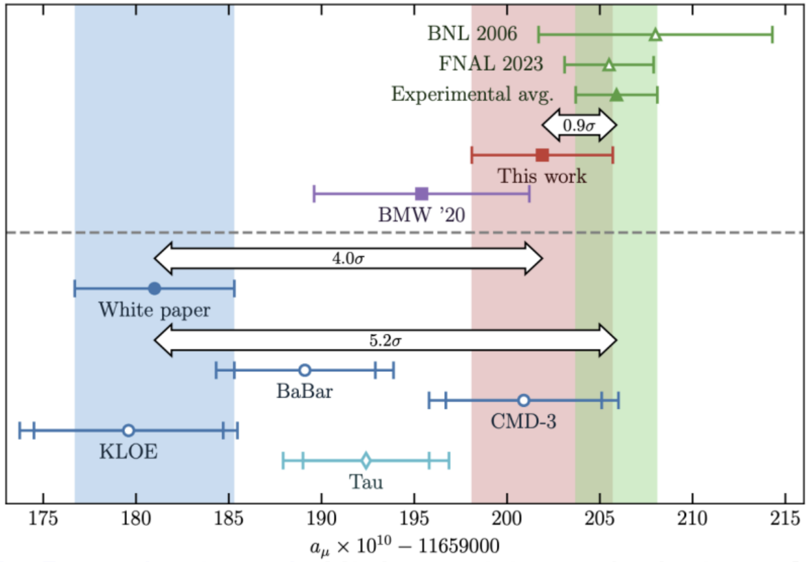
$$a_\mu^{\mathrm{HLO\ (IV)}} = \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty}\frac{ds}{s}[K(s)-\tilde{K}_1(s)]R(s)$$

Time-like data
+
pQCD

Tau Data



"… at the required precision to match the e^+e^- data, the present understanding of the IB [isospin breaking] corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals." - TI White Paper, 2020

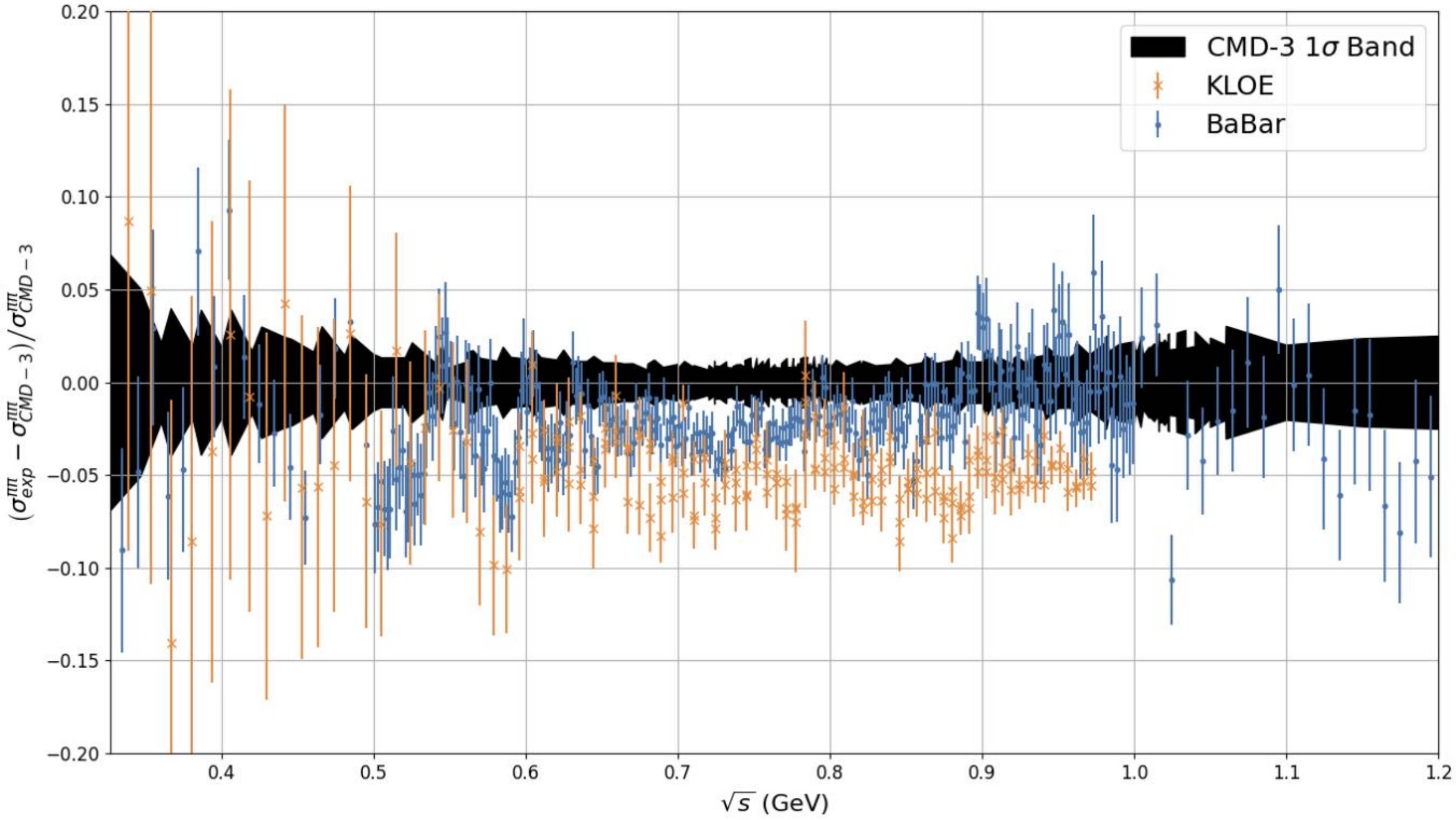


A. Boccaletti et al, High precision calculation of the hadronic vacuum polarisation contribution to the muon anomaly, Figure 3 (arXiv:2407.10913).

- Historically data from hadronic tau decays used to supplement lacking or low accuracy cross section data.
- Poor understanding of the scale of systematic uncertainties associated with IB corrections meant these data was no longer to be included.
- More accurate calculations of IB corrections are in process, supplemented by lattice QCD and ChPT.
- DHMZ argue for the re-inclusion of τ data due to the existence of greater discrepancies among the cross section datasets.

Data Tensions

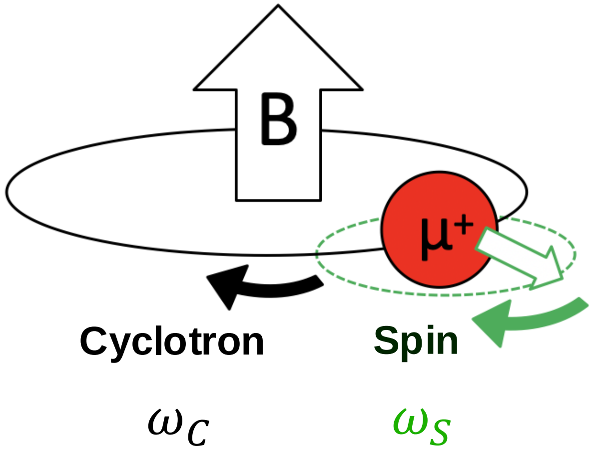
- Particularly on the ρ, CMD-3 2π significantly in excess of all previous data.



- A potential KLOE-CMD-3 systematic is too large for a_μ^{HVP} to be useable.
- Similarity at low energy used to motivate hybrid approaches...
- Belle-II see a similar excess on the 3π resonances but there are potential issues with the data.

Experimental Principle at J-PARC

Muon Precession in the Magnetic Field



$$\vec{\omega}_u = -\frac{e}{m}\left[a_\mu \vec{B} - \cancel{\left(a_\mu - \frac{1}{\gamma^2-1}\right)}\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B}+\frac{\vec{E}}{c}\right)\right]$$

Magic "γ": $a_\mu = \frac{1}{\gamma^2-1}$

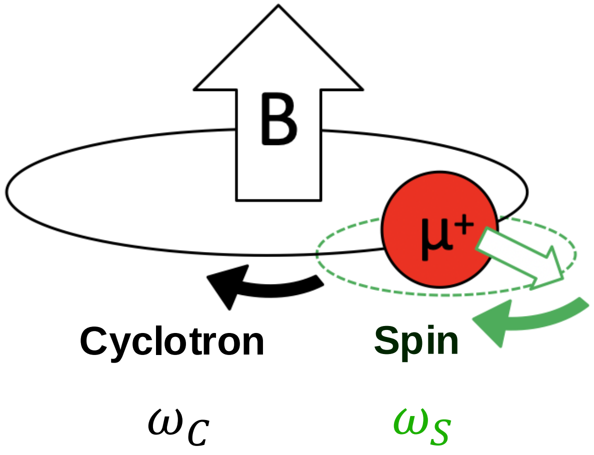
BNL E821 approach
γ=30 (P=3 GeV/c)

$$\boxed{\vec{\omega} = -\frac{e}{m}\Big[a_\mu \vec{B}} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B}+\frac{\vec{E}}{c}\right)\Big]$$

FNAL E989

Experimental Principle at J-PARC

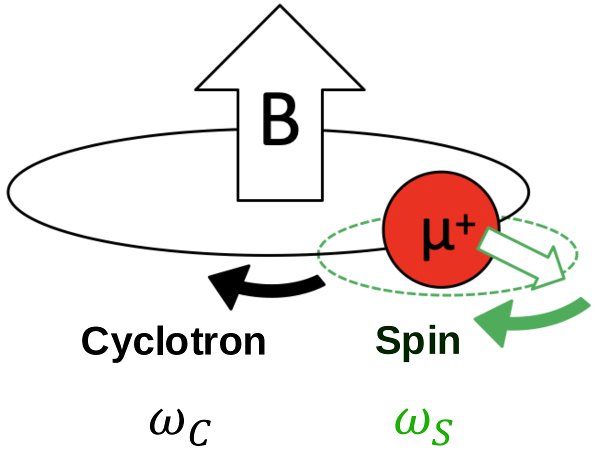
Muon Precession in the Magnetic Field



$$\vec{\omega}_u = -\frac{e}{m}\left[a_\mu \vec{B} - \underline{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

Experimental Principle at J-PARC

Muon Precession in the Magnetic Field



$$\vec{\omega}_u = -\frac{e}{m}\left[a_\mu \vec{B} - \cancel{\left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

Directly remove E field

J-PARC approach

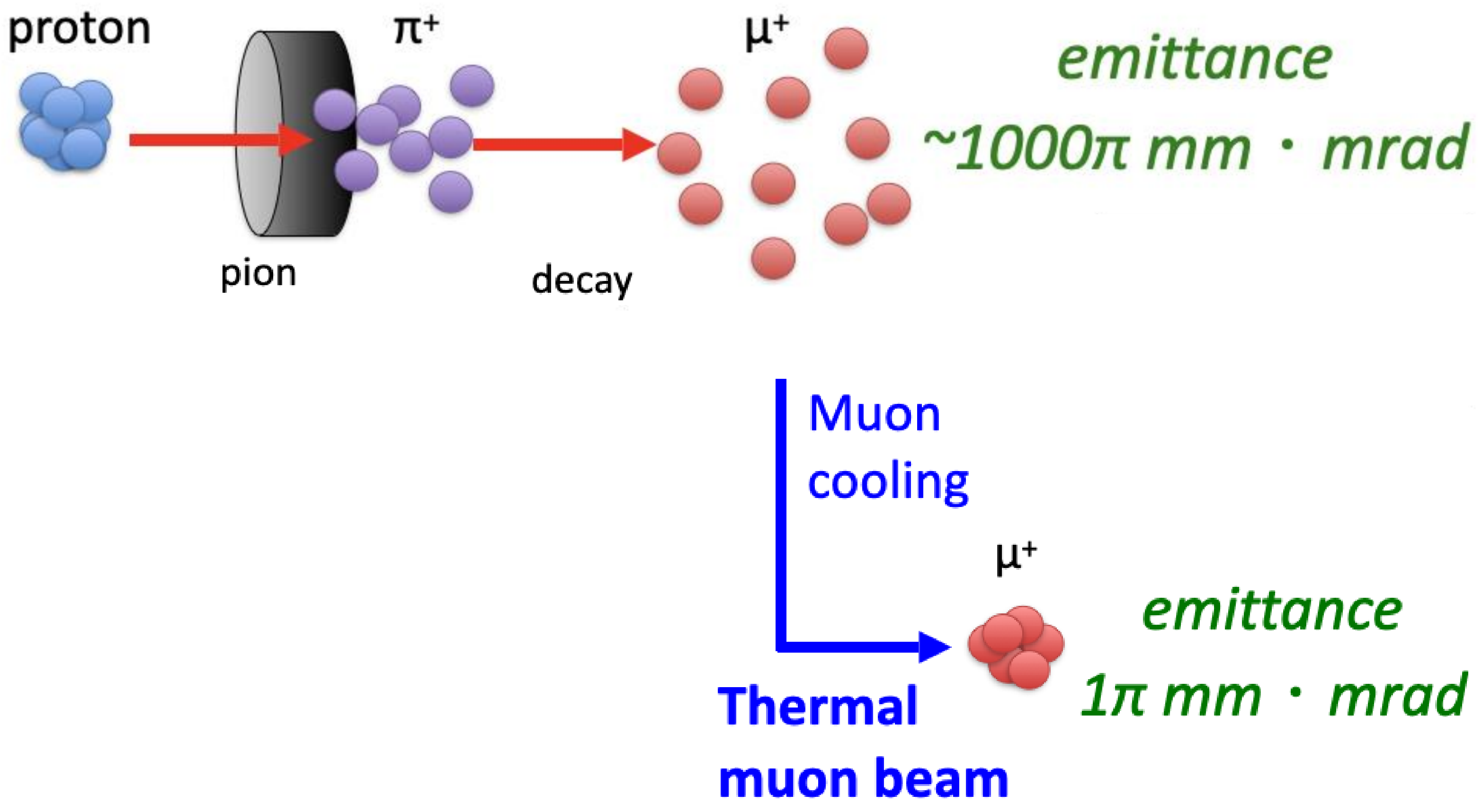
E = 0 at any γ

$$\vec{\omega}_a = -\frac{e}{m}\left[a_\mu \vec{B} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B}\right)\right]$$

J-PARC E34

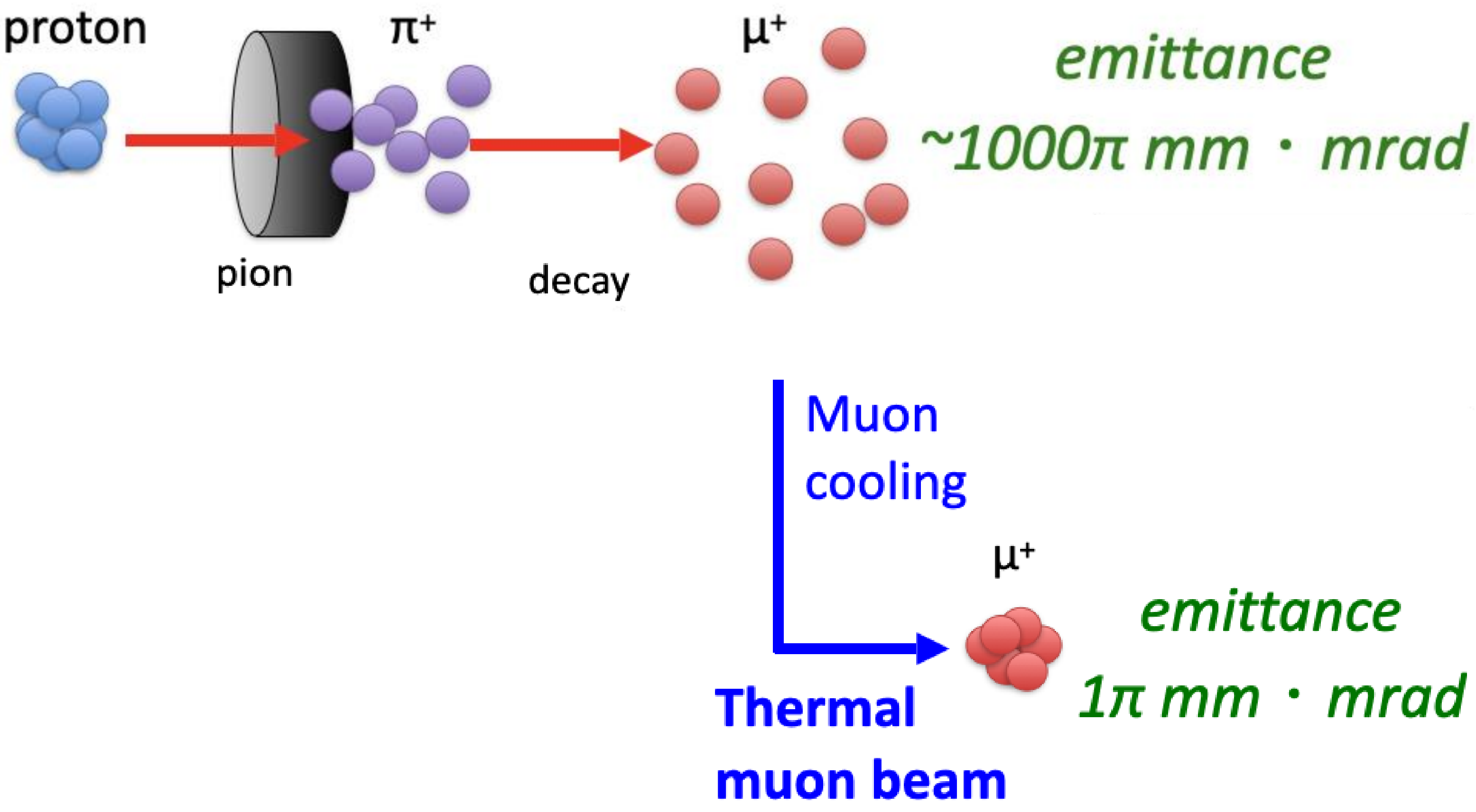
Experimental Principle at J-PARC

Low-emittance muons needed



Experimental Principle at J-PARC

Low-emittance muons needed



proton

pion

π^+

decay

μ^+

emittance

$\sim 1000\pi$ mm · mrad

Muon cooling

μ^+

Thermal muon beam

emittance

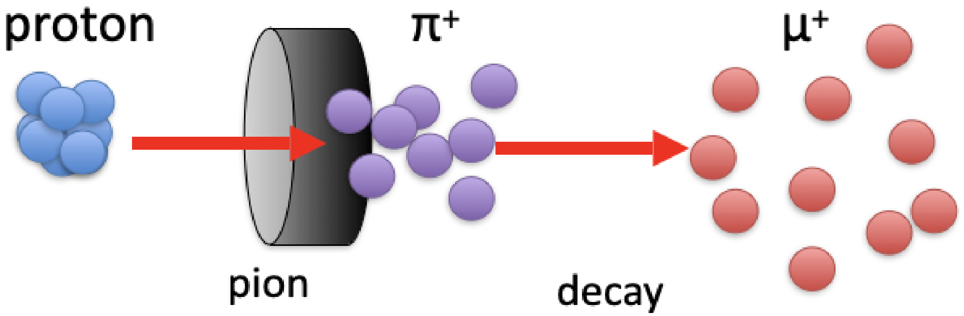
1π mm · mrad

Ordinary flashlight

Laser light!

Experimental Principle at J-PARC

Low-emittance muons needed



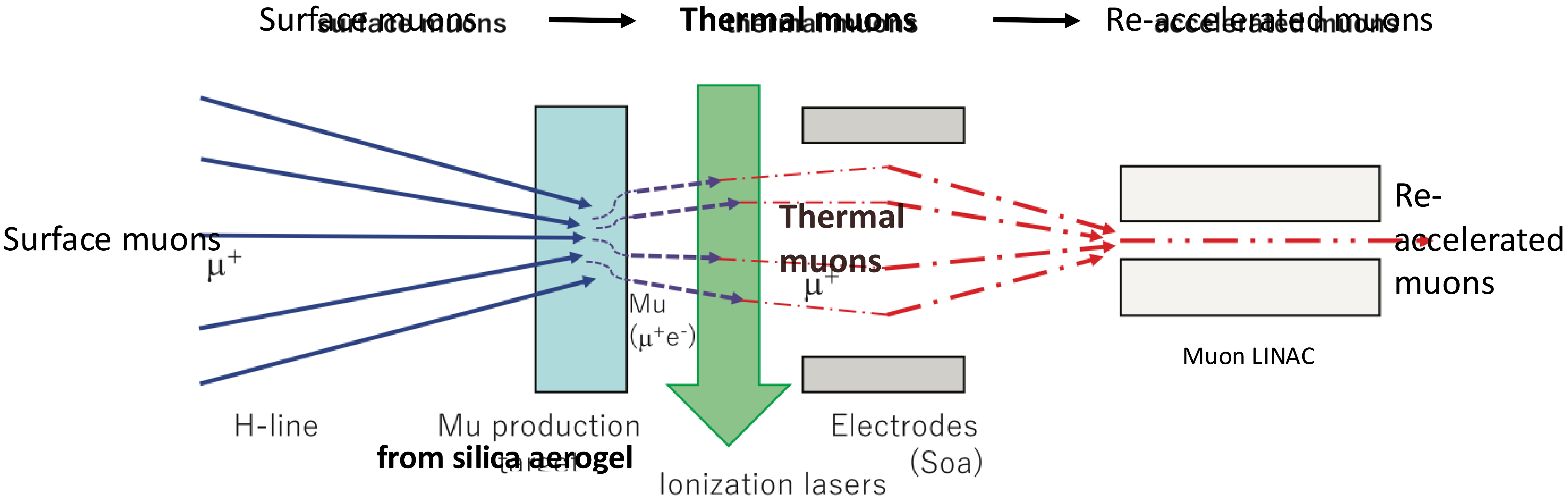
emittance

~1000π mm・mrad

	Fermilab Muon g-2	**J-PARC Muon g-2/EDM**
Focusing field	Electric quadrupole	**E = 0,** very weak magnetic
Muon momentum	3.09 GeV/c	300 MeV/c
Cyclotron period	149 ns	7.4 ns
Muon orbit diameter	**14 m**	**66 cm**
Storage Field	B = 1.45 T	B = 3 T (Solenoidal)
Polarization	100%	50%

Thermal Muon Source

- Surface muon cooling by laser ionization of muonium (Mu) to thermal muon

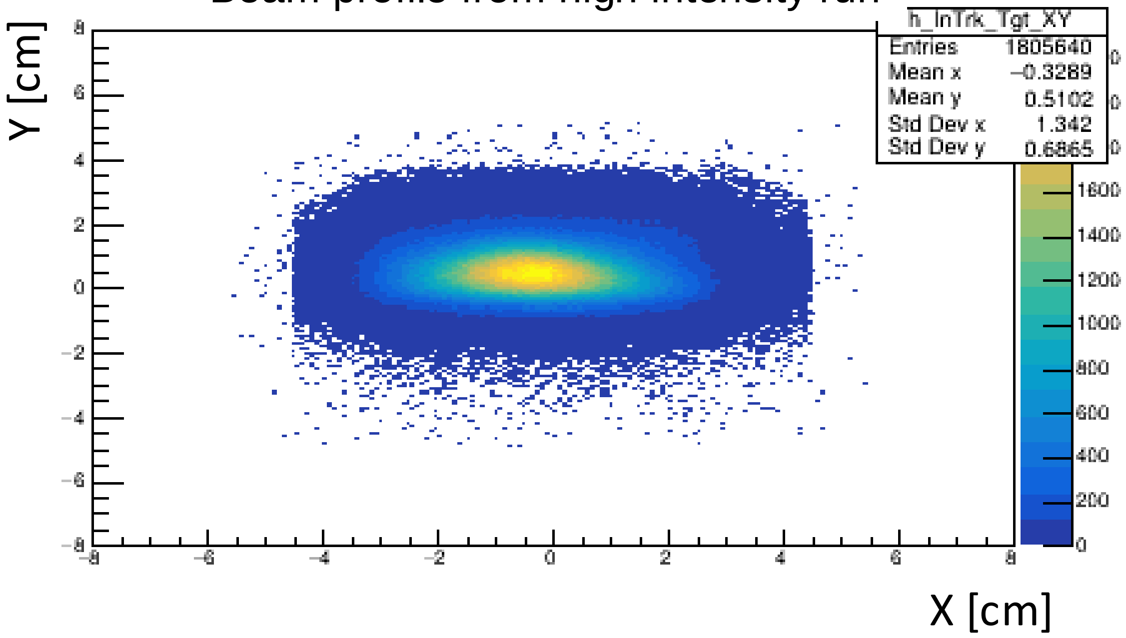


Test Run 2023

Muon beam profile & intensity

- Silicon strip helps us ‘see’ the beam profile at the target position

Beam profile from high intensity run



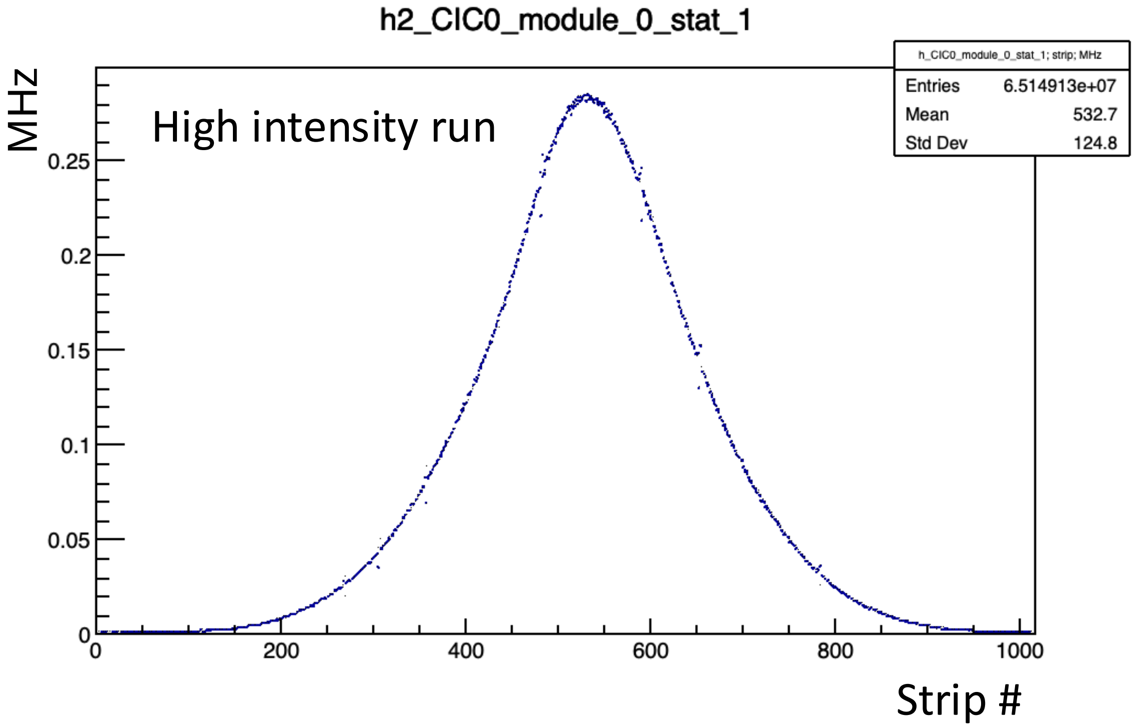
$\sigma_x = 1.34$ cm
$\sigma_y = 0.68$ cm

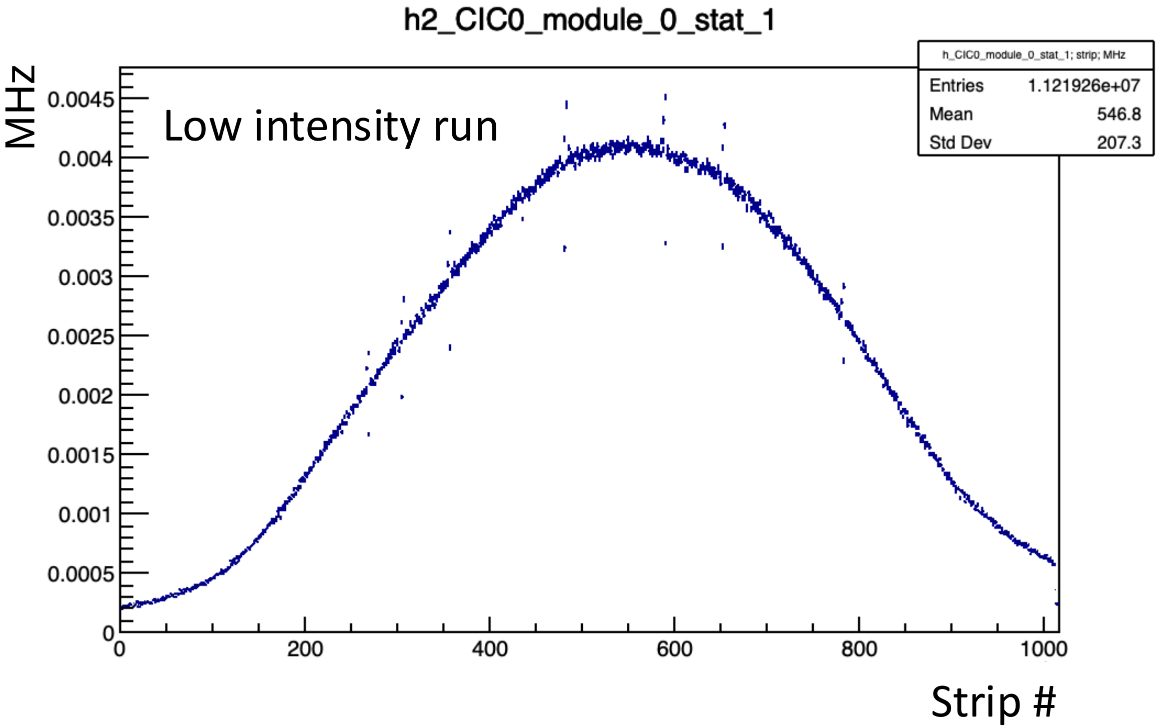
Confirmed the beam profile fits our detector dimensions

Test Run 2023

Muon beam profile & intensity

- Silicon strip helps us ‘see’ the beam profile at the target position

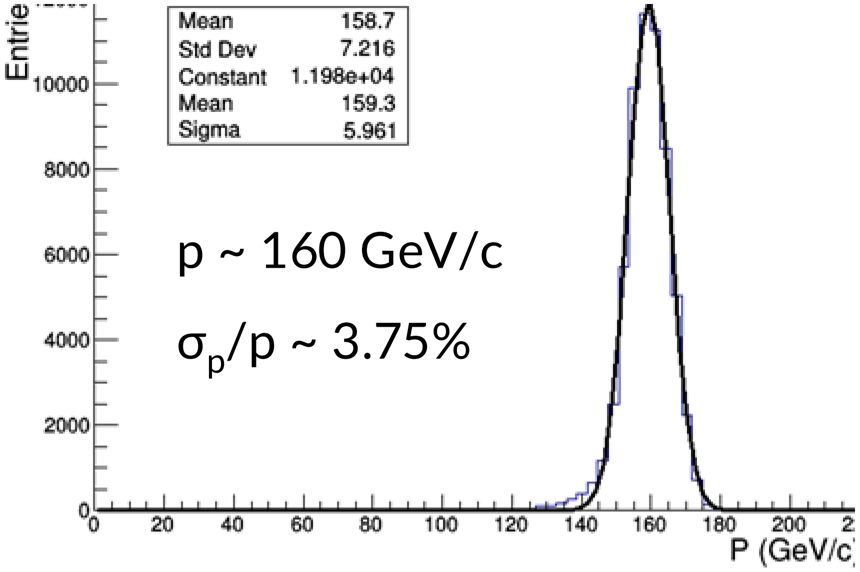




Test run 2023

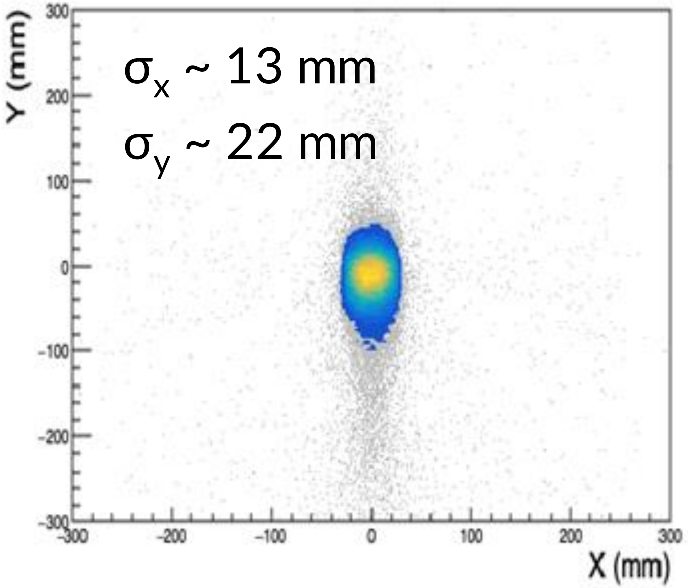
Muon (M2) beam-line at CERN Prévessin site

- CERN North Area M2: upstream of the COMPASS detector
  - Maximum 50 MHz (2-3×10^8 μ^+/spill) for 10^{12} 400 GeV/c incident protons



$p \sim 160$ GeV/c

$\sigma_p/p \sim 3.75\%$



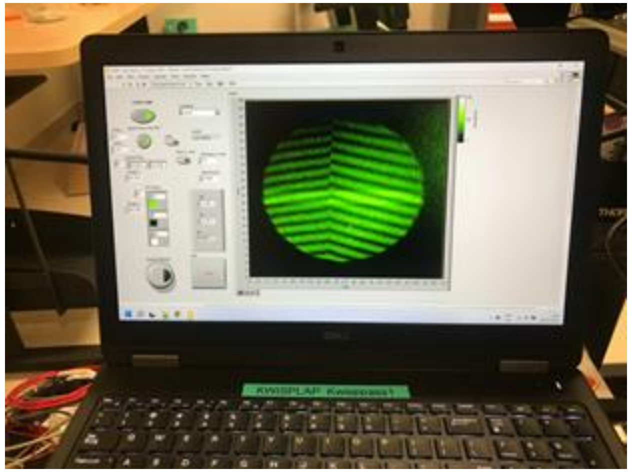
$\sigma_x \sim 13$ mm

$\sigma_y \sim 22$ mm

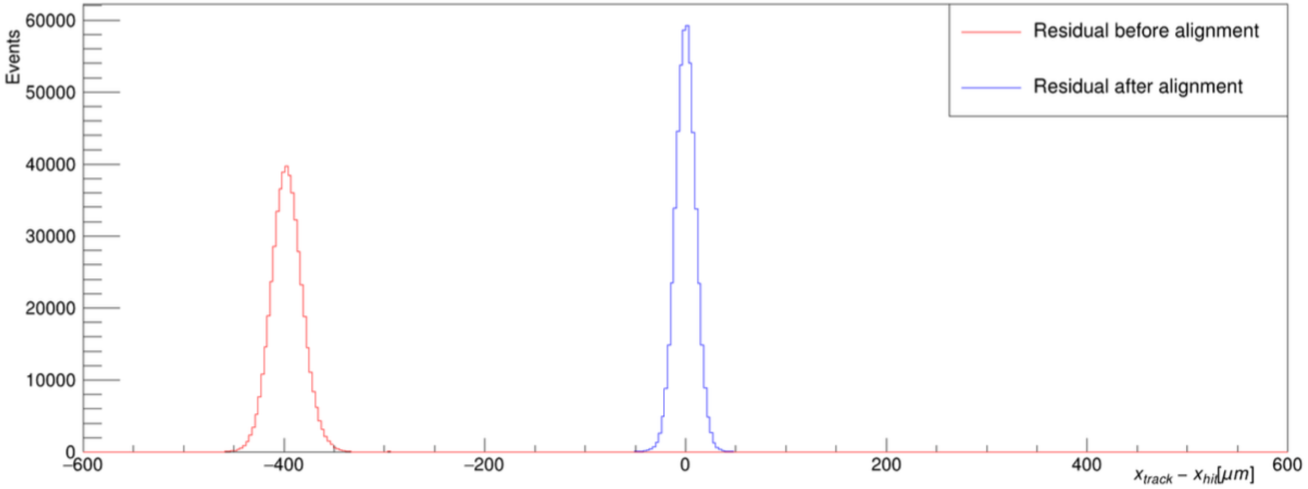
Test run 2023

Module alignment, resolution and efficiency

- It is extremely challenging for MUonE to achieve precise alignment of less than **1 um** transversely and **~10 μm** longitudinally for the modules and stations.
  - Hardware level: metrology measurements using **laser survey**
  - Software level: implemented with FairMUonE



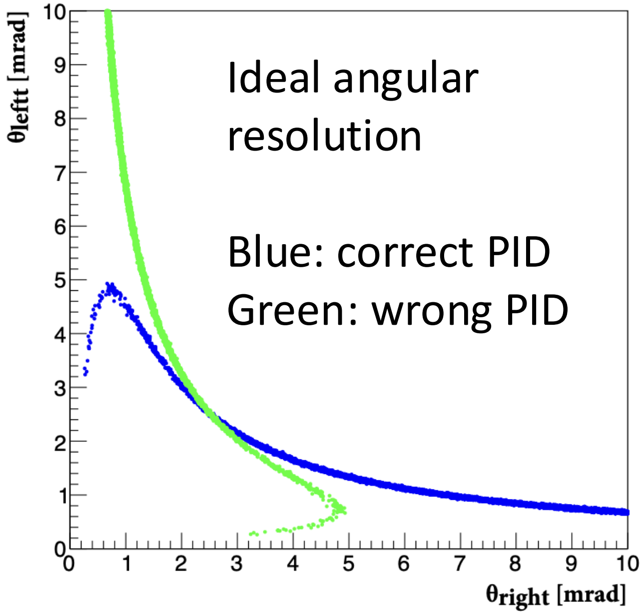
a laser holographic system developed at INFN



Analysis for test run 2023

Event reconstruction & selections

- Some basic criteria
  - Track candidate quality (χ^2)
  - Vertex position in the target
  - Acoplanarity
- Kinematic considerations
  - $E_{\mu(\mathrm{beam})},\ \theta_\mu,\ \theta_e$:
    - θ_μ : tune background of e^+e^- pairs
    - θ_e : tune acceptance
    - $E_{\mu(\mathrm{beam})}$ is in principle described by two angles
  - PID: muons can be distinguished from electrons using solely the angular information



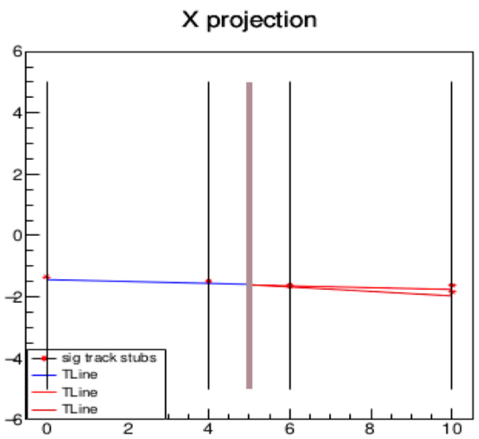
Ideal angular resolution

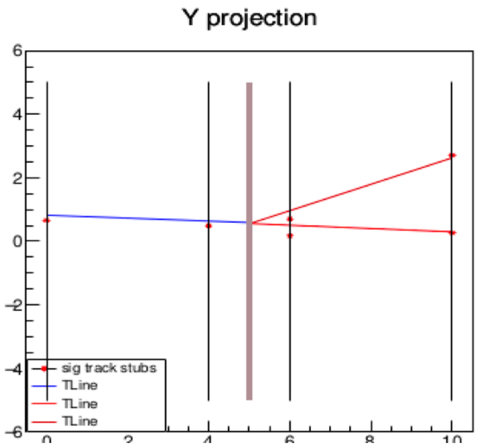
Blue: correct PID
Green: wrong PID

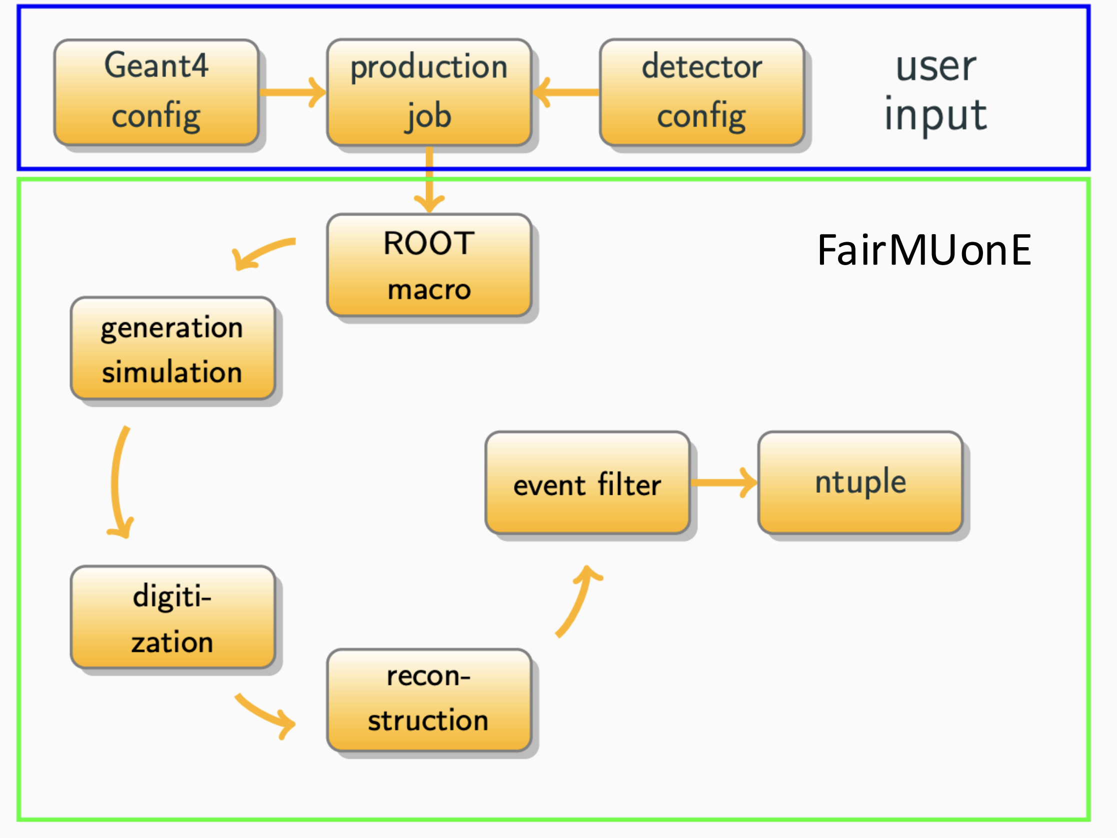
Elastic Scattering Analysis

‘FairMUonE’

- The package developed dedicated to MUonE
- Both simulation and track reconstruction in the same package
- Digitization of tracker & calo are implemented

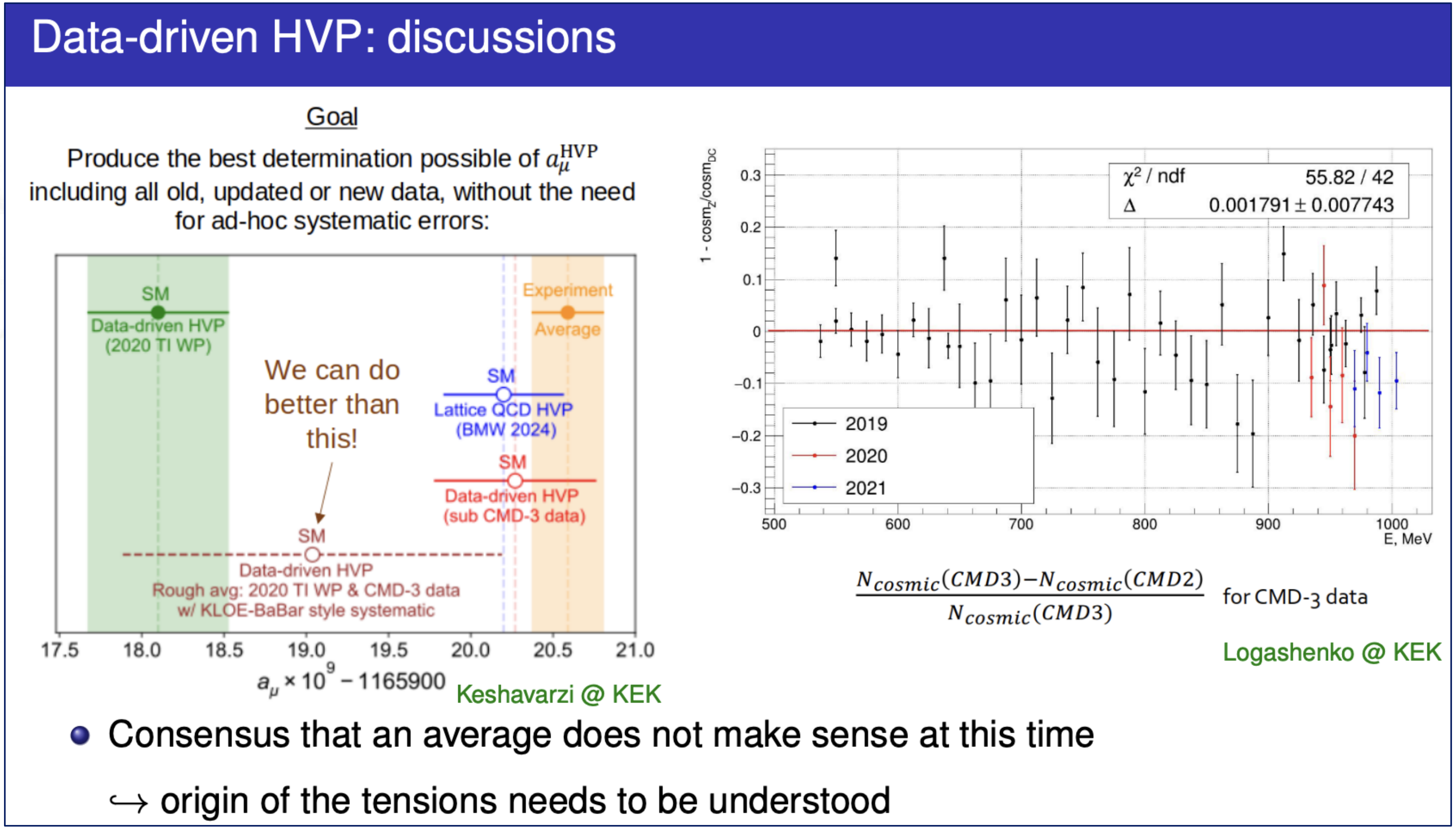






Muon g-2 theory initiative; Seventh workshop at KEK (Sep 9-13, 2024)

https://conference-indico.kek.jp/event/257/



Data-driven HVP: discussions

Goal

Produce the best determination possible of a_μ^{HVP} including all old, updated or new data, without the need for ad-hoc systematic errors:

Keshavarzi @ KEK

$$\frac{N_{cosmic}(CMD3)-N_{cosmic}(CMD2)}{N_{cosmic}(CMD3)}$$ for CMD-3 data

Logashenko @ KEK

- Consensus that an average does not make sense at this time

  ↪ origin of the tensions needs to be understood

Review slides by
Martin Hoferichter

Towards the ultimate muon anomaly test

Slides by T. Mibe, inspired by K. Jungmann's slide

Fermilab Muon g-2 (E98

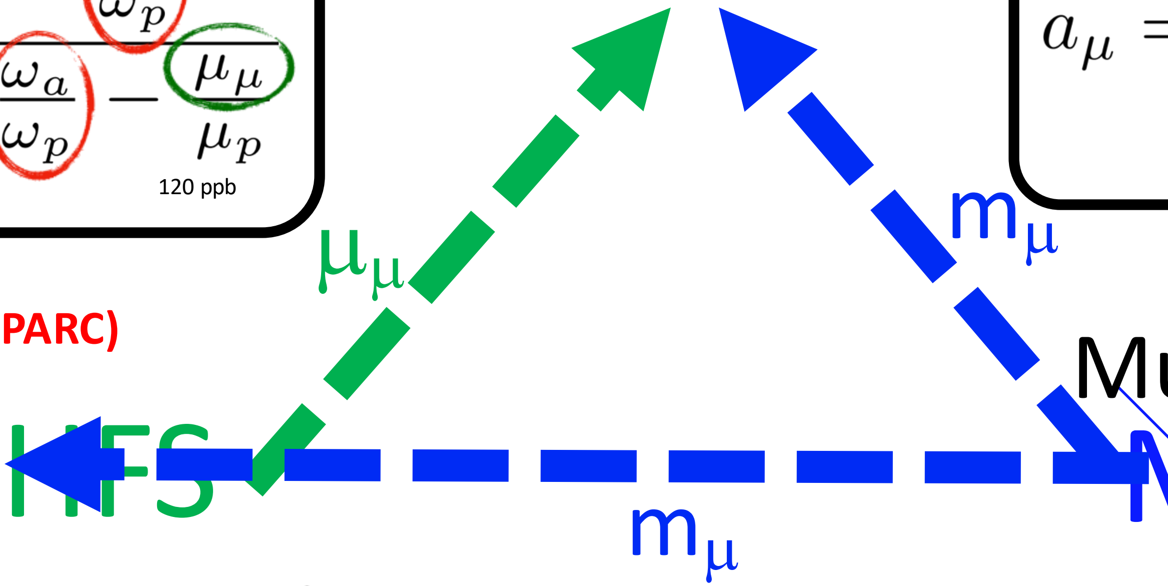
Muon $g-2$

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\omega_a}{\omega_p} - \frac{\mu_\mu}{\mu_p}}$$

120 ppb

$$a_\mu = \frac{\omega_a}{\omega_p}\frac{\mu_p}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2}$$

8 ppb 120 ppb 0.3 ppt



MuSEUM(J-PARC)

$\nu_{34} - \nu_{12} \propto \frac{\mu_\mu}{\mu_p}$ Mu HFS

Mu-MASS(PSI), **J-PARC**

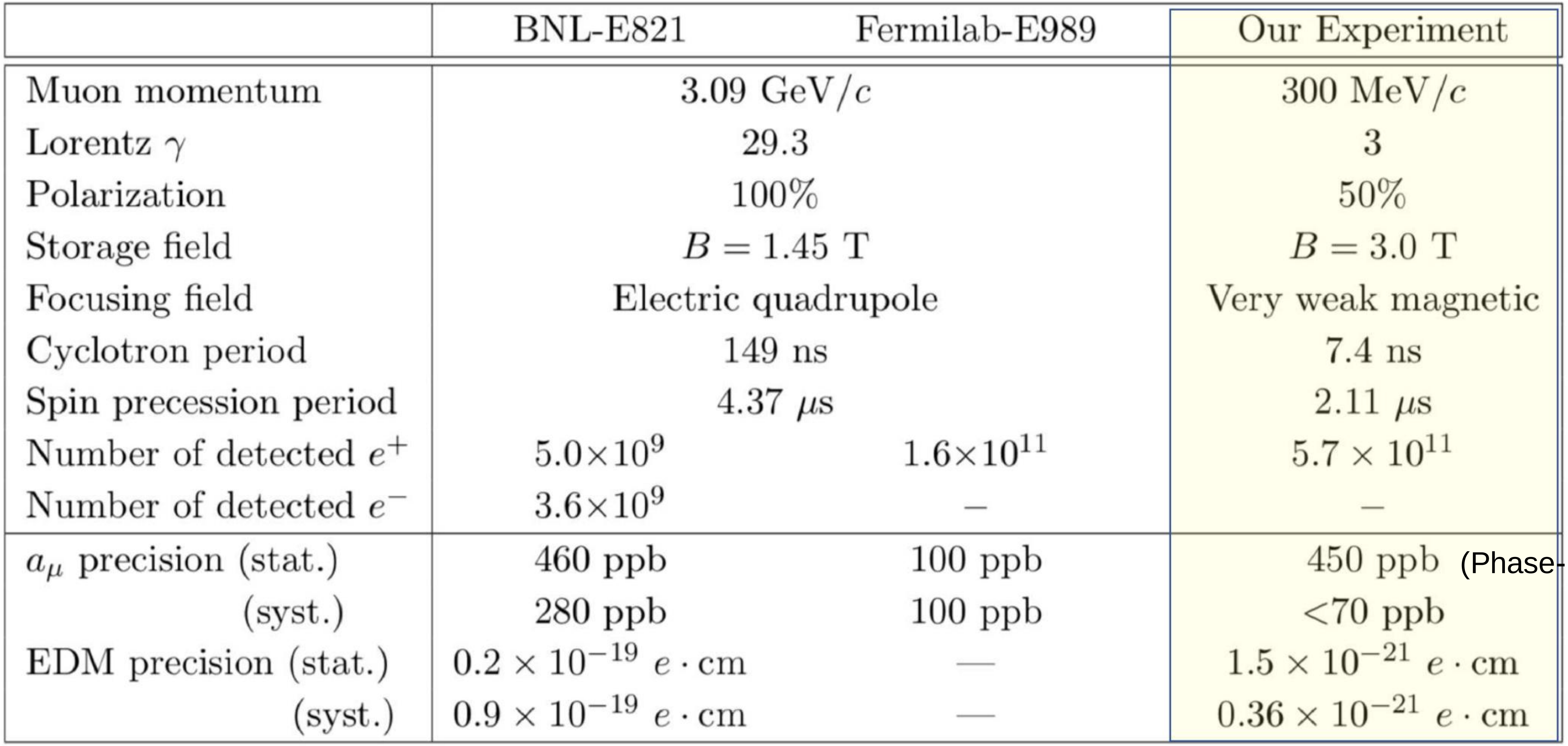
Mu 1S-2S

$$\Delta\nu_{\text{1S-HFS}} \simeq \frac{16}{3}\alpha^2 R_\infty \frac{\mu_\mu}{\mu_B}\left(1 + \frac{m_e}{m_\mu}\right)^{-3}$$

$$\Delta\nu_{1S2S} \simeq \frac{3\alpha^2}{8h} m_e c^2 \left(1 + \frac{m_e}{m_\mu}\right)^{-1}$$

Precision comparison

J-PARC E34



	BNL-E821	Fermilab-E989	Our Experiment
Muon momentum	3.09 GeV/c		300 MeV/c
Lorentz γ	29.3		3
Polarization	100%		50%
Storage field	$B = 1.45$ T		$B = 3.0$ T
Focusing field	Electric quadrupole		Very weak magnetic
Cyclotron period	149 ns		7.4 ns
Spin precession period	4.37 μs		2.11 μs
Number of detected e^+	5.0×10^9	1.6×10^{11}	5.7×10^{11}
Number of detected e^-	3.6×10^9	–	–
a_μ precision (stat.)	460 ppb	100 ppb	450 ppb (Phase-1)
(syst.)	280 ppb	100 ppb	<70 ppb
EDM precision (stat.)	0.2×10^{-19} $e \cdot$ cm	—	1.5×10^{-21} $e \cdot$ cm
(syst.)	0.9×10^{-19} $e \cdot$ cm	—	0.36×10^{-21} $e \cdot$ cm