Three-body study of the $T_{cc}(3875)^+$ tetraquark in lattice QCD

André Baião Raposo, Ruhr-Universität Bochum

in collaboration with

H. Alharazin, J. Bulava, S. Dawid, J. Green, D. Laudicina, C. Morningstar,
F. Romero-López, M. Salg, S. Sharpe, A. Stump

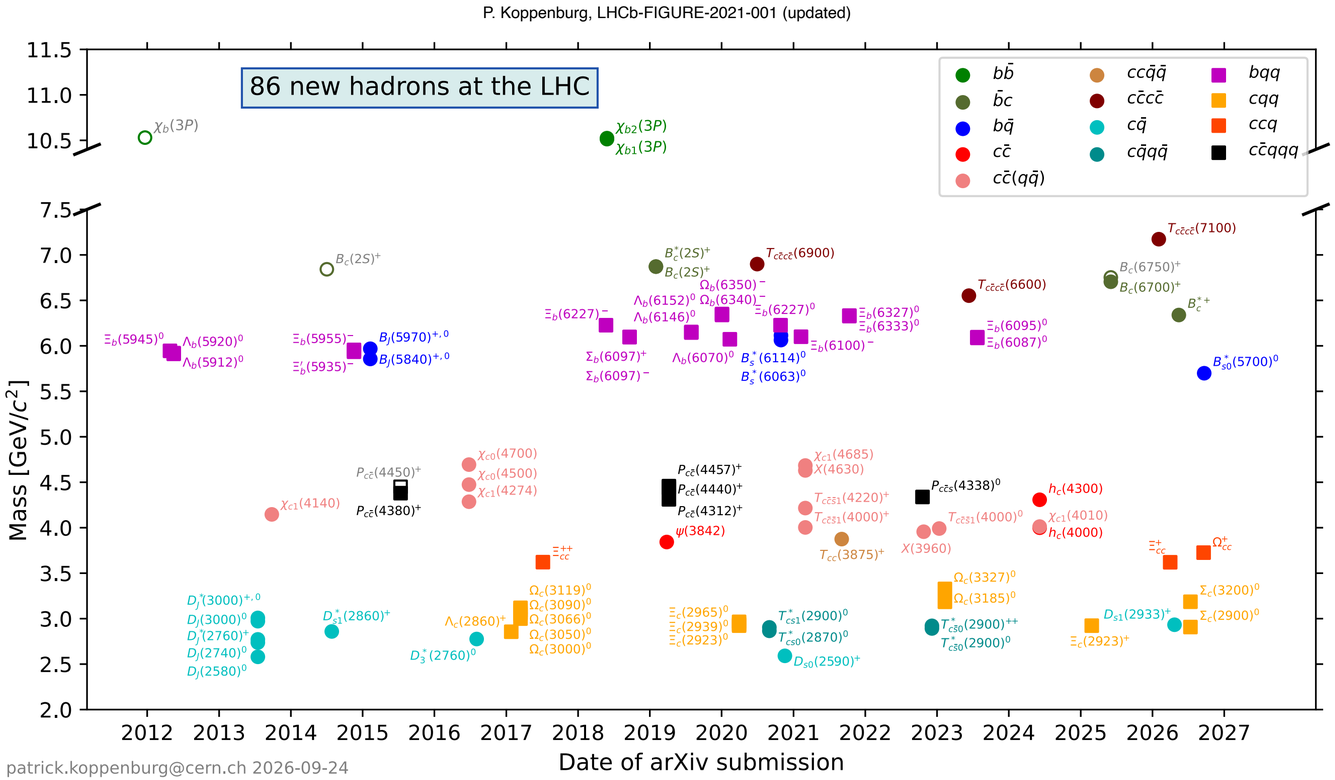
Ruhr Hadron Seminar

Bochum, 10th November 2026

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Hadron scattering and interactions from QCD

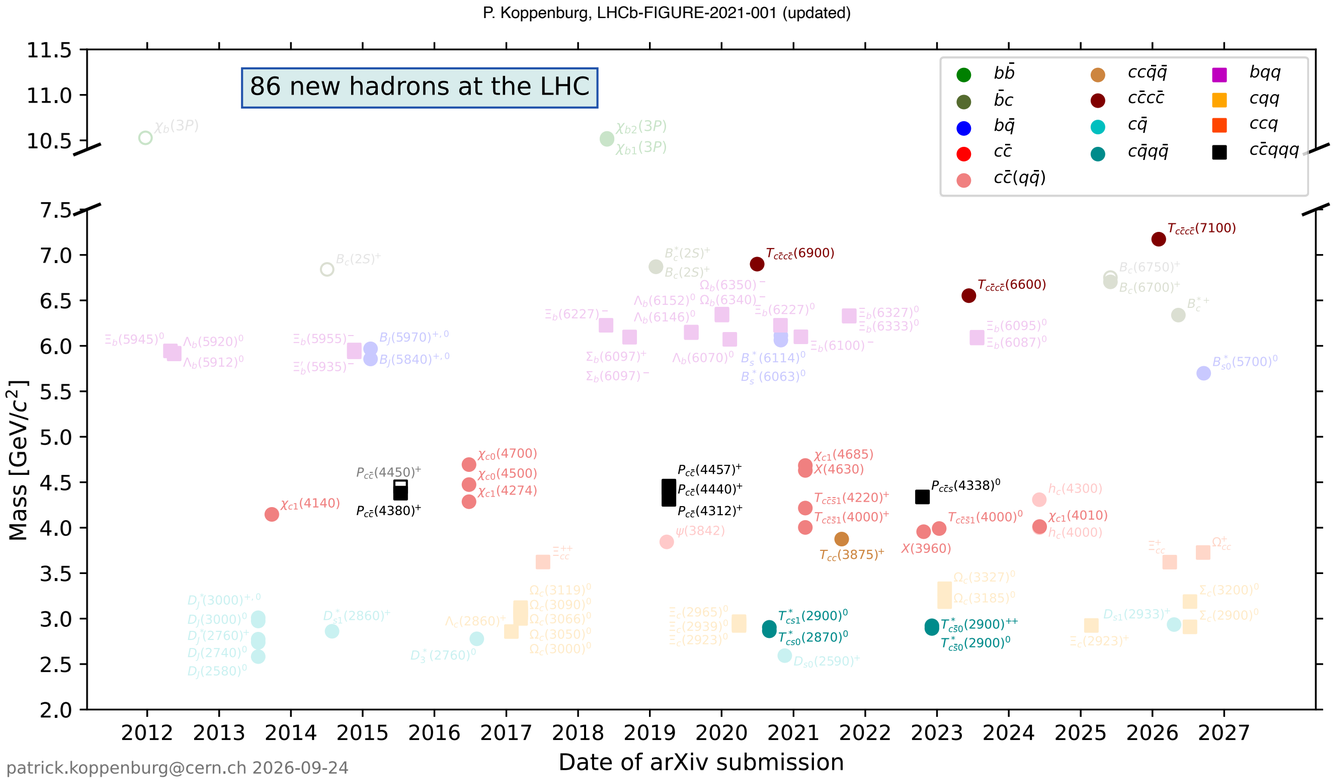
- Obtaining theoretical predictions from QCD in the low-energy regime is extremely challenging
- Predictions for multi-hadron scattering and interactions are valuable to many fields, e.g.
  - explaining the observed hadron spectrum, in particular exotic states
  - tests of the Standard Model and flavour physics (e.g. hadronic contributions to the muon $g-2$)
  - inputs for EFTs and in nuclear physics and astrophysics (e.g. LECs for NN or YN interactions)
- Lattice QCD is increasingly capable of providing reliable, systematically improvable predictions directly from QCD, complementing experiment



new hadrons found at the LHC, by year of discovery

Exotic hadron spectroscopy

- Many new hadrons discovered in the last two decades, at experiments like Belle or LHCb
- Several are conventional — fit quark model picture as excited mesons or baryons
- A growing share are exotic, e.g. XYZ states — their structure and binding mechanism still debated
- Many sit close to production thresholds: do they form as hadron-hadron molecules, compact multiquark structures, or something else?
- Input from lattice QCD could help shed light into these questions

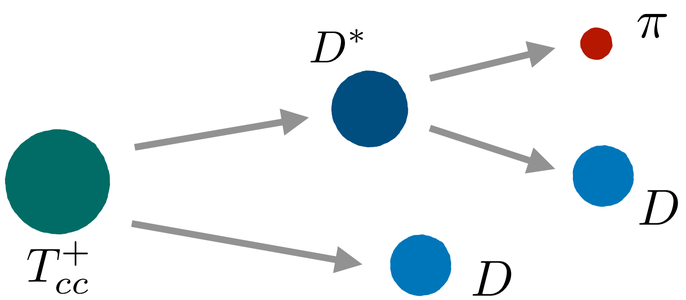


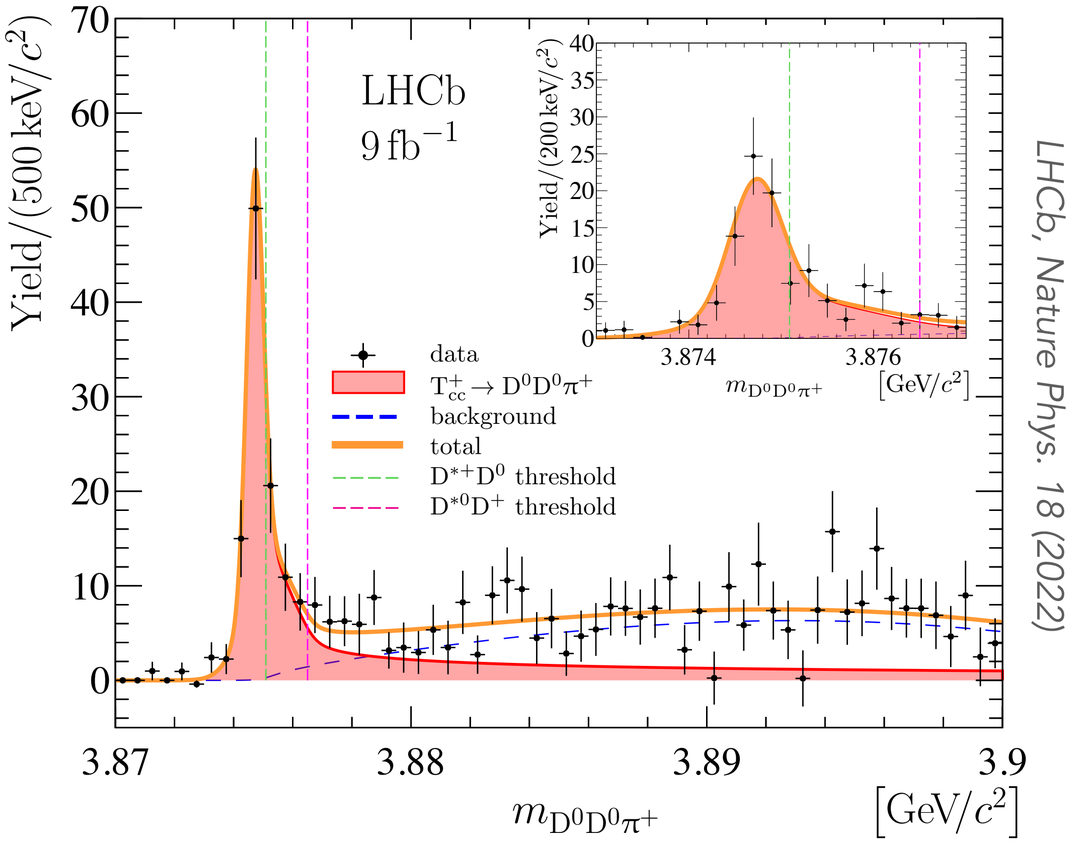
new exotic hadrons found at the LHC highlighted

A popular exotic: the $T_{cc}^{+}(3875)$ tetraquark

- $J^P = 1^+$, $I = 0$ resonance with quark content $cc\bar{u}\bar{d}$, first observed by the LHCb collaboration in 2021
- Seen as narrow peak in $D^0D^0\pi^+$ mass spectrum, located very close to DD^* thresholds
- Longest-lived known exotic hadron, decays in nature mainly to three-hadron $DD\pi$ final state, via intermediate DD^* state:

$$T_{cc}^{+} \to D^0D^{*+} \to D^0D^+\pi^0, D^0D^0\pi^+$$





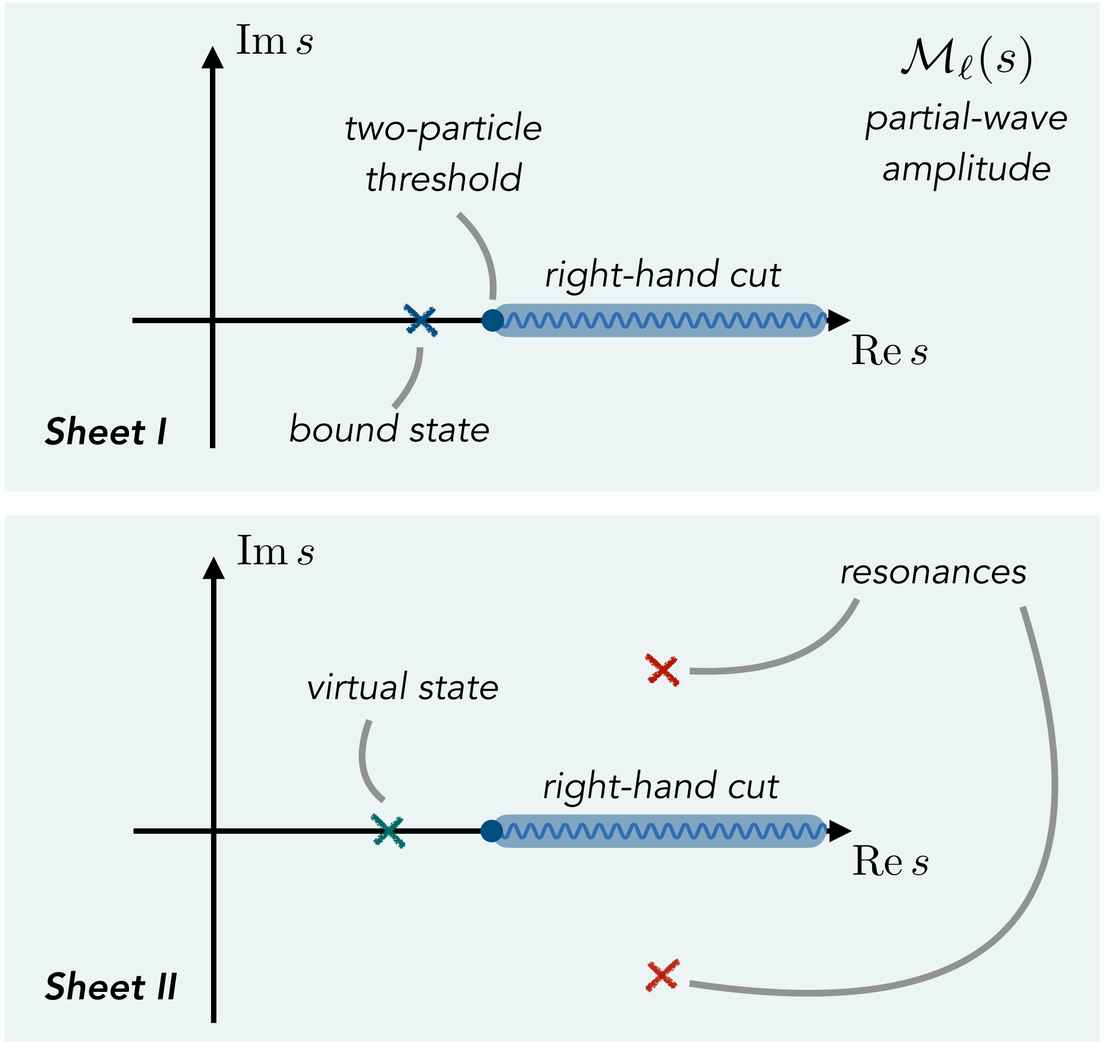
LHCb, Nature Phys. 18 (2022)

Hadronic states from amplitudes

- Like the T_{cc}^{+}, most states in the QCD spectrum are unstable, i.e. **resonances**, observed only as enhancements or peaks in scattering cross-sections
- Rigorously understood as poles in hadron scattering amplitudes
- Position in complex energy (or momentum) plane and the Riemann sheet determine type of state

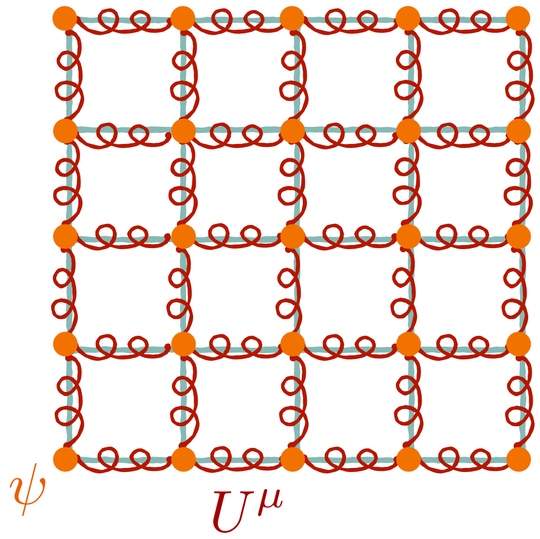
e.g. for two-particle elastic scattering, partial-wave amplitude has a square-root branch cut (right-hand cut) and two-sheet structure

→ goal of spectroscopy on the lattice is to determine the amplitude and find location of the poles



Lattice QCD

- Lattice QCD is a first-principles non-perturbative numerical approach to QCD
- QCD path integral implemented in a finite, discretised Euclidean spacetime — the **lattice**
- Field configurations generated by Monte Carlo importance sampling, with probability distribution $\propto e^{-S_E}$ (S_E is the Euclidean action)
- Observables computed by averaging over the ensemble of sampled field configurations
- Infinite-volume, continuum and physical quark mass extrapolations often necessary for meaningful predictions

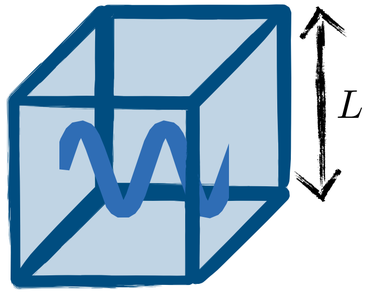


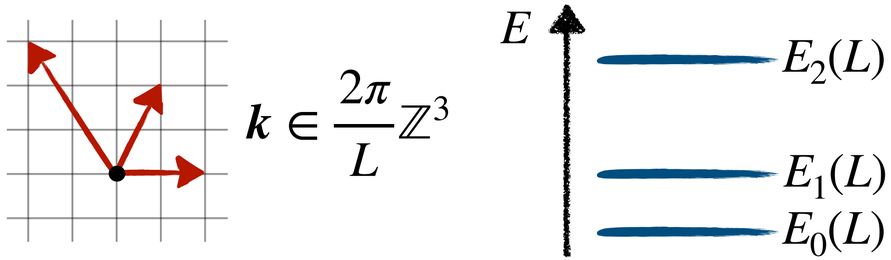
$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \, \mathcal{D}U \;\; \mathcal{O}[\psi, \bar{\psi}, U] \;\; e^{-S_E[\psi, \bar{\psi}, U]}$$

$$\mathcal{Z} = \int \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \, \mathcal{D}U \;\; e^{-S_E[\psi, \bar{\psi}, U]}$$

Scattering from lattice QCD

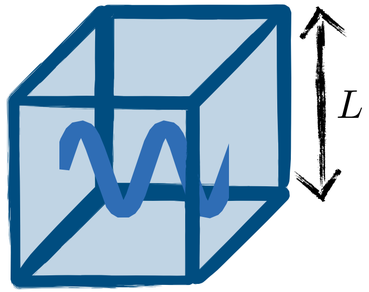
- Direct study of scattering on the lattice is not possible
  - *Euclidean signature* — no access to real-time dynamics
  - *finite extent of the box* — no asymptotic states
- We need indirect methods to connect finite-volume observables to scattering quantities, *e.g.*
  - *finite-volume methods* — exploit the volume dependence
  - *spectral reconstruction*





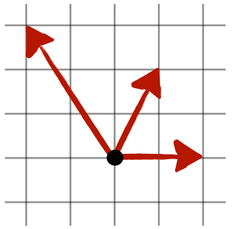
discretised momenta and energies in a periodic box

Scattering from lattice QCD

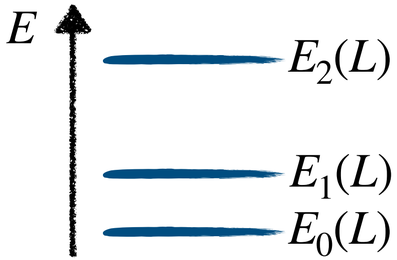


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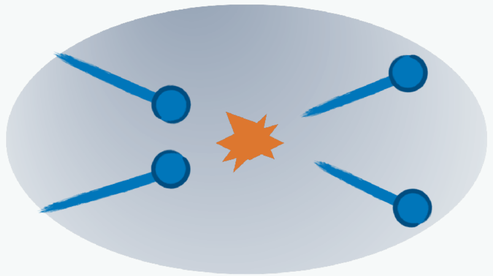
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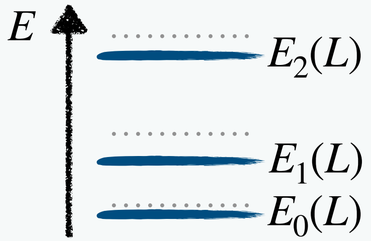


$$\boldsymbol{k} \in \frac{2\pi}{L}\mathbb{Z}^3$$



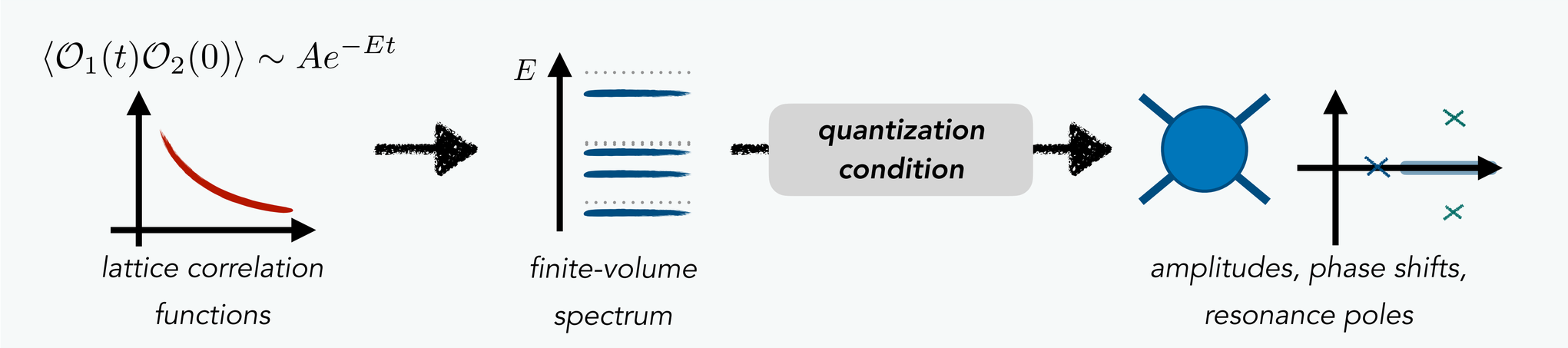
interactions leave an imprint on the discrete finite-volume spectrum



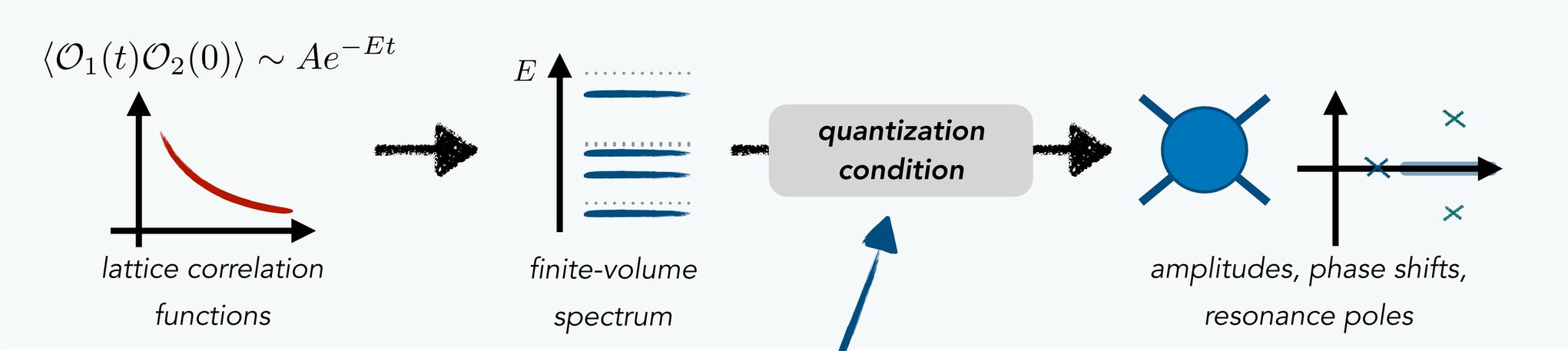


M. Lüscher, 1986 and many others since

Scattering from lattice QCD: typical pipeline



Scattering from lattice QCD: typical pipeline



e.g. for 2-body scattering:

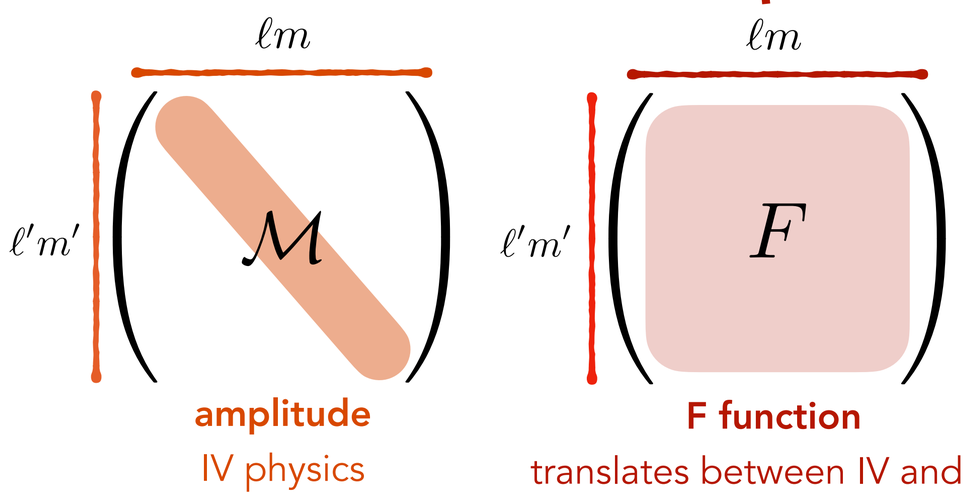
$$\det_{\ell m}\left[\mathcal{M}(E_{\rm cm})^{-1} + F(E, \boldsymbol{P}, L)\right] = 0$$

at the finite-volume energies

Lüscher quantisation condition

The 2-body Lüscher condition

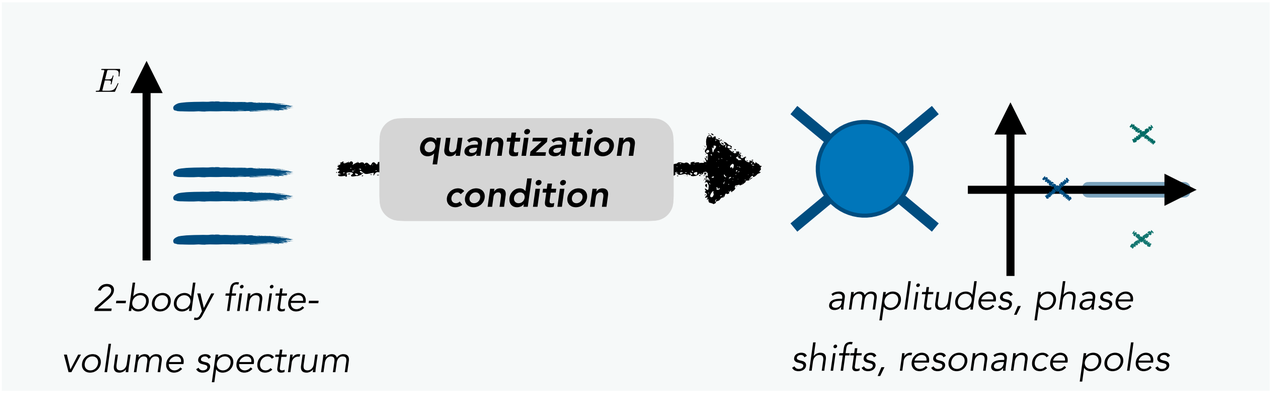
$$\det_{\ell m}\left[\mathcal{M}(E_{\mathrm{cm}})^{-1} + F(E, \boldsymbol{P}, L)\right] = 0$$



amplitude
IV physics

F function
translates between IV and FV

building blocks are matrices in angular momentum ℓm index space
(plus channel and spin indices in more complicated systems)



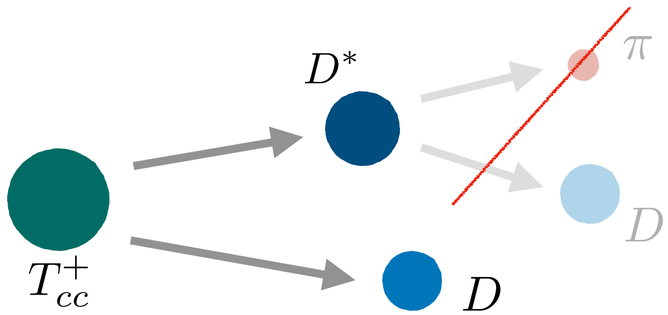
…in practice:

- truncate in ℓ — lowest waves dominate near threshold
- parameterise $\mathscr{M}_\ell$ via a K matrix or phase shift.
  (e.g. an effective range expansion)
- fit the finite-volume levels

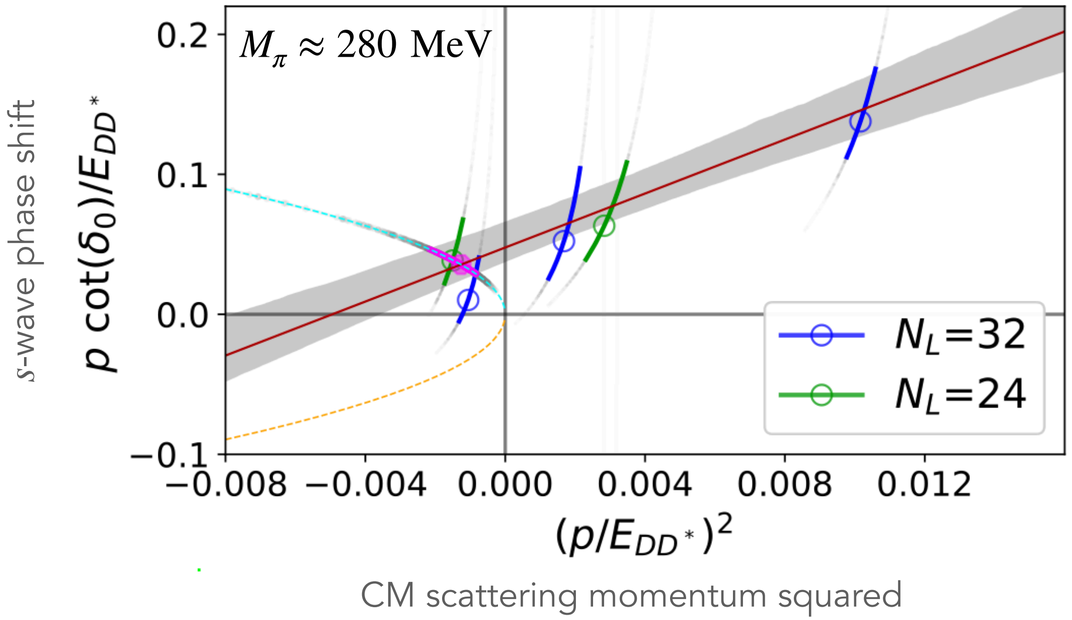
T_{cc}^+ via a 2-body approach

- At heavier-than-physical light-quark masses, D^* meson becomes stable ($D^* \to D\pi$ kinematically forbidden)

- We can study the T_{cc}^+ via $I = 0$ DD^* scattering, as has been done in several lattice calculations to date:



Padmanath, Prelovšek PRL 129 (2022)



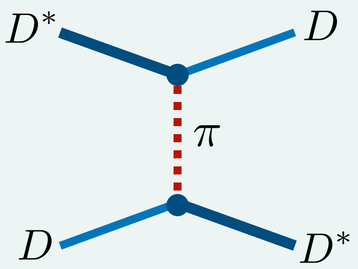
Chen et al. PLB 833 (2022)
Lyu et al. PRL 131 (2023)
Collins et al. PRD 109 (2024)
Whyte, Wilson, Thomas PRD 111 (2025)

Analyses using the standard Lüscher method find a virtual bound state at heavier-than-physical pion masses

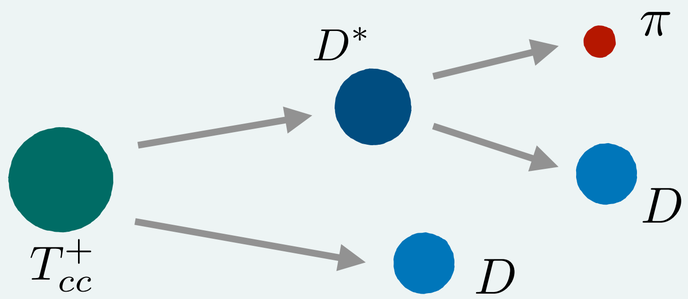
Seems fine but…

Limitations of the 2-body approach

There are two interconnected issues affecting a standard Lüscher 2-body analysis of the DD^* system:



neglects the effect of one-pion exchange and the corresponding left-hand cut

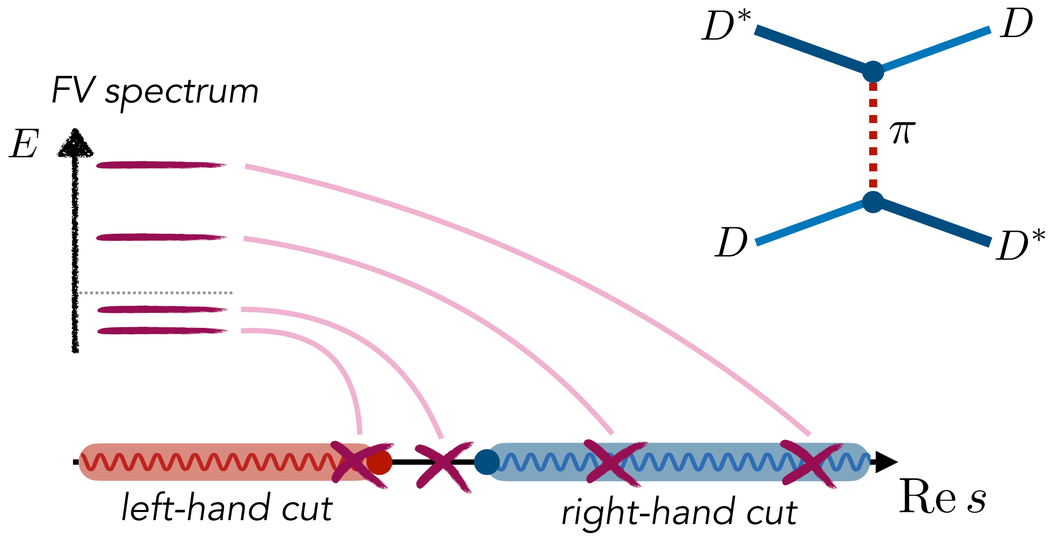


neglects 3-particle ($DD\pi$) effects, increasingly relevant as one approaches physical light quark masses

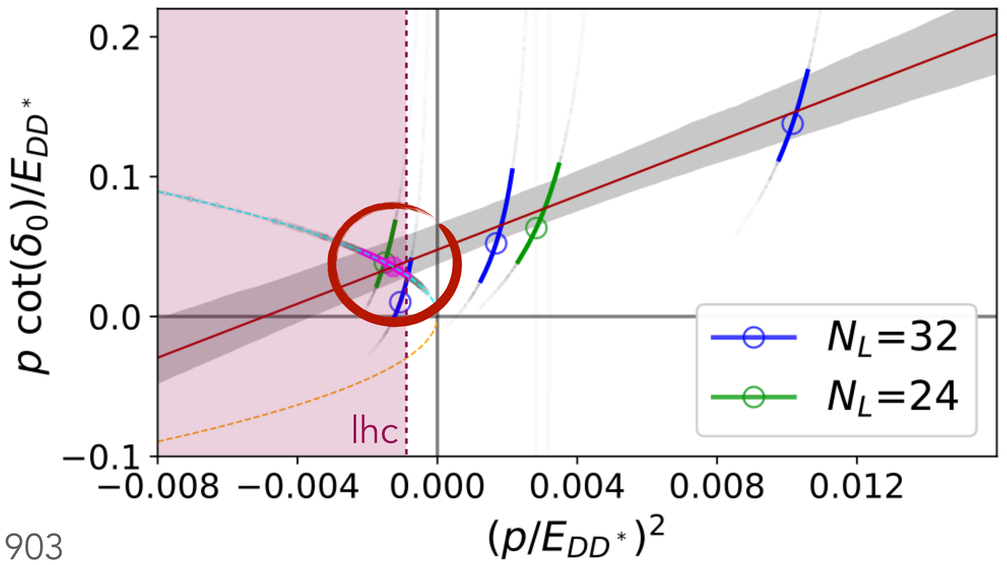
The left-hand cut problem

Near-threshold left-hand cuts can arise from partial-wave projecting light-particle exchanges between two heavier scatterers…

- Systems where they occur, such as DD^*, can have finite-volume levels that are near to or overlap with the cut
- Derivation of the Lüscher condition does not take this into account — the QC breaks down near to/on the cut
  - *we risk neglecting potentially strong finite-volume effects from the one-particle exchange*
- Typical parameterisations of the phase shift (or K matrix), such as the effective range expansions (EREs), also break down
  - *output amplitude fails to reproduce known analytic structure, e.g. real amplitude on the cut instead of complex*



Padmanath, Prelovšek PRL 129 (2022)



Importance for the T_{cc}^+ first pointed out in Du, Filin et al. *PRL* 131 (2023) 13, 131903

How to fix the issue?

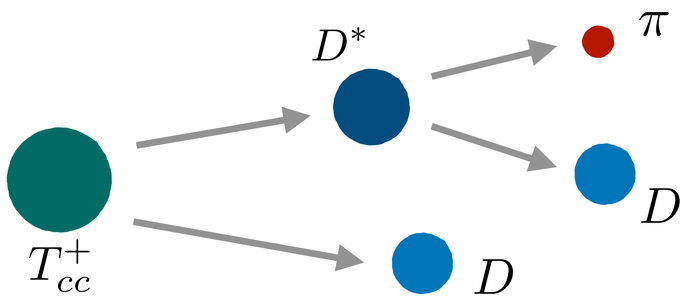
- **Two-body route:** several proposals to resolve the issue in 2-body methods

ABR, Hansen JHEP 08 (2024) — relativistic left-hand cut method
Meng, Baru et al. PRD 109 (2024) — plane wave method
Bubna, Hammer et al. JHEP 05 (2024)
Yu, Wang et al. JHEP 04 (2025)
ABR, Briceño, Hansen, Jackura JHEP 06 (2025)
Dawid, Jackura, Szczepaniak PLB 864 (2025)

- **Three-body route:** discussed in this talk!

T_{cc}^{+} via a 3-body approach

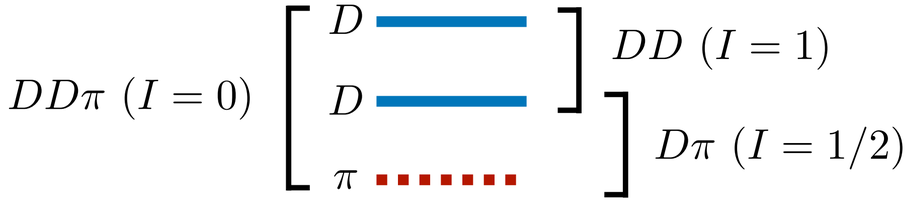
- The DD^* system presents an extra complication if one wants to study it at near-physical quark masses:
  - 3-body effects play an increasingly important role as we approach the physical point
  - D^* ultimately becomes a resonance, thus a fully 3-body treatment of the $DD\pi$ system is unavoidable
- 2-body formalism is inadequate in that situation, but **3-body formalism can provide a unified framework for both stable and unstable D^***
- **Left-hand cut is built in** — the formalism automatically takes it into account

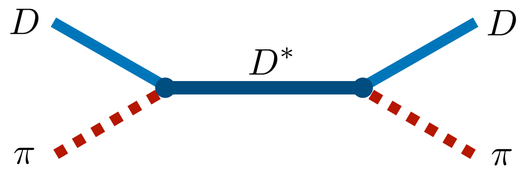


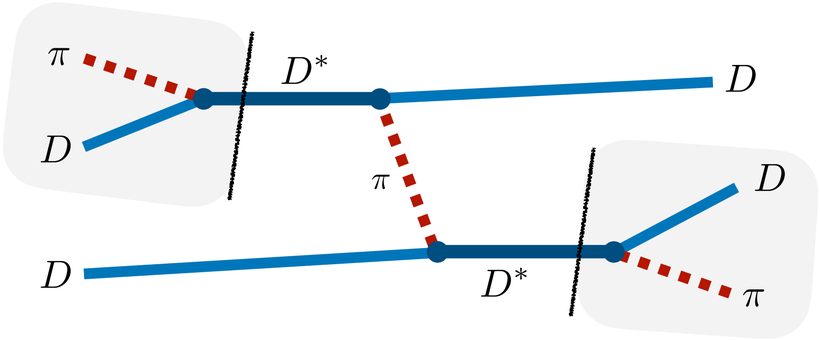
T_{cc}^{+} via a 3-body approach: setup

For T_{cc}^{+} , the relevant case is $I = 0$:

- two 2-particle sub-channels to consider,
  $DD\ (I = 1)$, $D\pi\ (I = 1/2)$
- D^* included as a **pole in the p-wave $D\pi$ amplitude** (as a bound state or resonance)
- For stable D^*, the DD^* amplitude is accessed by **LSZ-reducing the $DD\pi$ amplitude** and **analytically continuing** below $DD\pi$ threshold

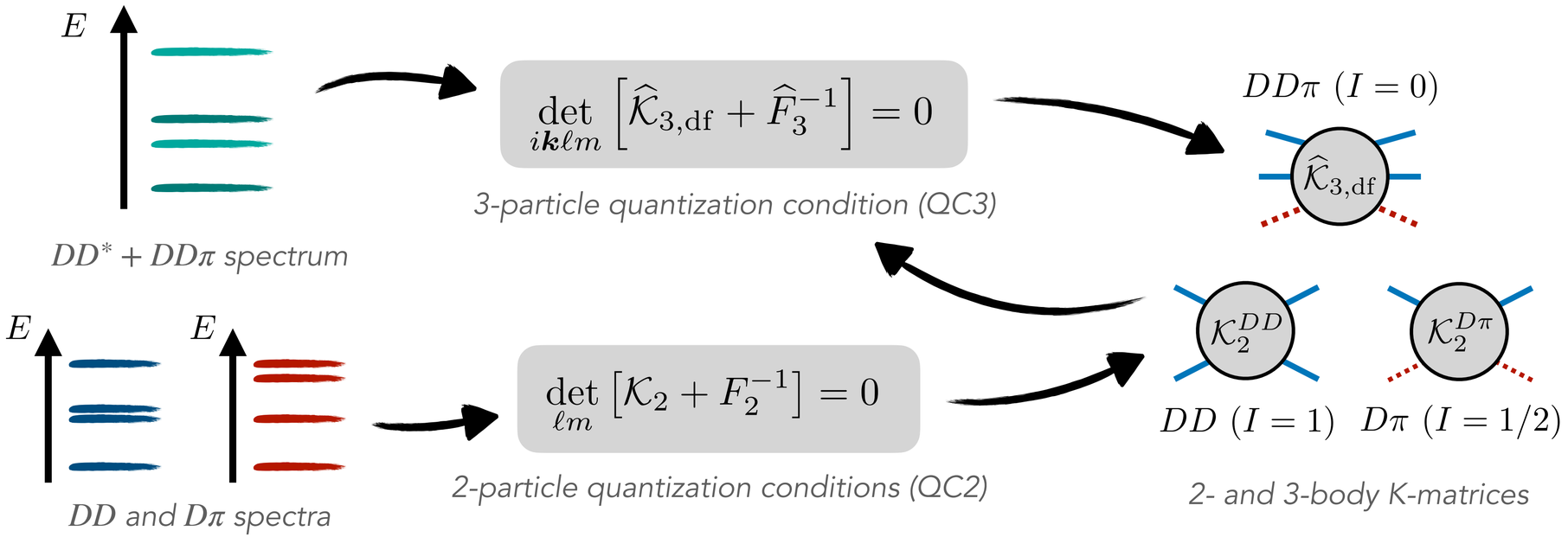






Hansen, Romero-López, Sharpe JHEP 06 (2024)

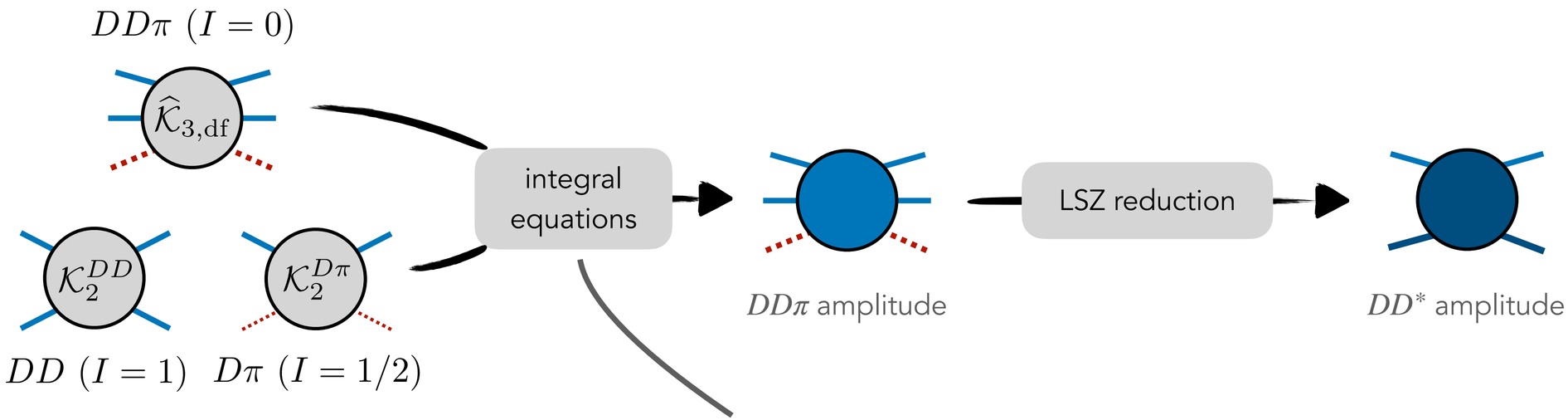
Finite-volume analysis



Simultaneous fits of finite-volume spectra of two-body subsystems and three-body main system allow us to constrain K-matrices

Hansen, Romero-López, Sharpe JHEP 06 (2024)

Connecting to infinite volume



… we feed the K-matrices through **integral equations for $DD\pi$ amplitude**, **LSZ-reduce** and **analytically continue** to obtain DD^* amplitude and find T_{cc}^+ poles.

Hansen, Romero-López, Sharpe JHEP 06 (2024)

Our lattice setup

- We have used two ensembles from the CLS collaboration
- $N_f = 2 + 1$ ensembles with same pion mass $m_\pi \approx 280\,\mathrm{MeV}$ and lattice spacing, but two different volumes
- c valence quark mass tuned so that D meson mass is average of physical D^0, D^+, D_s^+meson masses
- At this pion mass, the D^* **meson is stable**

	L/a	T/a	$m_\pi L$	$m_\pi[\mathrm{MeV}]$	$a[\mathrm{fm}]$
X252	36	128	3.4	280	0.064
X253	40	128	3.7	280	0.064

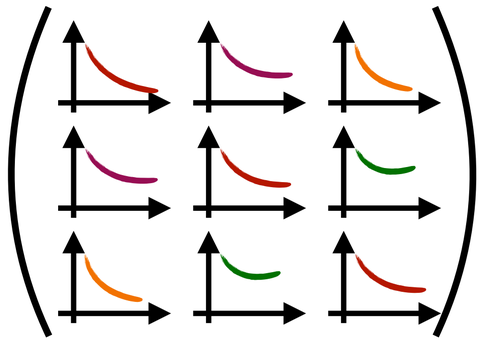
Spectrum extractions

- To extract multihadron energies, we first form large bases of interpolating operators for each system:
  - DD in $I = 1$: $DD\,,\ DD^*\,,\ D^*D^*\,,\ D_0^*D_0^*$
  - $D\pi$ in $I = 1/2$: $D\pi\,,\ D^*\,,\ D^*\pi\,,\ D_0^*$
  - $DD^* + DD\pi$ in $I = 0$: $DD^*\,,\ DD\pi\,,\ D^*D^*\,,\ D_0^*D\,,\ D_0^*D^*$

    \+ local tetraquark operators,

    e.g. $T^i_{DD^*}(t,\boldsymbol{x}) = \left(\bar{u}\gamma^i c\,\bar{d}\gamma^5 c - \bar{d}\gamma^i c\,\bar{u}\gamma^5 c\right)(t,\boldsymbol{x})$

- We build matrices of correlators $C_{ij}(t) = \langle \mathcal{O}_i(t)\,\mathcal{O}_j(0)^\dagger \rangle$ in each frame and finite-volume irrep, from the bases above



$C(t)$

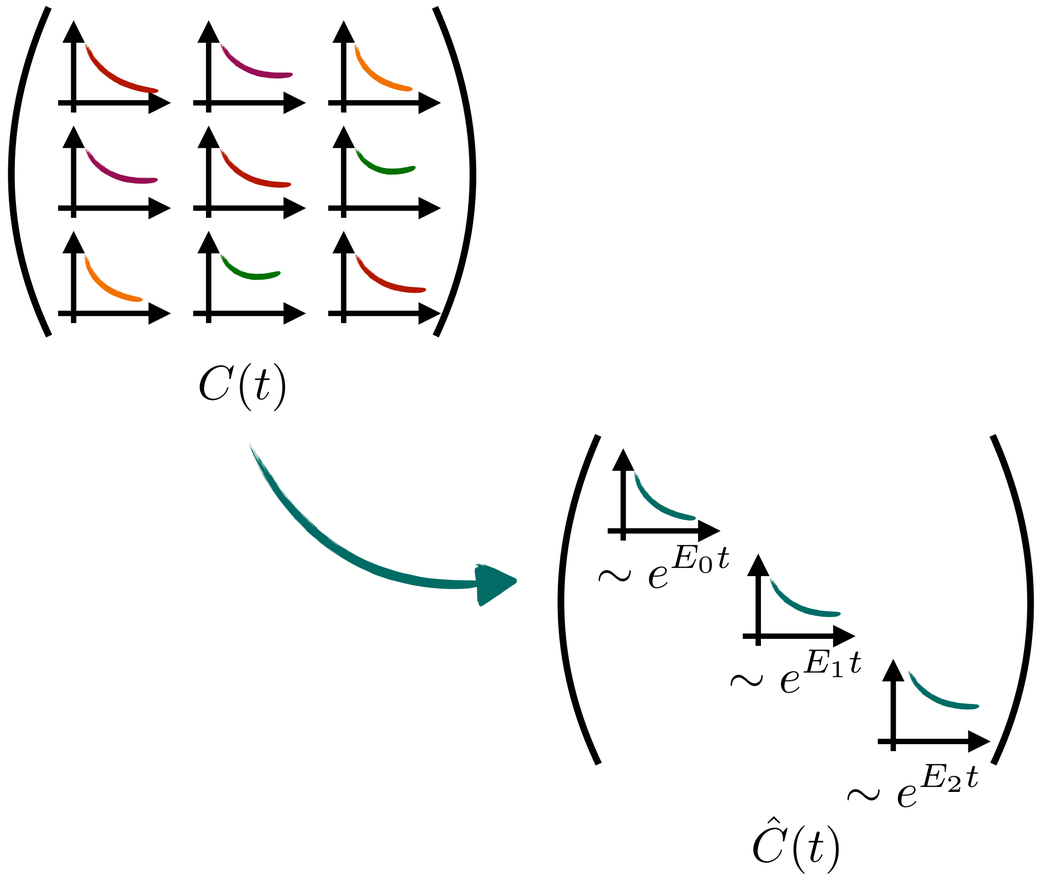
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- We build matrices of correlators $C_{ij}(t) = \langle \mathcal{O}_i(t)\,\mathcal{O}_j(0)^\dagger \rangle$ in each frame and finite-volume irrep, from the bases above
- Solve a generalised eigenvalue problem (GEVP) to diagonalise the matrix — n-th diagonal entry falls off as n-th level $\hat{C}(t)_{nn} \sim e^{-E_n t}$
- We fit ratios of rotated correlators to single-hadron correlators to obtain energy shifts between interacting and noninteracting levels



2-body subsystems: isospin-1 DD

- We fit total of 16 levels, across 4 different moving frames on the two ensembles
- An s-wave Lüscher analysis on DD levels gives a good fit using a 3-parameter effective range expansion:

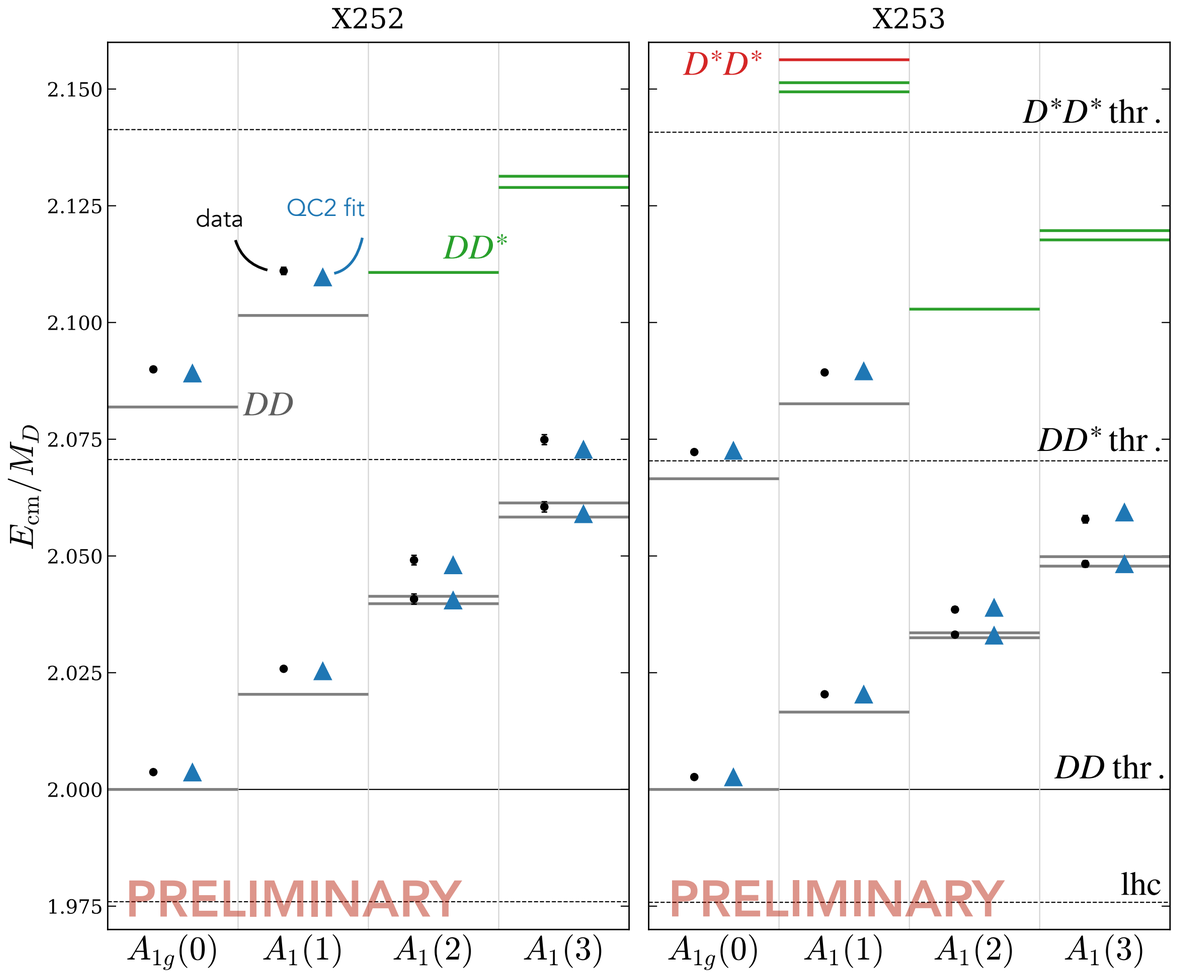
$$\frac{q}{M_D}\cot\delta_{DD,s} = C_1 + C_2\left(\frac{q}{M_D}\right)^2 + C_3\left(\frac{q}{M_D}\right)^4$$

$$C_1 = -0.437(12),\ C_2 = -4.48(85),\ C_3 = -14.2(9.8)$$

$$\chi^2/\mathrm{dof} = 13.02/13$$

- as expected, weakly repulsive interaction

$$a_0 = 2.29(6)/M_D\,, \quad r_0 = 8.9(1.7)M_D$$



Spectra on the two ensembles and best fit result

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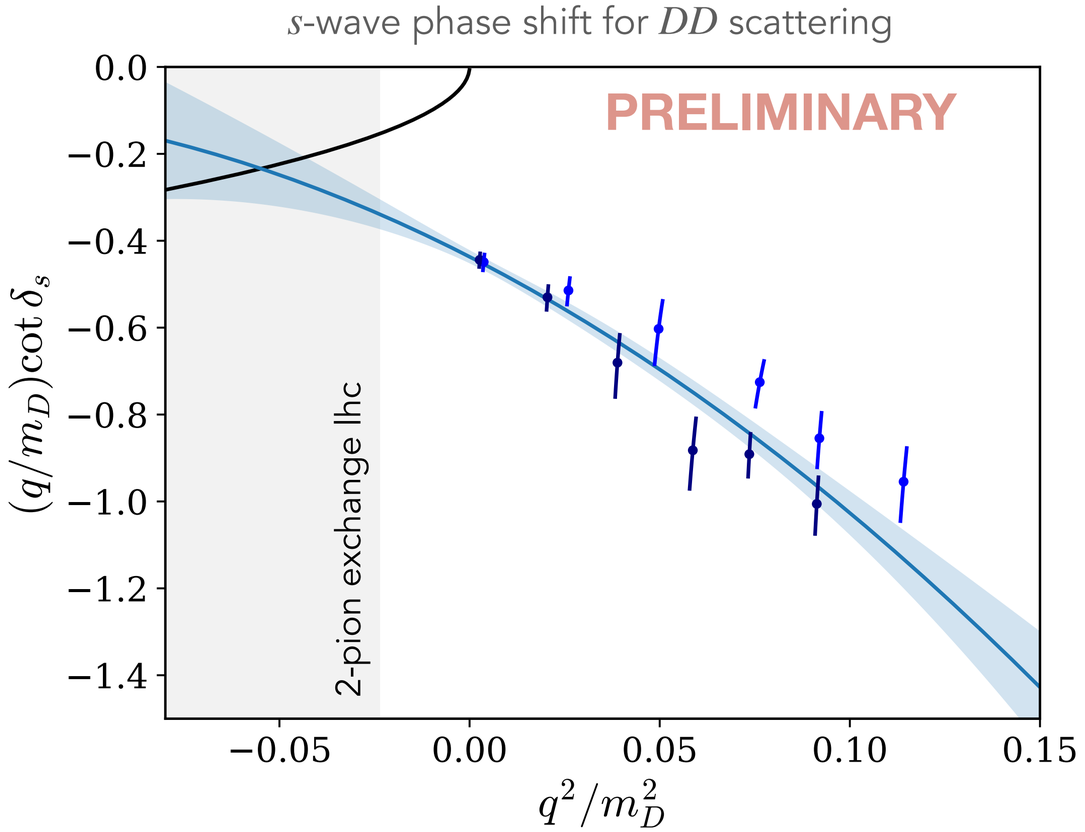
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2-body subsystems: isospin-1/2 $D\pi$

- Lüscher fit to 32 levels across the two ensembles, four different frames, keeping s- and p-waves
- Several parameterisations tested, best descriptions provided by simple 2-parameter modified EREs
- Heavy-quark effective theory $D^*D\pi$ coupling g, and D^* mass used as p-wave fit parameters
- M_{D^*} very well constrained by the data, g very poorly constrained — we used a Bayesian constraint $g = 0.55(10)$ (experimental value ~ 0.57 and lattice-determined values ranging $\sim 0.50 - 0.65$)

s-wave: $$q \cot \delta_{D\pi,s} = \frac{1}{2} r_{0,s} \left(q^2 - q_{0,s}^2\right)$$

p-wave: $$q^3 \cot \delta_{D\pi,p} = \frac{1}{2} r_{0,p} \left(q^2 - q_{0,p}^2\right)$$

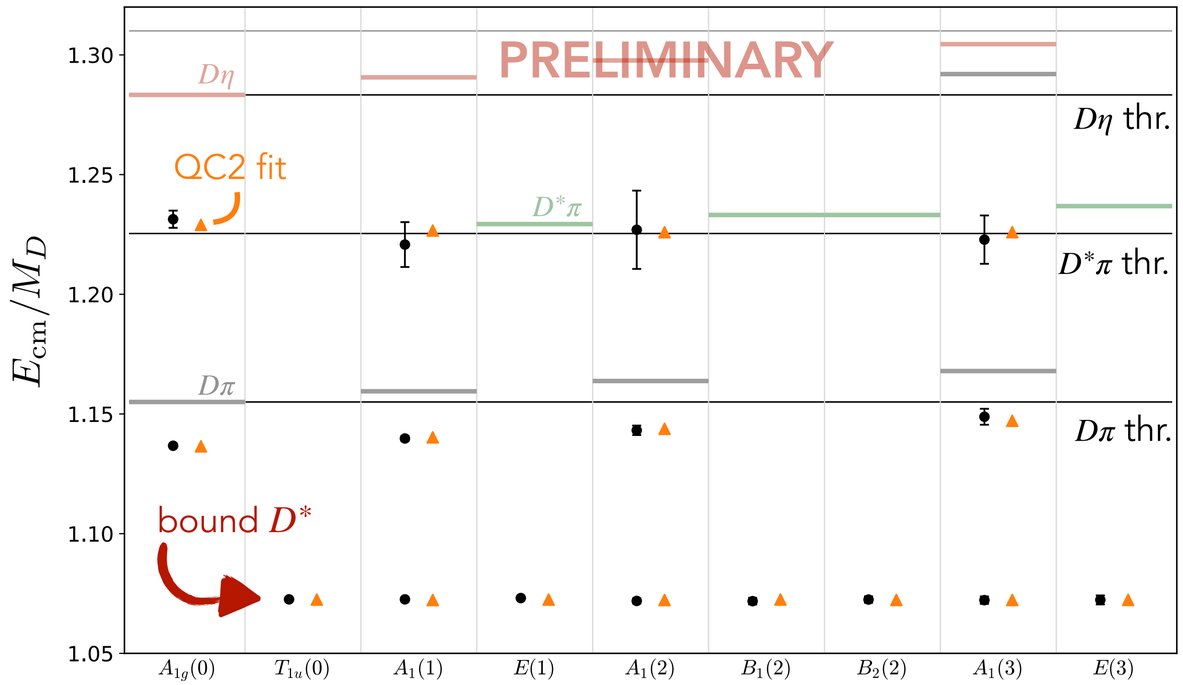
$$q_{0,s}^2/M_D^2 = 0.00801(182)\,,\ r_{0,s}M_D = -7.91(80)$$

$$M_{D^*}/M_D = 1.07041(63)\,,\ gM_D = 0.542(96)$$

$$\chi^2/\text{d.o.f.} = 36.50/29 = 1.26$$

2-body subsystems: isospin-1/2 $D\pi$

spectrum and best fit results on X253 (fit includes the two ensembles)

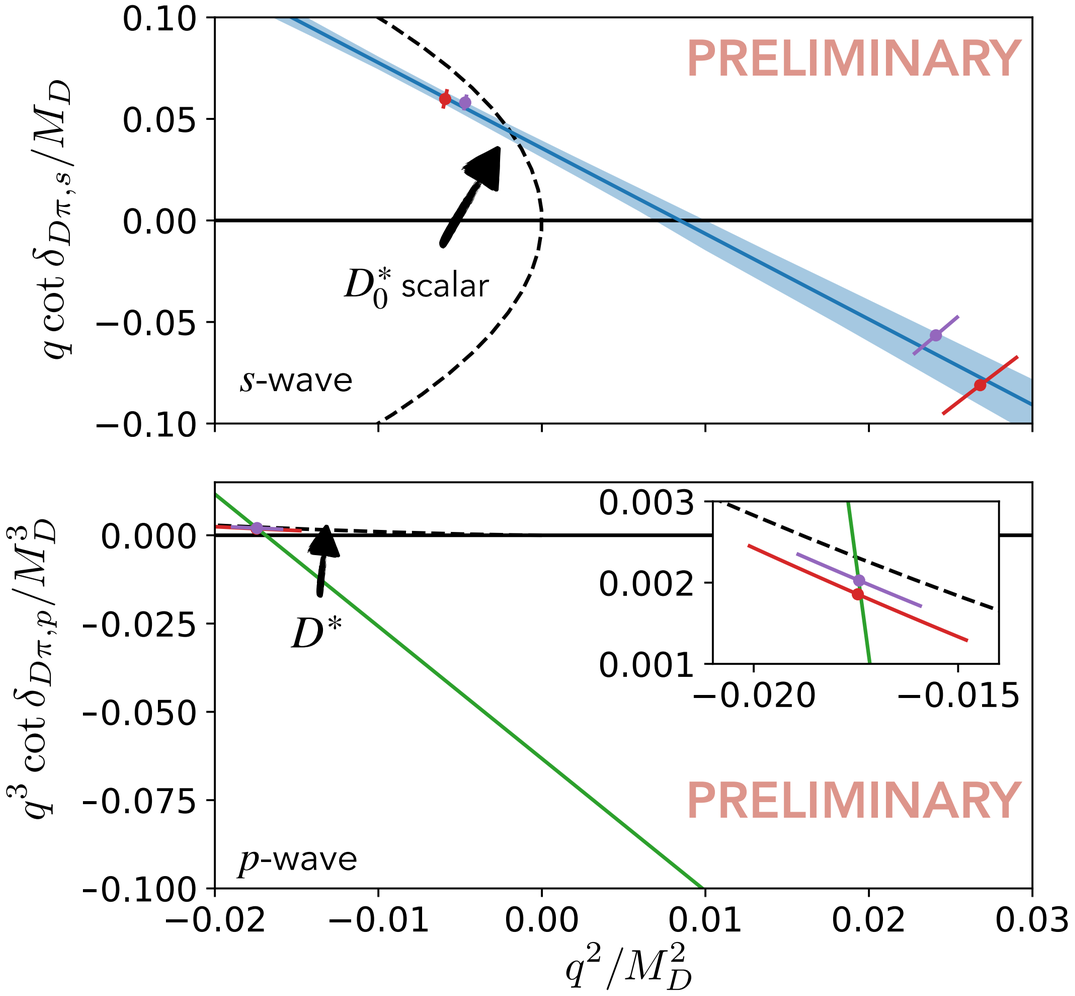


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Phase shifts plots for best fit

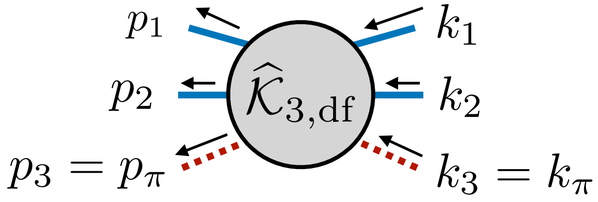


Simultaneous fits of $DD^* + DD\pi$, DD and $D\pi$

- 2-body subsystems produced reasonable results, we moved on to the simultaneous fits of the main system and subsystems
- Several parameterisations for the 3-particle K matrix and different selections of levels explored
- Can obtain satisfactory fits using a minimal 2-parameter description of the K matrix:

$$\mathcal{K}_{3,\mathrm{df}}(\{\boldsymbol{p}\},\{\boldsymbol{k}\}) = \mathcal{K}_3^{\mathrm{iso},0} + \mathcal{K}_3^{E}\frac{(k_\pi - p_\pi)^2}{M^2}$$

$$M = 2M_D + M_\pi$$



PRELIMINARY

Fit	levels	χ^2/dof
X252	31	$34.61/25 = 1.38$
X253	31	$38.64/25 = 1.55$
X252 + X253	62	$88.51/56 = 1.58$

Fits with 16 $D\pi$ + 15 $DD^*/DD\pi$ levels on each ensemble
(DD levels effectively decoupled due to cutoff function used)

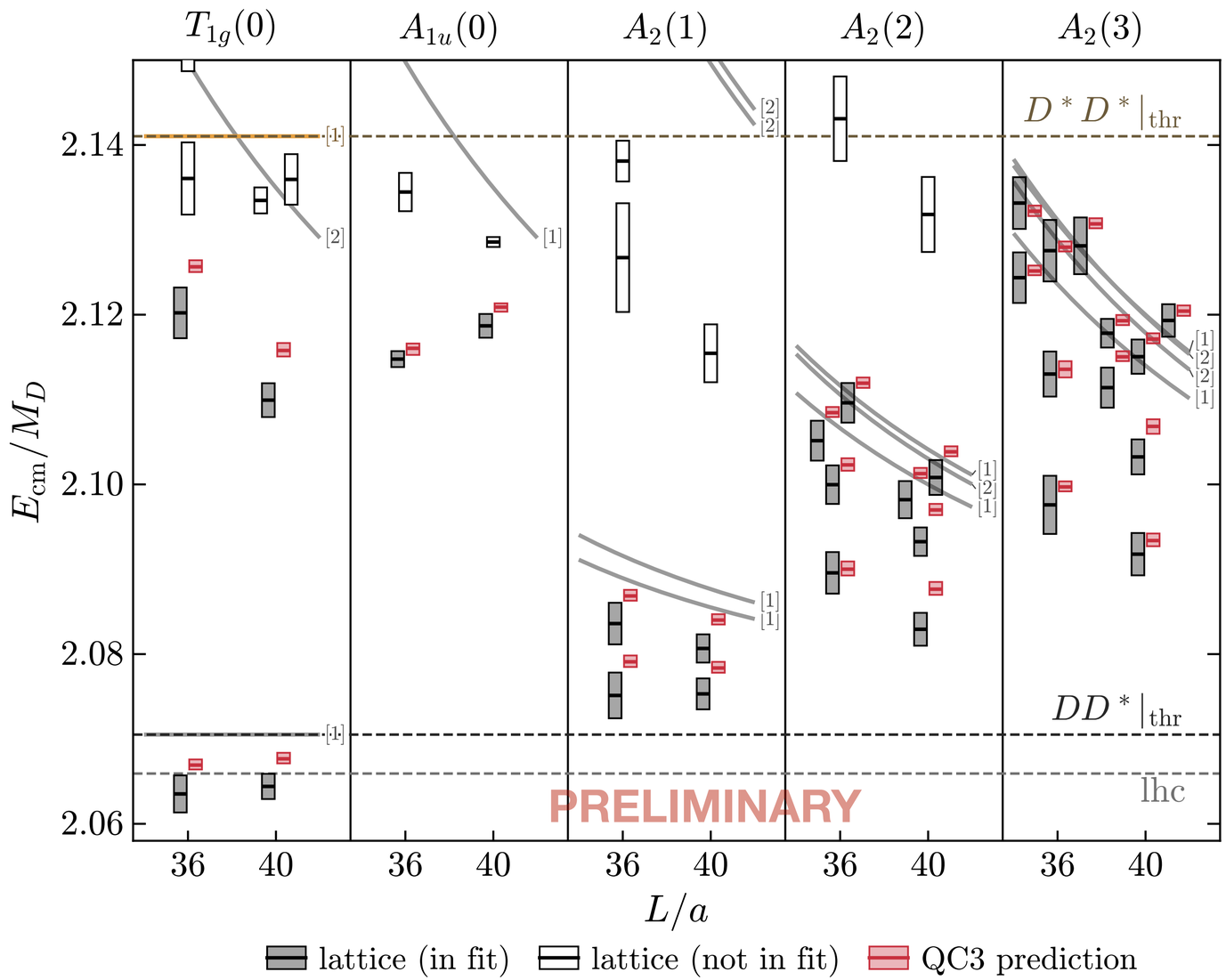
$J(\sigma)$ regulator/cutoff function smoothly turns off 2-body subsystems below their thresholds, all levels in this fit are below where DD turns fully off

Simultaneous fits of $DD^* + DD\pi$, DD and $D\pi$

- Looking at the $DD^* + DD\pi$ spectrum plots and best fit results for minimal combined fit:

$$\chi^2_{D\pi,\,\mathrm{X252}} = 16.30 \quad (16 \text{ levels})$$
$$\chi^2_{D\pi,\,\mathrm{X253}} = 7.14 \quad (16 \text{ levels})$$
$$\chi^2_{DD\pi,\,\mathrm{X252}} = 19.07 \quad (15 \text{ levels})$$
$$\chi^2_{DD\pi,\,\mathrm{X253}} = 22.63 \quad (15 \text{ levels})$$
$$\chi^2_{\mathrm{total}} = 88.51 \quad (62 \text{ levels})$$

- Good overall description of the spectrum, across both ensembles and finite-volume irreps
- Significant contributions of correlations to final χ^2
- Should we expect $\chi^2/\mathrm{dof} \approx 1$? Potentially neglected discretisation effects, finite-volume effects from small boxes, model choices?



The DD^* amplitude

- With results for the 2- and 3-particle K matrices in hand, we run the integral equation machinery to get the DD^* amplitude
- $J^P = 1^+$ has two coupled partial waves, ${}^3S_1 - {}^3D_1$, we use the Blatt-Biedenharn representation of the amplitude

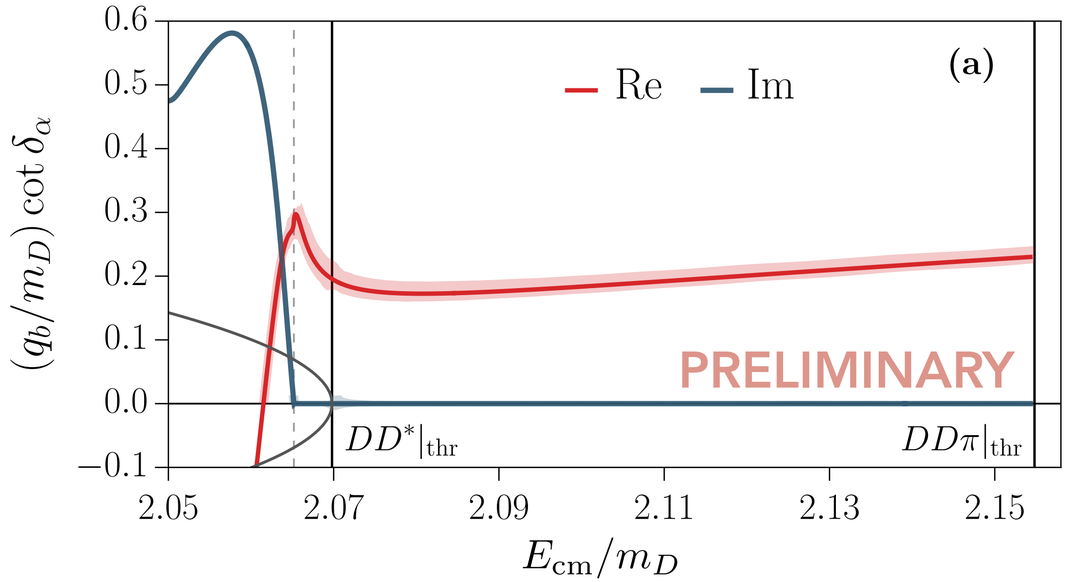
$$\mathcal{M}^{-1}_{DD^*} = \mathcal{K}^{-1}_{DD^*} - \frac{iq_b}{8\pi E_{\mathrm{cm}}}$$

$$(8\pi E_{\mathrm{cm}})\left[\mathcal{K}^{-1}_{DD^*}\right] =$$

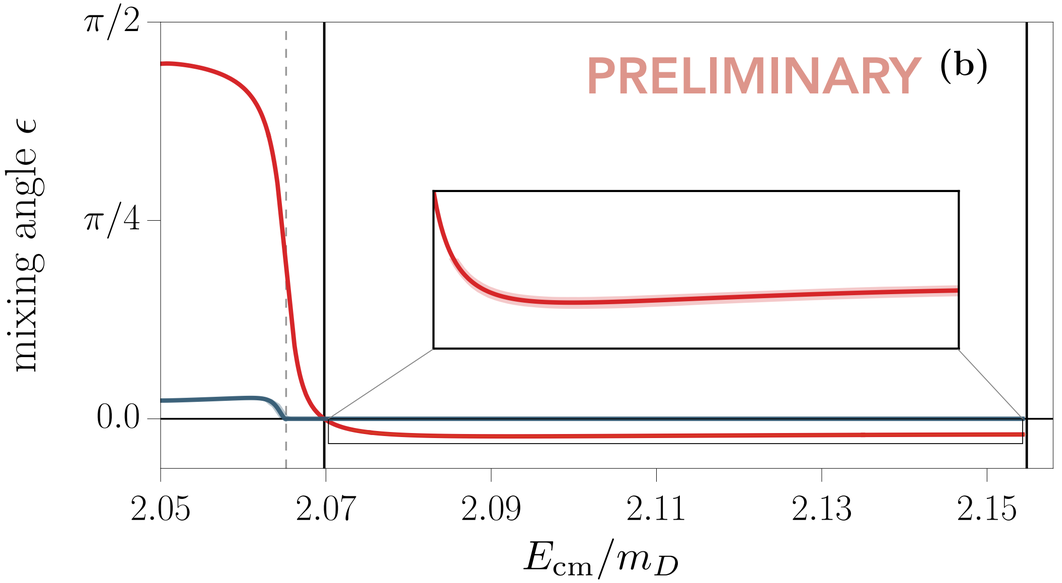
$$\begin{pmatrix} \cos\epsilon & -\sin\epsilon \\ \sin\epsilon & \cos\epsilon \end{pmatrix} \begin{pmatrix} q_b \cot\delta_\alpha & 0 \\ 0 & q_b \cot\delta_\beta \end{pmatrix} \begin{pmatrix} \cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon \end{pmatrix}$$

α phase shift; mixing angle; β phase shift

α-wave phase shift



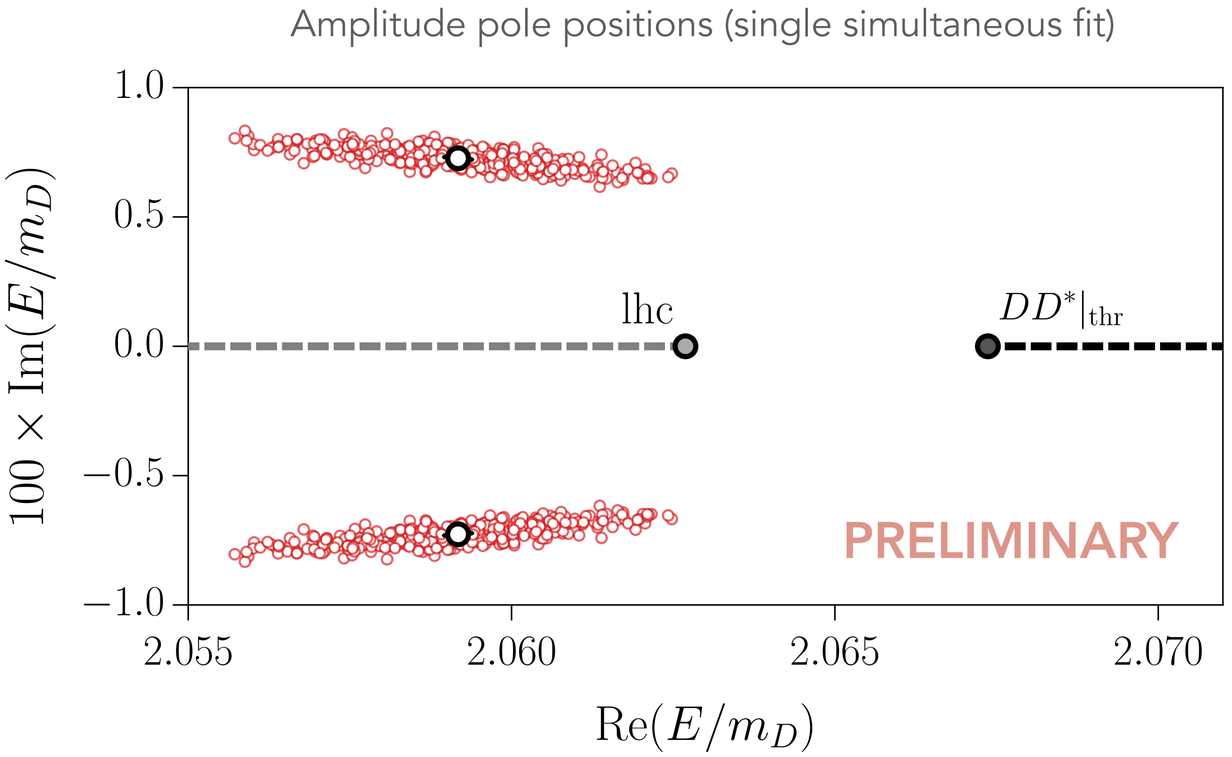
mixing angle



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- $J^P = 1^+$ has two coupled partial waves, ${}^3S_1 - {}^3D_1$, we use the Blatt-Biedenharn representation of the amplitude
- Analytically continue the amplitude into second sheet to find pole positions

… we find a **complex-conjugate pair of poles**, with real parts **below DD^* threshold and left-hand cut branch point**



Results in this and previous slide are for a single simultaneous fit, we are testing stability under parameterisations and other choices!

Robustness of results and systematics

- To test the robustness of the fit results and amplitude determinations, we explored:
  - several parameterisations for the 3-particle K matrix
  - different choices in cutoff functions
  - fit level selection (stability under inclusion/exclusion of higher-lying levels)
  - inclusion of d-waves in $D\pi$ subsystem

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  - several parameterisations for the 3-particle K matrix
  - different parameter choices in cutoff functions
  - fit level selection (stability under inclusion/exclusion of higher-lying levels)
  - inclusion of d-waves in $D\pi$ subsystem
- Fit parameters remain generally stable across all choices above…
  - physical parameters like $D^*D\pi$ coupling and other 2-body parameters show little variation
  - three-particle K matrix parameters, which are scheme-dependent, generally stable but vary to compensate changes in the cutoff
  - fit quality deteriorates when including further higher levels, more complicated parameterisation of 3-body K matrix preferred
- Propagation through integral equations to test stability of pole determination ongoing

Work still exploratory — more sophisticated model-averaging (e.g. AIC) in the future?

Comparison to other methods

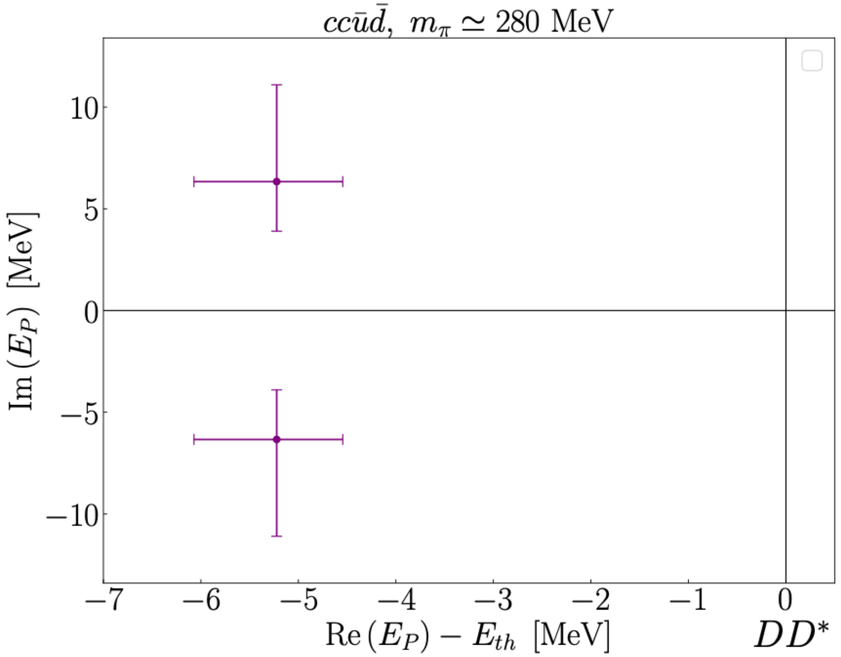
- At our value light-quark masses, the D^* is stable and comparison to two-body methods that incorporate left-hand cut effects is possible

- Several applications to T_{cc}^+ already exist, e.g.

Prelovsek, Ortiz-Pacheco et al. PRD 112 (2025) — plane wave approach
Pitanga Lachini, Briceño 2607.19009 (2026) — relativistic lhc approach

- Their findings seem to qualitatively agree with our result from the three-body approach, and disagree with standard Lüscher

  … complex-conjugate poles in the second sheet, with real part below threshold and below the top of the left-hand cut



Prelovsek, Ortiz-Pacheco et al. PRD 112 (2025)

Comparison to other methods

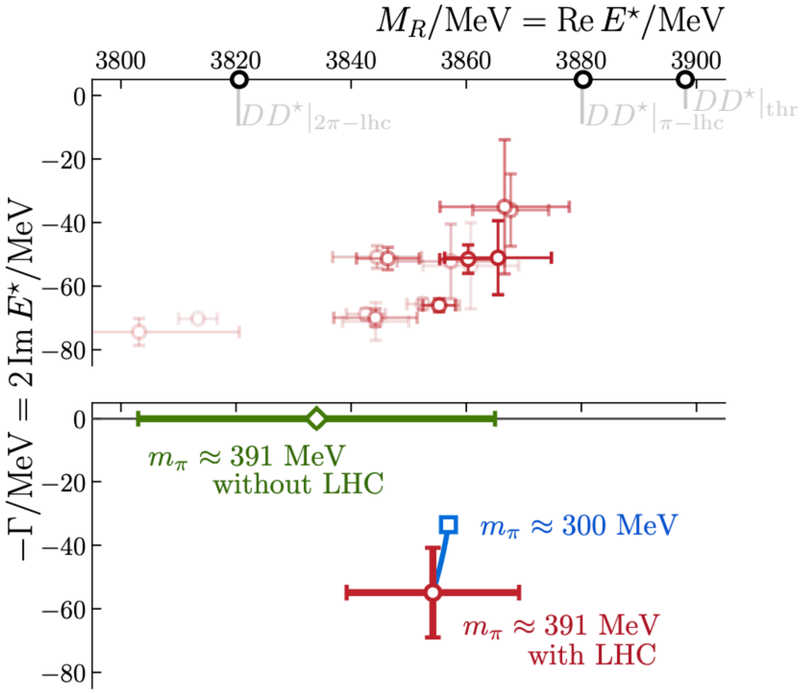
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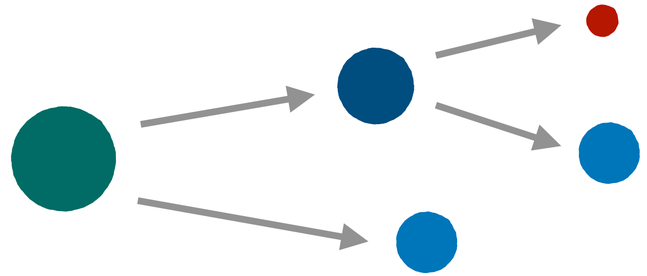
  … complex-conjugate poles in the second sheet, with real part below threshold and below the top of the left-hand cut



Original data from *Whyte, Wilson, Thomas (HadSpec) PRD 111 (2025)*, reanalysed in Pitanga Lachini, Briceño (Hadspec) arXiv:2607.19009 (2026)

Summary and Outlook

- I have shown preliminary results for the first lattice study of the T_{cc}^{+} via 3-body finite-volume formalism — it avoids the left-hand cut issue from the 2-body approach and provides a unified framework down to the physical point
- We used two lattices at $m_\pi \approx 280$ MeV, with same spacing but different volumes, and performed simultaneous fits of finite-volume spectrum of the main $DD^* + DD\pi$ system and its two-particle subsystems DD and $D\pi$
- We obtained the DD^* amplitude and T_{cc}^{+} pole position
- Paper with this analysis will be out in the next few months (hopefully!)
- **Next steps:** lighter quark masses, nearer to where the D^* becomes unstable and only the three-body treatment is valid



Thank you for your attention!

Back-up slides…

Scattering amplitudes: quick recap

- Hadron-hadron scattering amplitudes are the mathematical objects we seek to determine from lattice QCD

- Non-trivial analytic structure in complex energy plane from unitarity and analyticity, e.g. for a single channel the **partial-wave amplitudes** take the form

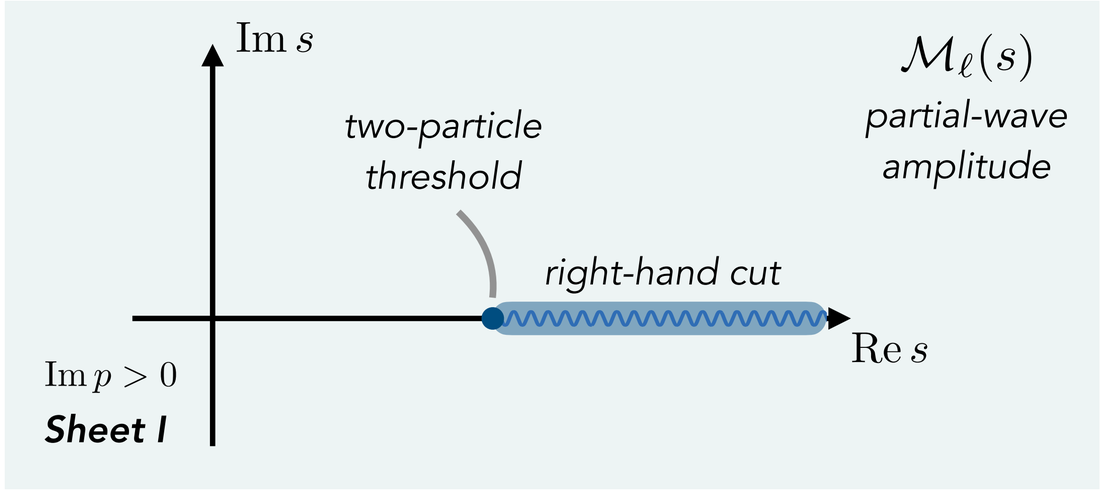
$$\mathcal{M}_\ell(s) = \frac{8\pi\sqrt{s}}{p\cot\delta_\ell - ip}$$

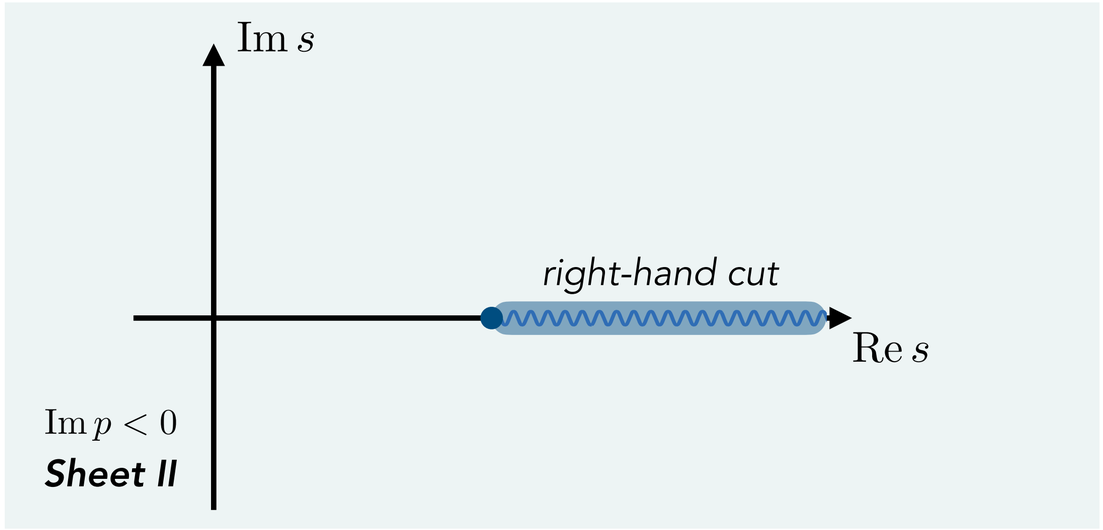
$s = E_{\mathrm{cm}}^2$ — *usual Mandelstam invariant*

δ_ℓ — *phase shift*

$p = \frac{1}{2}\sqrt{s - 4M^2}$ — *cm scattering momentum (identical particles)*

- Square root branch cut, the **right-hand cut**, leads to two Riemann sheets, distinguished by the sign of $\mathrm{Im}\, p$





Scattering amplitudes: quick recap

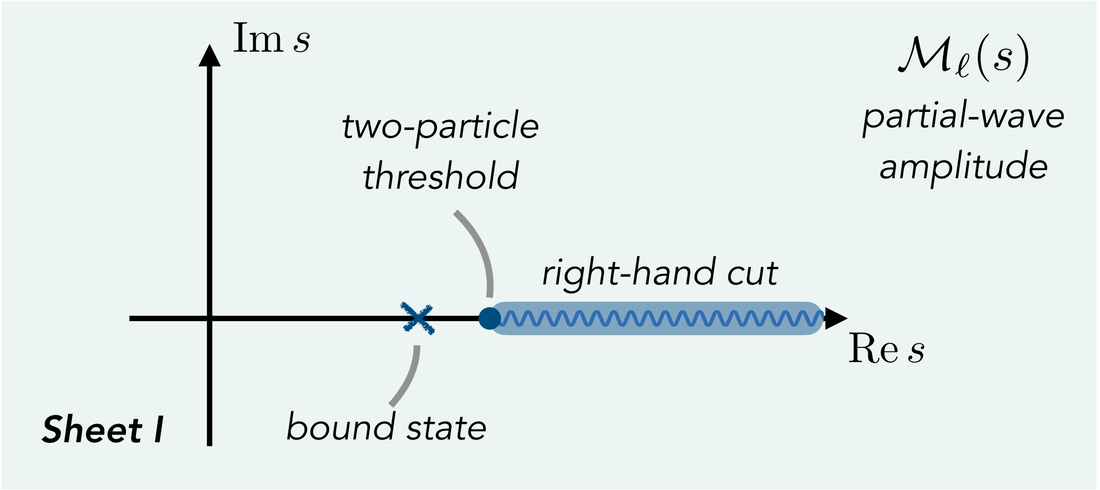
- Near-threshold behaviour of the amplitude tells us a lot about the underlying interaction, *e.g. scattering length, effective range, …*

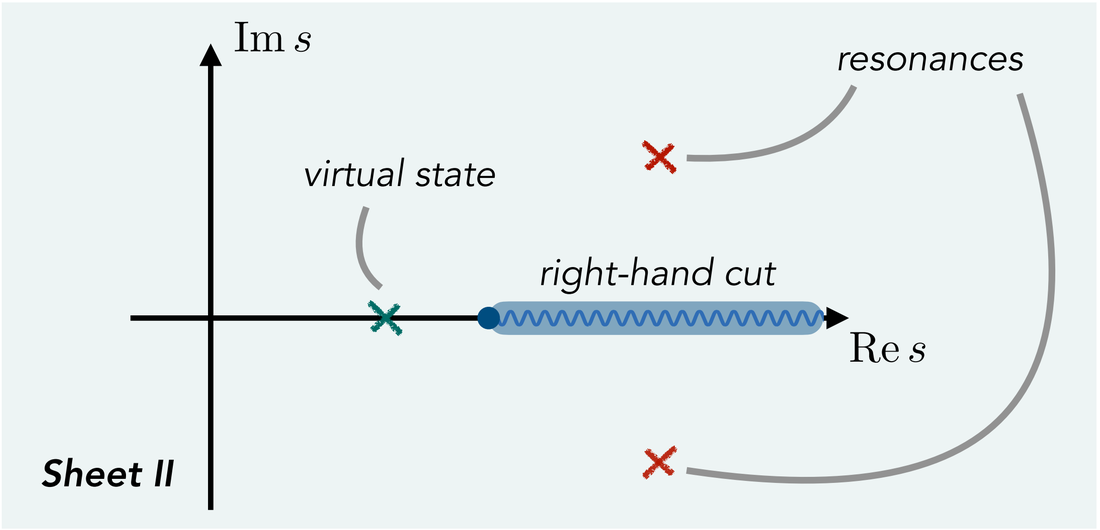
- Amplitude poles provide a rigorous definition of hadronic states:

  - poles on the real line below threshold correspond to bound states (Sheet I) or virtual bound states (Sheet II)

  - resonances correspond to complex-conjugate pairs of poles in Sheet II

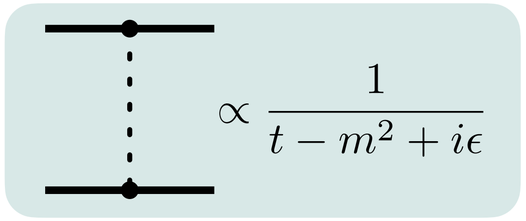
Not the full story… crossing symmetry implies more analytic structure emerges from cross-channel processes (t- and u -channels)





The left-hand cut

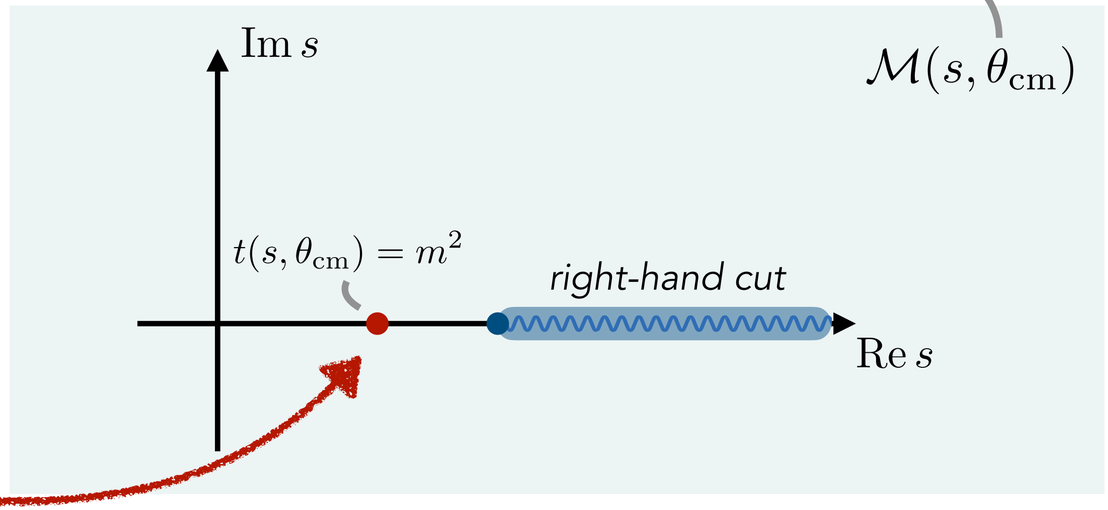
- What we call **"left-hand cut"** arises from partial-wave projecting single-particle exchanges
- Looking at a simple case of scalar scatterers of mass M exchanging a light scalar of mass m



$$\propto \frac{1}{t - m^2 + i\epsilon}$$

fixed cm scattering angle

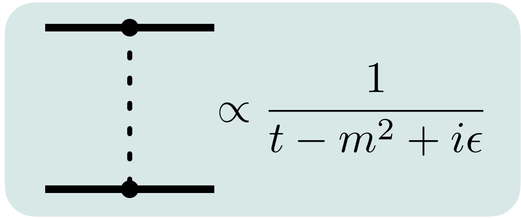
$\mathcal{M}(s, \theta_{\rm cm})$



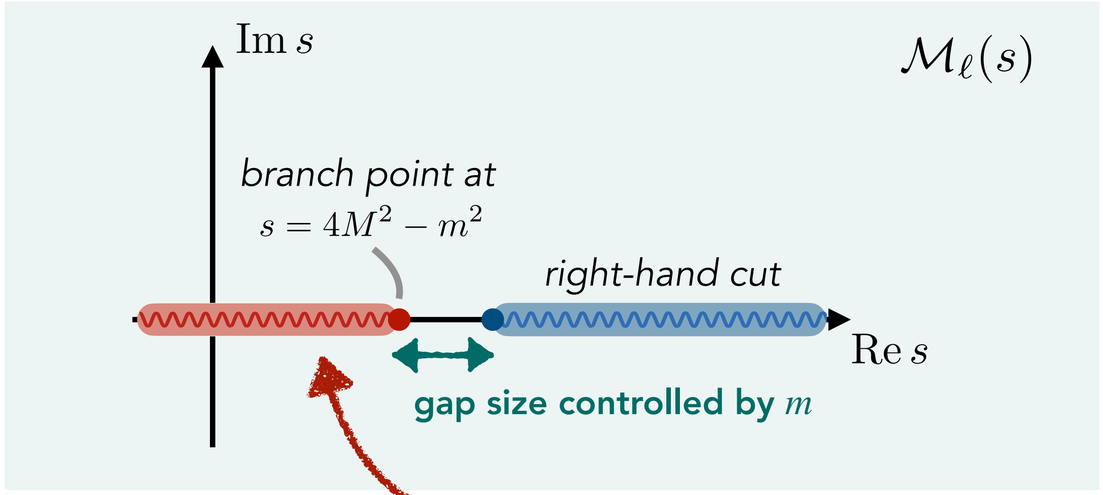
$$t(s, \theta_{\rm cm}) = -\frac{1}{2}(s - 4M^2)(1 - \cos\theta_{\rm cm})$$

The left-hand cut

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$$\propto \frac{1}{t - m^2 + i\epsilon}$$



- Partial-wave projection via…

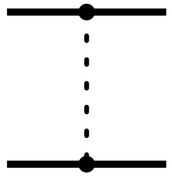
$$\mathcal{M}(s, \theta_{\rm cm}) \longrightarrow \mathcal{M}_\ell(s) = \frac{1}{2} \int_{-1}^{+1} \mathrm{d}\cos\theta_{\rm cm}\, P_\ell(\cos\theta_{\rm cm})\, \mathcal{M}(s, \theta_{\rm cm})$$

cm scattering angle — *Legendre polynomial*

… turns these simple poles into logarithms,

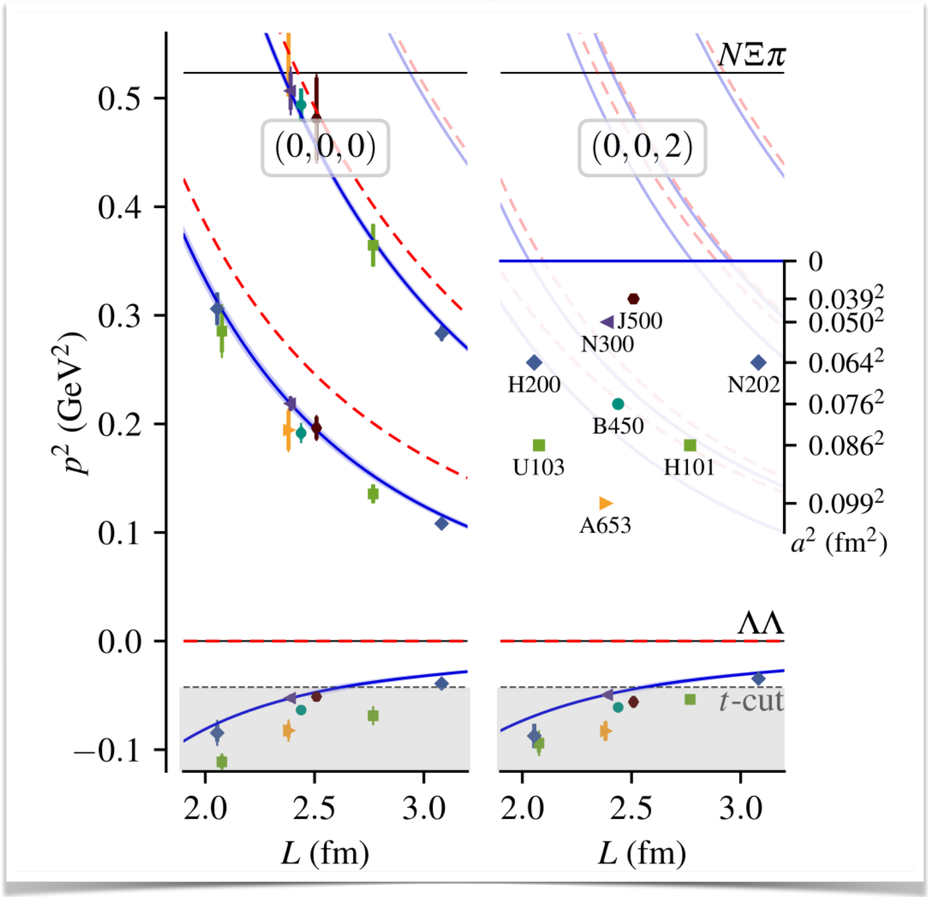
e.g. for $\ell = 0$:

$$\int_{-1}^{+1} \mathrm{d}\cos\theta_{\rm cm}$$



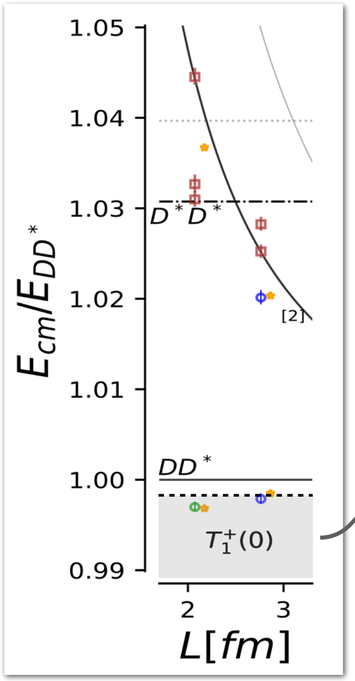
$$\propto \frac{1}{s - 4M^2} \log\left[\frac{s - (4M^2 - m^2)}{m^2} - i\epsilon\right]$$

The early warnings...



Study of the H dibaryon at the SU(3) flavor-symmetric point

Green, Hanlon, Junnarkar, Wittig PRL 127 (2021), arXiv 2103.01054



Spectrum for DD^* system in $T_{1g}(0)$ irrep

adapted from

Padmanath, Prelovsek PRL 129 (2022)

arXiv 2202.10110

left-hand cut

Role of left-hand cut contributions on pole extractions from lattice data: Case study for $T_{cc}(3875)^+$

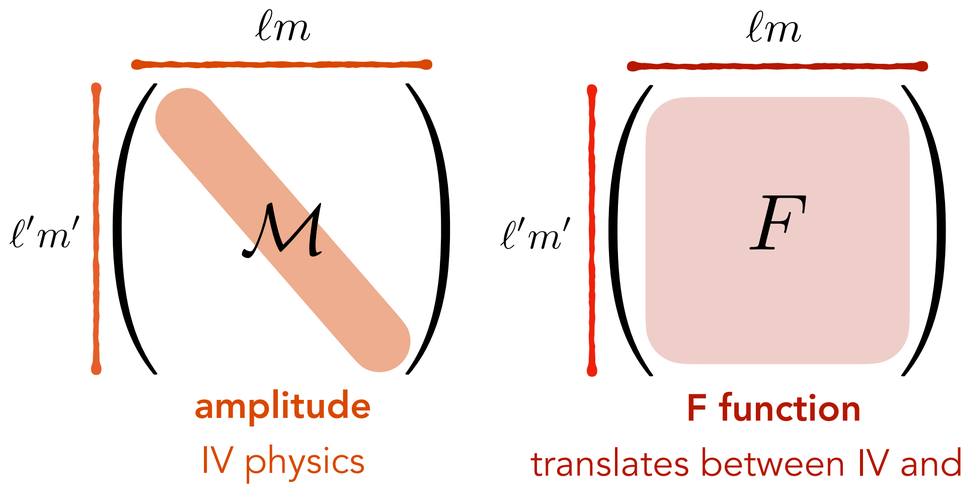
Meng-Lin Du,[1] Arseniy Filin,[2] Vadim Baru,[2] Xiang-Kun Dong,[3,4] Evgeny Epelbaum,[2] Feng-Kun Guo,[3,4,5] Christoph Hanhart,[6] Alexey Nefediev,[7,8] Juan Nieves,[9] and Qian Wang[10,11,12]

We discuss recent lattice data for the $T_{cc}(3875)^+$ state to stress, for the first time, a potentially strong impact of left-hand cuts from the one-pion exchange on the pole extraction for near-threshold exotic states. In particular, if the left-hand cut is located close to the two-particle threshold, which happens naturally in the DD^* system for the pion mass exceeding its physical value, the effective-range expansion is valid only in a very limited energy range up to the cut and as such is of little use to reliably extract the poles. Then, an accurate extraction of the pole locations requires the

PRL 131 (2023), arXiv 2303.09441

The 2-body Lüscher condition

$$\det_{\ell m}\left[\mathcal{M}(E_{\rm cm})^{-1} + F(E, \boldsymbol{P}, L)\right] = 0$$



amplitude
IV physics

F function
translates between IV and FV

building blocks are matrices in angular momentum ℓm index space
(plus channel and spin indices in more complicated systems)



2-body finite-volume spectrum

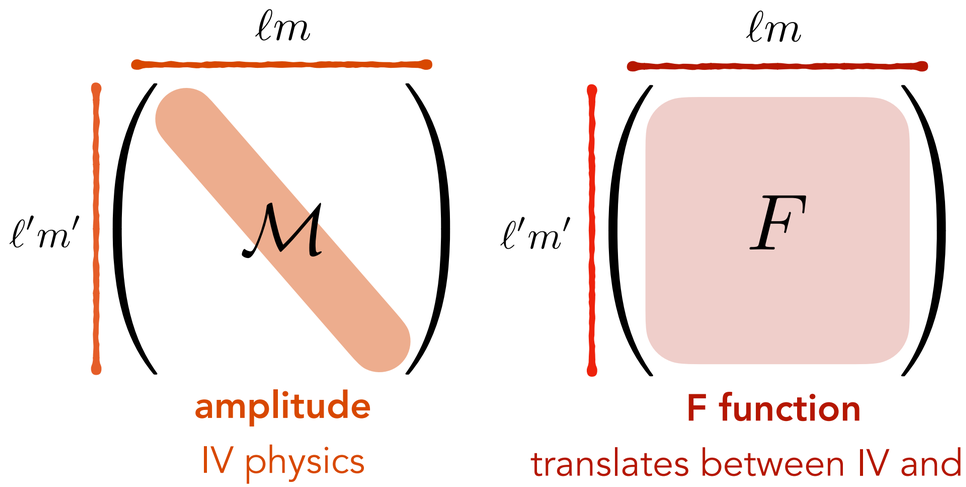
amplitudes, phase shifts, resonance poles

…in practice:

- truncate in ℓ — lowest waves dominate near threshold
- parameterise $\mathscr{M}_\ell$ via a K matrix or phase shift.
  (e.g. an effective range expansion)
- fit the finite-volume levels

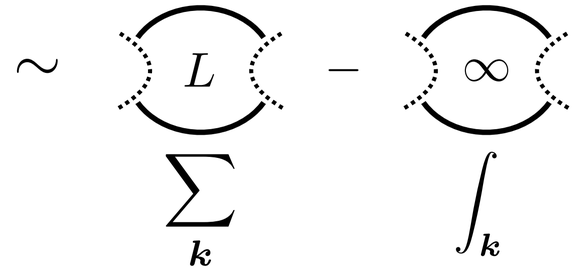
Scattering from lattice QCD: the 2-body pipeline

$$\det_{\ell m}\left[\mathcal{M}(E_{\mathrm{cm}})^{-1} + F(E, \boldsymbol{P}, L)\right] = 0$$

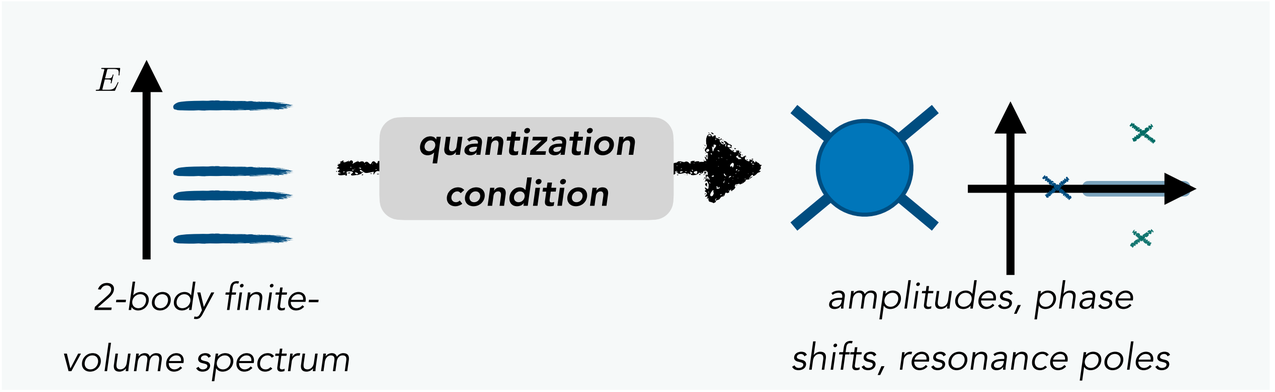


amplitude
IV physics

F function
translates between IV and FV



difference between 2-particle bubble diagram in finite and infinite volume



building blocks are matrices in angular momentum ℓm index space
(plus channel and spin indices in more complicated systems)