From Anomaly to Agreement: Did we Close the Door on New Physics?

Z. Fodor: Budapest-Marseille-Wuppertal (BMW) Collaboration

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(1) Nature 653 (2026) 8114, 373 (2021)
(2) Nature 593 (2021) 7857, 51 (2026)

Universita di Milano-Bicocca, November 2, 2026



Muon magnet: Worldwide 'Largest' Scientific Prize April 18, 2026

In the tradition of Alfred Nobel: Breakthrough Prize or Fundamental Physics

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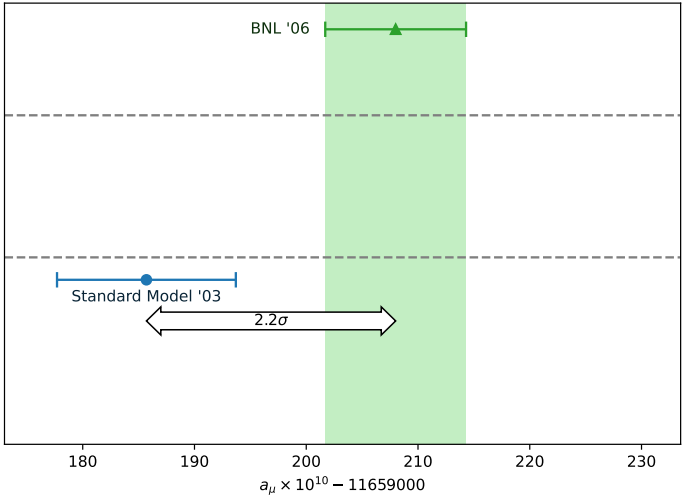
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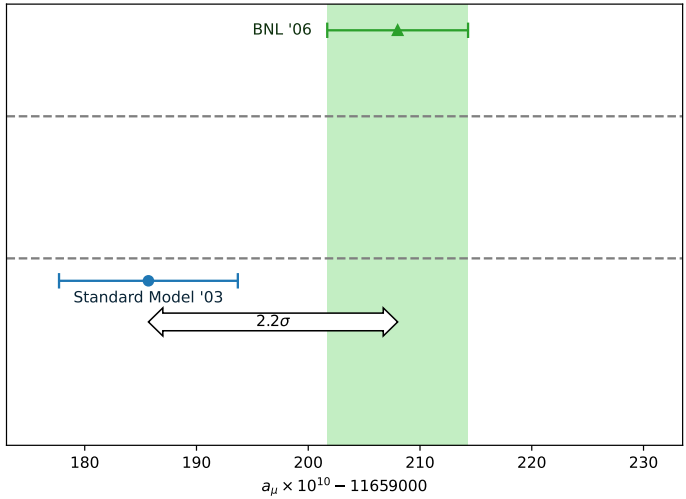
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Relevant finding: decisively relies on the theory results (unfortunately, no prize)

Take-home message: magnetic moment of the muon 2006

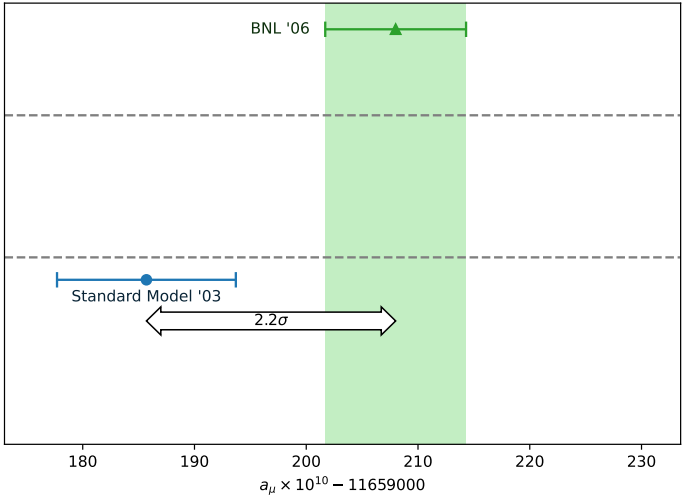


Take-home message: magnetic moment of the muon 2006



New physics needs $> 5\sigma$, new level of accuracies: both for theory/experiment

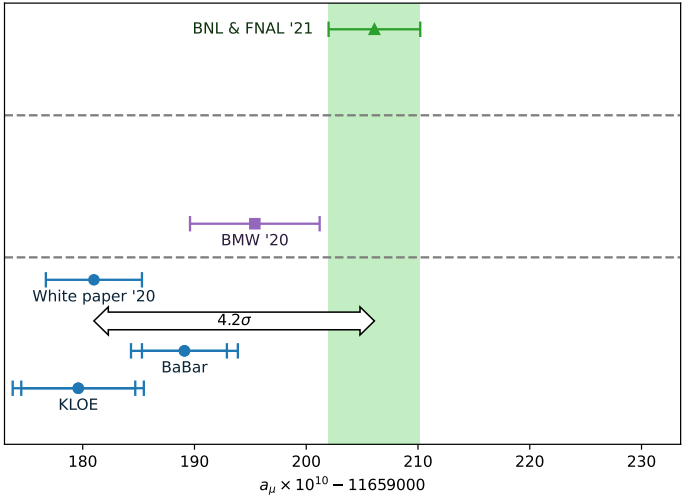
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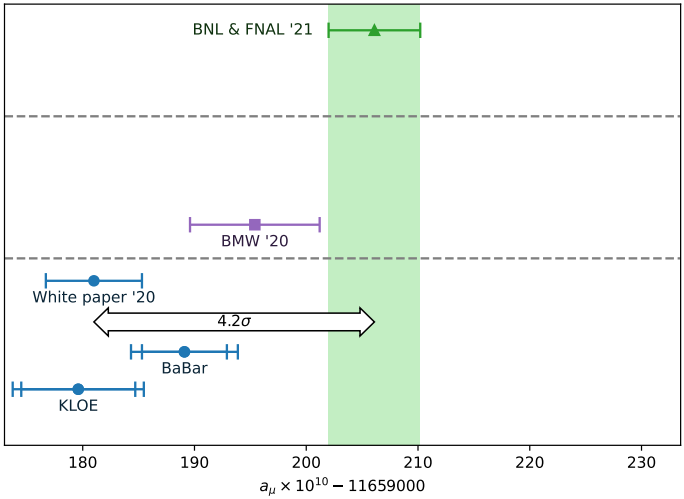
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Theory needs $e^+e^- \to$ *hadrons* data, experimental errors of them give theory error

Take-home message: magnetic moment of the muon 2021

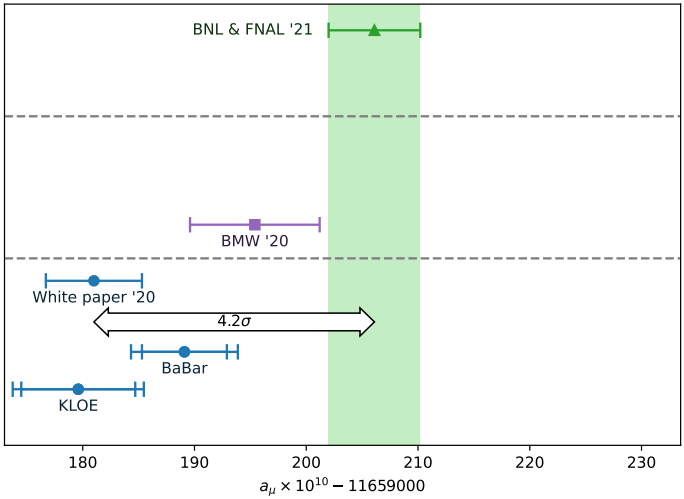


Take-home message: magnetic moment of the muon 2021



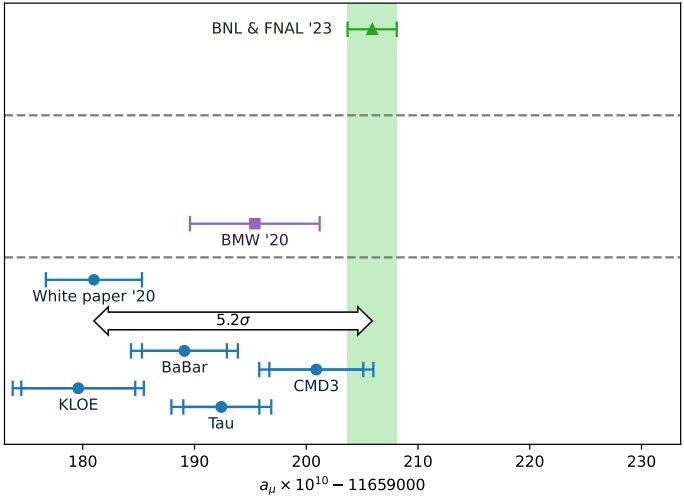
WP: $e^+e^- \to had$. R-ratio, data driven, dispersion, theory initiative, theory consensus

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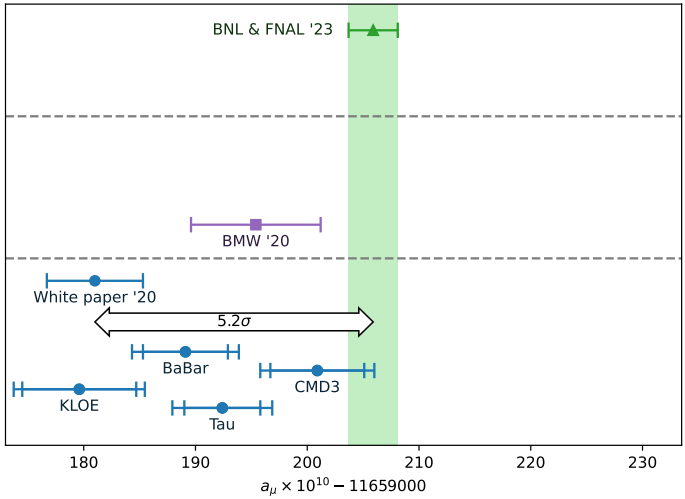


WP: $e^+e^- \to had$. R-ratio, data driven, dispersion, theory initiative, theory consensus
Warning signs: BMW vs. Exp. 1.5σ agreement & tensions in the e^+e^- data (inflation)

Take-home message: magnetic moment of the muon 2023

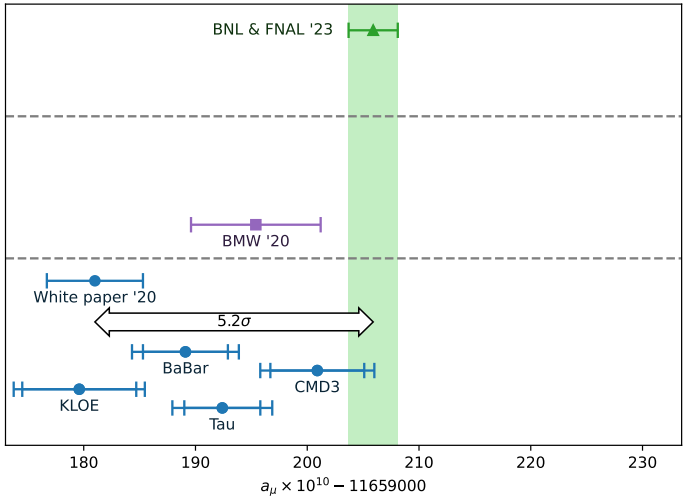


Take-home message: magnetic moment of the muon 2023



White Paper'20: theory error is the decisive factor

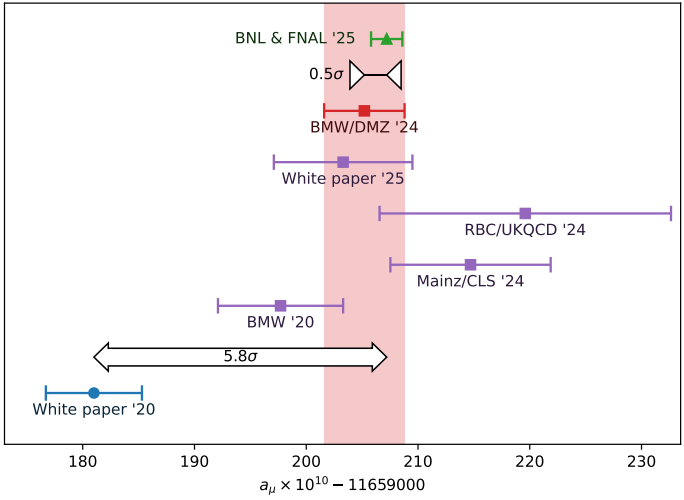
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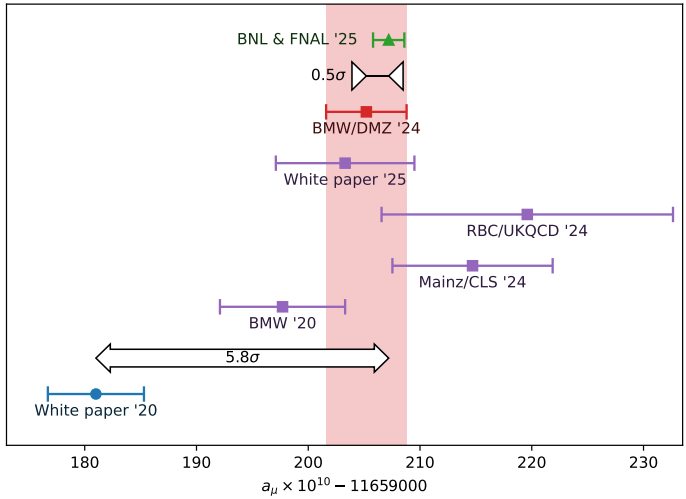
White Paper'20: theory error is the decisive factor

Although 5.2σ discrepancy with WP no announcement (further warning sign: CMD-3

Take-home message: 2025/26

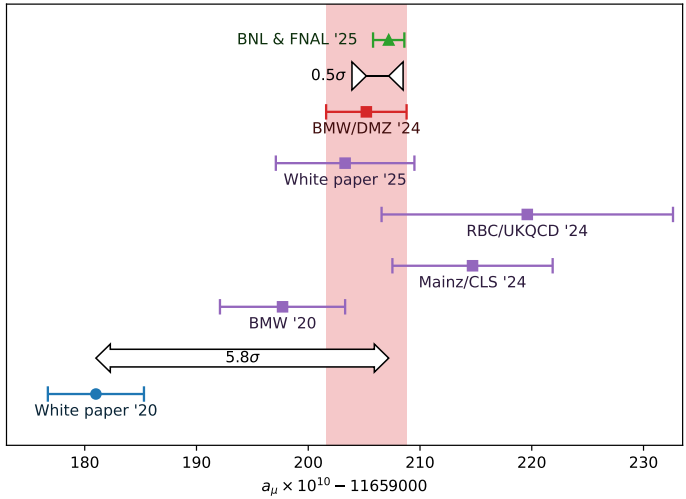


Take-home message: 2025/26



Tiny experimental error $\Rightarrow 5.8\sigma$ discrepancy with WP'20

Take-home message: 2025/26



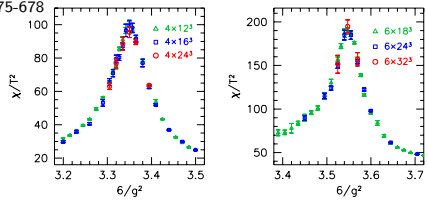
Tiny experimental error $\Rightarrow 5.8\sigma$ discrepancy with WP'20

BMW/DMZ complete agreement, also other lattice results (WP'25 lattice averages)

Lattice QCD: examples

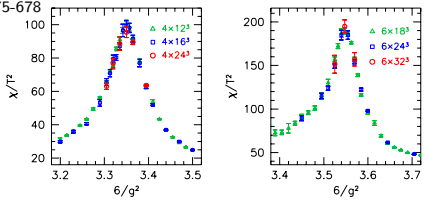
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  The order of the quantum chromodynamics transition predicted by the standard model of particle physics, Nature 443 (2006) 675-678

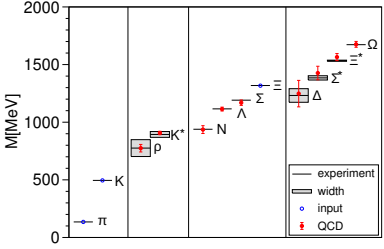


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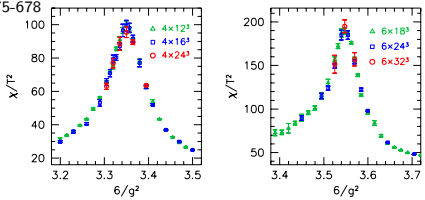


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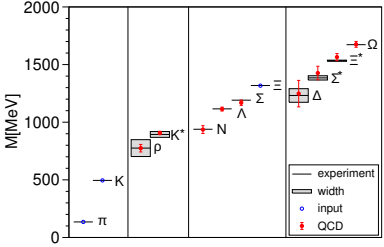


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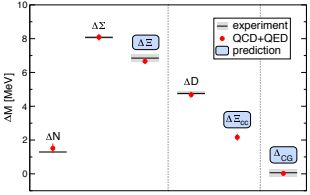
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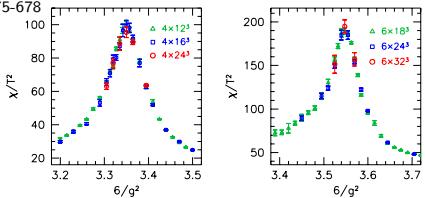


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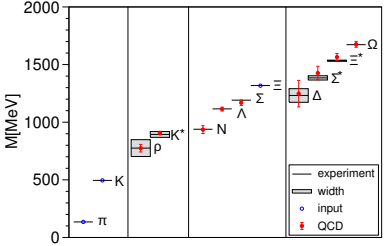


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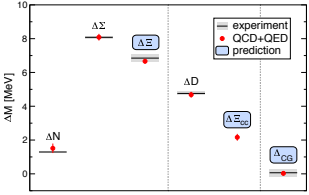
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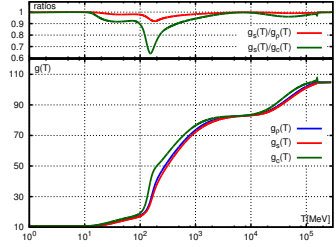
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Outline and Inputs

- Experiment
  - Input: Brookhaven & Fermilab complete agreement
  - Statistics dominated total error
  - Final error of Fermilab has been reached
- Data driven method
  - Input: hadronic e^+e^- R-ratios: thousands of data points ($>$4549)
  - Systematics dominated error (4-times larger than statistical)
- Lattice
  - Input: Lagrangian with 4 quark masses (pion, kaon, Omega, D_s), α
  - Systematic uncertainties are about the same as statistical ones

History of the $(g-2)_\mu$ experiments

Experiment	Jahr	$a_\mu \times 10^{10}$
Columbia	1957	10% on g_μ
CERN I	1961	11 450 000(220 000)
CERN II	1962-1968	11 661 600(3100)
CERN III	1974-1976	11 659 100(110)
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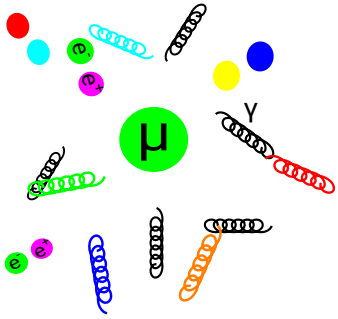
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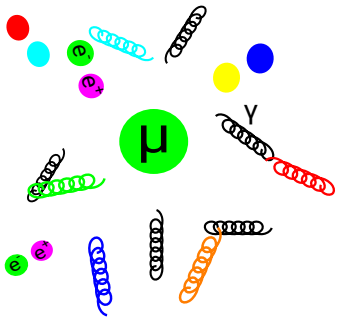
$\Longrightarrow$ Bring the 14 meter diameter magnet ring to Fermilab

Theory: Standard Model



Virtual particle contributions:

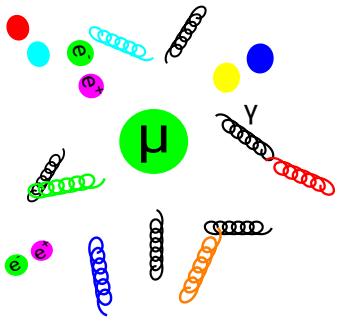
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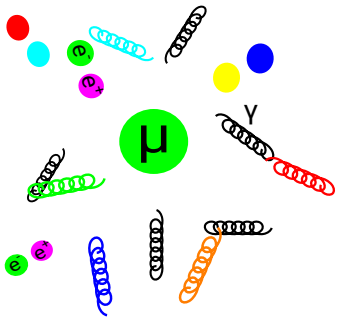


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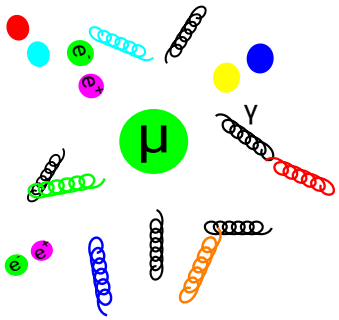
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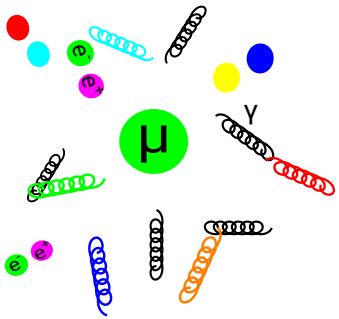


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134($\rightarrow$220) physicists: arXiv:2006.04822 White Paper (WP'20): Muon g-2 Theory Initiative"

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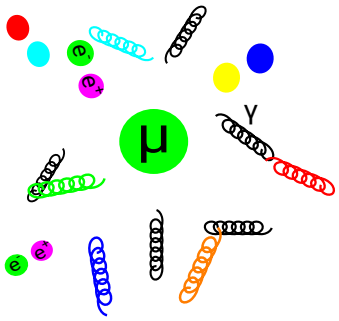
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Total	11659181.0(4.3)

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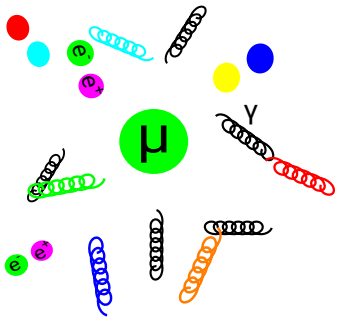
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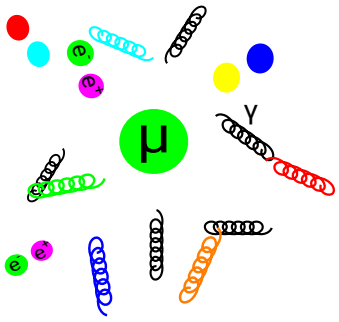
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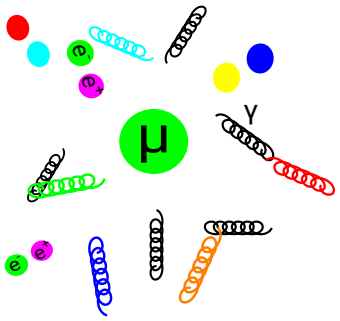
1. Quantum Electrodynamics (QED): photons, leptons (e.g. electrons);
2. Weak interactions (EW): W and Z Bosons, neutrinos, Higgs;
3. Strong interaction (QCD): quarks and gluons.

134(→220) physicists: arXiv:2006.04822 White Paper (WP'20): Muon g-2 Theory Initiative"

	$a_\mu \times 10^{-10}$
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EW	15.4(0.1)
QCD	693.7(4.3)
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R-Ratio: e^+e^- annihillation $\rightarrow$ photon $\rightarrow$ hadrons

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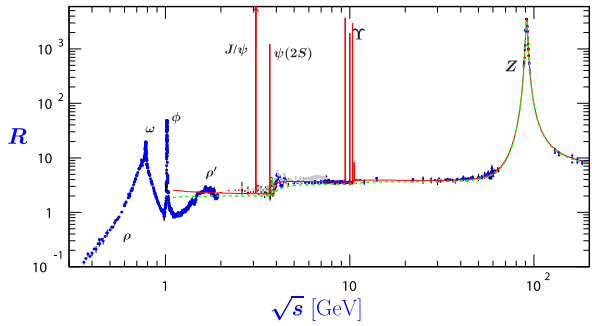
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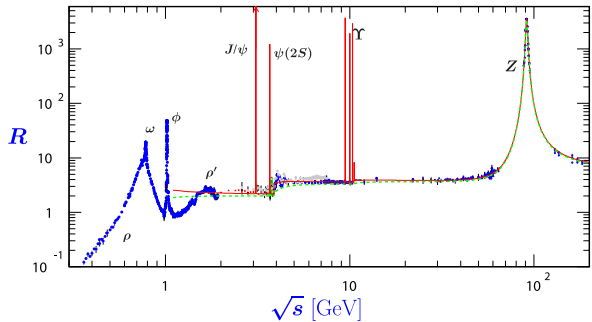
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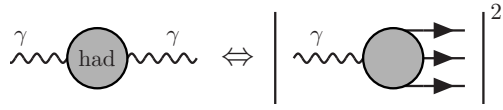
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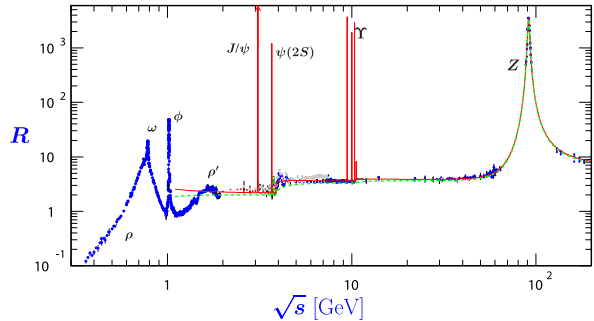
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2025 White Paper result: 713.2(6.1), lattice window averages

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systematic non-perturbative approach (numerical solution):

quantum fields on the lattice

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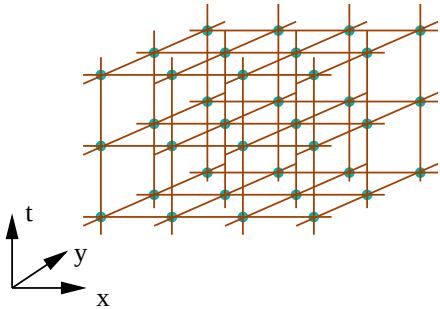
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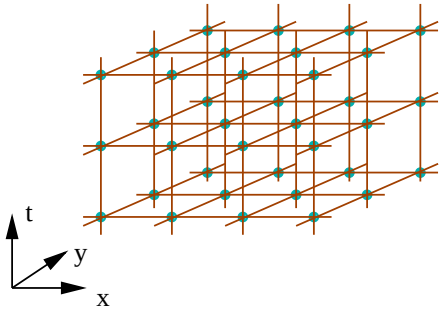
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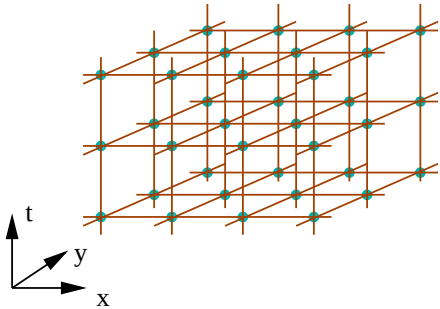
$\Rightarrow$ stochastic approach, with reasonable spacing/size: solvable theory, algorithm, CPU;
easy to make it $1000\times$ more expensive



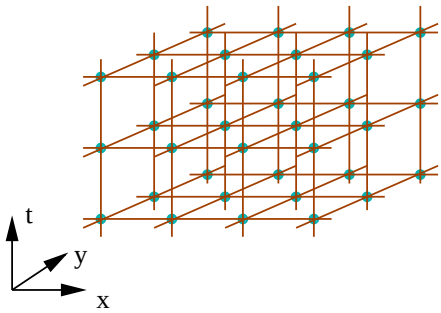
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few fm size is needed
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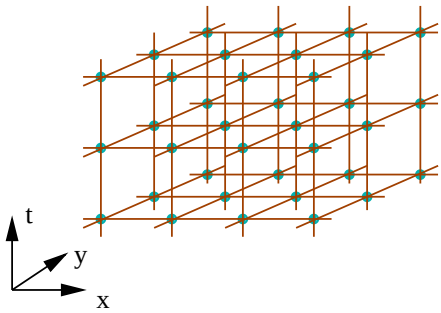
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advanced techniques,
good balance: discretization,
math, coding, setup, physics and
O(20) Pflops (10^{15}) needed (24/7)

Department of Physics, University of Adelaide, Australia

Magnetic moment on the lattice Nature 593 (2021) 7857, 51

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current-current correlator:

$C(t) = \langle J_\mu(t) J_\nu(0) \rangle$

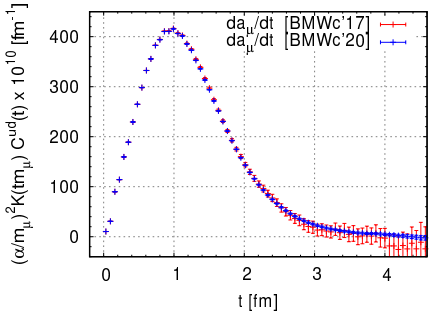
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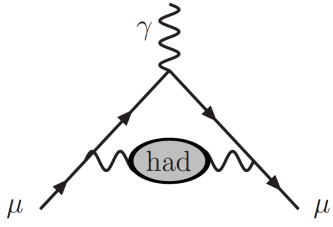
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$$\Rightarrow \alpha^2 \int_0^\infty dt\ K(t)\ C(t)$$

K(t) describes the muon-line:





BMW simulation setup

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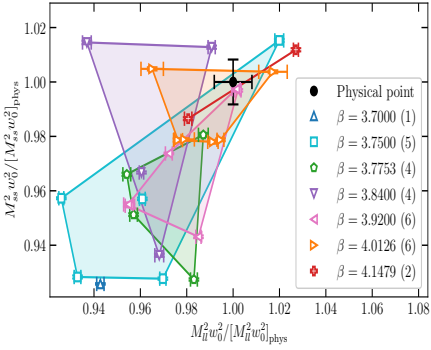
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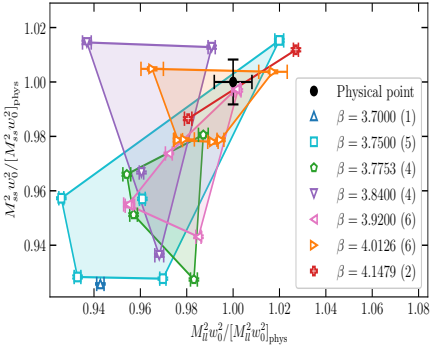
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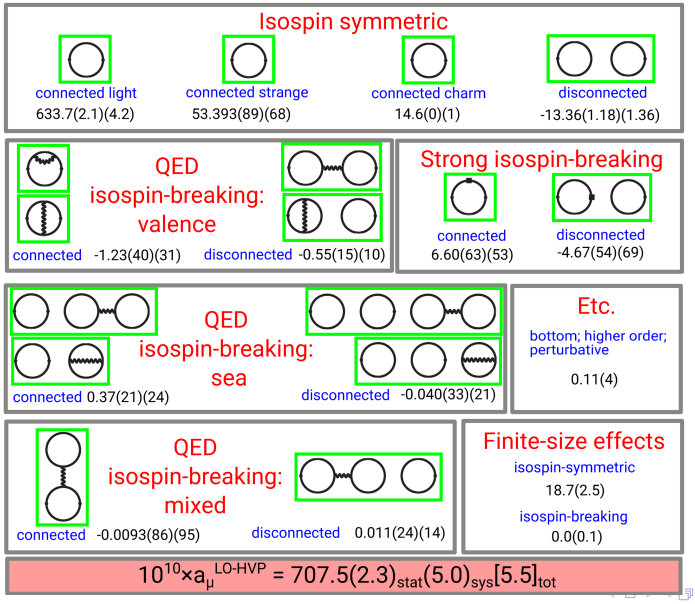
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β	a[fm]	$L \times T$	#conf
3.7000	0.1315	48×64	904
3.7500	0.1191	56×96	2072
3.7753	0.1116	56×84	1907
3.8400	0.0952	64×96	3139
3.9200	0.0787	80×128	4296
4.0126	0.0640	96×144	6980
4.1479	0.0480	128×192	5017

CPU demand scales as $\approx a^{-8}$:
very careful planning needed

Final result: 2020



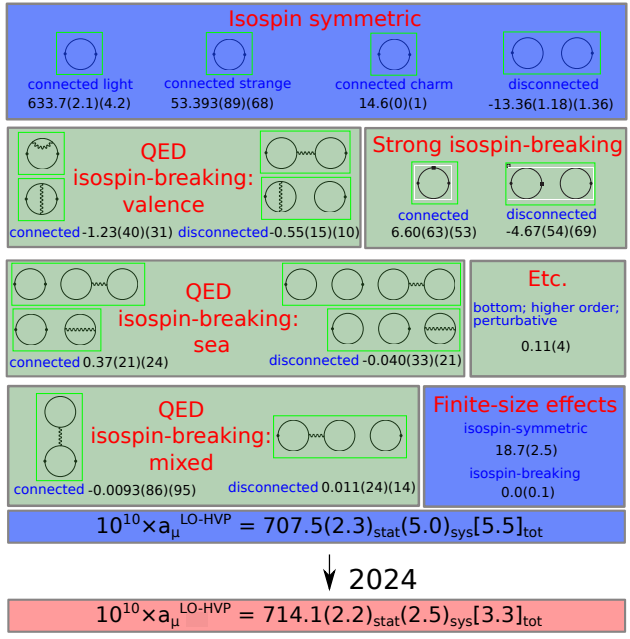
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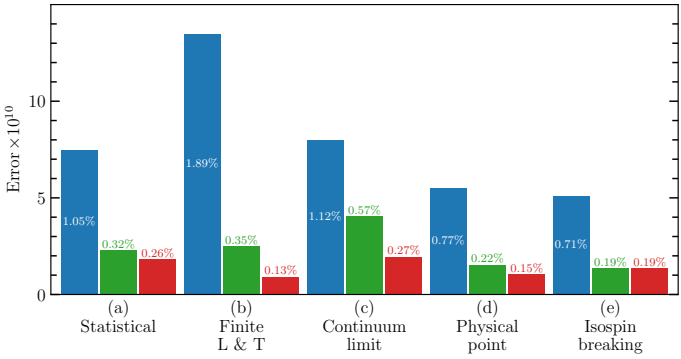
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Improved in new work

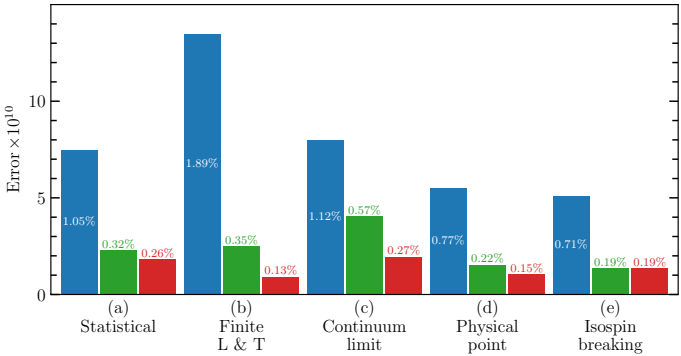
Some checks in new work



Error reduction 2017 (blue) – 2020 (green) - 2024 (red)



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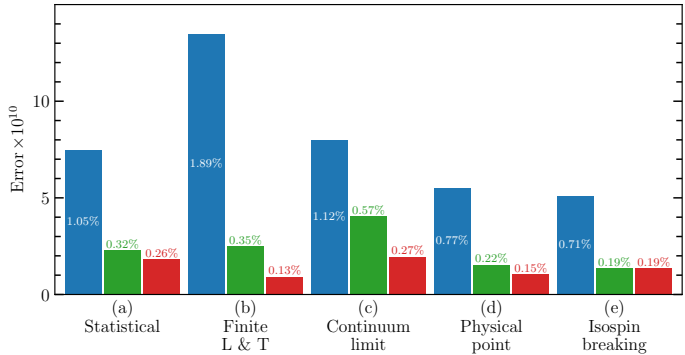
- In 2017 the dominant uncertainty: finite volume

$\Longrightarrow$ in 2020 dedicated large volume simulation

earlier only 6 fm box, increased to 11 fm

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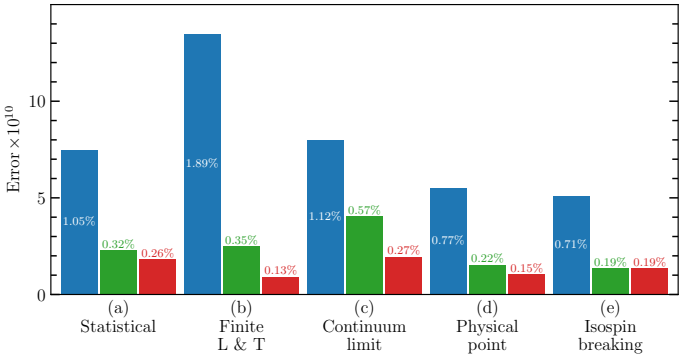
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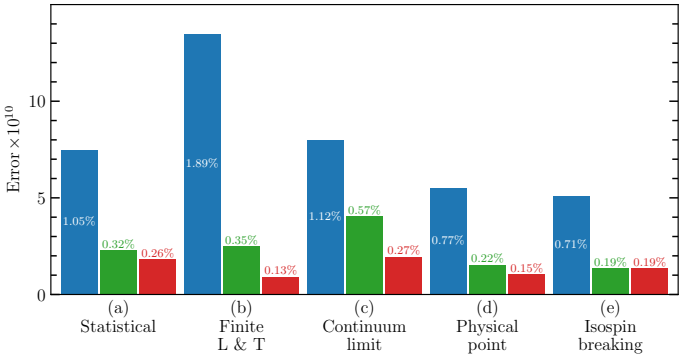
- In 2020 the dominant uncertainty: continuum extrapolation

$\Longrightarrow$ new, even finer lattice with 0.048 fm lattice spacing (alltogether seven)

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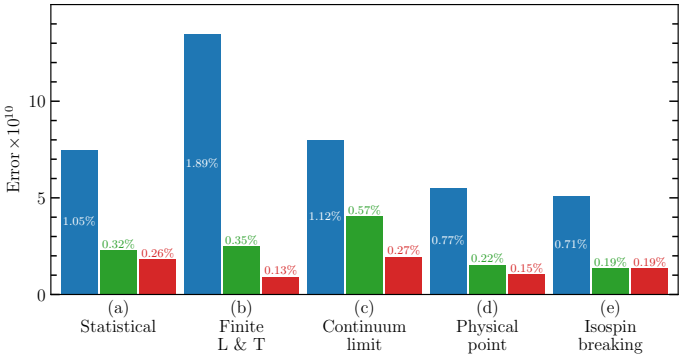


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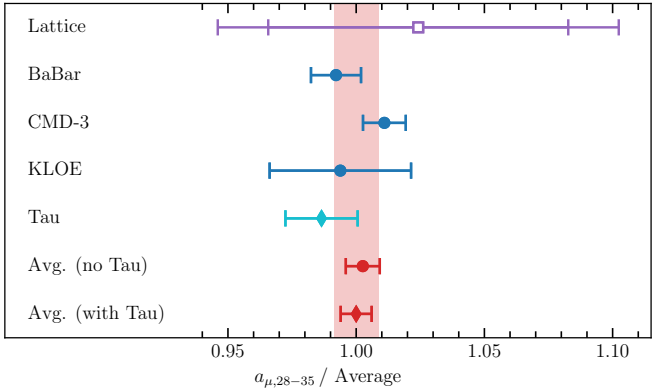
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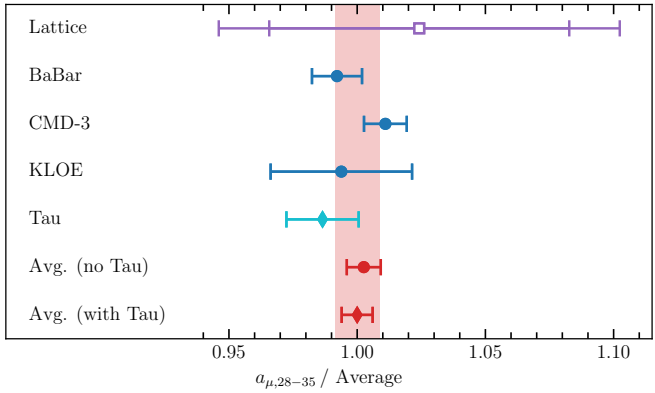
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- Scale for physical point & dedicated isospin breaking analysis

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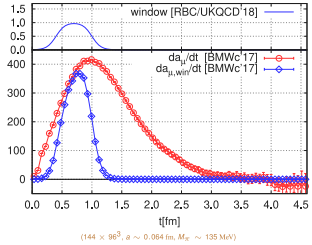


- $t > 2.8$ fm contributes only less than 5%
- data driven uncertainty: an order of magnitude better than lattice
- low energy part (below ρ) all agree
- we could be generous with errors: even 5 times larger wouldn't change the final error

Window observable

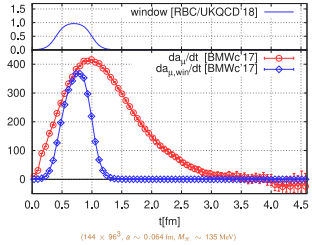
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[RBC/UKQCD'18]



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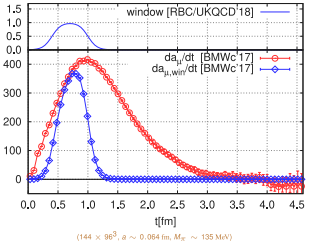


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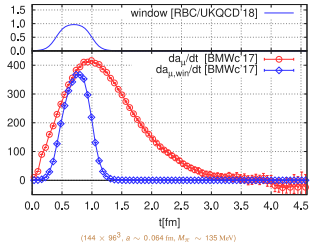


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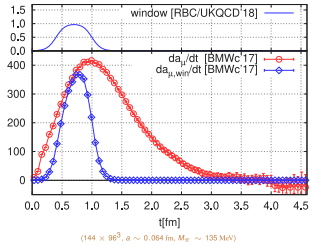
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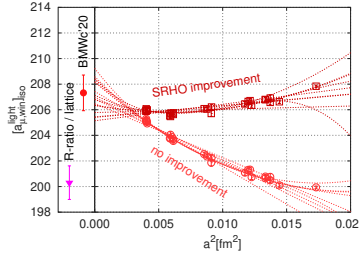


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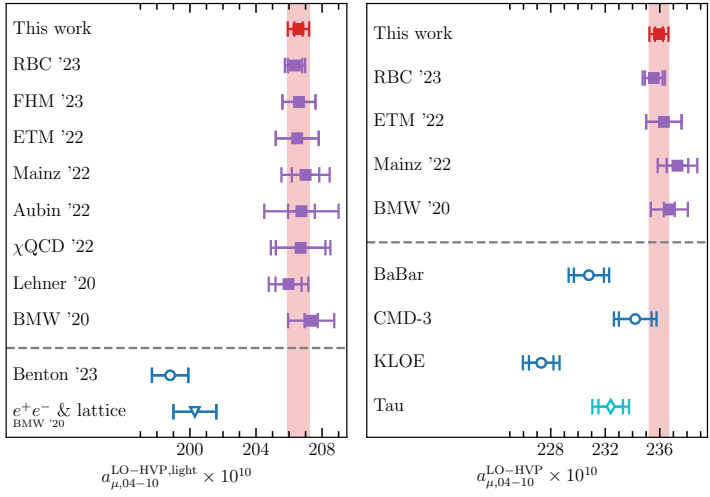
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histogram of 250,000 fits
with and without improvements
3.7 σ tension (already in 2020)

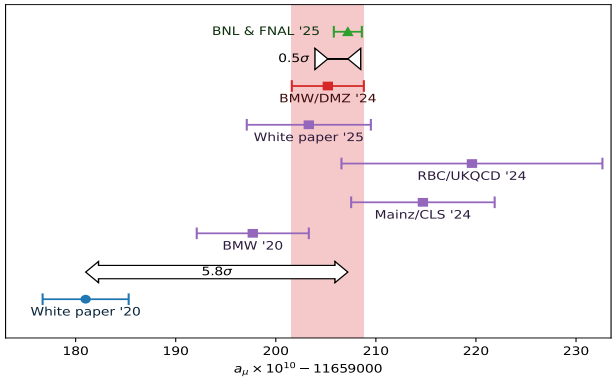


Window observable tension: other groups



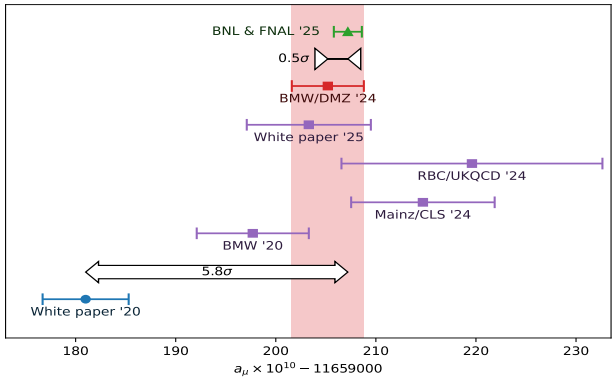
Huge tension between e^+e^- & our lattice result (with the average even more)

Take-home message: success of Quantum Field Theory up to 11 digits



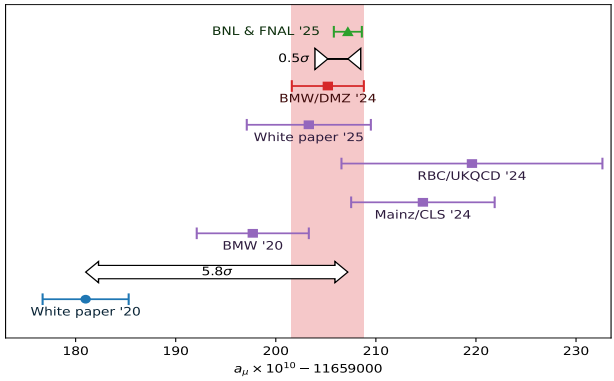
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  weak theory: renormalization and spontaneous symmetry breaking
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- BMW just one result: others confirm/refute $\Rightarrow$ precision era/bright future