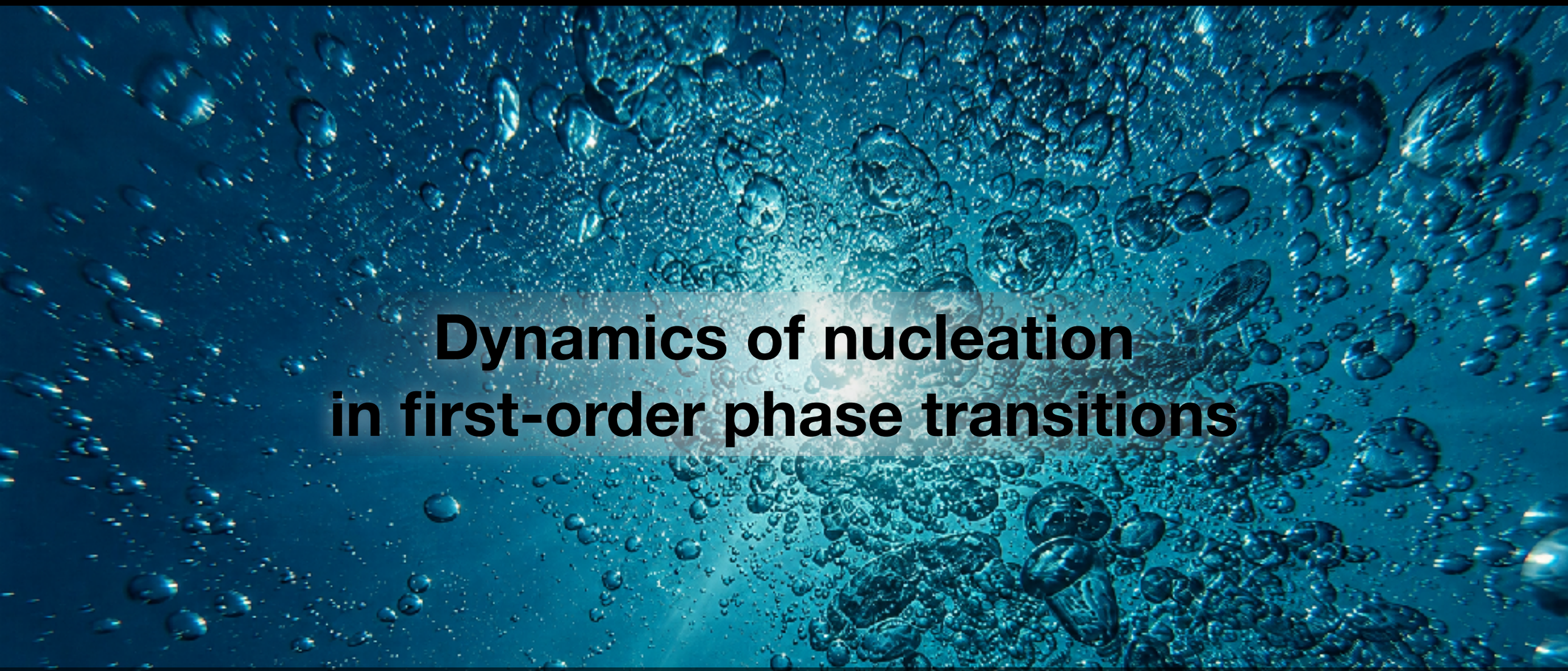


Dynamics of nucleation in first-order phase transitions

Andrey Shkerin

Perimeter Institute for theoretical physics

2608.00169

Oliver Gould

Joonas Hirvonen

Sergey Sibiryakov

HEP Theory Seminar

FTPI

October 8, 2026

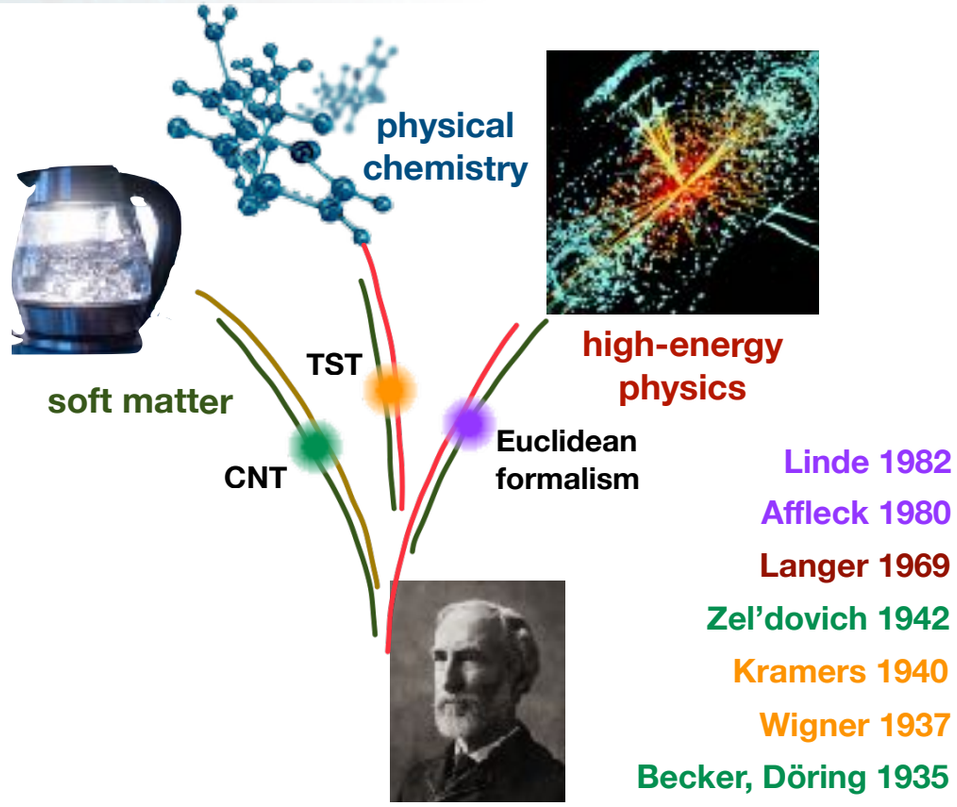
First-order phase transitions…

- play a fundamental role in various branches of physics

  from quantum cosmology to quantum matter

- have been studied for ~150 years

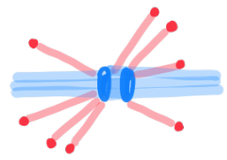
- *dynamical* aspects are still not fully understood



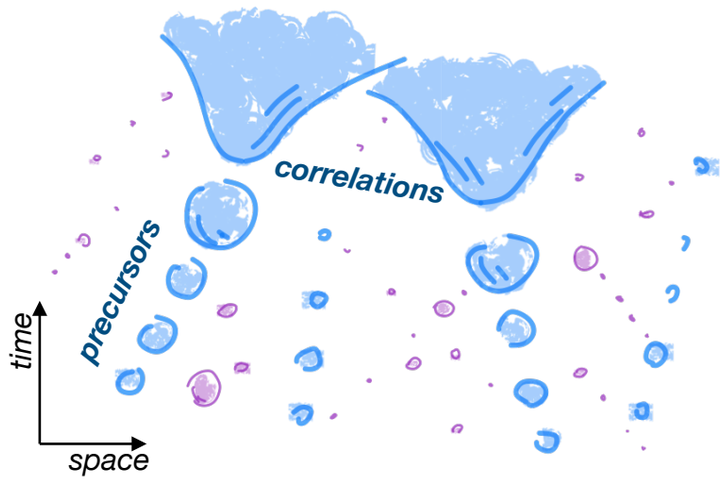
First-order phase transition — false vacuum decay — is a non-equilibrium process.

- non-equilibrium initial state


Evaporating black hole


Particle collisions

- dynamics of nucleation



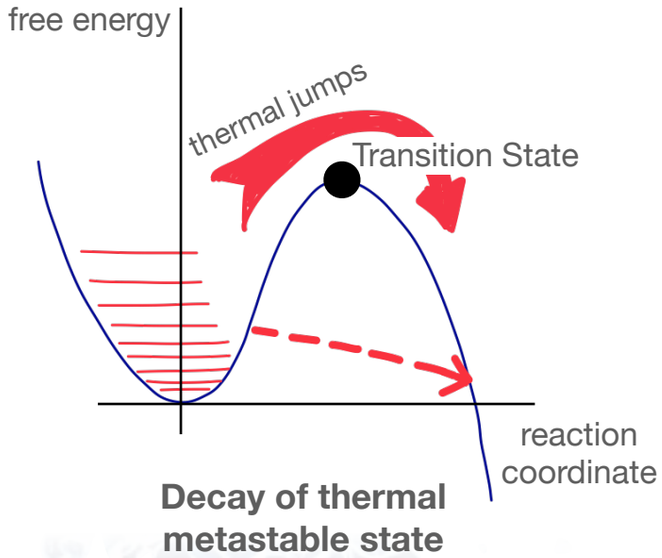
Pirvu, Johnson, Sibiryakov 23

Pirvu, AS, Sibiryakov 24

Consider **thermal phase transitions** in the classical regime.

The **decay rate** is given by

$$\Gamma \propto e^{-E_{\rm ts}/T}$$



Decay of thermal metastable state

How to define decay rate

- **Decay rate ~ Flux** (in phase space) out of the metastable region.

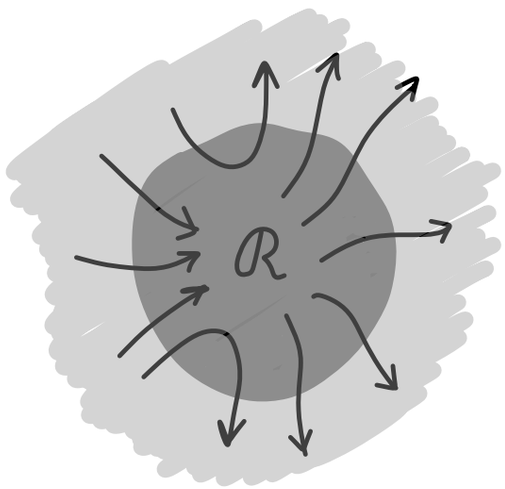
Equilibrium approach

~ statistical, Euclidean, **Transition State Theory**

$$\Gamma_{\rm TST} = \frac{1}{Z_{\mathcal{R}}} \int_{\partial\mathcal{R}} {\rm d}S\, \mathbf{n}\cdot\dot{\mathbf{z}}\, \theta(\mathbf{n}\cdot\dot{\mathbf{z}})\, {\rm e}^{-H(\mathbf{z})/T}$$

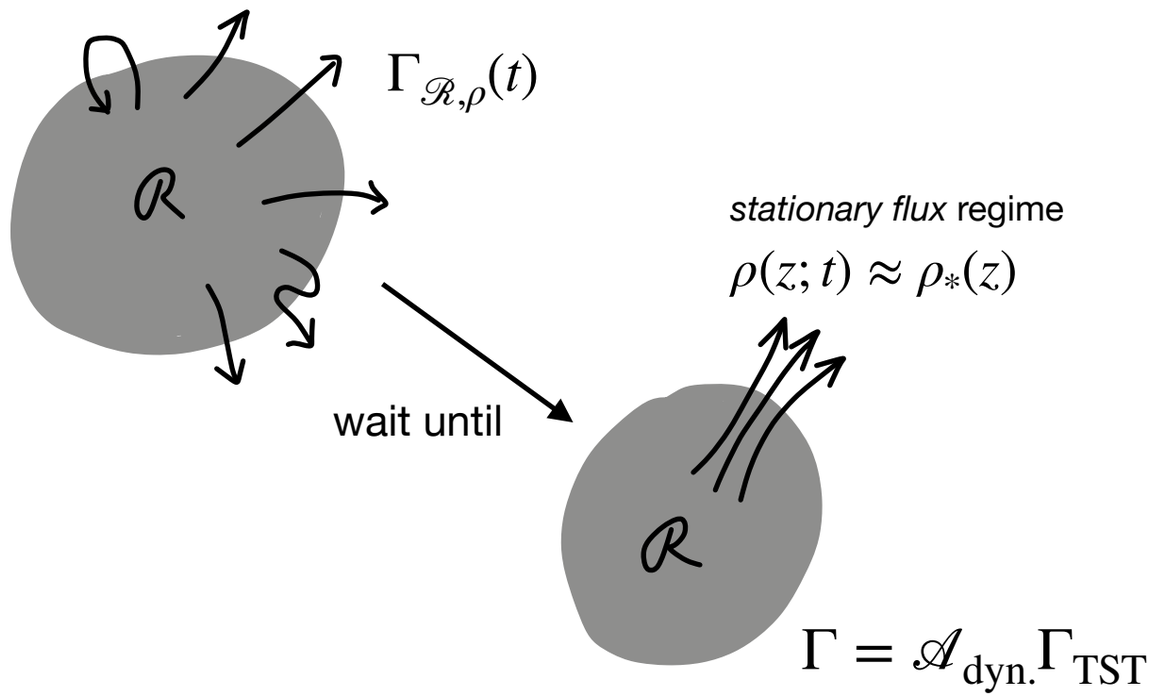
SPA!

$$\Gamma_{\rm TST} \overset{\text{1-loop}}{=} \frac{|\omega_-|}{\pi T}\cdot {\rm Im}\, F$$



What happens in real life

$$\rho(\mathbf{z}; t=0) = \frac{1}{Z_{\mathcal{R}}} {\rm e}^{-H(\mathbf{z})/T} \theta(\mathbf{z}\in\mathcal{R})$$



Questions:

- Does $\rho_*(z)$ exist?
- How to compute $\mathscr{A}_{\rm dyn.}$?
  - How is Γ independent of the choice of $\mathscr{R}$ and (in the semiclassical regime) TS?

Langer 69; Affleck 81; Arnold, McLerran 87; Hirvonen 25
Moore, Rummukainen, Tranberg 01

Thermal decay rate

Our derivations are similar to the reactive-flux method in chemistry

Chandler 77
P. Hanggi, P. Talkner and M. Borkovec 90

and to the MRT method

Moore, Rummukainen 00
Moore, Rummukainen, Tranberg 01

- Consider a classical system with the Hamiltonian H quadratic in momenta:

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \ , \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i} - \sum_j \eta_{ij}\dot{q}_j + \xi_i$$

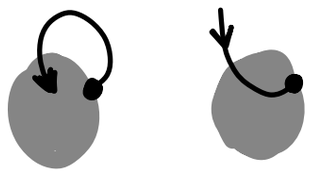
$$\langle \xi_i(t)\xi_j(t')\rangle = 2\eta_{ij}T\,\delta(t-t')$$

$$i = 1, \ldots, N$$

- In the *stationary flux* regime (more on its existence later)

$$\Gamma = \Gamma_{\rm TST}\cdot(1-R)$$

$R = R^{(+)} + R^{(-)}$ — re-crossing probability



Thermal decay rate

$$\Gamma = \Gamma_{\mathrm{TST}} \cdot (1 - R)$$

$$R = R^{(+)} + R^{(-)}$$

- This formula is exact (up to corrections of order $\mathcal{O}(e^{-E_{\mathrm{ts}}/T})$)
- R and Γ_{TST} depend on $\mathscr{R}$; Γ does not.
- Γ does not depend on the initial state preparation.

Our derivations are similar to the reactive-flux method in chemistry

Chandler 77
P. Hanggi, P. Talkner and M. Borkovec 90

and to the MRT method

Moore, Rummukainen 00
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Thermal decay rate

$$\Gamma = \Gamma_{\rm TST} \cdot (1 - R)$$

$$R = R^{(+)} + R^{(-)}$$

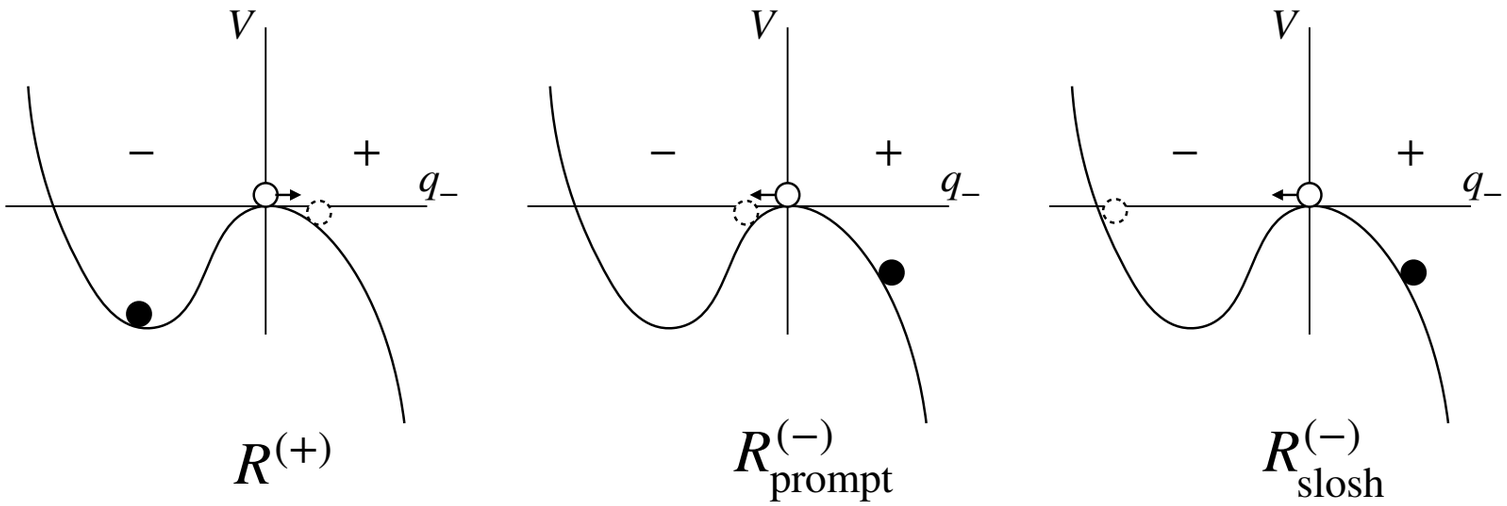
- It makes sense to split $R = R_{\rm prompt} + R_{\rm slosh}$.

Our derivations are similar to the reactive-flux method in chemistry

Chandler 77
P. Hanggi, P. Talkner and M. Borkovec 90

and to the MRT method

Moore, Rummukainen 00
Moore, Rummukainen, Tranberg 01



- $R_{\rm prompt}$ can be systematically evaluated (at weak coupling)

  $R_{\rm slosh}$ can give a big contribution! (due to oscillons)

Thermal decay rate

$$\Gamma = \Gamma_{\mathrm{TST}} \cdot (1 - R)$$

$$R = R^{(+)} + R^{(-)}$$

Our derivations are similar to the reactive-flux method in chemistry

Chandler 77
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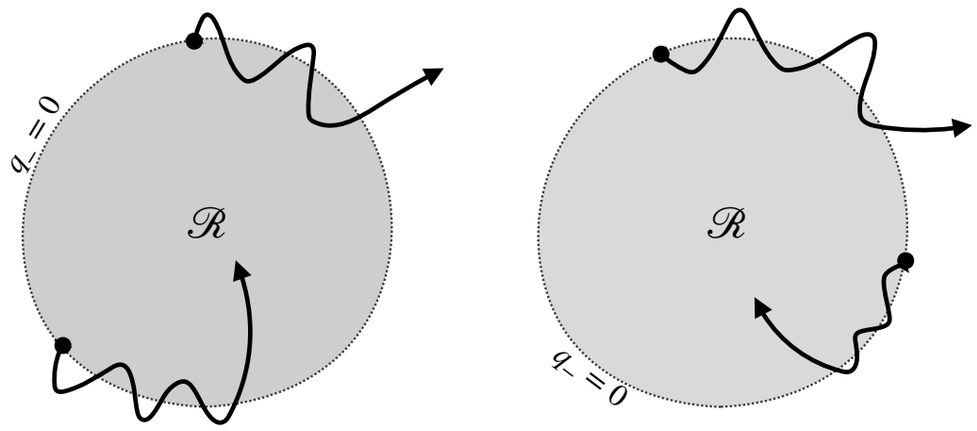
- R can be found from simulations; no exponentially long simulation time required.

Practically, choose $q_- = 0$ (amplitude of the negative mode of the TS) as a dividing surface,

sample $\rho^{(\pm)}(\mathbf{q}, \mathbf{p}; t = 0) = \mathcal{N}\delta(q_-)\theta(\pm\dot{q}_-)|\dot{q}_-|\mathrm{e}^{-H(\mathbf{q},\mathbf{p})/T}$

then $R^{(\pm)} = \lim_{t_{\mathrm{plat.}} \ll t \ll t_{\mathrm{dec.}}} \int \mathrm{d}\mathbf{q}\mathrm{d}\mathbf{p}\, \rho^{(\pm)}(\mathbf{q}, \mathbf{p}; t)\theta(\mp q_-)$ where $t_{\mathrm{dyn.}} < t_{\mathrm{plat.}} < t_{\mathrm{therm.}}$





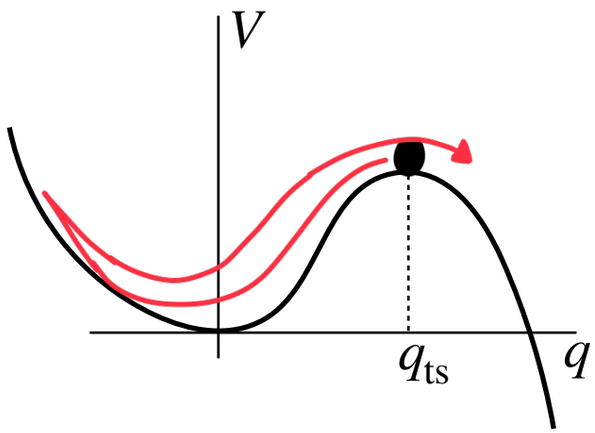
Two rates

Mel'nikov, Meshkov 86

Kramers

$N = 1$; weak damping

$$R = R_{\text{slosh}} + \dots$$



$$R^{(-)} = 1 - \exp\left[\int_{-\infty}^{\infty} \frac{\mathrm{d}z}{\pi(z^2+1)} \ln\left(1 - \mathrm{e}^{-(z^2+1)\varepsilon/4T}\right)\right]$$

$$R^{(+)} = 0$$

where $\varepsilon = \eta \oint \dot{\bar{q}}^2(t)\,\mathrm{d}t = \eta \oint p\,\mathrm{d}q$

oscillons?..

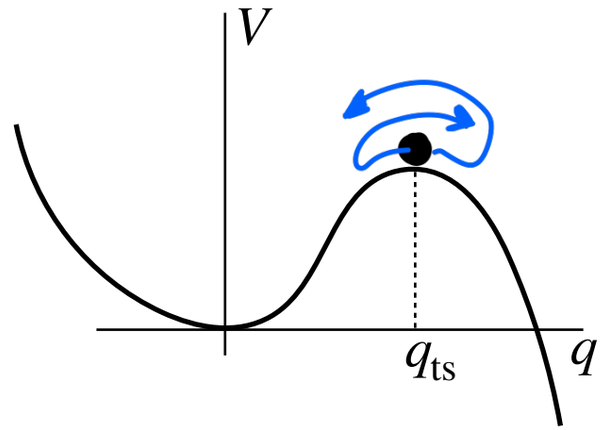
N.B.: $t_{\text{plat.}} \sim t_{\text{slosh}} \sim \dfrac{1}{\eta}\dfrac{T}{E_{\text{ts}}}$

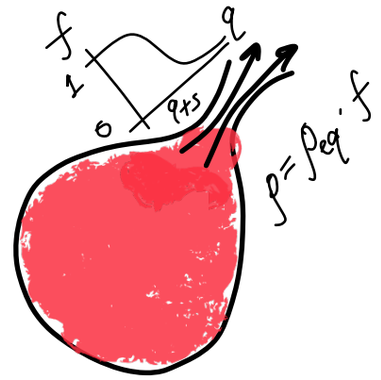
Langer 69

Langer

$N =$ many; moderate to strong damping

$$R = R_{\text{prompt}} + \dots$$
$$= R_{\text{pert}} + \dots$$





To LO in T/E_{ts}:

$$R^{(-)} = R^{(+)} = \frac{1}{2}\left(1 - \sqrt{1 + \frac{\eta^2}{4\omega_-^2}} + \frac{\eta}{2\omega_-}\right)$$

where ω_-^2 is the negative e.v. of the critical bubble

$$\Gamma_{\text{L}} = \left(\sqrt{1 + \frac{\eta^2}{4\omega_-^2}} - \frac{\eta}{2\omega_-}\right)\Gamma_{\text{TST}}$$

N.B.: $t_{\text{plat.}} \sim t_{\text{dyn.}}$

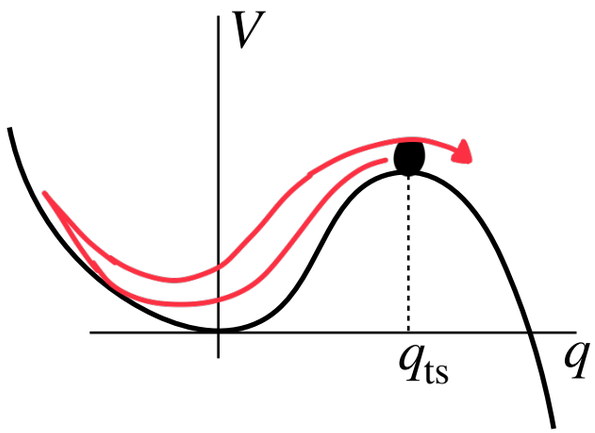
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$$R^{(-)} = 1 - \exp\left[\int_{-\infty}^{\infty} \frac{\mathrm{d}z}{\pi(z^2+1)} \ln\left(1 - \mathrm{e}^{-(z^2+}\right.\right.$$

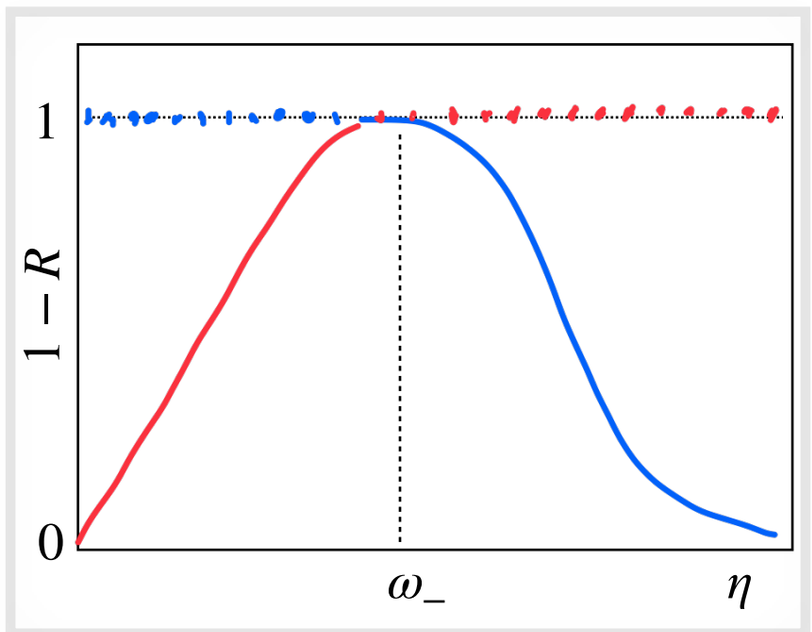
$$R^{(+)} = 0$$

where $\varepsilon = \eta \oint \dot{\bar{q}}^2(t)\,\mathrm{d}t = \eta \oint p\,\mathrm{d}q$

oscillons?..

N.B.: $t_{\text{plat.}} \sim t_{\text{slosh}} \sim \dfrac{1}{\eta}\dfrac{T}{E_{\text{ts}}}$

Dynamical factor R



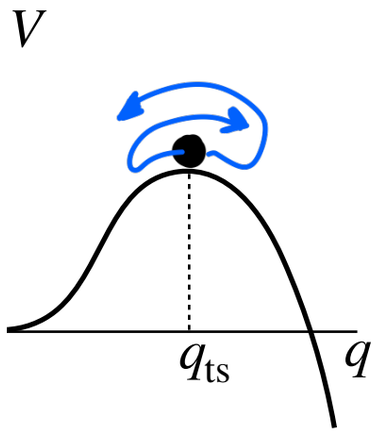
Langer 69

Langer

N = many; moderate to strong damping

$$R = R_{\text{prompt}} + \ldots$$

$_{\text{rt}} + \ldots$



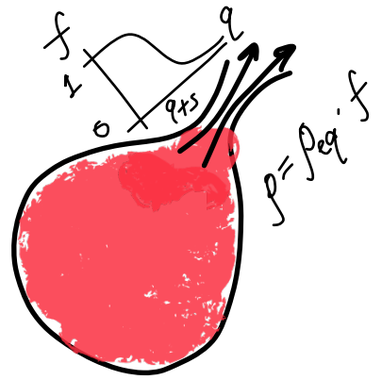
$/E_{\text{ts}}$:

$$= R^{(+)} = \frac{1}{2}\left(1 - \sqrt{1 + \frac{\eta^2}{4\omega_-^2}} + \frac{\eta}{2\omega_-}\right)$$

where ω_-^2 is the negative e.v. of the critical bubble

$$\Gamma_{\text{L}} = \left(\sqrt{1 + \frac{\eta^2}{4\omega_-^2}} - \frac{\eta}{2\omega_-}\right)\Gamma_{\text{TST}}$$

N.B.: $t_{\text{plat.}} \sim t_{\text{dyn.}}$



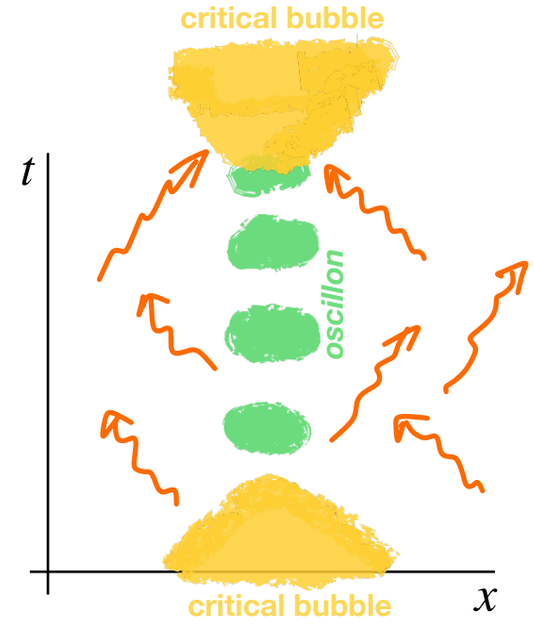
Structure of re-crossings

$$R = R_{\text{prompt}} + R_{\text{slosh}}$$

$$\overset{\text{weak coupling}}{=} R_{\text{pert}} + R_{\text{non-pert}}$$

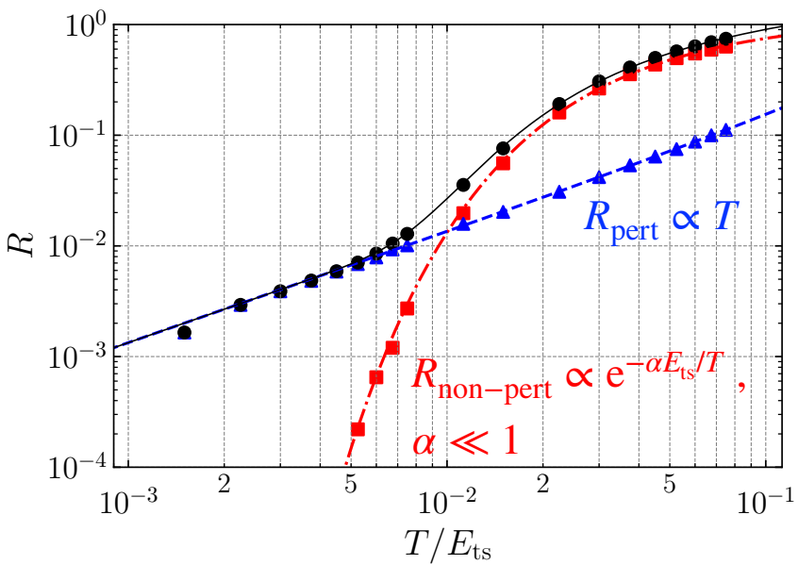
Sloshing re-crossings can be important in field theory!

They are amplified in theories with **oscillons**.

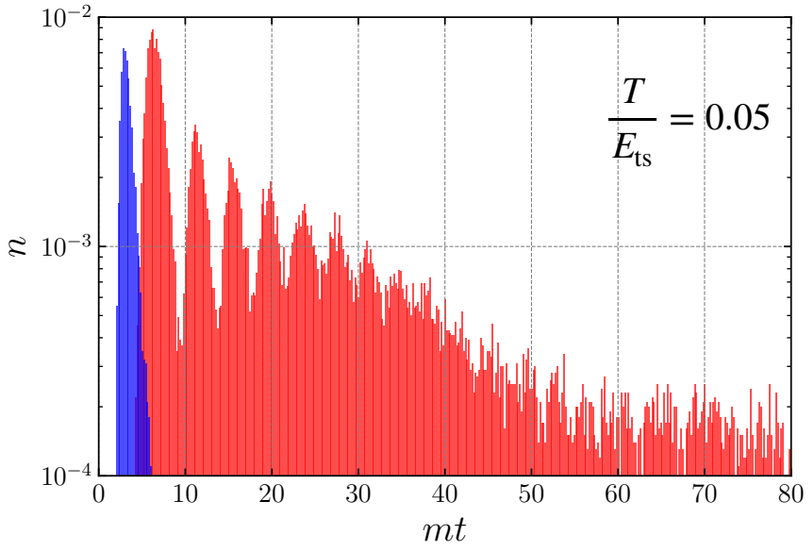


Example: scalar field with unstable quartic potential in (1+1)d,
Hamiltonian dynamics

Dynamical factor R from simulations

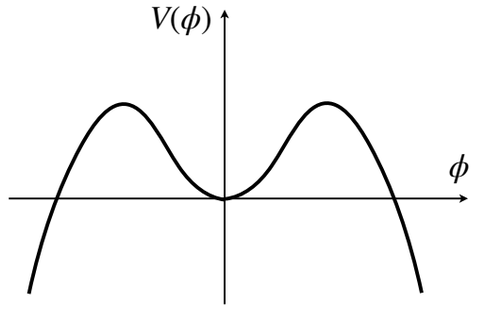


Re-crossings as a function of time



$$S = \int \mathrm{d}t\, \mathrm{d}x \left(\frac{(\partial_\mu \phi)^2}{2} - V(\phi) \right)$$

$$V(\phi) = \frac{m^2 \phi^2}{2} - \frac{\lambda \phi^4}{4}$$

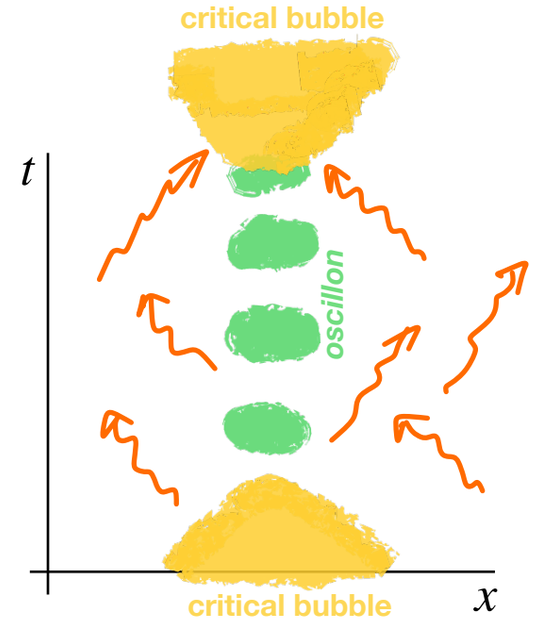


Thus: $R \to 0$ at $T/E_{\text{ts}} \to 0$.

Structure of re-crossings

$$R = R_{\text{prompt}} + R_{\text{slosh}}$$

$$\overset{\text{weak coupling}}{=} R_{\text{pert}} + R_{\text{non-pert}}$$

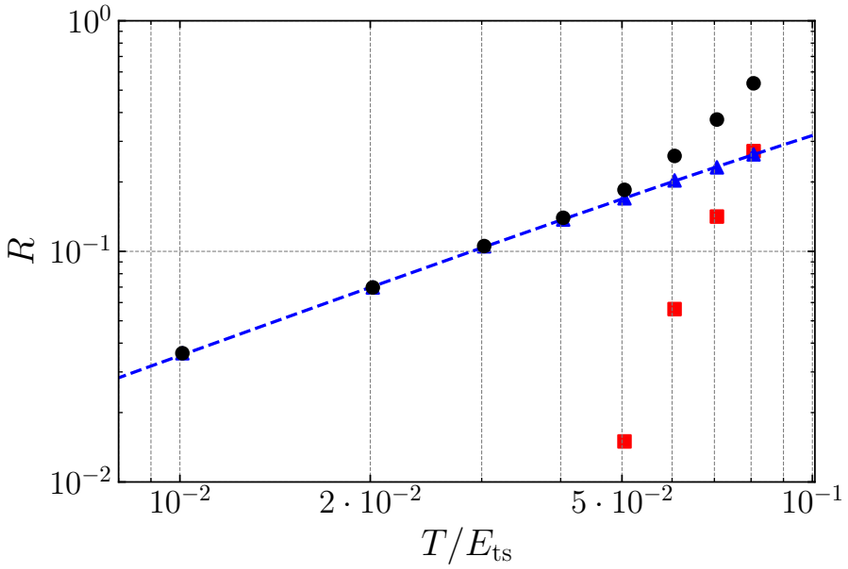


Sloshing re-crossings can be important in field theory!

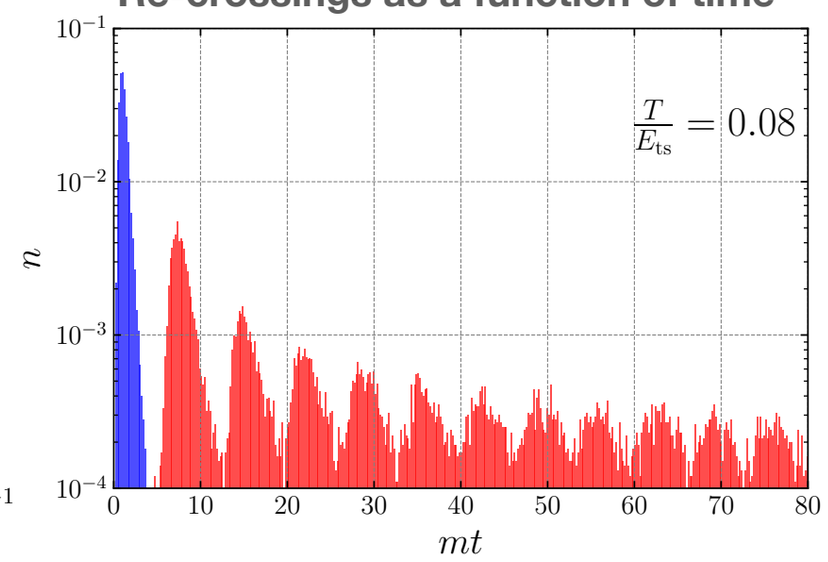
They are amplified in theories with **oscillons**.

Example: scalar field with unstable exponential potential in (1+1)d,
Hamiltonian dynamics

Dynamical factor R from simulations

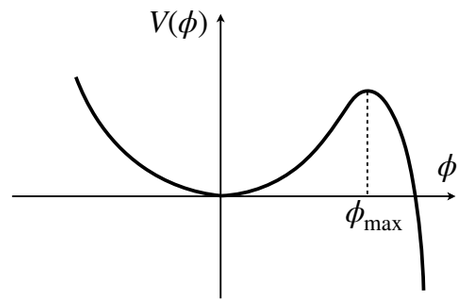


Re-crossings as a function of time



$$S = \int \mathrm{d}t\, \mathrm{d}x \left(\frac{(\partial_\mu \phi)^2}{2} - V(\phi) \right)$$

$$V(\phi) = \frac{m^2 \phi^2}{2} - \kappa \mathrm{e}^{\phi}$$



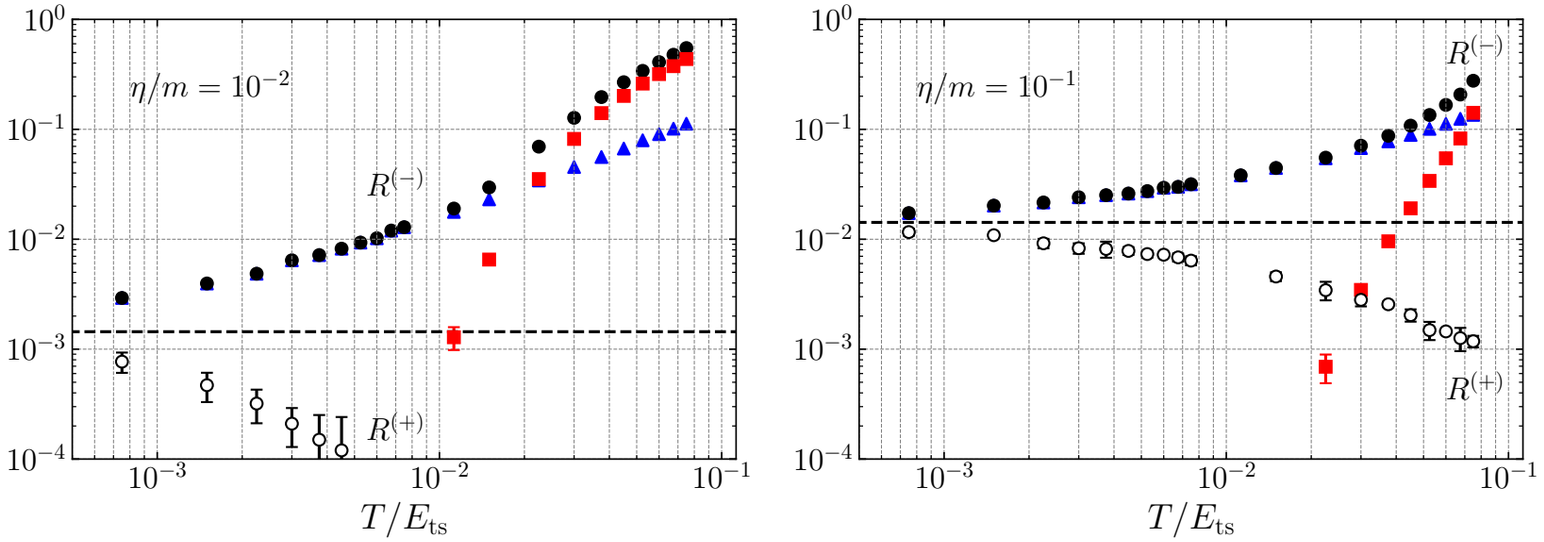
Thus: No oscillons → sloshing re-crossings are suppressed, but still there.

Structure of re-crossings

$$R = R_{\text{prompt}} + R_{\text{slosh}}$$

$$\overset{\text{weak coupling}}{=} R_{\text{pert}} + R_{\text{non-pert}}$$

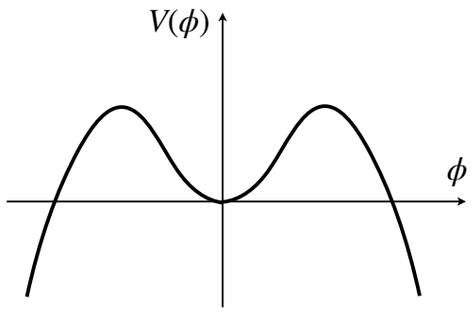
Dynamical factor R with dissipation



- T/E_{ts}-corrections to Langer
- Sloshings are damped by damping

$$S = \int \mathrm{d}t\, \mathrm{d}x \left(\frac{(\partial_\mu \phi)^2}{2} - V(\phi) \right)$$

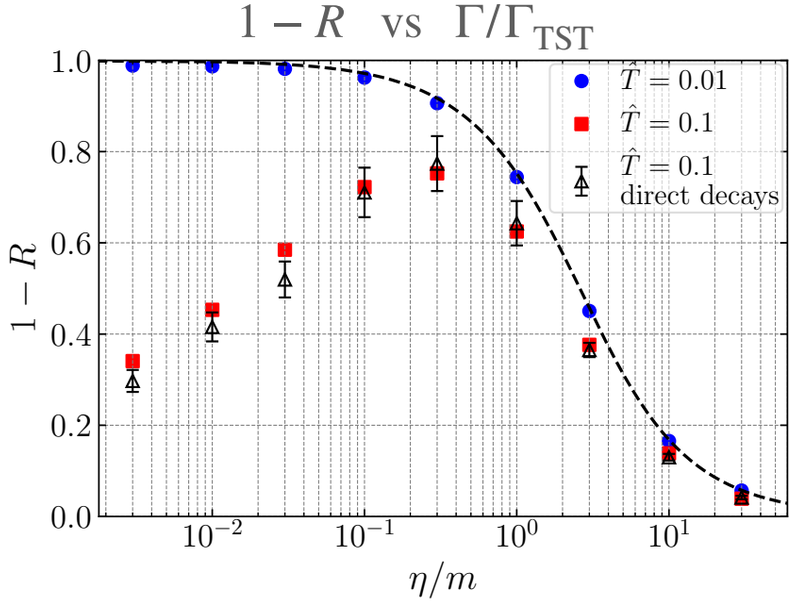
$$V(\phi) = \frac{m^2 \phi^2}{2} - \frac{\lambda \phi^4}{4}$$



At $T/E_{\text{ts}} \sim 10^{-1}$, we can compare with **direct** simulations of the decays

~10% discrepancy is due to higher-loop corrections to R and Γ_{TST}

Gould, Hirvonen, AS, Sibiryakov, *coming soon*



Small Systems

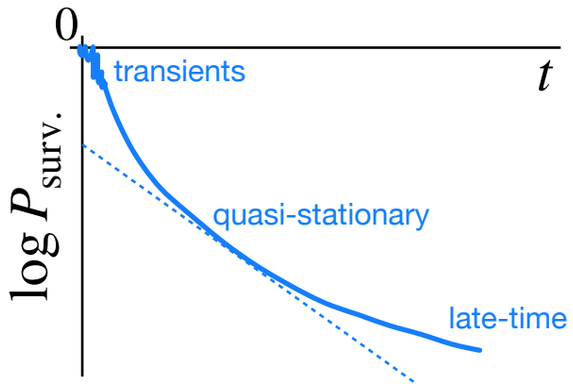
- System is **big** if $\mathscr{E} \gg E_{\rm ts}$ **and** $\delta\mathscr{E} \gg E_{\rm ts} \Leftrightarrow \sqrt{\mathscr{C}T} \gg E_{\rm ts}$.

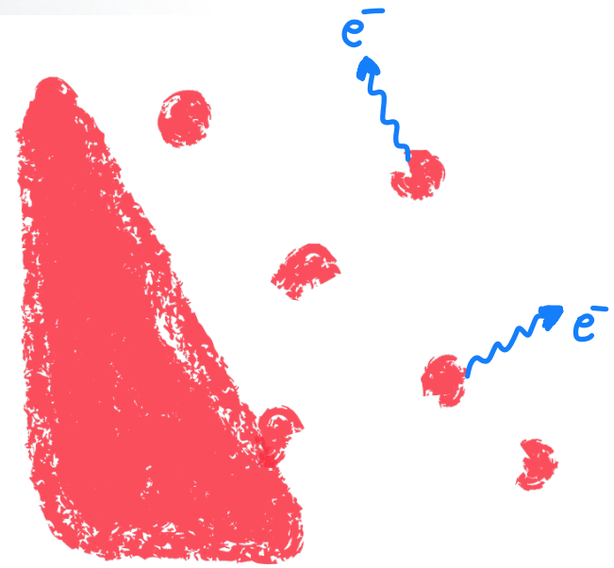
In small systems, there can be no steady-state decay

— the decay rate *drifts*.

$$t_{\rm drift} \sim t_{\rm dec.}\, {\rm e}^{-\beta}, \qquad \beta = \frac{E_{\rm ts}^2}{\mathscr{C}T^2}$$

Moreover, $\Gamma_{\rm canonical} = e^{\beta/2} \cdot \Gamma_{\rm microcanonical}$





Thermionic decay in molecular clusters

Klots '85

Hansen, Andersen et al '01

- Big systems with poor thermalization ($t_{\rm therm.} > t_{\rm dec.}$) are small.

We see this in numerical simulations of direct decays.

Can be important in quantum simulators. Not in cosmology, perhaps.

Discussion

➡ Application to cosmological phase transitions

Caprini et al 20

$$\Gamma = \mathscr{A}_{\text{dyn.}}\Gamma_{\text{TST}}\,, \quad \mathscr{A}_{\text{dyn.}} = 1 - R_{\text{pert.}} - R_{\text{non-pert.}}$$

? Perturbation theory for $R_{\text{pert.}}$

? Investigation of $R_{\text{non-pert.}}$ in (3+1)d

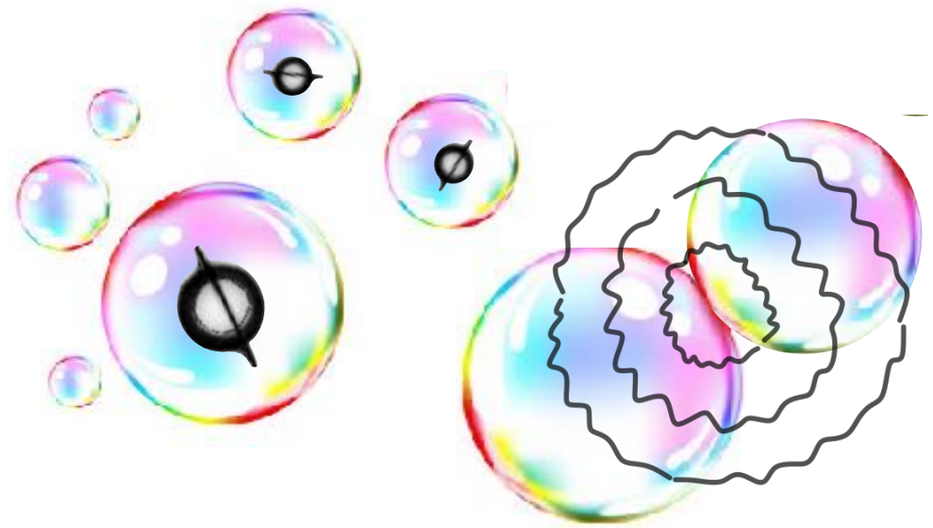
Gould, Hirvonen 21

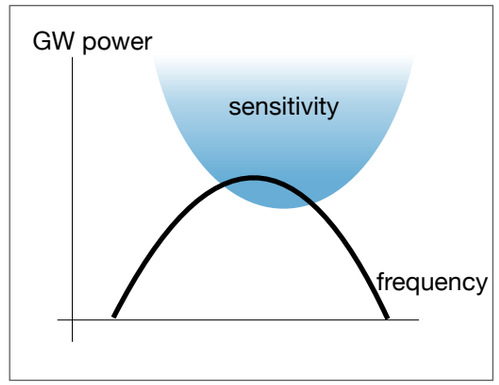
Ekstedt 22

Kierkla, Schicho, Swiezewska, Tenkanen, van de Vis 25

? Non-local effective theory for bubble nucleation

(if the barrier is generated radiatively by light fields)





➡ Application to BECs, cold atom systems,…

Song et al 21 (Nature Phys. 18 (2022) no.3, 259-264)

Zenesini et al 23 (Nature Phys. 20, no.4, 558-563)

➡ Quantum corrections

Garriga, Kanno, Sasaki, Soda, Vilenkin 12

Turok 13

Hirvonen 26

? Quantum corrections to the dynamical prefactor

? Real-time effects in thermally-assisted tunneling

➡ Other N^3 processes

? Γ_{TST} vs $\Gamma_{\text{diff.}}$ in sphaleron transitions at all temperatures

Bödeker 99

Moore, Rummukainen 01

Mancha, Moore 22