Inflation and primordial black holes

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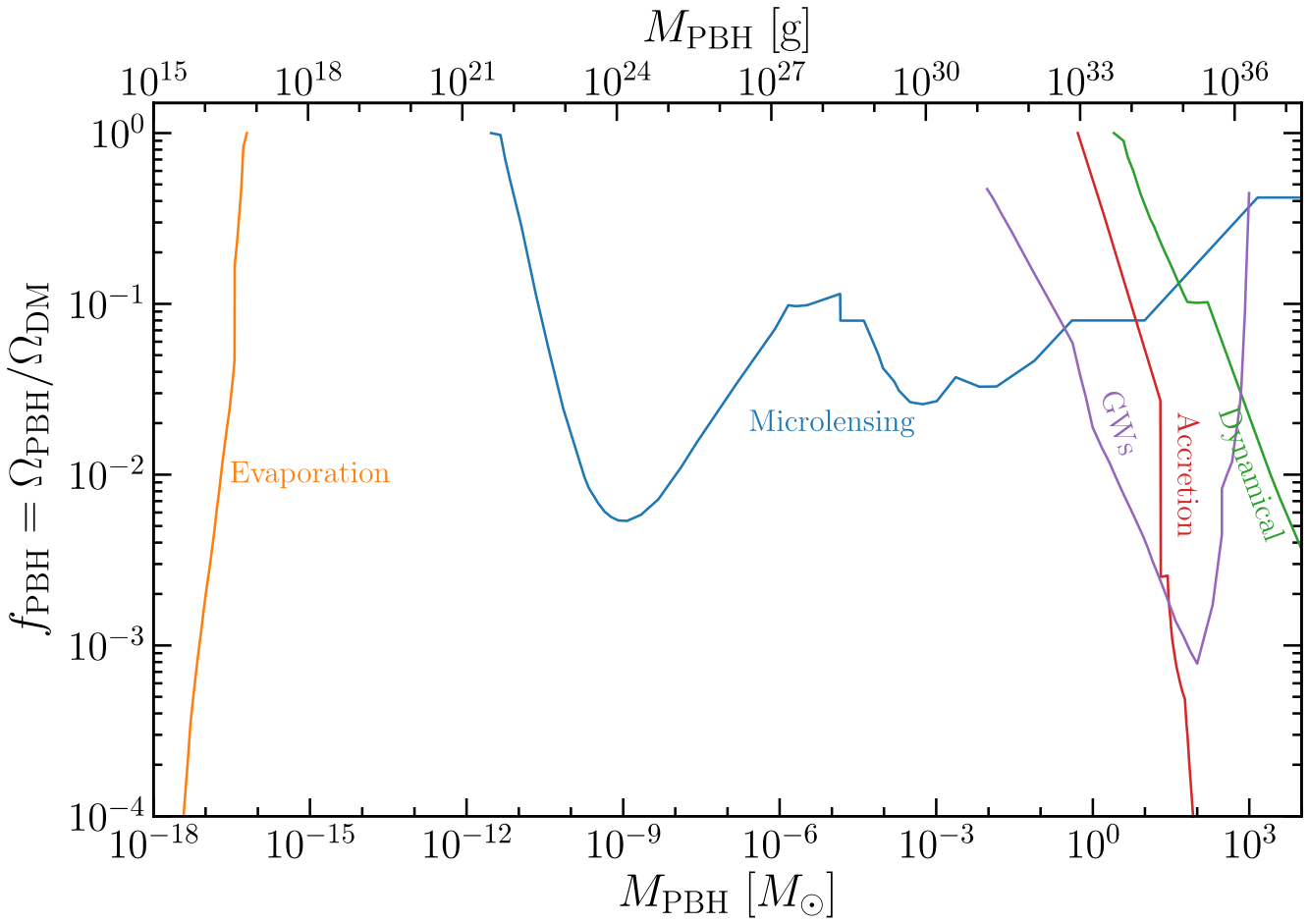
Dark matter without dark matter

- Primordial black holes (PBHs) are a candidate for dark matter. (Hawking 1971)

- PBHs could also seed supermassive black holes in centres of galaxies.
  - Since the 1990s, more and earlier supermassive black holes have been observed.
  - Now also Little Red Dots.
  - Their formation can be kickstarted by PBHs with initial mass $\sim 10^4\ M_\odot$.

Asteroid-mass or Planck scale



Green and Kavanagh: 2007.10722

Slow means large

- PBHs require large (~1) fluctuations on small scales.
- They could be generated by the same process as the small (~10^{-5}) fluctuations on large scales.
- The simplest and most successful scenario is inflation.
  (Dolgov and Silk 1993, Ivanov, Naselsky and Novikov 1994)
- The curvature perturbation is $\zeta = -H\delta\phi/\dot{\phi} \sim H^2/\dot{\phi}$.
- The slower the field, the larger the perturbations.

The road to black holes

- During inflation, $\vec{k}$-modes stretch to super-Hubble scales and freeze.
- After inflation, they cross back inside the Hubble radius and start evolving.
- If a Hubble patch is overmassive enough, it forms a PBH.
  - PBH mass is close to the mass inside the Hubble patch.

Large means stochastic

- If PBH seeds are generated by inflation, stochastic effects cannot be neglected.
  - Perturbations are large.
  - PBHs are rare, and the tail of the distribution is sensitive to small corrections.

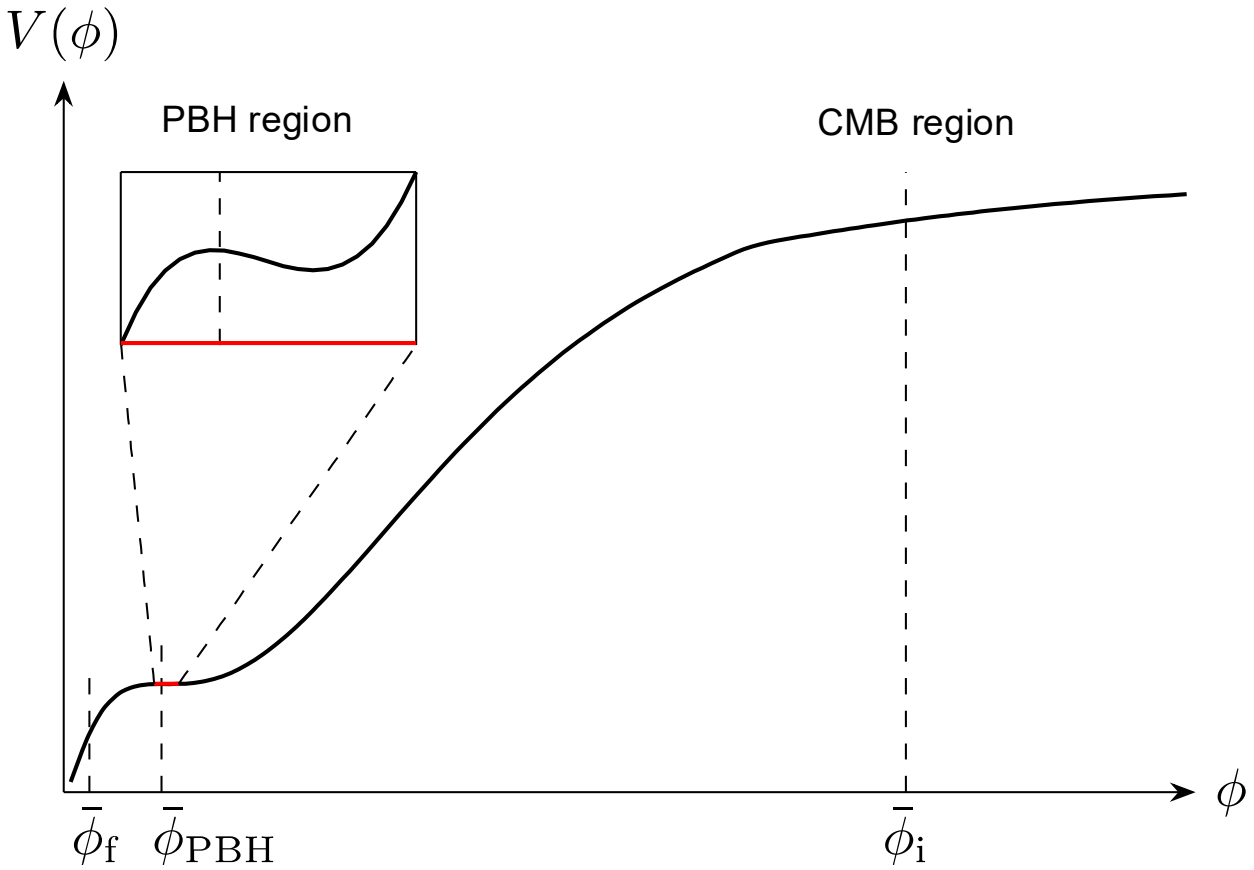
Stochastic inflation

- Inflaton evolution is stochastic when the coupling of small- and long-wavelength modes is taken into account. (Starobinsky 1986)

- As modes become super-Hubble and classicalise, they change the background in which shorter modes evolve.

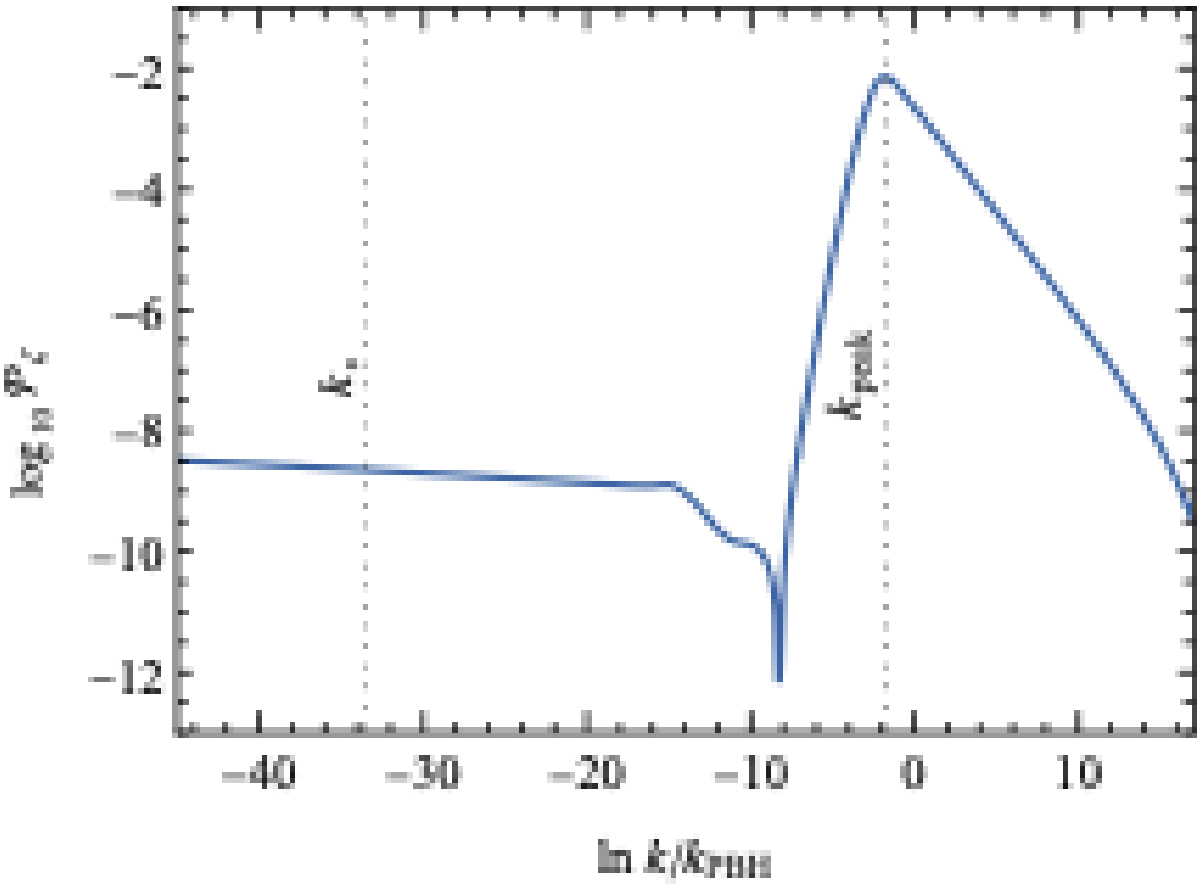
- Amplitude of every $\vec{k}$-mode is independently drawn from a Gaussian distribution, so the background is subject to Gaussian white noise.

Potential for asteroid-mass PBHs



SR and Tomberg: 1810.12608

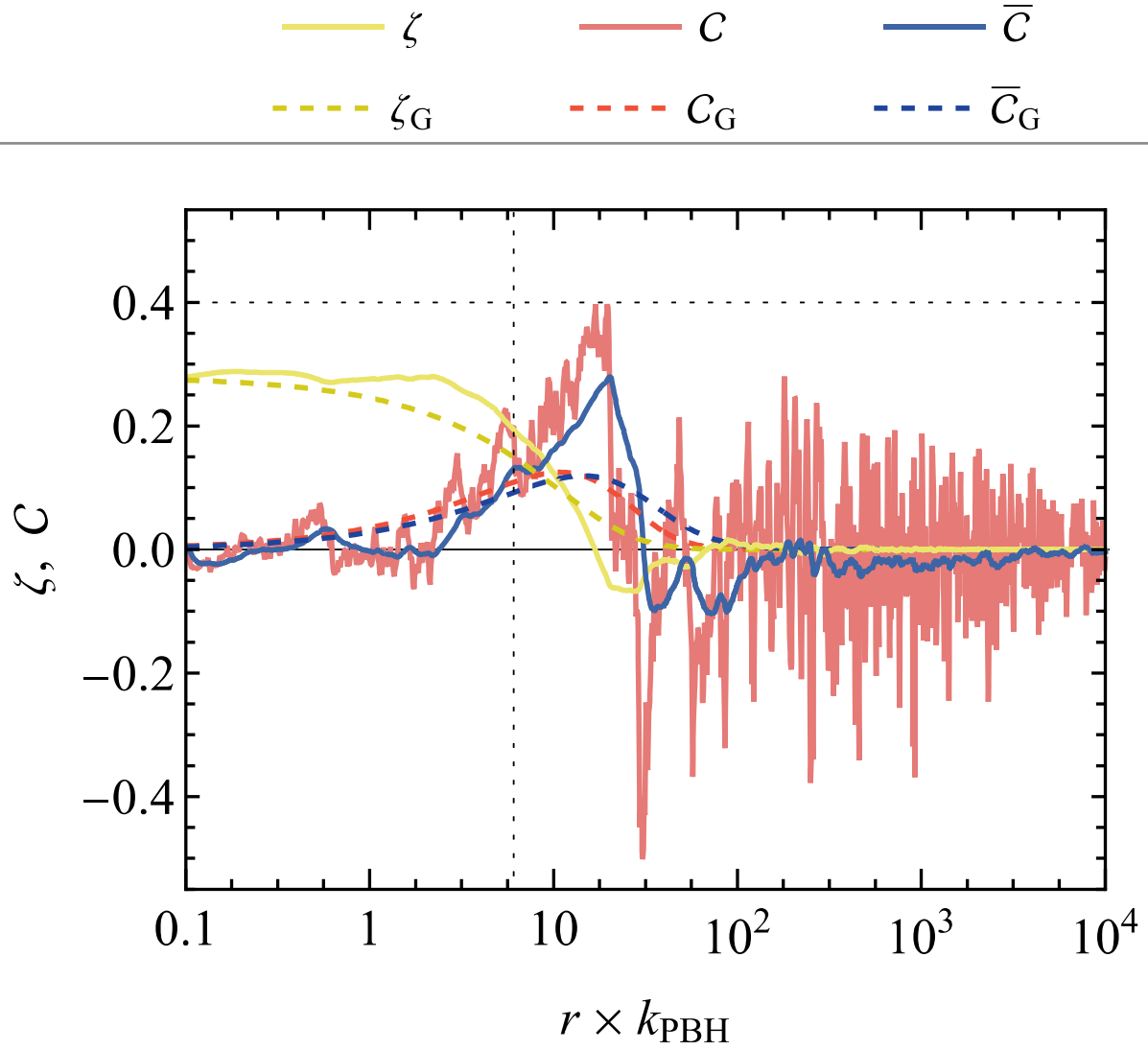
Curvature power spectrum



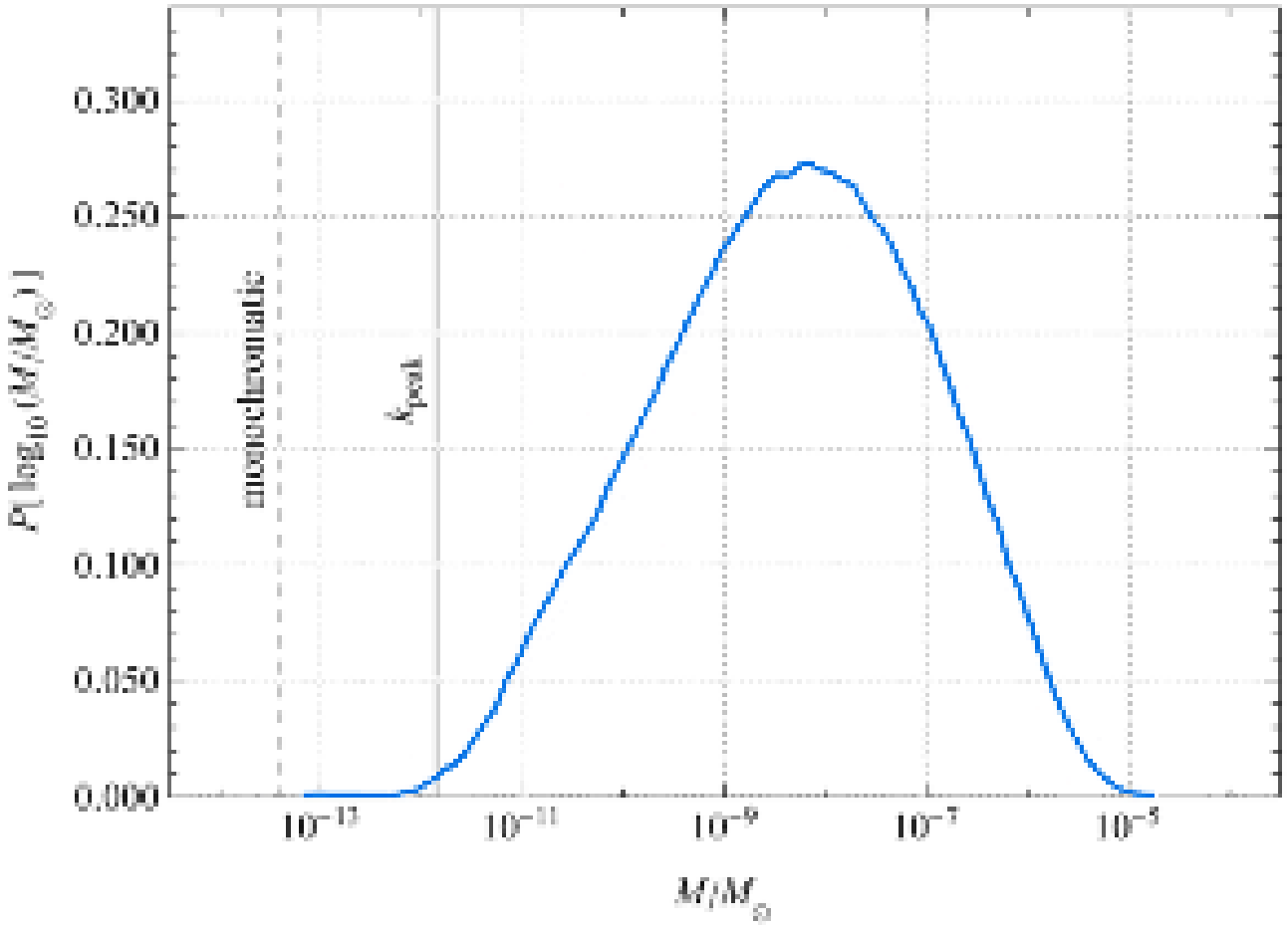
One step at a time

- We solve the coupled background + perturbation evolution.
  - Under the assumption that a patch is spherically symmetric, we find the radial profile from first principles.
  - Repeat $\sim 10^{11}$ times to get statistics.

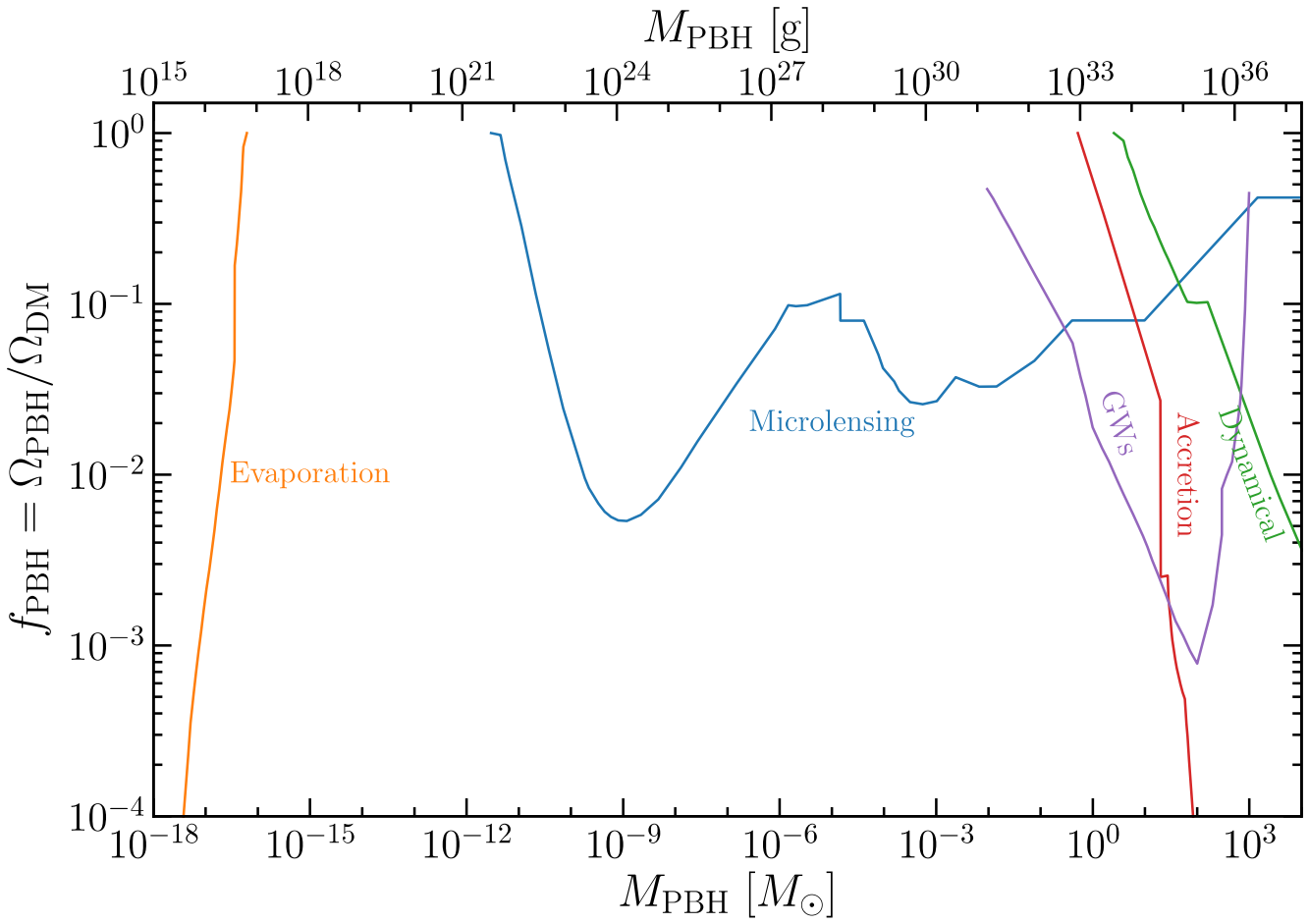
One realisation



PBH mass is stochastic



Window with four orders of magnitude



Green and Kavanagh: 2007.10722

Stochastic spikes

- Stochasticity of the individual patches naively enhances PBH abundance by 10^{14}.
  - Very different from the case of smooth ad hoc profiles or Gaussian mean profile.
- Caveat: inside Hubble radius the spikes lead to large pressure gradients that smoothen the profile.
- Validity of spherical symmetry is unclear: the profile is not expanded around a maximum.

Spiking the conclusions

- PBHs can be dark matter or seed supermassive BHs.
  - Producing PBH seeds in single-field inflation requires large perturbations, so stochastic kicks are important.
- We calculate the density profile starting from the inflaton potential, realisation by realisation.
  - Patches are spiky, far from the Gaussian mean profile.
  - PBH abundance and mass enhanced by orders of magnitude.
- Need to redo collapse simulations with spiky profiles, check spherical symmetry, and use new techniques to get to the tail of the distribution.

Backup slides

Split the system and environment

- The inflaton is split into long and short wavelength modes:

$$\begin{aligned}\phi(t,\mathbf{x}) &= \int \frac{\mathrm{d}^3k}{(2\pi)^{\frac{3}{2}}}\phi_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}} \\ &= \int_{k<\sigma aH} \frac{\mathrm{d}^3k}{(2\pi)^{\frac{3}{2}}}\phi_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}} + \int_{k>\sigma aH} \frac{\mathrm{d}^3k}{(2\pi)^{\frac{3}{2}}}\phi_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}} \\ &\equiv \bar{\phi}(t,\mathbf{x}) + \delta\phi(\mathbf{t},\mathbf{x})\end{aligned}$$

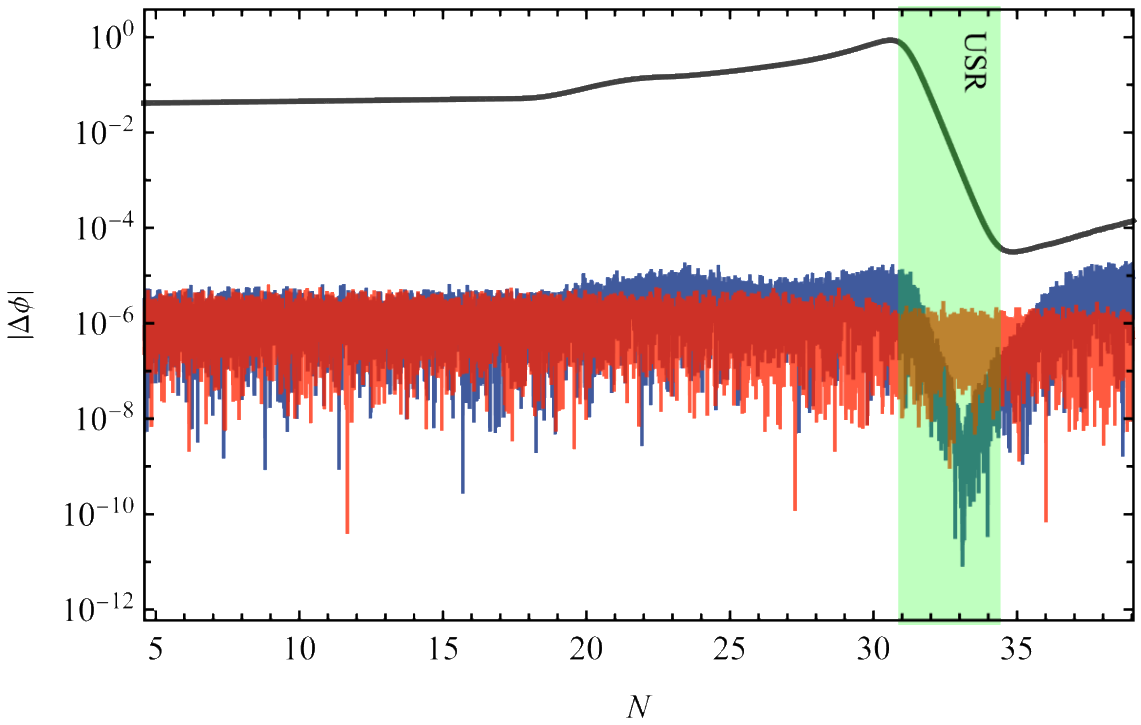
- Here σ<<1 defines the coarse-graining scale at which perturbations join the background.

- As modes join the background, they give stochastic kicks.
  - Noise is white and Gaussian, mean amplitude is determined by the power spectrum of $\delta\phi_{\mathbf{k}}$.

Far down the tail

- PBH density today is $\Omega_{\rm PBH} \approx 9 \times 10^7 \beta \left(\frac{M}{M_\odot} \right)^{-\frac{1}{2}}$.
- The initial fraction of PBHs is β<<1.
- The model of inflation determines β and M.

Classical force vs quantum kicks

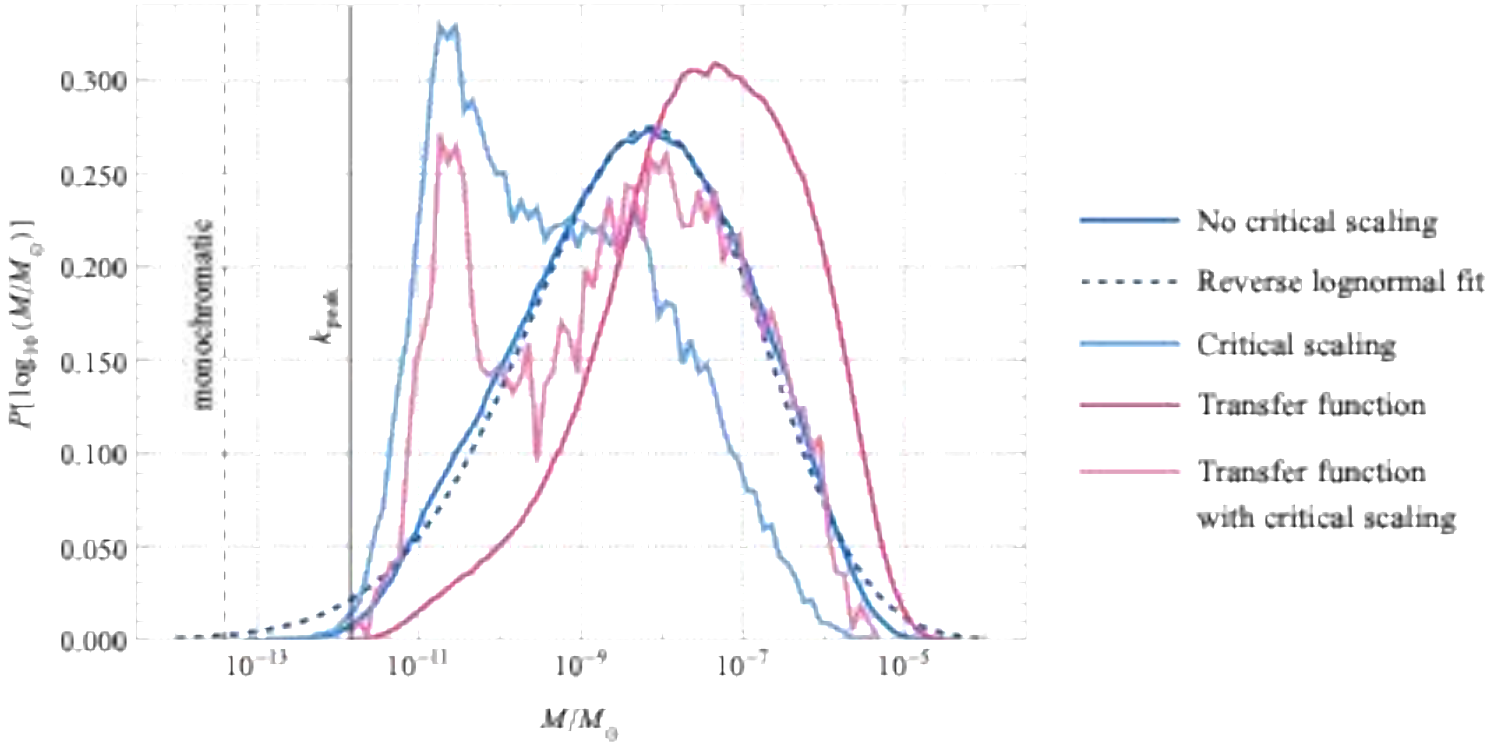


Classical

Full

Simplified

Effects of transfer function and critical collapse



Closing the windows

- Various astrophysics can be used to probe asteroid-mass and supermassive seed PBHs.
  - Planck-mass relics difficult.

- Asteroid-mass: improved lensing, ultra-faint dwarf stellar mass functions (Esser et al: 2503.03352), planetary orbits (Thoss and Burkert: 2409.04518), ...

- Supermassive seeds: cosmic microwave background spectral distortions, ...