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DEPARTMENT OF PHYSICS AND ASTRONOMY

Toponium at the LHC

Singlet *and* octet threshold effects in $pp \to t\bar{t}$

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People who hear of some extraordinary phenomenon start proposing to explain it with improbable hypotheses. First consider the simplest explanation: that it's all nonsense.

— L. D. Landau

ATLAS and CMS: more $t\bar{t}$ than expected at low $m_{t\bar{t}}$

CMS [arXiv:2503.22382]

Dilepton $t\bar{t}$, 13 TeV, 138 fb^{-1}

$m_{t\bar{t}}$ + two spin-correlation angles

excess $> 5\sigma$ over the standard prediction (pQCD)

Fitted signal: $\sigma = 8.8^{+1.2}_{-1.4}$ pb

(fit assumes a spin-0, colour-neutral bound state)

ATLAS [ATLAS-CONF-2025-008]

Dilepton $t\bar{t}$, 13 TeV, 140 fb^{-1}

more events near threshold

7.7σ over the standard prediction (pQCD)

Fitted signal: $\sigma = 9.0 \pm 1.3$ pb

(fit assumes colour-neutral S-wave bound states)

- In both, the extra events sit at the lowest $m_{t\bar{t}}$, where the top pair is only just produced
- Both compare with a 6.4 pb signal estimate with no quoted theory uncertainty. The ATLAS fit gives a signal cross section $(40 \pm 20)\%$ above that estimate.

Why the threshold is special

- At $m_{t\bar{t}} \approx 2m_t \approx 346$ GeV the tops are produced at rest
- Slow tops exchange many gluons: a Coulomb-like attraction
- Fixed-order NLO + parton shower treats the tops as free particles
- Each exchange costs α_s/β (β: relative velocity), not small as $\beta \to 0$: the exchanges must be resummed
- In global fits these bins probe the gluon PDF in the threshold region and help constrain α_s and m_t

Numbers

$m_t = 173$ GeV

top width $\Gamma_t = 1.49$ GeV (lifetime $\sim 10^{-25}$ s)

binding energy $\approx$ 1–2 GeV $\approx \Gamma_t$: the bound state decays as it forms

Toponium: a quasi-bound $t\bar{t}$ state

- Colour-singlet pair: attractive potential $V_1(r) = -C_F\,\alpha_s/r$ at short distance
- $\Gamma_t \approx$ binding energy: the bound states smear into *one broad bump* just below $2m_t$
- Everything is in the Green's function $G(E, p)$; G_0: free propagation ($V = 0$)

Green's function [JKT (2)–(6)]

$$G(\vec{p}, E) = \left\langle \vec{p} \left| \frac{1}{E + i\Gamma_t - H} \right| \vec{r} = 0 \right\rangle$$

$$H = \frac{p^2}{m_t} + V(r), \qquad E = m_{t\bar{t}} - 2m_t$$

$$G_0 = \frac{1}{E + i\Gamma_t - p^2/m_t} \quad (V = 0)$$

How to put toponium into a simulation

- Per event: $E = m_{t\bar{t}} - 2m_t$ and p^*, the top momentum in the $t\bar{t}$ rest frame
- $|G/G_0|^2$: the effect of the potential at that E and p^*
- Multiply $|\mathcal{M}|^2$ by $|G/G_0|^2$ near threshold

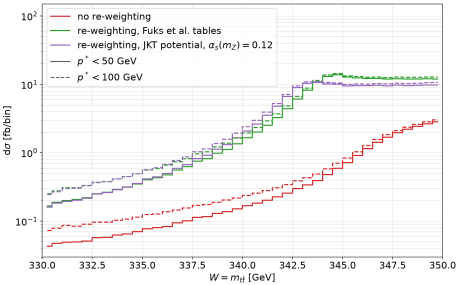
Matrix-element reweighting [Fuks et al., arXiv:2411.18962, (15)]

$$|\mathcal{M}|^2 \longrightarrow \left|\frac{G(E,p^*)}{G_0(E,p^*)}\right|^2 |\mathcal{M}|^2 \qquad \text{near threshold}$$

How the experiments model it today

- **CMS**: adds a new spin-0 particle (η_t) just below $2m_t$, a stand-in for the toponium ground state
  [Fuks, Hagiwara, Ma, Zheng, PRD 104 (2021) 034023; Maltoni et al., arXiv:2401.08751]
- **ATLAS**: multiplies events by $|G/G_0|^2$, but only for colour-neutral pairs made from two gluons
  [Fuks, Hagiwara, Ma, Zheng, arXiv:2411.18962]
- In both, G comes from the QCD potential, solved with the method of Jeżabek, Kühn, Teubner
  [JKT, Z. Phys. C 56 (1992) 653]

Fuks et al.: the colour-singlet result, reproduced



$gg \to t\bar{t} \to b\ell\nu\,\bar{b}\ell\nu$, colour singlet only, 13 TeV: the setup of Fuks et al., Fig. 3

$p^* < 50$ GeV	Fuks et al.	reproduced
peak of $d\sigma$	≈ 14 fb/bin	13.8 fb/bin
peak position	≈ 345 GeV	344.75 GeV
$W = 350$, with/without	$\approx 12/\approx 3$	12.0/2.8
$\langle p^* \rangle$ at $E = -2$ GeV	≈ 20 GeV	20.7 GeV

Fuks et al.: read off Fig. 3; $\langle p^* \rangle$ from Eq. (22)

- With the Fuks et al. G tables: same curves as Fuks et al., Fig. 3
- We could match the Fuks et al. tables *only* with a fixed Coulomb force at $\alpha_s = 0.15$, not with the $\alpha_s = 0.12$ or JKT potential stated in the paper
- Purple: the same events with G from the full JKT potential at $\alpha_s(m_Z) = 0.12$ (slide 15): peak ≈ 1 GeV lower, lower above it

What this recipe leaves out

Singlet only

[Fuks et al., (16), (19)–(20)]

$$gg: \quad |\mathcal{M}|^2 \;\to\; |\mathcal{M}|^2 + \left(\left| \frac{G_1}{G_0} \right|^2 - 1 \right) |\mathcal{M}_1|^2$$

- Fuks et al.'s signal sample: **only** $gg \to t\bar{t}$, **only** its colour singlet, reweighted (Fuks et al., Figs. 2–5 show this sample alone)
- No octet pairs in Figs. 2–5: "we ignored colour-octet contributions that gain importance with increasing W" [Fuks et al., Sec. 4]. At leading order near threshold, the octet share of $gg \to t\bar{t}$ is $\approx 72\%$.
- ATLAS and CMS do not directly tag the $t\bar{t}$ colour state. Inclusive bins generally receive contributions from both channels.
- Octets enter only as a standard $t\bar{t}$ background, without repulsion (Fuks et al., Sec. 5; ATLAS: "does not include contributions from P-wave and colour-octet states" [ATLAS-CONF-2025-008, Sec. 3.2])

Colour: the pair is either neutral or coloured

Final state

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

singlet (colour-neutral): attract, $C_F = \frac{4}{3}$
octet (coloured): repel, $C_F - \frac{C_A}{2} = -\frac{1}{6}$

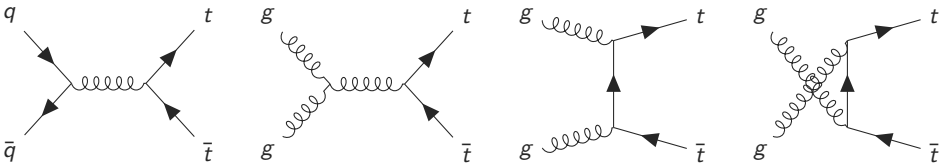
Initial states

$$q\bar{q}: \ \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

$$gg: \ \mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$$

- Colour is conserved: whatever goes in, the $t\bar{t}$ pair comes out as a singlet ($\mathbf{1}$) or an octet ($\mathbf{8}$)
- $q\bar{q} \to$ one gluon $\to t\bar{t}$: a single gluon carries colour, so the pair is **always an octet**
- $gg \to t\bar{t}$ can give either

The four tree-level diagrams



$q\bar{q}$, s-channel	gg, s-channel	gg, t-channel	gg, u-channel
octet only	octet only	singlet + octet	

- Through a single gluon (s-channel): the pair is always coloured (octet)
- Exchanging a top (t- and u-channel): the pair can come out colour-neutral (singlet)

Singlet and octet contributions to the matrix element

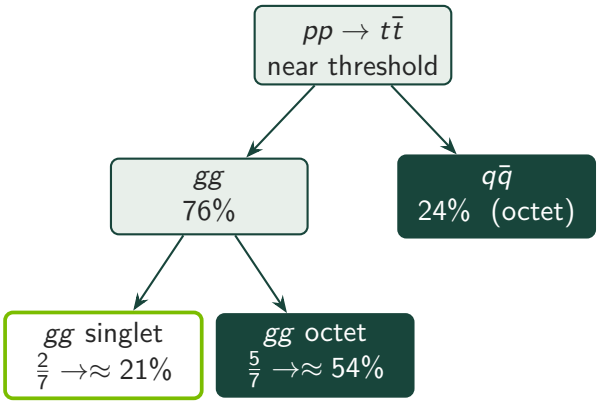
MadGraph writes the gg amplitude in two colour pieces; its colour matrix splits exactly into a singlet and an octet part:

$$C = \tfrac{1}{3}\begin{pmatrix} 16 & -2 \\ -2 & 16 \end{pmatrix} = \underbrace{\tfrac{2}{3}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}}_{C_1 \text{ singlet}} + \underbrace{\tfrac{1}{3}\begin{pmatrix} 14 & -4 \\ -4 & 14 \end{pmatrix}}_{C_8 \text{ octet}}$$

$$|\mathcal{M}|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_8|^2$$

- The two parts never mix, so each can get its own Green's function
- At leading order in the threshold limit, the singlet-to-octet ratio is $2:5$ and the singlet share of gg is $2/7$

Who makes the threshold rate?



Colour: a $t\bar{t}$ pair is a singlet (attract) or an octet (repel); $q\bar{q}$ pairs are always octets

Cross sections near threshold (LO, 13 TeV, $m_{t\bar{t}} \leq 350$ GeV, $p^* \leq 100$ GeV):
gg: 0.122 pb, $q\bar{q}$: 0.039 pb

singlet share of gg: $2/7$ (0.28 in the simulated events)

$\approx$ **4**/**5** of the pairs at threshold are colour **octets**

From 23% to 4.2%: adding $q\bar{q}$ and the octet

first CMS $m_{t\bar{t}}$ bin $[250, 400]$ GeV; same MadGraph setup and G in every row

every row: LO $|\mathcal{M}|^2 \times |G/G_0|^2$, the same order as the ATLAS template

Δ: extra cross section from G LO: the bin's cross section without G

sample	G applied to	Δ	LO	$K = 1 + \Delta/\text{LO}$
gg singlet events (Fuks et al.'s sample)	singlet	0.195 pb	0.85 pb	1.229
gg, all colours	singlet	0.195 pb	3.20 pb	1.061
gg, all colours	singlet + octet	0.177 pb	3.20 pb	1.055
pp ($gg + q\bar{q}$)	singlet	0.195 pb	4.01 pb	1.049
pp ($gg + q\bar{q}$)	singlet + octet	0.169 pb	4.01 pb	**1.042**

- Adding the octet gg pairs and the $q\bar{q}$ pairs makes LO bigger, with the same Δ: K drops
- The octet repulsion lowers Δ itself (0.195 $\rightarrow$ 0.169 pb), but only for octets near threshold
- The Fuks et al. signal added to the full pp prediction (as ATLAS uses it) gives the $K = 1.049$ row; 1.229 divides by the singlet events alone

MC errors ≤ 0.001 on every K; Δ from $326 \leq m_{t\bar{t}} \leq 350$ GeV, $p^* < 100$ GeV

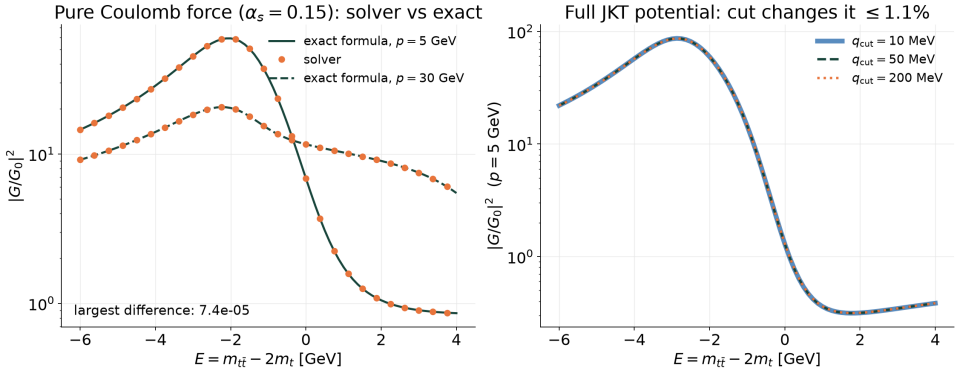
What we did: the Green's function, following JKT

- **Potential**: perturbative QCD with running α_s above 5 GeV, linear confinement below [JKT (13)–(15)]
- **Regularisation**: we cut the potential below $q_{\rm cut} = 50$ MeV and match its short-distance normalisation [JKT (19)–(20)]. The reweighting factor is numerically stable at the $\sim 1\%$ level (slide 16).
- **Lippmann–Schwinger equation** for $G = G_0 K$, solved on a momentum grid; JKT's subtraction removes the $q = p$ singularity of the kernel [JKT (16)–(23)]

$$K(p) = 1 + \int_0^\infty dq\, \hat{V}(p,q)\, G_0(q)\, K(q) \qquad (\hat{V}:\ \text{S-wave potential})$$

- **Octet**: the same equation with colour factor $C_F \to -\frac{1}{6}$ (repulsive) [HSY (9)–(10)]

Is the solver right?



- Plotted: $|G/G_0|^2$, the factor applied to each event, against E at fixed top momentum p
- Left: a Coulomb potential ($\alpha_s = 0.15$) has an exact analytic G; solver (dots) and formula (lines) differ by less than 10^{-4}
- Right: full JKT potential. At $q_{\rm cut} = 10$, 50 and 200 MeV, the curves agree within 1.1%.
- Also reproduced: JKT's published figures, and the Fuks et al. tables (Coulomb, $\alpha_s = 0.15$)

Both colour states in MadGraph, and K-factors

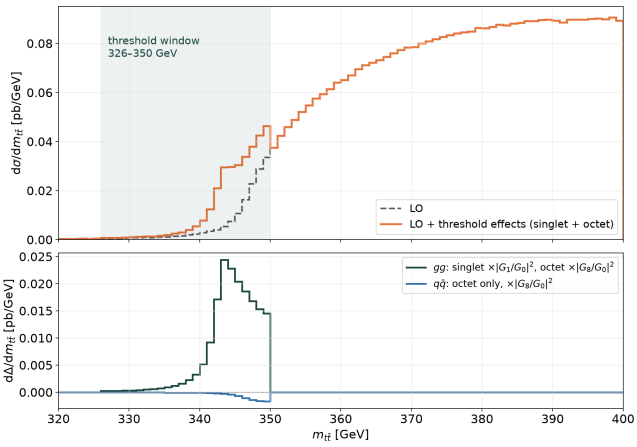
$$gg:\ |\mathcal{M}|^2 \to \left|\frac{G_1}{G_0}\right|^2 |\mathcal{M}_1|^2 + \left|\frac{G_8}{G_0}\right|^2 |\mathcal{M}_8|^2 \qquad q\bar{q}:\ |\mathcal{M}|^2 \to \left|\frac{G_8}{G_0}\right|^2 |\mathcal{M}|^2$$

- Only near threshold: $m_{t\bar{t}} \leq 350$ GeV, $p^* < 80$ GeV (K-factors: $p^* < 100$ GeV); the same α_s (CT25NNLO) everywhere

$$K(\text{bin}) = \frac{\sigma_{\text{with attraction}}(\text{bin})}{\sigma_{\text{LO}}(\text{bin})}$$

- K multiplies the standard prediction in each measured bin
- Extra samples made only near threshold, so K is precise to 10^{-4}; cross-checked against full samples and an independent calculation by M. Yáñez Reyes

Where the enhancement comes from: the threshold region



- Orange vs grey: the toponium bump just below 346 GeV
- Green: gg pairs, dominated by the singlet bump ($+0.177$ pb)
- Blue: $q\bar{q}$ pairs (all octet) go *down* (-0.008 pb)
- ATLAS and CMS bins are wide and average over this peak, which is switched on only below 350 GeV

First $m_{t\bar{t}}$ bin

CMS $[250, 400]$: $K = 1.042$
ATLAS $[325, 400]$: $K = 1.042$
± 0.003 (model); $K = 1$ above 400 GeV

Why this matters for PDF, α_s and m_t fits

- Global fits use measured $t\bar{t}$ distributions to constrain the gluon PDF in the kinematic region probed by top pair production, as well as α_s and m_t
- The threshold bins carry most of the sensitivity to m_t
- If the fit's prediction lacks the threshold effect, the fit absorbs it into $g(x)$, α_s or m_t
- The K-factors are given in the ATLAS and CMS binnings, to multiply the fit's prediction bin by bin; the 23% of singlet events alone would overcorrect, the first-bin correction is 4.2%

Summary

- The dilepton fits give toponium-like signal cross sections of ≈ 9 pb in both experiments
- Toponium is the simplest explanation: plain Standard Model, once the attraction between slow tops is included
- The signal templates model colour-singlet states. Colour-singlet gg pairs make up about $1/5$ of our near-threshold LO sample.
- With both channels and both colour states at the same order: singlet signal divided by singlet events alone 23%; on top of the full pp prediction 4.9%; with the octet 4.2% (first $m_{t\bar{t}}$ bin)
- We calculated the Green's functions for both colour states (following JKT) and put both into MadGraph
- Including the octet removes 13.5% of the toponium excess; first H_T bin: $+6.4\%$
- Next: refit the gluon and α_s with these K-factors; top masses 170–175 GeV on the MSU cluster for m_t

Backup

The story

1. ATLAS and CMS see more top–antitop pairs than expected near threshold
2. Could toponium, a short-lived top–antitop "atom", explain it?
3. How it is simulated today: only some of the pairs get the attraction
4. What the template leaves out: most pairs at threshold
5. What we did: the full calculation for both colour states
6. The result: how much each measured bin goes up, and what it means for PDF, α_s and m_t fits

What about the octet?

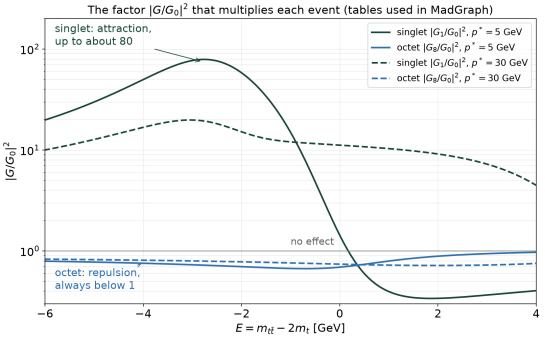
- Octets are $\approx 4/5$ of the threshold LO sample. Repulsion gives $\Delta_8 = -0.026$ pb.
- Singlets are $\approx 1/5$ of the sample. Attraction gives $\Delta_1 = +0.195$ pb.
- So including the octet removes 13.5% of the toponium excess

  Δ from $326 \leq m_{t\bar{t}} \leq 350$ GeV, $p^* < 100$ GeV

MadGraph, first CMS $m_{t\bar{t}}$ bin $[250, 400]$

singlet only: $K = 1.049$
singlet + octet: $K = 1.042$
$\Delta K = -0.0066 \pm 0.0001$

Small in K, but far above the simulation error (0.0001) and larger than the octet-model spread (≤ 0.001): a consistent treatment includes it



The force between the tops

Short distance: one-gluon exchange, with a coupling that weakens at high momentum ($\alpha_s(m_Z) = 0.12 \Rightarrow \Lambda = 223$ MeV) [JKT (14)]

$$\alpha_{\overline{\rm MS}}(Q^2) = \frac{4\pi}{b_0 \ln(Q^2/\Lambda^2) + \frac{b_1}{b_0} \ln\ln(Q^2/\Lambda^2)}, \qquad b_0 = 11 - \tfrac{2}{3} n_F,\ b_1 = 102 - \tfrac{38}{3} n_F$$

The potential, including its first quantum correction [JKT (13)]

$$\tilde{V}_{\rm pert}(Q^2) = -\frac{16\pi}{3} \frac{\alpha_{\overline{\rm MS}}(Q^2)}{Q^2} \left[1 + \left(\frac{31}{3} - \frac{10}{9} n_F\right) \frac{\alpha_{\overline{\rm MS}}(Q^2)}{4\pi}\right]$$

Long distance (below 5 GeV): a form that grows linearly with distance, i.e. confinement [JKT (15)]

$$\tilde{V}(p) = -\frac{4}{3} \frac{48\pi^2}{27} \frac{1}{p^2} \left(\frac{1}{\ln(1 + p^2/\Lambda_R^2)} + C\right), \qquad \Lambda_R = 400 \text{ MeV}$$

$C = -0.0262$ makes the two pieces join smoothly at 5 GeV

JKT's values. For the K-factors: the same potential with CT25NNLO's running α_s ($\alpha_s(m_Z) = 0.118$), $C = -0.0332$

Cutting off the long-distance part

[JKT (19)–(20)]

Problem: the confining part blows up at very small momentum (very large distance), which breaks the numerics. The short top lifetime limits sensitivity to that region. JKT therefore cut it off below $q_{\mathrm{cut}} = 50$ MeV:

$$\tilde{V}_{\mathrm{cut}}(p) = \begin{cases} \tilde{V}(p), & p > q_{\mathrm{cut}} \\ -\dfrac{C_1}{p^2 + q_{\mathrm{cut}}^2} + \tilde{V}_0\,\delta^3(\vec{p}), & p < q_{\mathrm{cut}} \end{cases}$$

- The two pieces are matched at q_{cut}, and the potential is normalised to $V(r = 1\ \mathrm{GeV}^{-1}) = -\frac{1}{4}$ GeV
- The normalisation adds a constant to the potential (here $\approx +5.7$ GeV)
- Check: at $q_{\mathrm{cut}} = 10$, 50 and 200 MeV, the $|G/G_0|^2$ curves agree within 1.1% (slide 16)

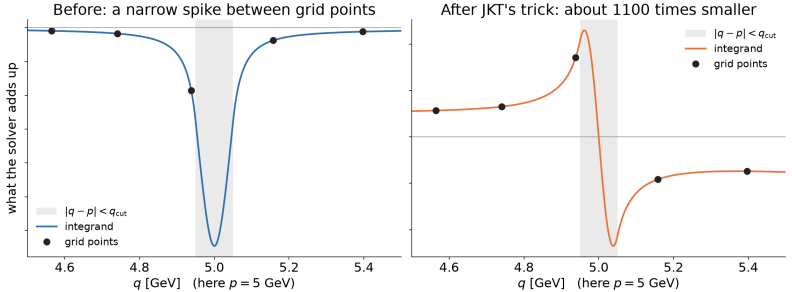
Solving for G [JKT Sec. 3]

G obeys an integral equation: free motion plus repeated scattering off the potential ($G = G_0 K$) [(16)–(18)]

$$K(p) = 1 + \int_0^\infty dq\, \hat{V}(p,q)\, G_0(q)\, K(q)$$

JKT's trick: add and subtract the spike at $q = p$, so it is integrated separately on a fine grid (exact) [(21)–(22)]

$$\big[1 - \mathcal{A}(p)\big] K(p) + \int_0^\infty dq\, \hat{V}\, G_0 \big[K(p) - K(q)\big] = 1$$



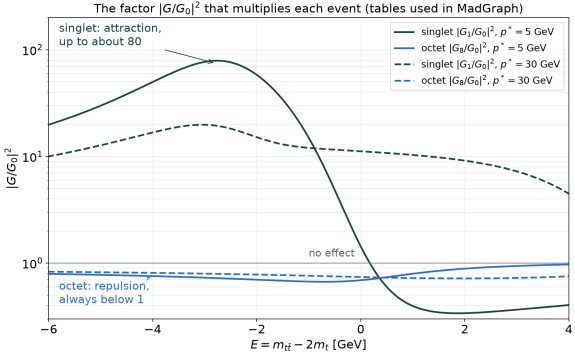
What the solver adds up near $q = p$ ($p = 5$ GeV, $E = -2$ GeV). Left: the spike (width $\approx q_{\rm cut}$) falls between grid points, so a plain sum misses it. Right: after the trick, what is left is about 1100 times smaller and its two sides cancel, so the grid handles it. Then: one linear system with 240 unknowns per energy [(23)]

The octet, with the same solver

Same potential, but the colour factor changes from $+\frac{4}{3}$ (attract) to $-\frac{1}{6}$ (repel) [HSY (9)–(10)]

$$\tilde{V}_8(q) = +\frac{1}{6}\,\frac{4\pi\alpha_s(Q^2)}{q^2}\left[1+\mathcal{O}(\alpha_s)\right]$$

- Repulsive: no bound states, the octet rate goes *down*
- No confinement problem, so no cut is needed
- Four versions of the octet potential give the uncertainty
- Near threshold: the singlet factor reaches about 80, the octet factor stays below 1, down to about 0.65



Both colour states inside MadGraph

- Each event's $|\mathcal{M}|^2$ is split into singlet and octet, and each part gets its own factor:

$$gg: \ |\mathcal{M}|^2 \to \left|\frac{G_1}{G_0}\right|^2 |\mathcal{M}_1|^2 + \left|\frac{G_8}{G_0}\right|^2 |\mathcal{M}_8|^2 \qquad q\bar{q}: \ |\mathcal{M}|^2 \to \left|\frac{G_8}{G_0}\right|^2 |\mathcal{M}|^2$$

- Only near threshold ($m_{t\bar{t}} \leq 350$ GeV, $p^* < 80$ GeV); everything else unchanged
- The same α_s everywhere (CT25NNLO): in the PDFs, the matrix element and both potentials
- MadGraph then generates events directly with the new $|\mathcal{M}|^2$

From events to K-factors

$$K(\text{bin}) = \frac{\sigma_{\text{with attraction}}(\text{bin})}{\sigma_{\text{LO}}(\text{bin})} = 1 + \frac{\Delta(\text{bin})}{\sigma_{\text{LO}}(\text{bin})}$$

- K = how much a measured bin goes up: multiply the standard prediction for that bin by K
- Standard prediction: 10^7 LO events, $pp \to t\bar{t} \to b\ell\nu\,\bar{b}\ell\nu$, 20.13 pb
- The change Δ: extra samples made only near threshold, where the effect is, so K is precise to 10^{-4}
- Cross-checked against full samples, and against an independent calculation by M. Yáñez Reyes (agreement to a few 10^{-4})
- Done in the ATLAS and CMS binnings of $m_{t\bar{t}}$, $y_{t\bar{t}}$, H_T, p_T^t, $|y_t|$, and $|y_t|$ vs p_T^t (2D)

Other measured distributions

K-factors (singlet + octet)	
$H_T^{t\bar{t}}$ (ATLAS), $[0, 90]$ GeV	1.064
$H_T^{t\bar{t}}$, $[90, 170]$ GeV	1.002
p_T^t, first bin $[0, 60]$ GeV	1.039
$y_{t\bar{t}}$ (CMS 2ℓ), all bins	1.008–1.010
$\lvert y_{t\bar{t}}\rvert$, $\lvert y_{\text{boost}}\rvert$ (ATLAS)	1.008–1.010
total cross section	1.008

Only bins with slow tops change; rapidity bins all go up by about 1%

- Near threshold ($m_{t\bar{t}} \leq 350$ GeV): gg $+0.177$ pb, $q\bar{q}$ -0.008 pb
- Including the octet removes 13.5% of the singlet-only excess
- Choice of octet potential: changes K by at most 0.001
- p^* selection 50 vs 100 GeV: changes Δ by 7%
- Simulation statistics: ± 0.0001 on K