Gravitational waves and primordial black holes from metastable vacuum decay in the dark sector



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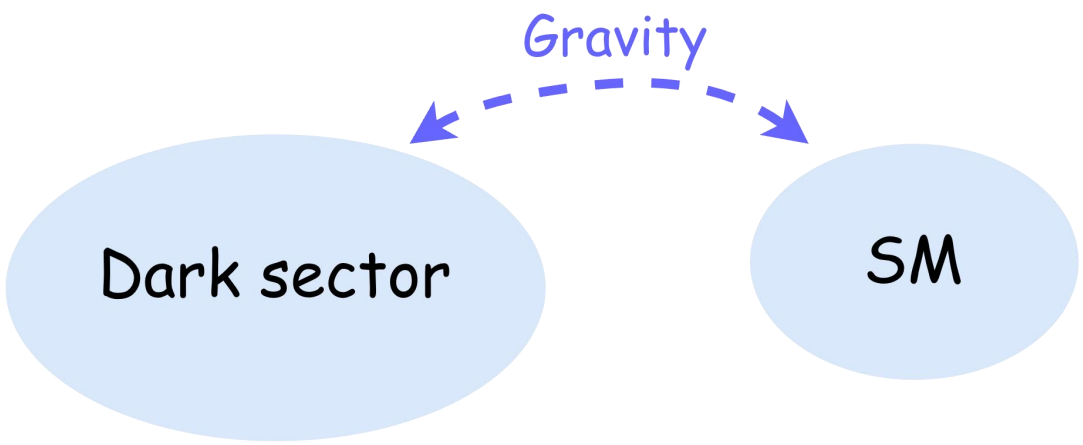
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PIKIMO 21
Pittsburgh, November 7, 2026

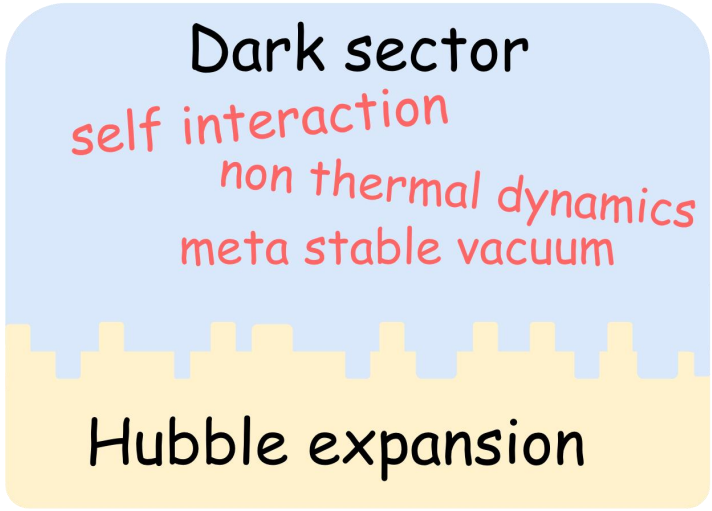
In collaboration with Haipeng An and Chen Yang
arXiv:2601.14366 and arXiv:2606.06587

Observable signals from a dark-sector transition

A dark sector coupled to the SM through gravity



Metastable vacuum decay

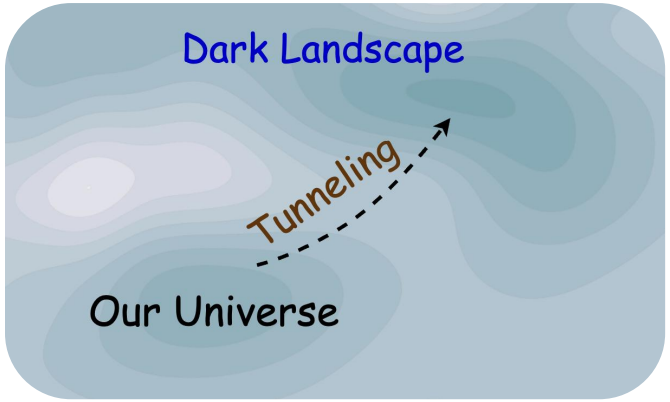


What observable signals can this transition produce?

Gravitational waves

Primordial black holes

Dark radiation and its cosmological constraint



Vacuum decay during radiation domination

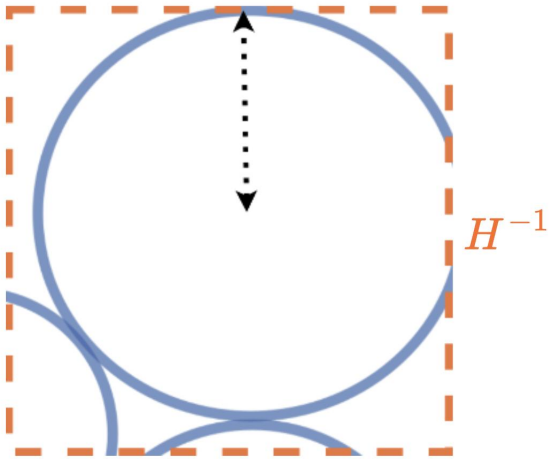
Radiation domination (RD)

Approximately constant nucleation rate

$$\frac{\Gamma}{\mathcal{V}} = constant$$ No coupled fluid or plasma

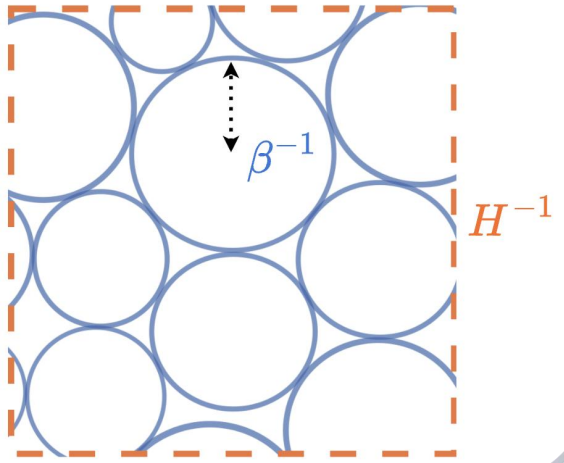
Transition completes when

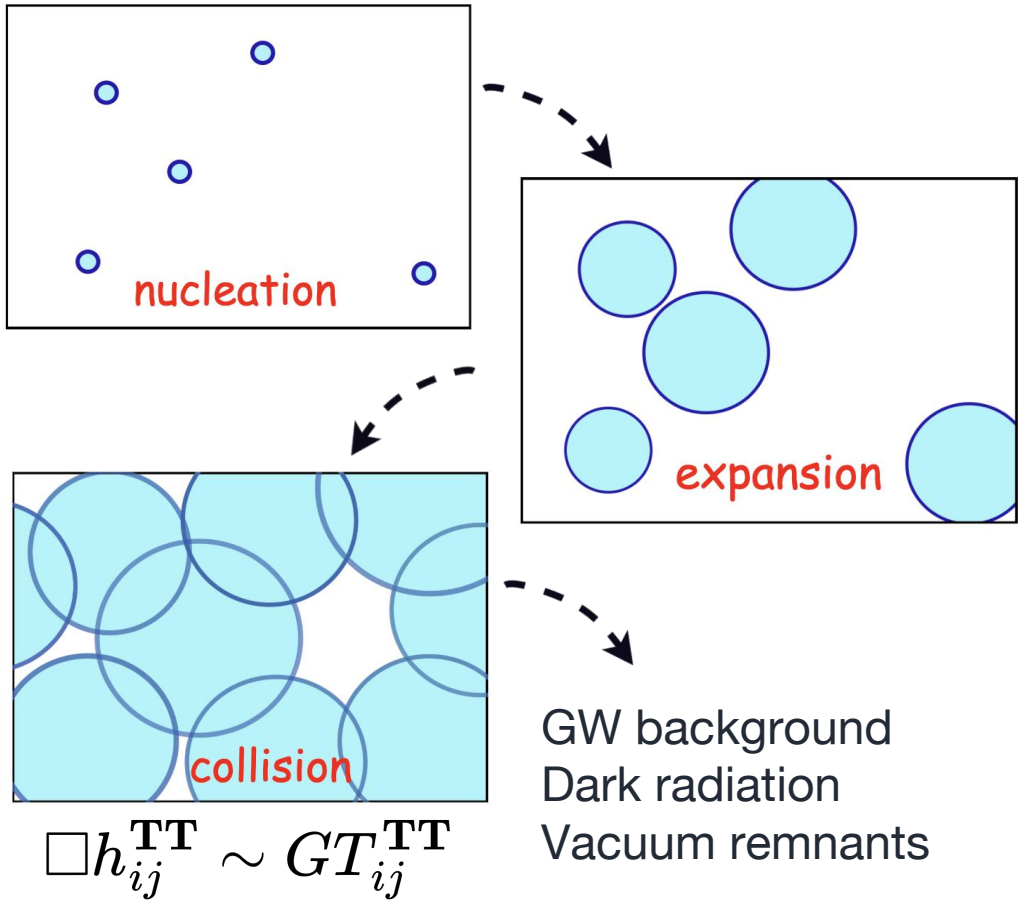
$$\frac{\Gamma}{\mathcal{V}} \sim H^4$$



Thermal transition

$$\frac{\Gamma}{\mathcal{V}} \sim \exp(\beta t)$$





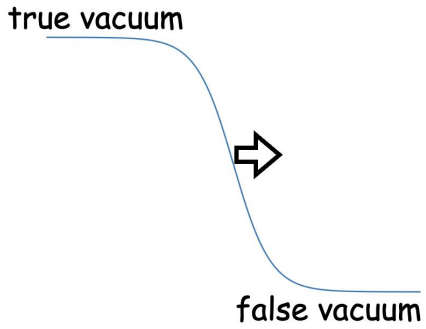
$$\Box h_{ij}^{\mathbf{TT}} \sim G T_{ij}^{\mathbf{TT}}$$

GW background
Dark radiation
Vacuum remnants

Bubble walls reach large Lorentz factors

No coupled fluid or plasma

Released vacuum energy accelerates the bubble walls



Effective bubble energy $$E_{bubble} = -\frac{4\pi}{3}(aR)^3\delta V + 4\pi(aR)^2\gamma\sigma$$

[Lewicki, Toczek et al., arXiv:2305.04924]

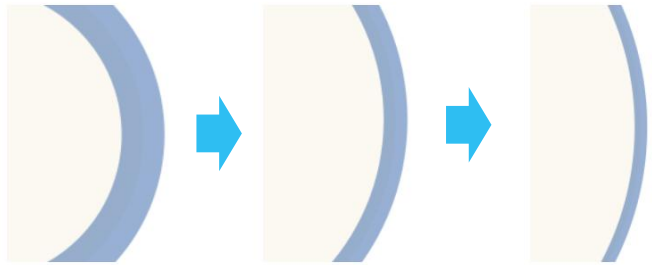
Bubble-wall Lorentz factor $$\gamma(R) = \frac{R}{R_0} \sim \frac{m}{H} \gg 1 \quad v_{wall} \approx 1$$

$\gamma \sim 10^{10}$

EW-scale example

Large Lorentz factor

Lorentz-contracted wall thickness

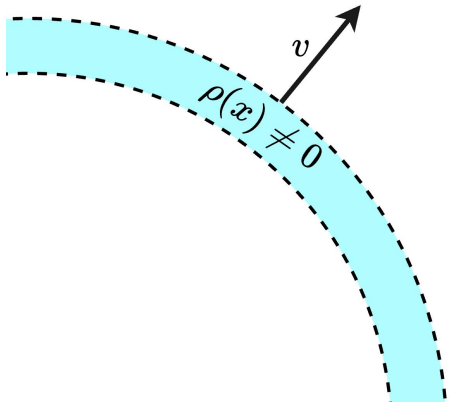


Resolving the contracted walls makes direct lattice simulations costly

Models for the GW source after collisions

How long does the GW source persist after bubble collisions?

Thin-wall approximation

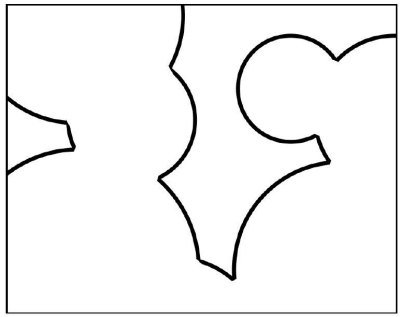


$$T_{ij} \approx \hat{x}_i \hat{x}_j \rho_{wall}$$

GW energy-density spectrum

$$\frac{d\rho_{\rm GW}}{d\ln k} \propto k^3 \sum_{ij} \left(|I_{c,ij}|^2 + |I_{s,ij}|^2 \right)$$

Both cosine and sine source integrals contribute

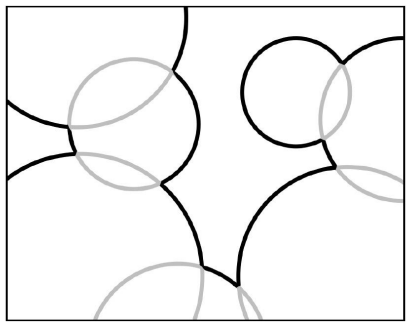


Envelope model

The model removes the wall contribution after collision

$$\tilde{T}_{ij}^{\rm TT}(\eta, k) = 0, \quad \eta > \eta_{end}$$

[Jinno et al., arXiv:1605.01403]



Bulk-flow model

The wall contribution persists and redshifts in an expanding universe

$$\tilde{T}_{ij}^{\rm TT} \propto \begin{cases} C & \text{Minkowski} \\ a^{-2} & \text{RD} \end{cases}$$

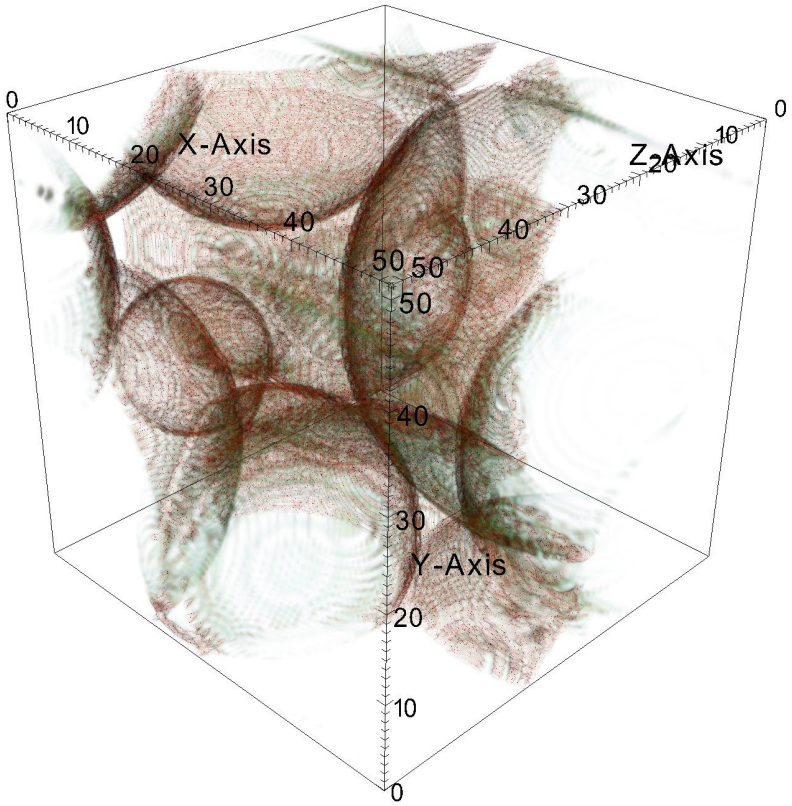
$$\eta > \eta_{\rm end}$$

[Konstandin, arXiv:1712.06869; Jinno et al., arXiv:1707.03111]

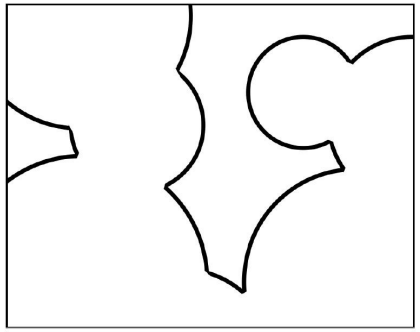
Post-collision dynamics favor a persistent source

Which model better describes the transition we consider?

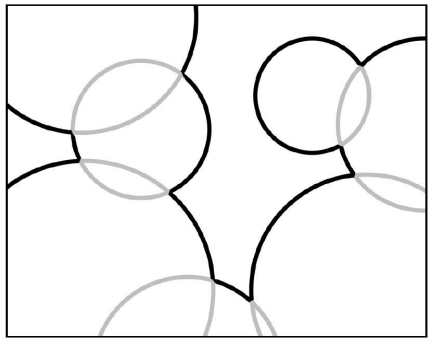
3+1D lattice simulation



Energy density after bubble collisions



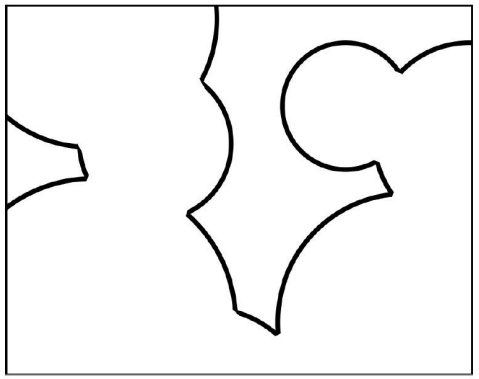
Envelope model



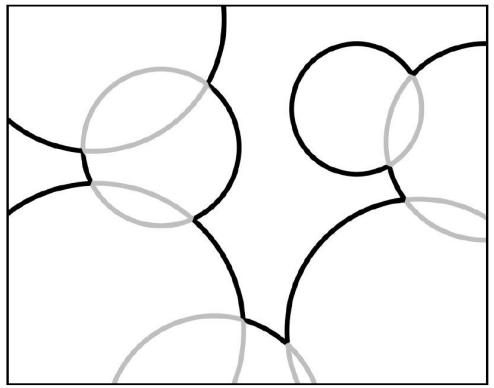
Bulk-flow model

At large Lorentz factors,
the simulated dynamics are closer to the bulk-flow description

Source lifetime and cosmic expansion shape the spectrum



Envelope model



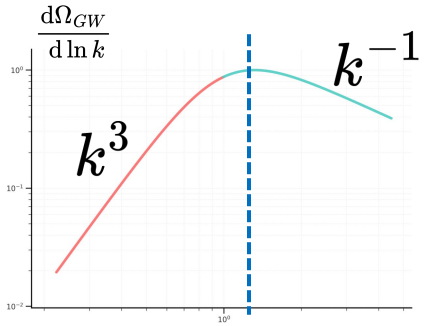
Bulk-flow model

Minkowski background

For $k \to 0$

$$\frac{\mathrm{d}\Omega_{GW}}{\mathrm{d}\ln k} \propto k^3 \cdot [C]^2$$

Causality

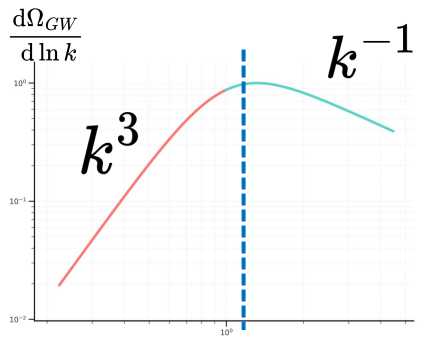


RD background

For $k \to 0$

$$\frac{\mathrm{d}\Omega_{GW}}{\mathrm{d}\ln k} \propto k^3 \cdot [C]^2$$

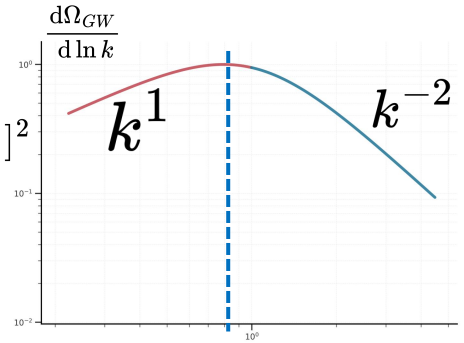
Causality



For $k \to 0$

$$\frac{\mathrm{d}\Omega_{GW}}{\mathrm{d}\ln k} \propto k^3 \cdot [\frac{1}{k}]^2$$

$$\propto k^1$$



Our case

Hubble friction
RD Green's function
Persistent source

$$\frac{\mathrm{d}\Omega_{GW}}{\mathrm{d}\ln k} \; ?$$

Infrared spectrum and source lifetime

Post-collision source in radiation domination

$$\widetilde{T}_{ij}^{\mathrm{TT}}(\eta, \mathbf{k}) \propto a^{-2}(\eta) \propto \eta^{-2} \quad (k \to 0)$$

Time-averaged GW spectrum

$$\frac{d\rho_{\mathrm{GW}}}{d\ln k} \propto k^3 \sum_{ij} \left(|I_{c,ij}|^2 + |I_{s,ij}|^2\right)$$

$$I_{c,ij} = \int_{\eta_i}^{\eta_f} d\eta\, a(\eta) \cos(k\eta) \widetilde{T}_{ij}^{\mathrm{TT}}(\eta, \mathbf{k})$$

GW spectrum behavior

$$k \ll \eta_f^{-1} : \qquad \frac{d\rho_{\mathrm{GW}}}{d\ln k} \propto k^3$$

$$\eta_f^{-1} \ll k \ll \eta_b^{-1} : \qquad \frac{d\rho_{\mathrm{GW}}}{d\ln k} \propto k^3 \ln^2(k\eta_b)$$

η_β: end of the main collisions ηf: end of the effective GW source

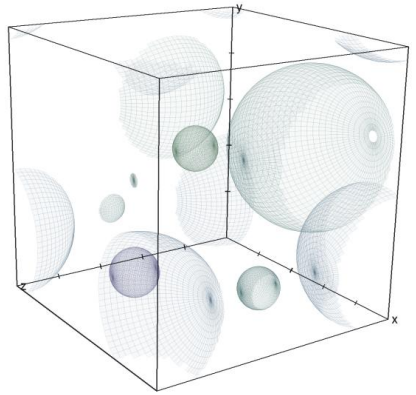
Two complementary simulation methods

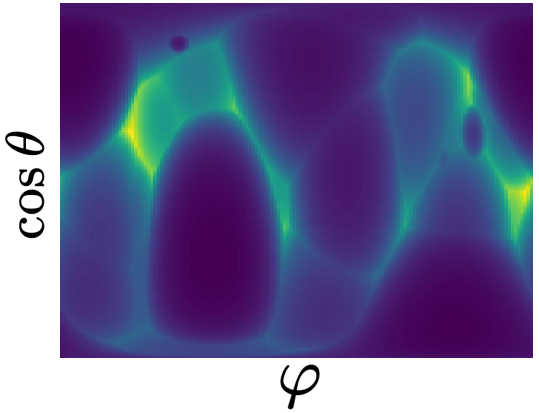
Thin-wall simulation

Sum the TT stress over all bubbles

$$\tilde{T}_{ij}^{\mathrm{TT}}(\eta,\mathbf{k}) = \sum_n e^{-i\mathbf{k}\cdot\mathbf{r}_n}\Lambda_{ij,kl}(\hat{\mathbf{k}})\int \mathrm{d}x^3 e^{-i\mathbf{k}\cdot\mathbf{x}} T^B_{ij,n}(\eta,\mathbf{x})$$

Treat each bubble as a two-dimensional surface in the thin-wall approximation



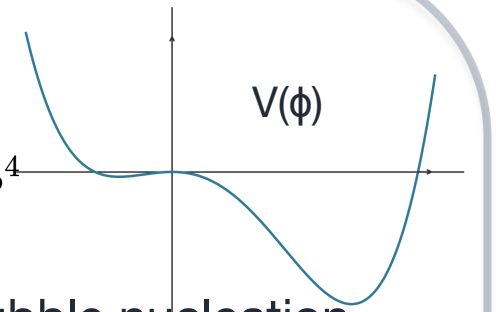


Energy density on the bubble surface

Lattice simulation

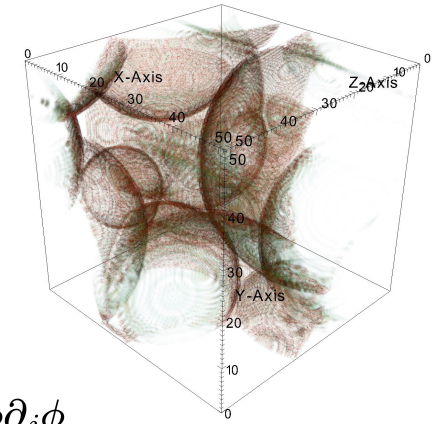
Scalar-field potential

$$V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{3}\kappa\phi^3 + \frac{1}{4}\lambda\phi^4$$



At each time step, sample bubble nucleation using a binomial distribution with probability p

$$p = \frac{\Gamma}{V_{phy}}a^4\Delta\eta_b(\Delta x)^3$$



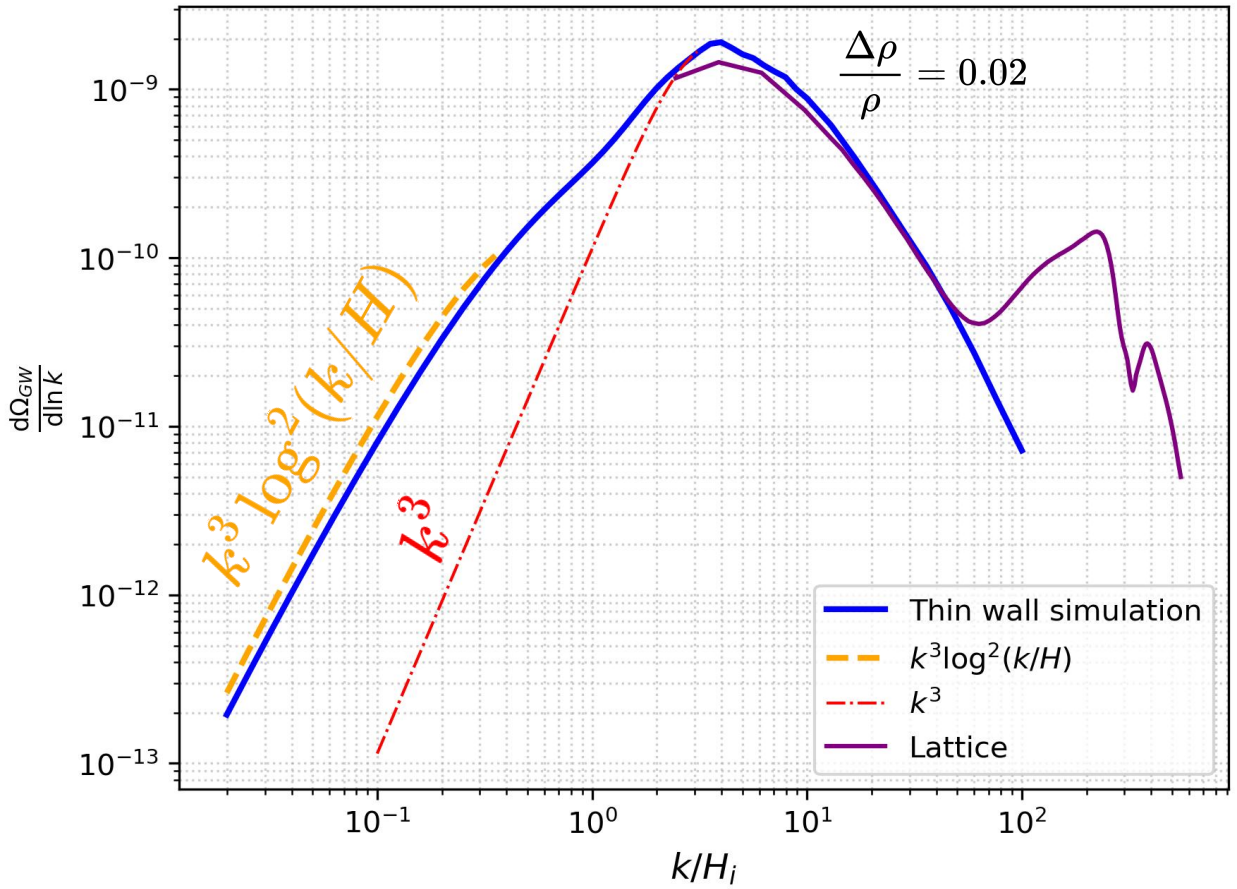
Energy–momentum tensor

$$\tilde{T}_{ij}^{\mathrm{TT}}(\eta,\mathbf{k}) = \Lambda_{ij,kl}(\hat{\mathbf{k}})\int \mathrm{d}x^3 e^{-i\mathbf{k}\cdot\mathbf{x}}\partial_i\phi\partial_j\phi$$

GW spectrum: simulation and infrared prediction

Thin-wall and lattice spectra

Comparison near the peak and in the infrared



Peak position

$$k_{\mathrm{peak}}/H_i \simeq 5$$

UV fit for the simulated model

$$\Omega_{\mathrm{GW}} \propto k^{-1.73}$$

The UV shape can depend on the model

Analytic IR prediction

$$\frac{d\rho_{\mathrm{GW}}}{d\ln k} \propto k^3 \ln^2(k\eta_b)$$

GW benchmarks and detector sensitivity

Present-day spectra for the benchmark parameters shown in the legend

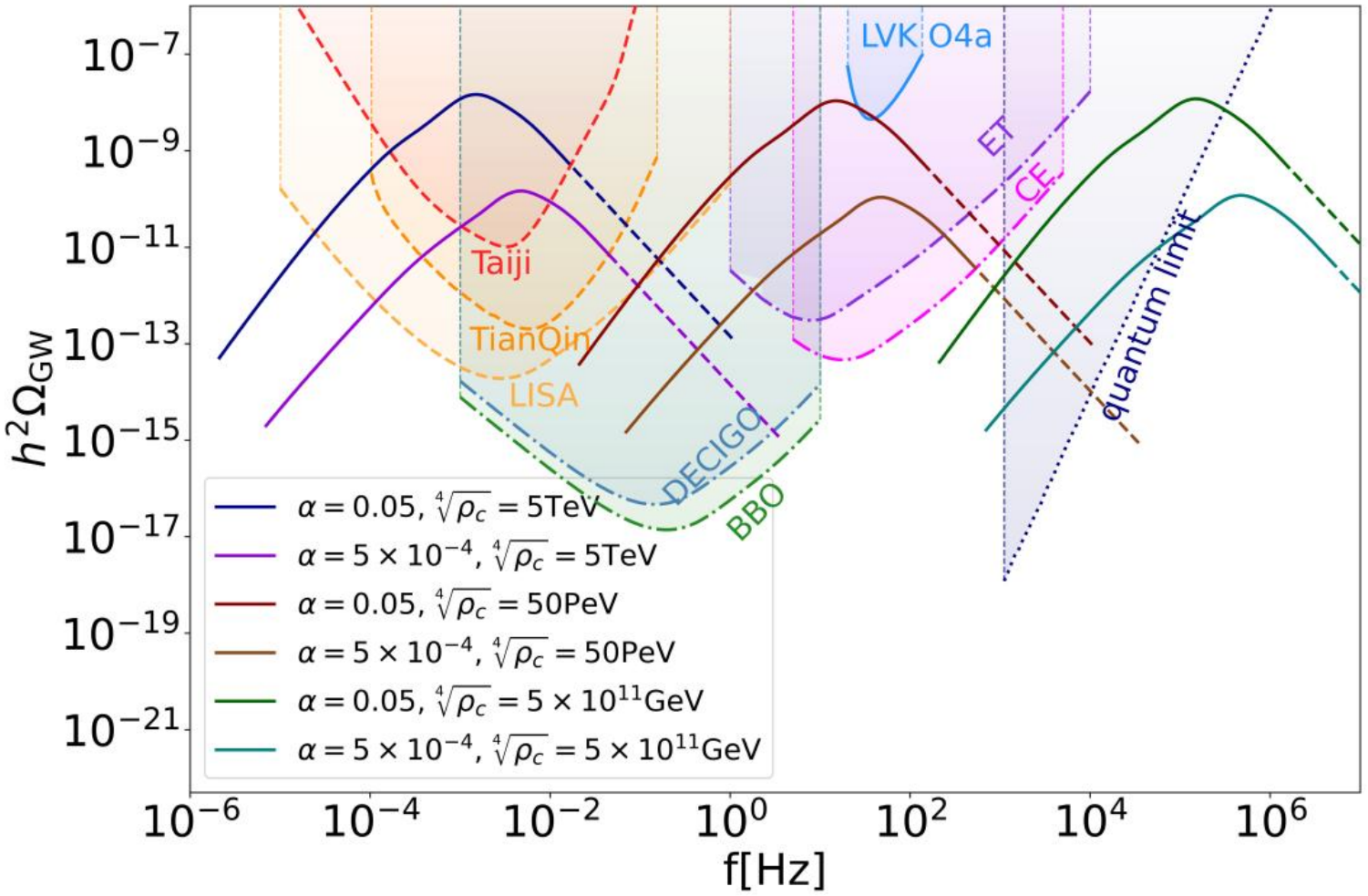
Transition scale

The transition scale shifts the peak frequency

Transition strength

Larger α gives a larger amplitude
at fixed transition scale

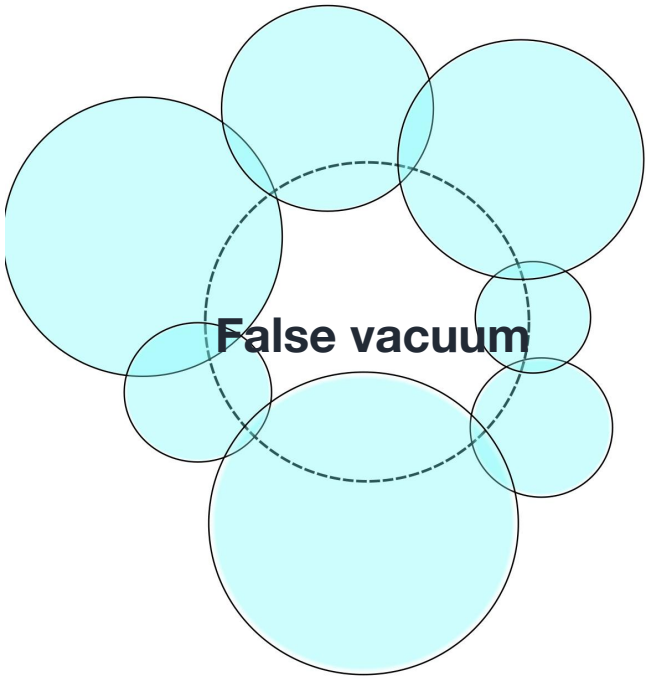
Sensitivity curves provide the observational comparison

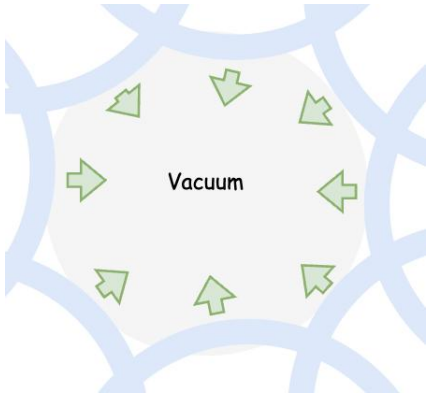


PBH formation from surviving vacuum islands

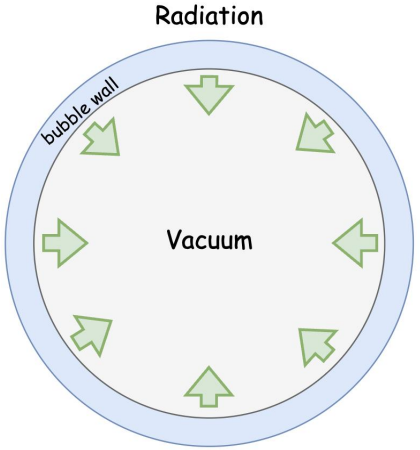
Stochastic nucleation leaves some regions in the false vacuum for longer

These regions can become overdense





Nonspherical remnant



Spherical approximation

Vacuum-dominated Type-II channel

$$M(r) \simeq \frac{4\pi}{3}\rho_{\rm false} r^3, \qquad \frac{2G_N M(r)}{r} = 1$$

Large islands can contain an inflating interior

Dark radiation suppresses Type-II PBH formation

Assumption: the released energy remains in the dark sector as radiation

Dark-radiation bound

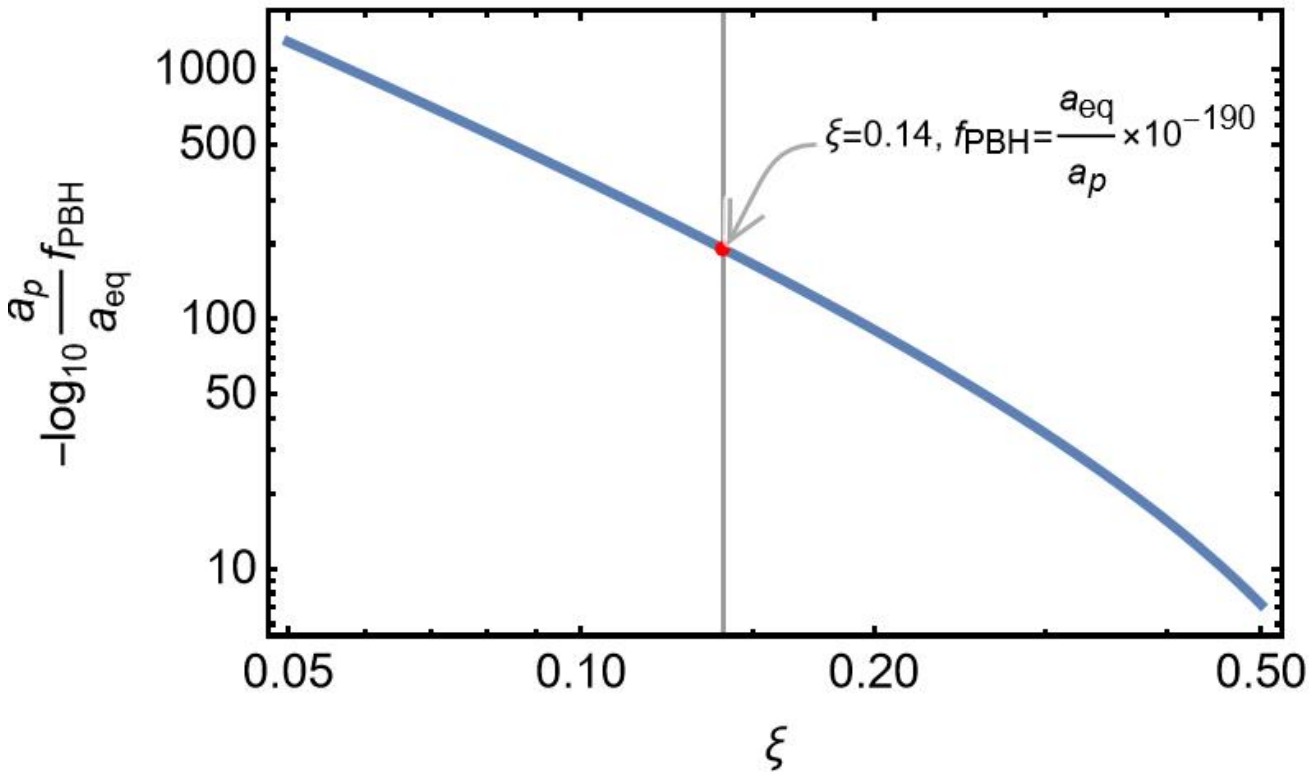
$$\xi = \sqrt{\frac{8\pi G_{\mathrm{N}}}{3}\Delta\rho}\left(\frac{\Gamma}{\mathcal{V}}\right)^{-\frac{1}{4}} \qquad \xi \leq 0.14$$

Minimum island size

$$R_{PBH} \simeq \left(\frac{\Gamma}{\mathcal{V}}\right)^{-\frac{1}{4}} \xi^{-1}$$

Large islands are exponentially rare

$$n(r) \sim \exp\left(-\left(\frac{\Gamma}{\mathcal{V}}\right)^{\frac{3}{4}} R^3\right)$$



The bound strongly suppresses this Type-II channel in the scenario studied

Summary and next steps

Bubble dynamics

Persistent post-collision shells sustain the GW source.
At large Lorentz factors, lattice simulations favor the bulk-flow description.

GW spectrum

Source lifetime and cosmic expansion set the IR behavior.

$$\frac{d\rho_{\mathrm{GW}}}{d\ln k} \propto k^3 \ln^2(k\eta_b)$$

Primordial black holes

If the released energy remains as dark radiation, its cosmological bound requires large vacuum islands, making Type-II PBH formation exponentially rare.

Next steps

Quantify the model dependence of the UV spectrum and test GW predictions with data.
Study particle production and its possible effect on the source lifetime.

Thank you

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Dark-sector evolution and the radiation bound

False-vacuum fraction

$$\mathcal{P}_{\rm false}\,(t) = \exp\left(-\frac{\Gamma}{\mathcal{V}}\frac{8\pi}{105}t^4\right)$$

Dimensionless transition strength

$$\xi = \sqrt{\frac{8\pi G_{\rm N}}{3}\Delta\rho}\left(\frac{\Gamma}{\mathcal{V}}\right)^{-\frac{1}{4}}$$

Energy transfer into dark radiation

$$\dot{\rho}_{\rm DR} + 4H\rho_{\rm DR} = -\dot{\rho}_v, \qquad \rho_v = \Delta\rho\,\mathcal{P}_{\rm false}$$

Adopted dark-radiation bound

$\Delta N_{\rm eff} < 0.4$ implies $\xi \leq 0.14$ [arXiv:2404.03002; arXiv:2404.15220]

Type-II PBH threshold and the role of wall energy

In the spherically symmetric case

Island mass

$$M = \frac{4\pi r^3}{3}\rho_{\text{false}} + 4\pi\sigma r^2\sqrt{1+\left(\frac{dr}{d\tau}\right)^2 - H_{\text{false}}^2 r^2} - 8\pi^2 G_{\text{N}}\sigma^2 r^3$$

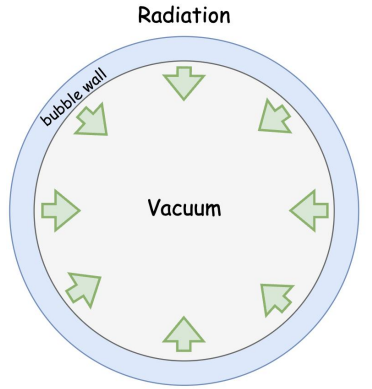
[Phys. Rev. D 35, 1747 (1987)]

The Type-II estimate neglects the wall energy for sufficiently large vacuum islands

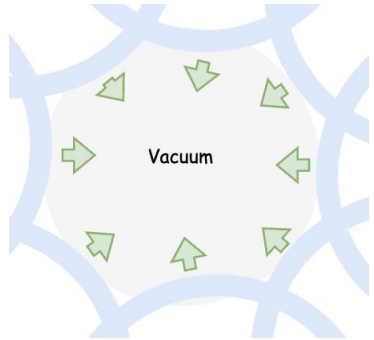
Schwarzschild criterion

$$\frac{2G_{\text{N}}M(r)}{r} = 1$$

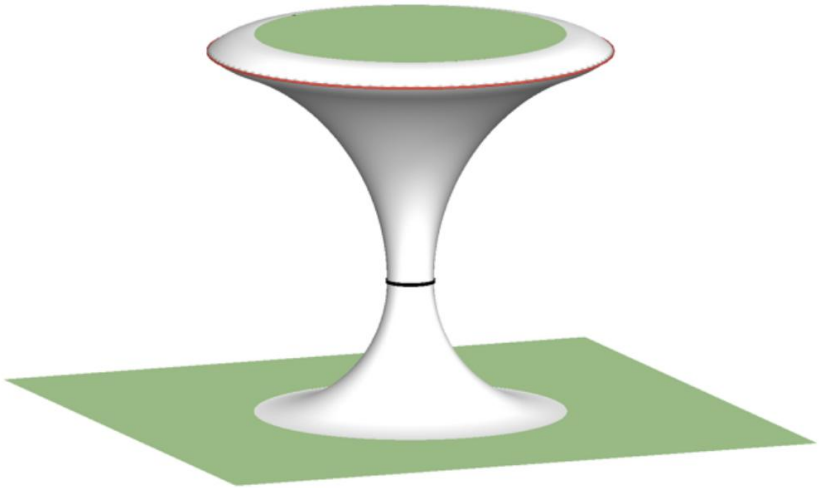
The interior can inflate into a baby universe



Spherical model



Nonspherical remnant



Type-II PBH [arXiv:1512.01819]

Source duration and infrared convergence

Post-collision TT source at small k

$$\tilde{T}_{ij}^{\mathrm{TT}}(\eta, \mathbf{k}) \propto \eta^{-2}$$

Metric perturbation from the retarded Green function

$$\tilde{\mathrm{h}}_{ij}(\eta, \mathbf{k}) = \frac{16\pi G_N}{k} \int d\eta' \sin k\left(\eta - \eta'\right) a\left(\eta'\right) \tilde{T}_{ij}^{\mathrm{TT}}\left(\eta', \mathbf{k}\right)$$

The relevant integration time depends on both the source and the mode

$$\eta_{\mathrm{eff}} \sim \min(\eta_f,\, k^{-1})$$

Long source durations are difficult to capture in direct 3D simulations

Free massless fields in radiation domination

Field equation

$$\partial_\eta^2 \phi(\eta, \mathbf{k}) + \frac{2a'}{a} \partial_\eta \phi(\eta, \mathbf{k}) + k^2 \phi(\eta, \mathbf{k}) = 0$$

Mode function

$$\phi(\eta, \mathbf{k}) = \frac{1}{a(\eta)} \left(\mathcal{A}_+(\mathbf{k}) e^{-ik(\eta - \eta_i)} + \mathcal{A}_-(\mathbf{k}) e^{ik(\eta - \eta_i)} \right)$$

TT energy–momentum tensor

$$T_{ij}^{TT}(\eta, \mathbf{k}) = \frac{\Lambda_{ij,kl}(\hat{\mathbf{k}})}{a^2(\eta)} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} p_k p_l \sum_{c_n = +,-} \mathcal{A}_{c_1}(\mathbf{p}) \mathcal{A}_{c_2}(\mathbf{k} - \mathbf{p}) e^{-ic_1 p(\eta - \eta_i) - ic_2 |\mathbf{k} - \mathbf{p}| (\eta - \eta_i)}$$

Cosine contribution to the metric perturbation

$$h_{ij}^c(\mathbf{k}) = \Lambda_{ij,kl}(\hat{\mathbf{k}}) \int \frac{\mathrm{d}^3 p}{(2\pi)^3} p_k p_l \sum_{c_1 = +,-} |\mathcal{A}_{c_1}(\mathbf{p})|^2 \left(\int_{\eta_i}^{\eta_f} \frac{\mathrm{d}\eta}{a(\eta)} \cos k\eta e^{-ic_1 \hat{\mathbf{p}} \cdot \mathbf{k} (\eta - \eta_i)} \right)$$

Analytic IR behavior and the radial-mode cutoff

Late-time cosine integral

$$\int_{\eta_i}^{\infty} \frac{\mathrm{d}\eta}{a(\eta)} \cos k\eta e^{-ic_1 \hat{\mathbf{p}}\cdot\mathbf{k}(\eta-\eta_i)} = -\frac{e^{ic_1\hat{\mathbf{p}}\cdot\mathbf{k}\eta_i}}{2H_0}\left(Ei\left(-i(\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}-1)k\eta_i\right)+Ei\left(-i(\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}+1)k\eta_i\right)\right)(\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}\neq 1)$$

Infrared limit

$$h_{ij}^{c}(\mathbf{k}) = -\frac{\log(k\eta_i)}{2H_0}\Lambda_{ij,kl}(\hat{\mathbf{k}})\int\frac{\mathrm{d}^3p}{(2\pi)^3}p_k p_l \sum_{c_1=+,-}|\mathcal{A}_{c_1}(\mathbf{p})|^2 = -\frac{\log(k\eta_i)}{2H_0}T_{ij}^{TT}(\eta_i,\mathbf{0})$$

GW spectrum in the long-source interval

$$\Omega_{\mathrm{GW}} \propto k^3 \ln^2(k\eta_i), \qquad \eta_f^{-1} \ll k \ll \eta_i^{-1}$$

The radial mode has a finite mass

$$\partial_\eta^2\rho(\eta,\mathbf{k}) + \frac{2a'}{a}\partial_\eta\rho(\eta,\mathbf{k}) + k^2\rho(\eta,\mathbf{k}) + a^2m^2\rho(\eta,\mathbf{k}) = 0$$

The radial contribution ends at $\eta \sim \gamma/H_0$; the Goldstone mode can persist longer

H_0 denotes the reference Hubble scale in these expressions, not today's value

Computational cost at large Lorentz factors

3+1D simulation

For a fixed model and number of bubbles

Physical box size $L_{box} \sim N_b^{1/3} R_c \sim N_b^{1/3} \gamma R_0$

Lattice spacing $\delta x \sim R_0/\gamma$

Grid points per spatial dimension $N = L_{box}/\delta x \propto \gamma^2$

Total computational cost $N^3 T \propto \gamma^8$

1+1D reduction with SO(2,1) symmetry

Bubbles nucleate along the z axis in Minkowski spacetime

Computational cost $NT \propto \gamma^4$

$$\Phi_i = \sum_{j=0}^{n} \Phi_{bounce}(\sqrt{t^2 - x^2 - y^2 - (z - z_j)^2})$$