Leggett-Garg Inequality Violation in Muon g-2 Experiments

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PIKIMO 21





Introduction

- Interest in quantum correlations in high energy physics [arXiv:2504.00086]
  - Spin correlations measurements, entanglement observations:

CMS [arXiv:2409.11067] [arXiv:2406.03976] ATLAS [arXiv:2311.07288]

    - Via angular distributions
    - Top → self analyzing Muon → self analyzing
  - These are 2 particle quantum correlations. Bell

What about 1 particle quantum correlations? Leggett-Garg

mesons [arXiv:1802.04265] neutrinos [arXiv:1602.00041]

Formulation

- Assumptions: [arXiv:1304.5133]

1.) Realism: observable has well defined value at all times, in absence of measurement

2.) Non-invasive measurability (NIM): performing a measurement does not disturb the system's evolution

$$Q = +1 \quad \text{heads} \qquad Q = -1 \quad \text{tails}$$



Formulation

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  1.) Realism: observable has well defined value at all times, in absence of measurement

  2.) Non-invasive measurability (NIM): performing a measurement does not disturb the system's evolution

$$C_{ij} = \sum_{Q_i,Q_j} Q_i Q_j P_{ij}(Q_i,Q_j) = \sum_{Q_i,Q_j} Q_i Q_j \sum_{Q_k;k\neq i,j} P_{ij}(Q_k,Q_i,Q_j) = \sum_{Q_i,Q_j} Q_i Q_j \sum_{Q_k;k\neq i,j} P(Q_k,Q_i,Q_j)$$

Realism NIM

Formulation

- Assumptions: [arXiv:1304.5133]

  1.) Realism: observable has well defined value at all times, in absence of measurement

  2.) Non-invasive measurability (NIM): performing a measurement does not disturb the system's evolution

$$K_3 = C_{21} + C_{32} - C_{31} = 1 - 4\left[P(+,-,+) + P(-,+,-)\right]$$

Formulation

- Assumptions: [arXiv:1304.5133]

1.) Realism: observable has well defined value at all times, in absence of measurement

2.) Non-invasive measurability (NIM): performing a measurement does not disturb the system's evolution

$$K_3 = C_{21} + C_{32} - C_{31} \leq 1$$

Qubits

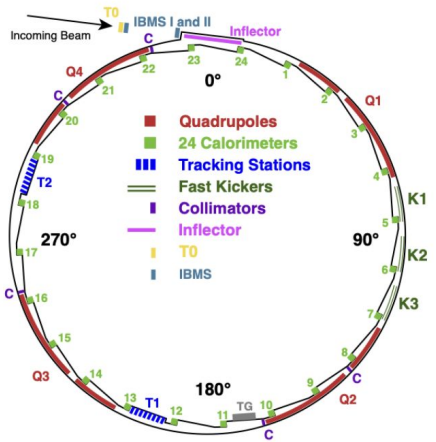
- For a qubit $Q_i \to \hat{Q}_i = U^\dagger(t_i, t_0)\hat{Q}(t_0)U(t_i, t_0) = \vec{a_i} \cdot \hat{\sigma}$

$$C_{ij} = \frac{1}{2}\langle\{\hat{Q}_i, \hat{Q}_j\}\rangle = \vec{a_i} \cdot \vec{a_j}$$

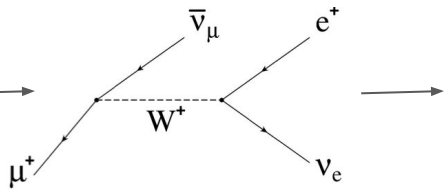
- So quantum theory predicts a lager upper bound

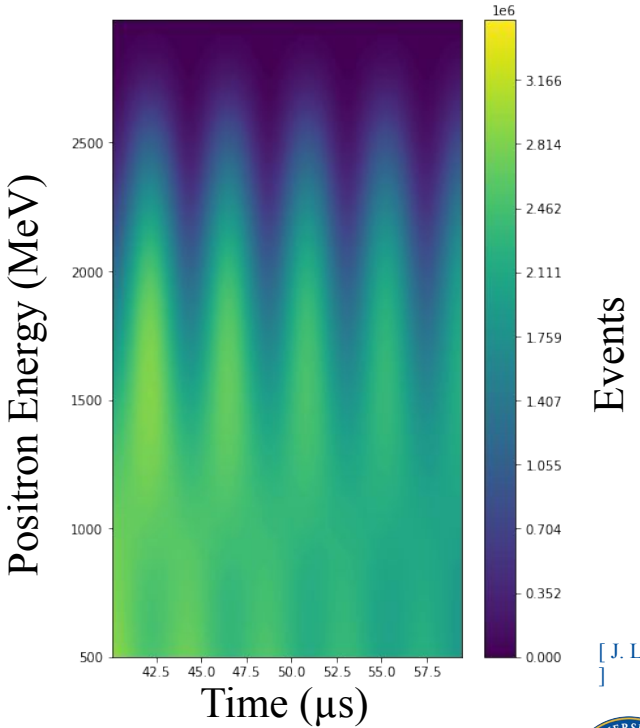
$$K_3 = \vec{a_2} \cdot \vec{a_1} + \vec{a_3} \cdot \vec{a_2} - \vec{a_3} \cdot \vec{a_1} \leq \frac{3}{2}$$

LGI at muon g-2



[arXiv:2606.17323]

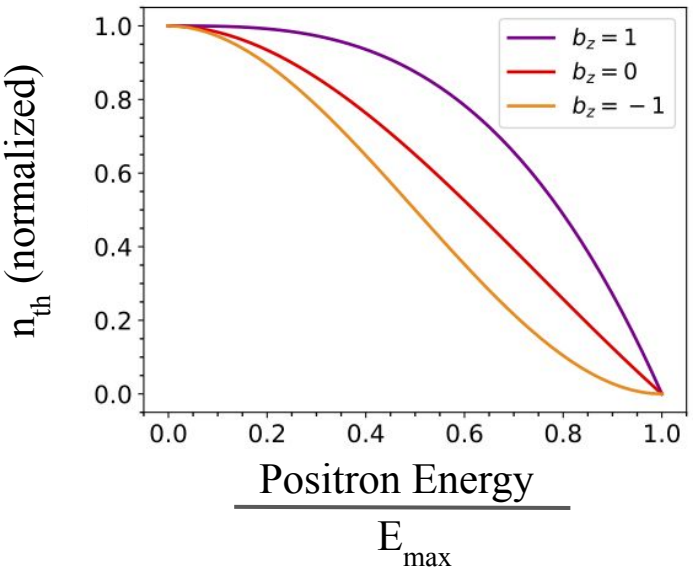


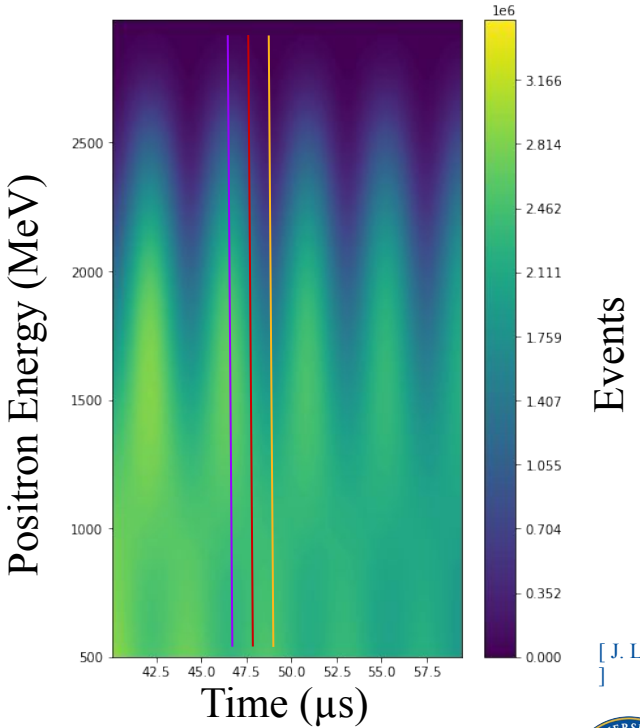


[J. LaBounty 2024]

LGI at muon g-2

theory prediction: $n_{\mathrm{th}}(E, b_z) = N_{\mathrm{th}}(E)\left[1 + b_z A_{\mathrm{th}}(E)\right]$

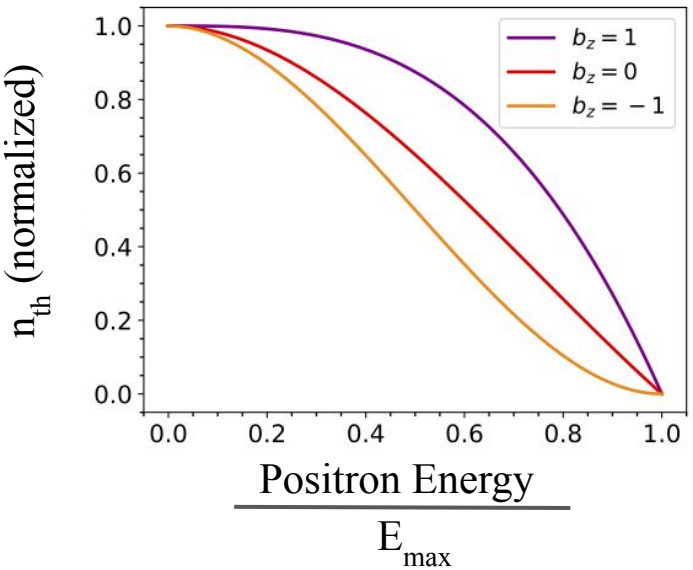


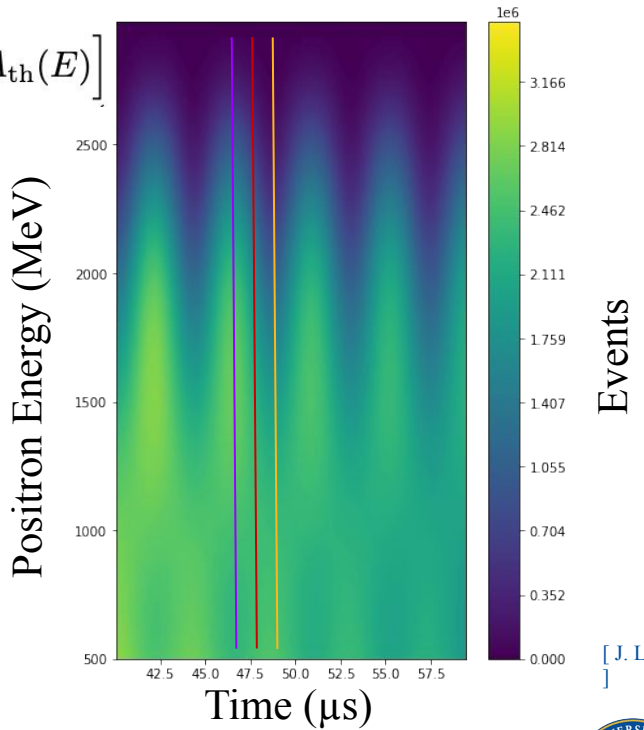


[J. LaBounty 2024]

LGI at muon g-2

theory prediction: $n_{\mathrm{th}}(E, b_z(t)) = N_{\mathrm{th}}(E)\left[1 + b_{\max}\cos(\omega_a t + \phi)A_{\mathrm{th}}(E)\right]$

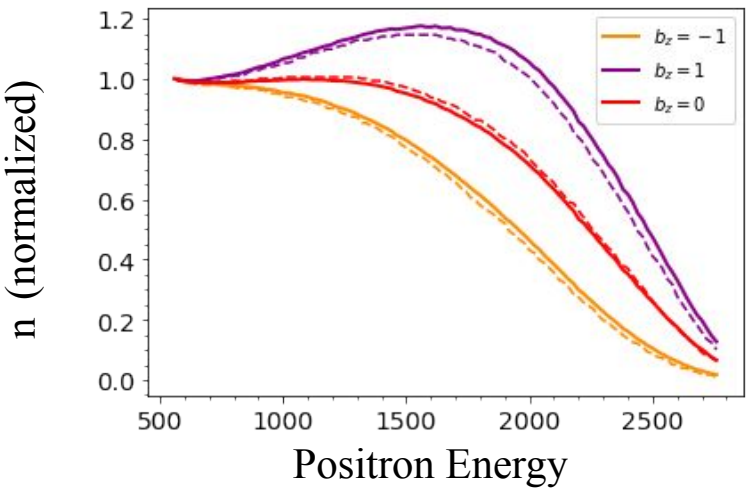


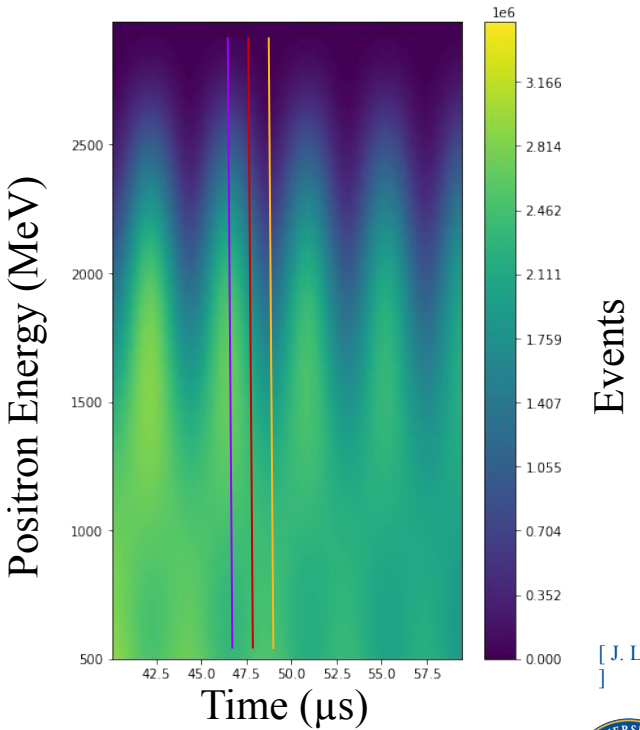


[J. LaBounty 2024]

LGI at muon g-2

Model detector efficiency: $\epsilon(E, b_z) = \epsilon_0(E) + b_z\epsilon_1(E)$





[J. LaBounty 2024]

LGI Construction

- Probabilities of spin up/down

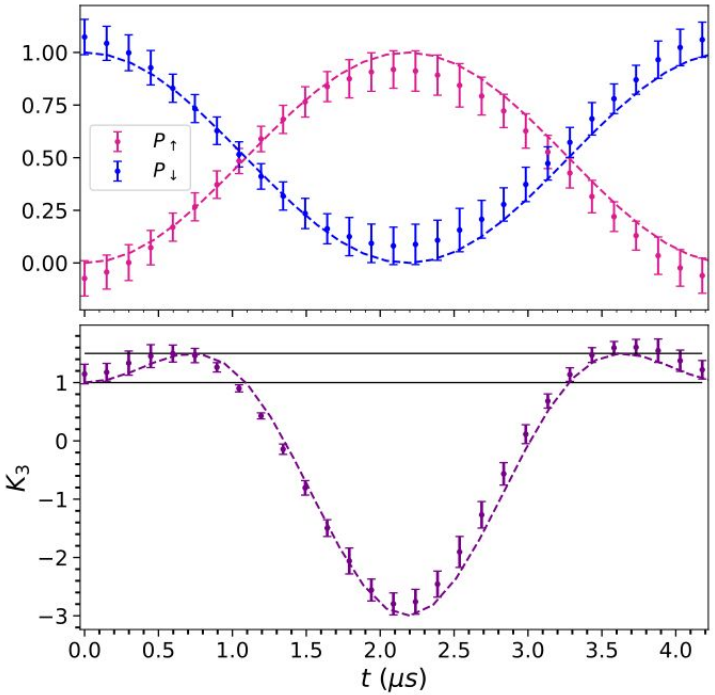
$$P_{\pm} = \frac{1 \pm b_z}{2}$$

- Correlation functions: (assume stationarity)

$$C(\Delta t) = P_{++}^{\Delta t} - P_{+-}^{\Delta t} = P_{--}^{\Delta t} - P_{-+}^{\Delta t}$$

- LG Observable

$$K_3 = 2C(\Delta t) - C(2\Delta t)$$

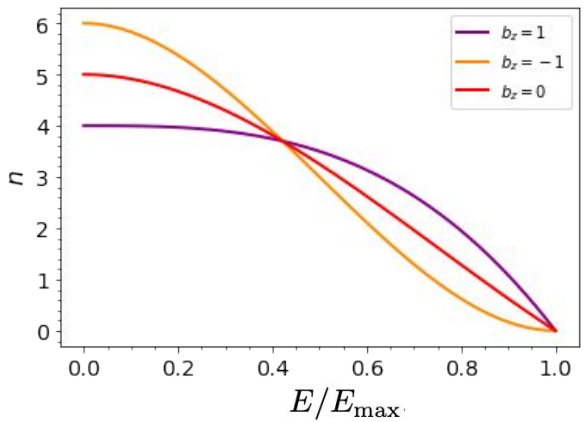


Conclusions

- First study of LGI violation for system of high energy muons
- Longitudinal polarization → correlation functions → Leggett-Garg variable
- Dominated by systematics
- Single bin significance of 5.5σ

extra

Muon g-2

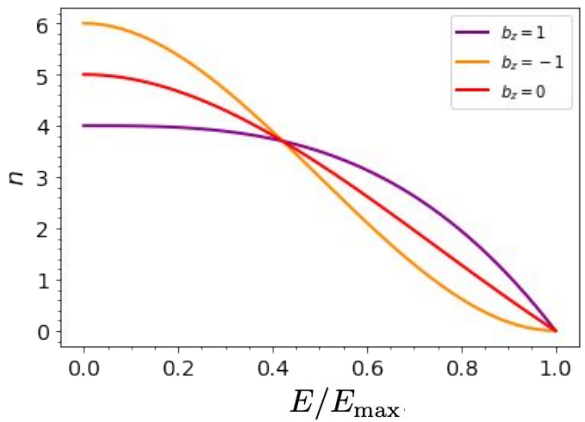


-Muon g-2 experiment provides opportunity to measure K_3 . First LG test with muons

-Muon spin precession around ring. The longitudinal polarization of the muon will oscillate in time.

-polarization can be determined from the energy distribution of decay products

Muon g-2



$$n_{\mathrm{th}}(E, b_z) = N(E)\Big(1 + b_z A(E)\Big)$$

Conclusions

- What are LGIs: family of inequalities for a system adhering to realism and non-invasive measurement.
- QM can violate these types inequalities
- Muon g-2 experiments can be used as a “Leggett-Garg” test (+ assumptions)
  - First test of LG with muons
- LGI violation consistent with QM predictions

extra

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[J. LaBounty 2024]

Get polarization at each time

$$\chi^2 = \sum_i \left(\frac{n_{\mathrm{exp}}(E_i, t) - n'_{\mathrm{th}}(E_i, b_z)}{\Delta n_i} \right)^2 \qquad P_{\pm} = \frac{1 \pm b_z}{2}$$

Assume stationarity:

- known initial state
- Time difference: $\Delta t \equiv t_j - t_i$

$$K_3 = 2C(\Delta t) - C(2\Delta t)$$

