Axions on a Hyperbolic Ride

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Geometric suppression of CMB isocurvature and a blue-tilted spectrum

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CMB limits on cold-dark-matter isocurvature are often interpreted as excluding the simultaneous realization of high-scale inflation and large QCD axion decay constants in preinflationary Peccei-Quinn (PQ) scenarios. This conclusion can be evaded by exploiting *field-space geometry*. For a minimal complex PQ scalar with a $U(1)$-symmetric potential and a nonlinear sigma-model kinetic term $d\sigma^2 = dR^2 + f^2(R)d\theta^2$, the observable axion fluctuation is $\delta\theta \sim H_{\star}/f(R)$, so an enhanced effective decay constant $f(R)$ suppresses isocurvature



Core setup: “flat” PQ field-space

Complex PQ field in radial/angular variables

$$\mathcal{L}_{\text{kin}} = -\left(\partial\Phi^{*}\partial\Phi\right)$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2}\left[\left(\partial R\right)^{2} + R^{2}\left(\partial\theta\right)^{2}\right]$$

$$ds^{2} = dR^{2} + R^{2}d\theta^{2}$$

Non-flat metric

Core setup: non-flat PQ field-space

Write the complex PQ field in radial/angular variables:

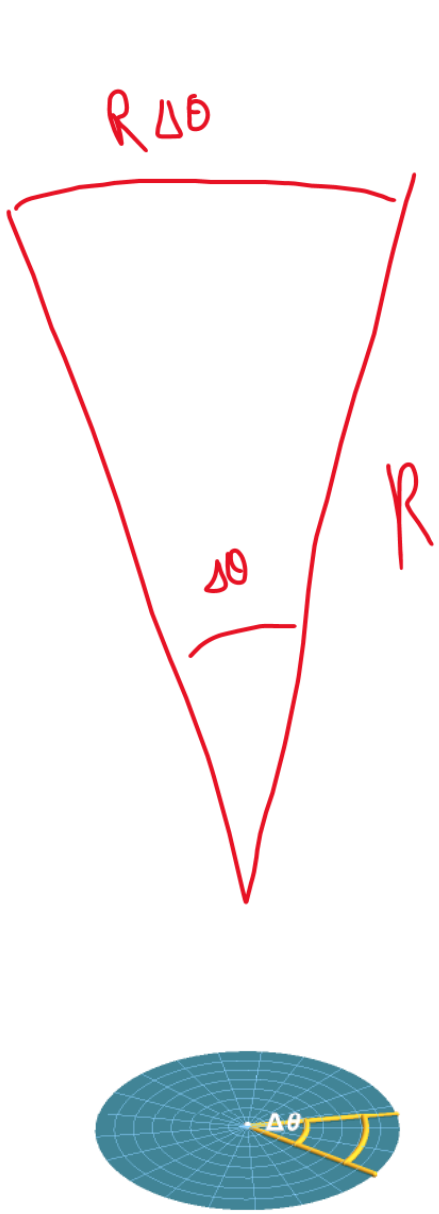
$$\mathcal{L}_{\text{kin}} = -\frac{1}{2}\left[(\partial R)^2 + f(R)^2 (\partial\theta)^2\right]$$

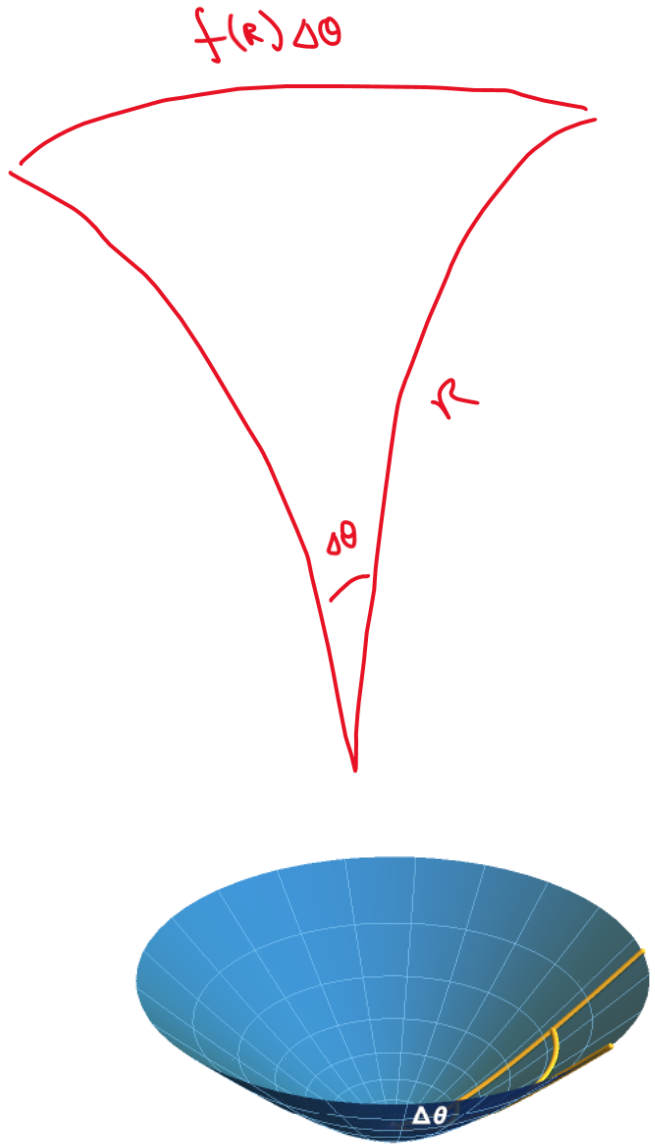
$$\chi^i = \{R, \theta\}$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2} g^{\mu\nu}\left[G_{ij}\partial_\mu \chi^i \partial_\nu \chi^j\right]$$

$$G_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & f(R)^2 \end{pmatrix}$$

Gij represents the internal field-space metric of the radial and theta real fields


R Δθ
R
Δθ
Δθ


f(R) Δθ
R
Δθ
Δθ

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial R)^2 - \frac{1}{2}f^2(R)(\partial\theta)^2 - V(R)\right]$$

1) At smaller radial displacements, the internal metric is expected to appear flat, i.e. f(R) → R, to preserve usual flat-QCD axion behavior.

2) At larger radial distances, f(R) deviates from R due to UV effects.

3) Gij also represents internal-geometry. Hence, it can be categorized as
   a. Flat (0 curvature)
   b. Closed (+ve curvature)
   c. Open (-ve curvature)

$$G_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & f(R)^2 \end{pmatrix}$$

Hyberbolic metric examples

N=1 SUPERGRAVITY (SUGRA)

$$K_{i\bar{j}} = \frac{\partial^2 K}{\partial\phi^i\,\partial\bar{\phi}^{\bar{j}}},$$

$$K(Z,\bar{Z}) = -3\alpha\, M_P^2\, \ln\left(1 - \frac{|Z|^2}{3M_P^2}\right), \quad \textbf{SU(1,1)}$$

$$K_{Z\bar{Z}} = \frac{\alpha}{\left(1 - |Z|^2/(3M_P^2)\right)^2}.$$

$$Z = \rho\, e^{i\theta}$$

$$R = \sqrt{6\alpha}\, M_P\, \tanh^{-1}\left(\frac{\rho}{\sqrt{3}\, M_P}\right).$$

$$d\sigma^2 = dR^2 + L^2 \sinh^2\left(\frac{R}{L}\right) d\theta^2, \qquad L = \sqrt{\frac{3\alpha}{2}}\, M_P.$$

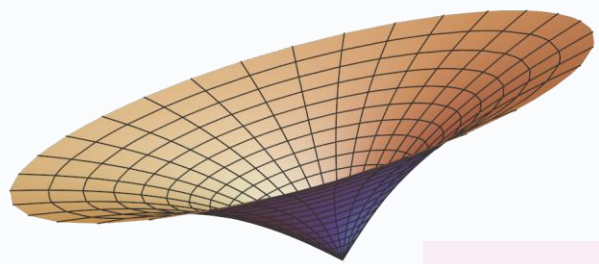
1. α-attractor/Pole inflation:
   $\mathcal{L} = \frac{|\partial Z|^2}{\left(1-|Z|^2/\Lambda^2\right)}$
2. None-linear sigma models:
   $\mathcal{L} = G^{ij}\partial\phi_i\partial\phi_j$
3. String compactification moduli spaces: $K = -3\ln\left(T+\bar{T}\right)$
4. Conformal field theories:
   $ds^2 = \frac{dz^2+dx^2}{z^2}$

Hyperbolic metric = exponential lever arm

$$f_{\text{hyp}}(R) = L \sinh\left(\frac{R}{L}\right)$$

L is the curvature of the field-space geometry.

A modest radial displacement in curvature units, R/L, gives exponential effects.



$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\partial R)^2 - \frac{1}{2} f^2(R)(\partial\theta)^2 - V(R) \right]$$

FLAT ROAD OR CURVED EARTH !!!

For radial displacements far smaller than the hyperbolic curvature scale L, the internal field-space metric is as good as **flat**.

Only when the radial displacements start probing distances of the order of L, does the internal metric appear curved.

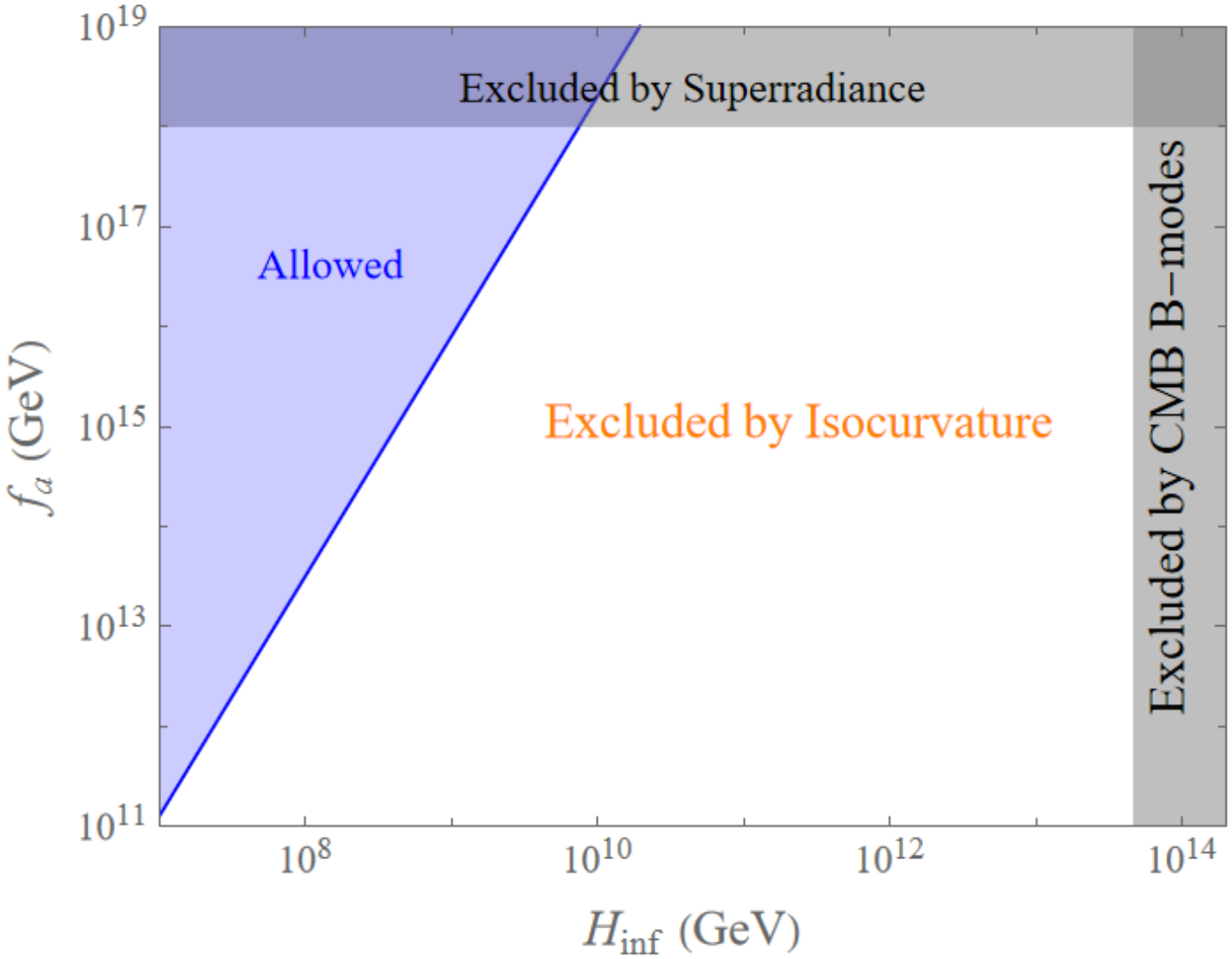
$$\lim_{R \gg L} f(R) = L \sinh\left(R/L\right) \to \frac{L}{2} e^{R/L}$$

$$\lim_{R \ll L} f(R) = L \sinh\left(R/L\right) \to R$$

Observational implications of non-flat metric

The axion isocurvature problem

Axion makes all of DM



Theory

$$\delta\theta \approx \frac{H_{\rm inf}}{R}$$

CMB

$$\frac{\delta\theta}{\theta} \lesssim 10^{-5}$$

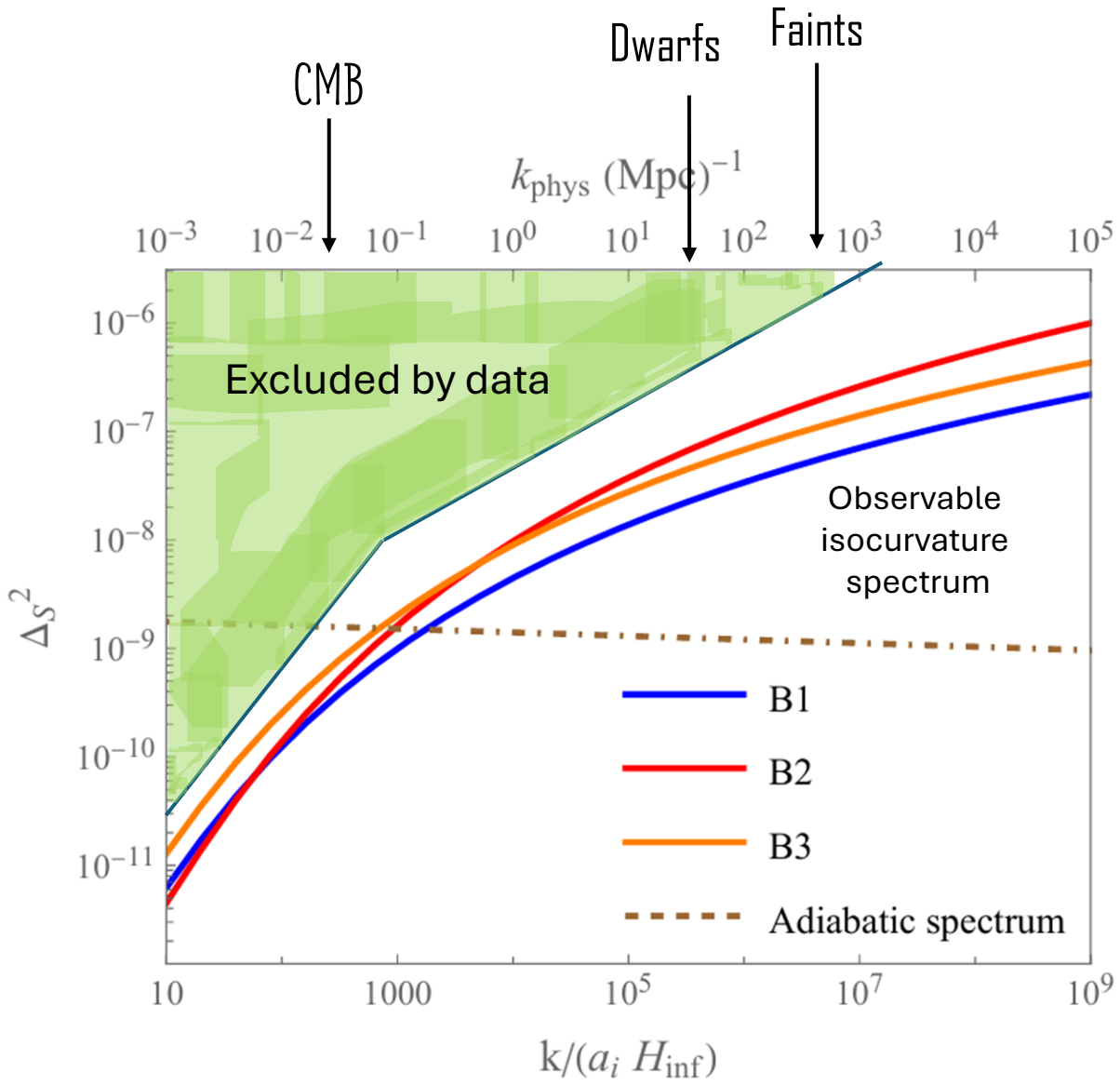
Axial Field equations

$$\delta\psi \equiv f(R_0)\,\delta\theta$$

$$\ddot{\delta\psi} + 3H\dot{\delta\psi} - \frac{\nabla^2}{a^2}\delta\psi + m_\psi^2(t)\,\delta\psi = 0,$$

crucial

$$m_\psi^2(t) = \frac{f_{,R}}{f}\,V_{,R}(R_0) - \frac{f_{,RR}}{f}\,\dot{R}_0^2.$$

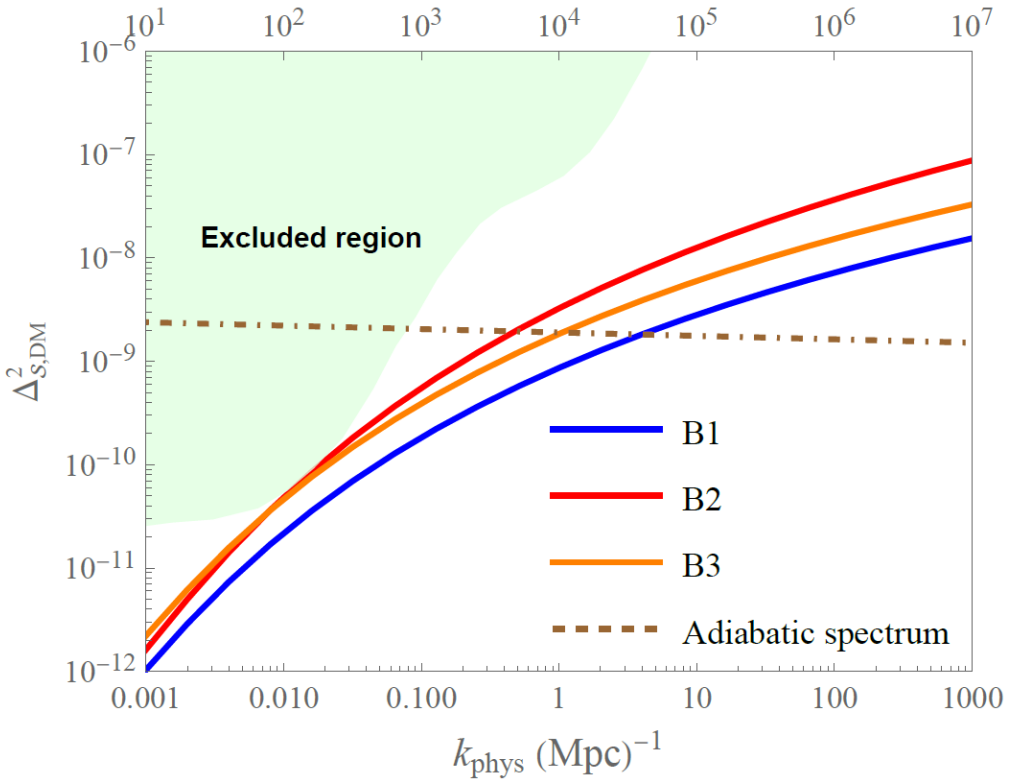

CMB
Dwarfs
Faints
$k_{\rm phys}$ (Mpc)$^{-1}$
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
Excluded by data
Observable isocurvature spectrum
B1
B2
B3
Adiabatic spectrum
Δ_S^2
10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 10^{-11}
10 1000 10^5 10^7 10^9
k/(a_i $H_{\rm inf}$)

Characteristic running

$$\Delta^2_{\mathcal{S}} = A_{\rm iso}\left(\frac{k}{k_p}\right)^{n_{\rm iso}(k)-1}$$

$$n_{\rm iso}(k_p) = 4-\sqrt{9-4\mu_p},$$

$$\alpha_{\rm iso}(k_p) = \left.\frac{dn_{\rm iso}(k)}{d\ln k}\right|_{k_p} = -\frac{4\mu_p^2}{\xi_p\sqrt{9-4\mu_p}}.$$



$$\xi = \frac{R}{L}, \qquad \mu \to\sim m_R^2$$

A few consequences of curved f(R):

1. Solves QCD-isocurvature problem
2. Gives a testable prediction of large primordial power spectrum on galactic scales
3. Tachyonic production of warm axion DM
4. Enhances isocurvature non-gaussianity