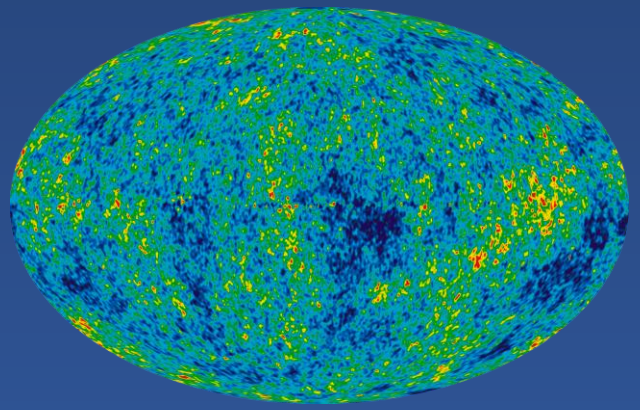
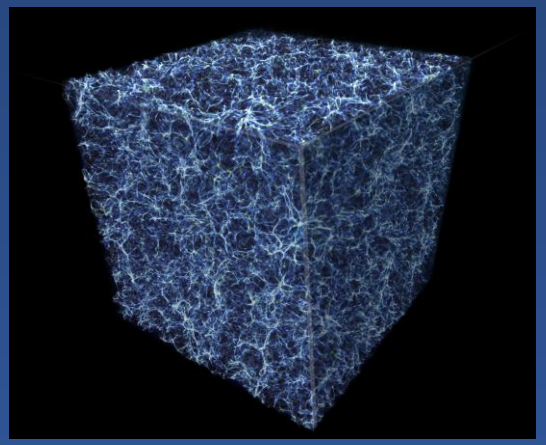
Intrinsic Non-Gaussian Perturbation Signals from Ultralight Dark Matter



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ULDM Field

ULDM has wavy behavior. For ULDM with $m \lesssim 10^{-20}\text{eV}$,

$$\lambda \sim 1/H \qquad \text{early universe}$$

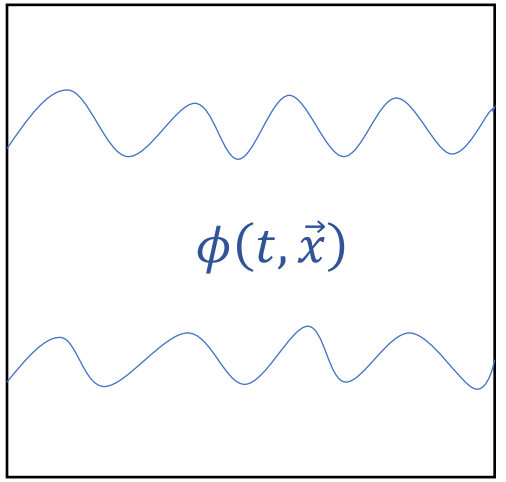
Expand ULDM field ϕ into its Fourier modes:

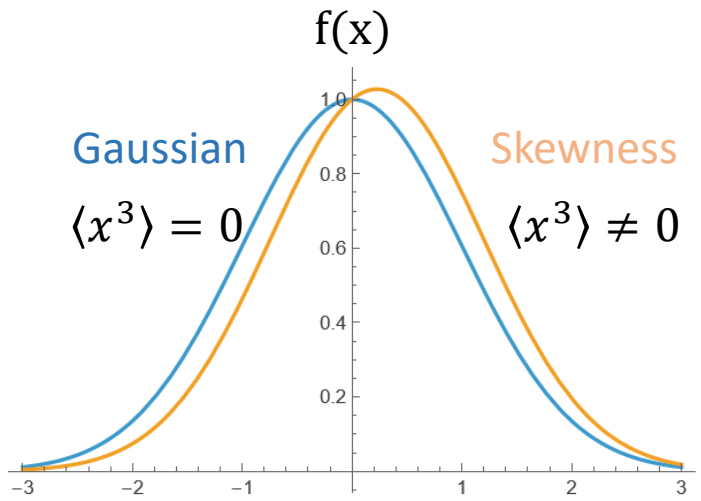
$$\phi(\tau, \mathbf{x}) = \sum_{\boldsymbol{k}} \frac{1}{\sqrt{V}} \left[\alpha_{\mathbf{k}} \phi_{\boldsymbol{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}} + \alpha^*_{\mathbf{k}} \phi^*_{\boldsymbol{k}}(\tau) e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

where $(\alpha_{\mathbf{k}}, \alpha^*_{\mathbf{k}})$ are Gaussian random fields:

$$\langle \alpha_{\mathbf{k}} \alpha^*_{\mathbf{p}} \rangle = \delta_{\mathbf{k},\mathbf{p}} f(\mathbf{k}) \,, \qquad \langle \alpha^3 \rangle = 0$$

Gaussian ***field*** 3-points function: $\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle \sim \langle \alpha_{k_1} \alpha_{k_2} \alpha_{k_3} \rangle = 0$





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Energy Density Bispectrum

However, the ***energy density*** $\rho \propto \phi^2$ has a non-zero 3-points function:

$$\langle \rho_{k_1} \rho_{k_2} \rho_{k_3} \rangle \sim \langle \alpha^*_{k_1 - p_1} \alpha_{p_1} \alpha^*_{k_2 - p_2} \alpha_{p_2} \alpha^*_{k_3 - p_3} \alpha_{p_3} \rangle \neq 0$$

Applying Wick's theorem, this generates a bispectrum:

$$\mathcal{B}_\delta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{(\mathrm{k}_1 \mathrm{k}_2 \mathrm{k}_3)^2}{(2\pi^2)^2} \frac{2m^3}{a^9 \bar{\rho}^3} \int \frac{\mathrm{d}^3 \mathrm{p}}{(2\pi)^3} f(\mathbf{p}) f(\mathbf{p} + \mathbf{k}_2) f(\mathbf{p} + \mathbf{k}_2 + \mathbf{k}_3)$$

From straightforward **dimension analysis**, one could find the following white-noise bispectrum:

$$\mathcal{B}_\delta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \approx \frac{(k_1 k_2 k_3)^2}{(2\pi)^2} B_0 \,, \quad B_0 \equiv \frac{2(2\pi^2)^2}{k_*^6} C \,, \qquad \text{when } k_i \ll k_*$$

C: $\mathcal{O}(1)$ numerical factor determined by the integral

k_*: ***Characteristic momentum*** for a given momentum distribution $\mathrm{f(k)}$

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White-noise Spectrum

For ***small*** *k*-modes, power(bi-) spectra always have power law behavior.

How can we utilize this to constrain a specific model?

Isocurvature perturbation

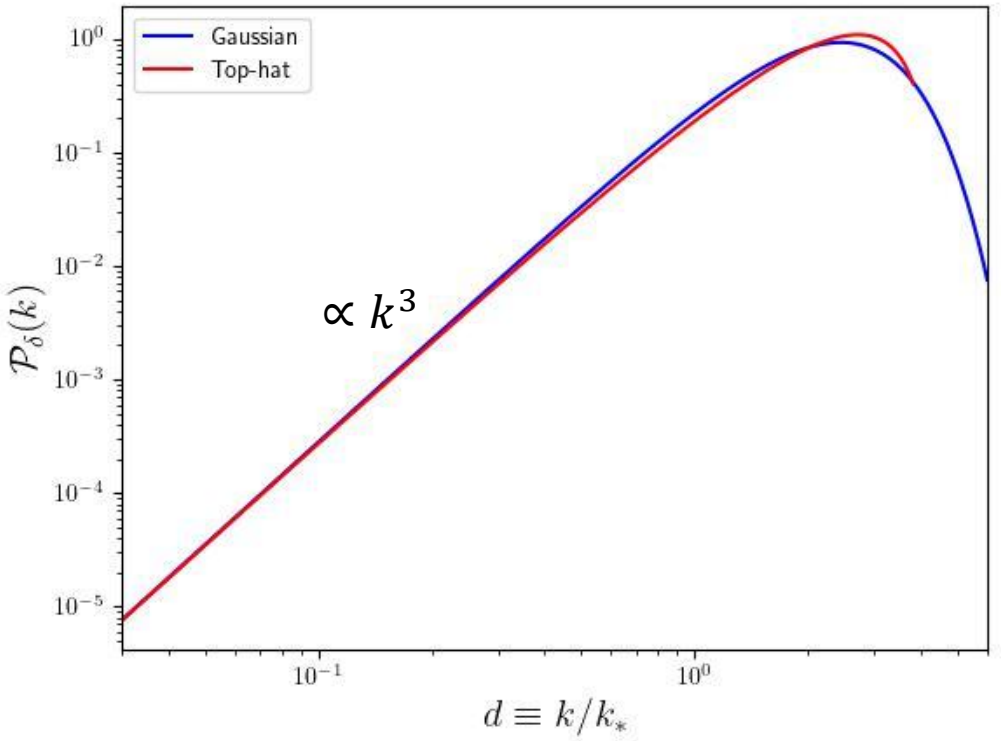
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White-noise spectrum

↓

Observational signals

(CMB and LSS)

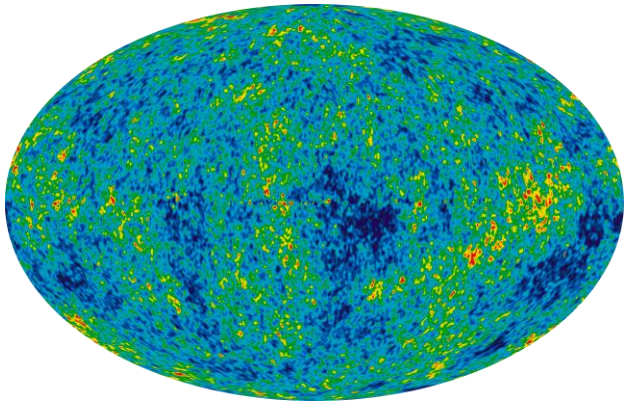


Power spectrum: [Buckley, Du, Fernandez, Weikert (2025)]

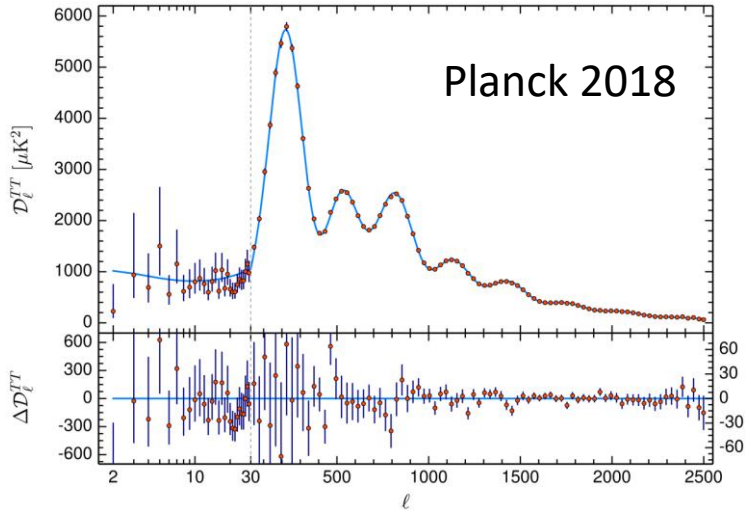
[Co, Tadepalli (2026)]

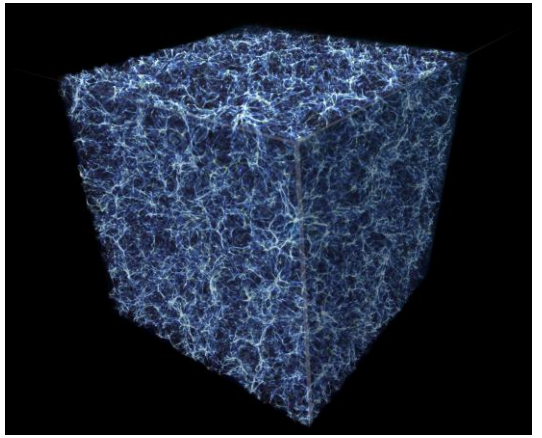
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Observational signals

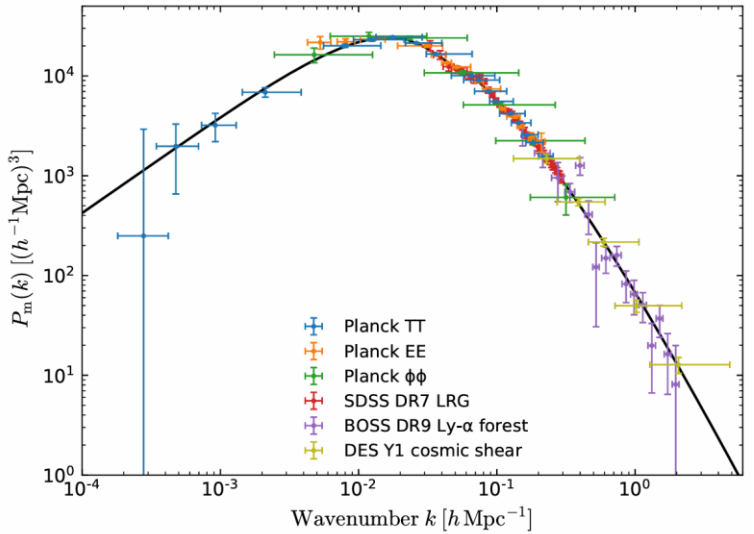


CMB anisotropy
of the last scattering surface





Large Scale Structure
clumpiness of the DM distribution

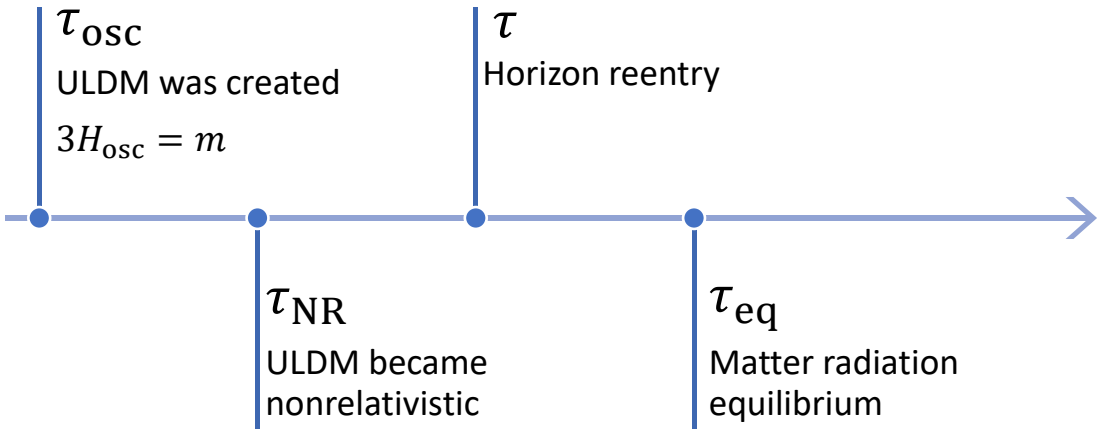


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Observational signals

$$f_{\mathrm{ULDM},0} = \rho_{\mathrm{ULDM},0}/\rho_{\mathrm{DM},0}$$

- CMB: Curvature perturbation ζ at horizon reentry



$$\zeta(k) \approx 1.6\times10^{-3} f_{\mathrm{ULDM},0} \left(\frac{\tau_{\mathrm{NR}}}{\mathrm{Mpc}}\right) (k\tau_{\mathrm{NR}})^{-1} \delta(k)$$

$\Rightarrow$ $\mathcal{B}_\zeta$

- LSS: DM perturbation generated by the ULDM: $\delta_{\mathrm{DM}}^{\mathrm{ULDM}}(z,k)$

$$\delta_{\mathrm{DM}}^{\mathrm{ULDM}}(z,k) = f_{\mathrm{ULDM},0}\, T_{\mathrm{iso}}(z,k)\delta(k)$$

T_{iso} is the isocurvature transfer function

Boltzmann solver: *CLASS*
arXiv:1104.2932 [astro-ph.IM]

$\Rightarrow$ $\mathcal{B}_{\mathrm{DM}}$

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Observational signals

- CMB

Package: *CMB-BEST*
[Sohn, Fergusso, Shellard (2023)]

Comparing $\mathcal{B}_\zeta$ with the Planck 2018 2σ bound

$$\Rightarrow \quad f_{\mathrm{ULDM},0} \leq 0.76\, C^{-\frac{1}{3}} \left(\frac{k_*}{1\mathrm{Mpc}^{-1}}\right)^2$$

- LSS

Signal-of-noise ratio (SNR) of $\mathcal{B}_{\mathrm{DM}}$

$$\Rightarrow \quad f_{\mathrm{ULDM},0} \leq 3.21\times10^{-9} C^{-\frac{1}{3}} \left(\frac{k_*}{1h\mathrm{Mpc}^{-1}}\right)^2 \times \left[\frac{V}{1(h^{-1}\mathrm{Gpc})^3}\right]^{-1/6} I(k_{\mathrm{max}})^{-1/6}$$

1. The final bound only loosely depend on the factor *C*
2. Current ULDM density bound solely depends on characteristic momentum k_*

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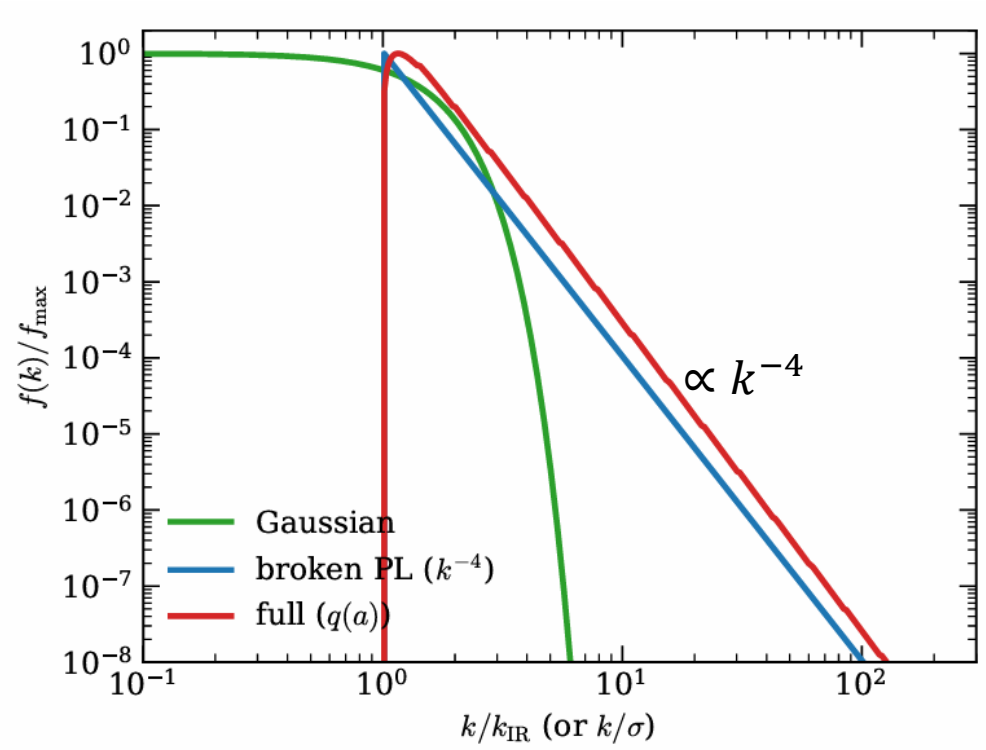
Applications: Two Scenarios

1. Gaussian distribution $f(\mathbf{k}) = \frac{\bar{\rho}_0}{m}\frac{(2\pi)^{3/2}}{k_{\rm osc}^3}\exp\left(-\frac{k^2}{2k_{\rm osc}^2}\right)$

- ULDM was created <u>nonrelativistically</u> $\Rightarrow\ \tau_{\rm NR} = \tau_{\rm osc}$
- $k_* = k_{\rm osc} \equiv \mathcal{H}_{\rm osc}$
- $C_G = 2/\left(3\sqrt{3}\pi\right)$

2. Cosmic string decay — Based on simulations

- ULDM was created <u>relativistically</u> $\Rightarrow\ \tau_{\rm NR} > \tau_{\rm osc}$
- $k_* = k_{\rm IR} = 10k_{\rm osc}$
- $C_{\rm cs} = 2.88\times10^{-2}$

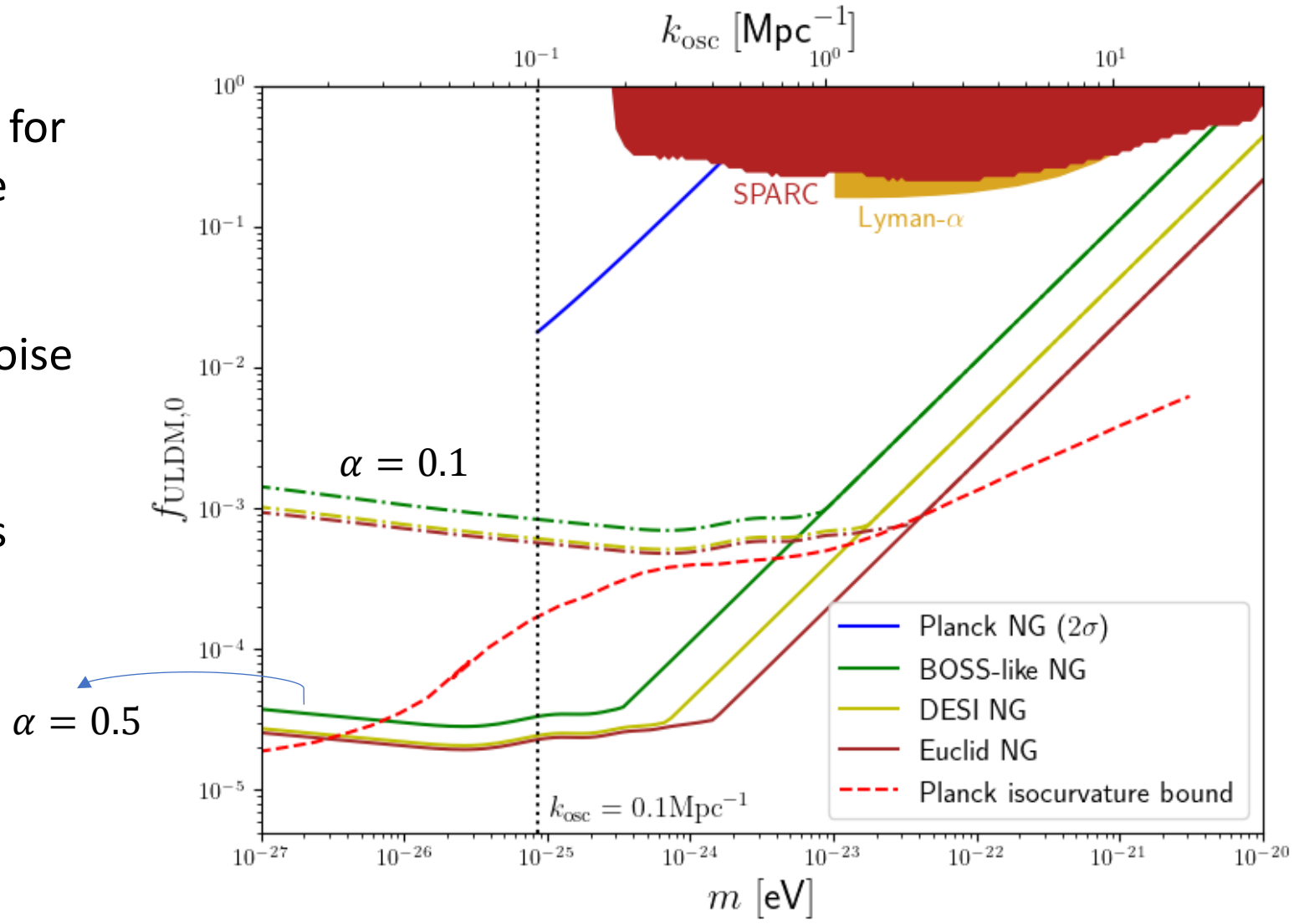


[Dror, Kyriazis (2026)]

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Applications: Gaussian Distribution

- We cutoff at $k_{\rm osc} = 0.1{\rm Mpc}^{-1}$ for Planck 2018 constraint to make sure $\tau_{osc} < \tau$
- Similarly, to avoid non-white-noise and nonlinear behaviors, we exclude k-modes larger than $\alpha k_{\rm osc}$. α can be some numbers smaller than 1.



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Applications: Cosmic String

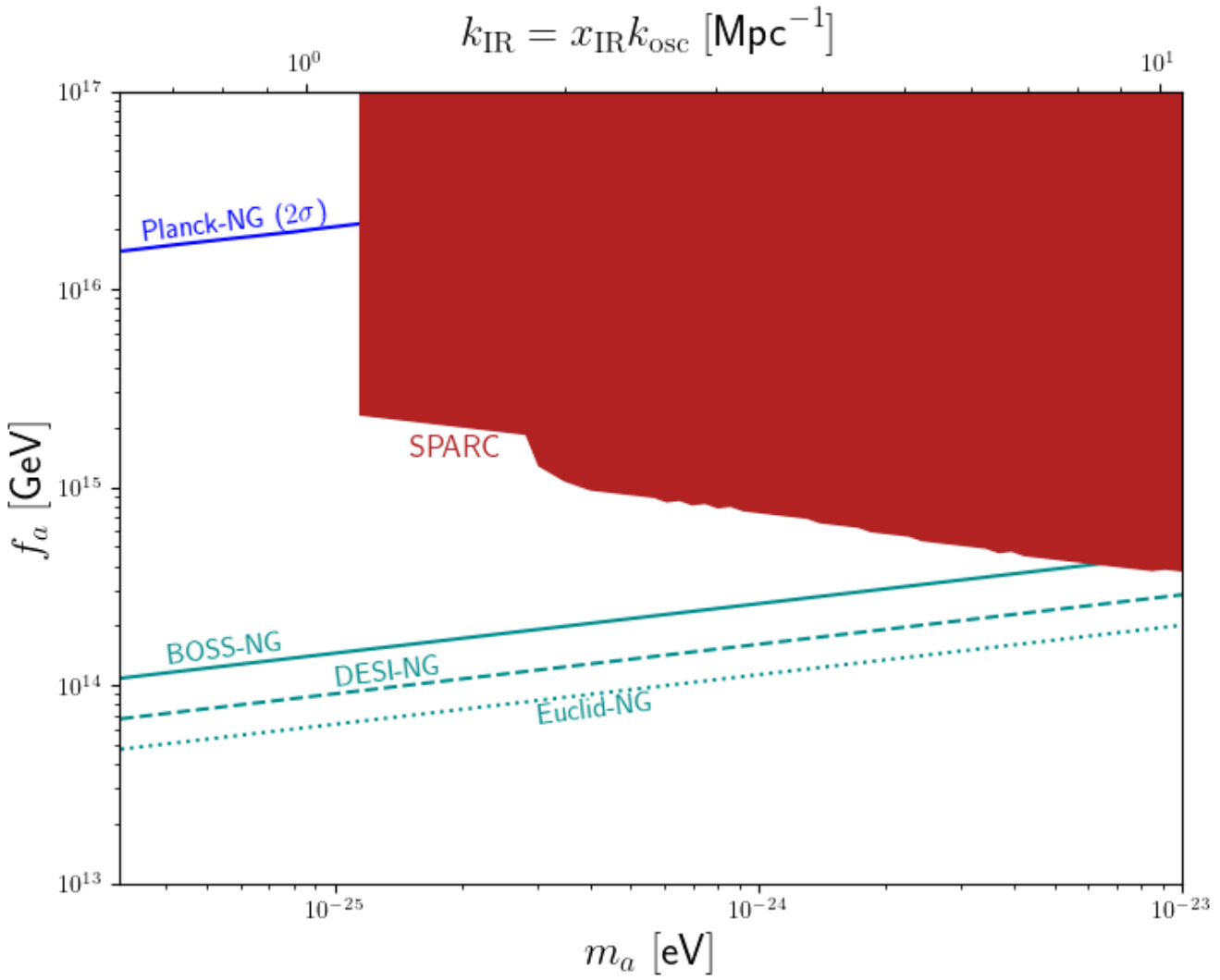
For cosmic string decay scenario, the constraint is commonly plot in $m_a - f_a$ plane

We can use the following two relations:

$$f_{\mathrm{ULDM},0} = 0.55 \left(\frac{f_a}{10^{15}\mathrm{GeV}}\right)^2 \left(\frac{m}{10^{-24}\mathrm{eV}}\right)^{\frac{1}{2}}$$

$$k_{\mathrm{IR}} = 10 k_{\mathrm{osc}} = 3.37 \times \left(\frac{m}{10^{-24}\mathrm{eV}}\right)^{\frac{1}{2}} \mathrm{Mpc}^{-1}$$

[Dror, Kyriazis (2026)]



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Conclusion

1. The ***ULDM energy density perturbations*** must be ***non-Gaussian***.
2. ULDM generates perturbations with ***non-Gaussianity*** on ***CMB*** and ***structure formation*** purely through ***gravitational effect***.
3. There is a simple relation between ***ULDM abundance constraint*** and ***ULDM characteristic momentum***.

Thanks!

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Backup

Observational signals: CMB

Bispectrum of curvature perturbation ζ:

$$\mathcal{B}_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \approx 7.45\times10^{-9} C f_{\mathrm{ULDM},0}^3 \frac{k_1k_2k_3}{k_*^6\,\mathrm{Mpc}^3}$$

→ $S \propto k_1k_2k_3$ Shape function

↓

Package: *CMB-BEST* [Sohn, Fergusso, Shellard (2023)]

←

Comparing this bispectrum with the Planck 2018 2σ bound:

$$\mathcal{B}_\zeta(k_1,k_2,k_3) \leq 3.21\times10^{-12} \frac{k_1k_2k_3}{(0.1\mathrm{Mpc}^{-1})^3}$$

We get the following generic bound

$$f_{\mathrm{ULDM},0} \leq 0.76\, C^{-\frac{1}{3}} \left(\frac{k_*}{1\mathrm{Mpc}^{-1}}\right)^2$$

- The final bound only loosely depend on the factor *C*
- Current ULDM density bound solely depends on characteristic momentum k_*

Observational signals: LSS

Using signal-to-noise ratio (SNR) to estimate future sensitivity:

$$\mathrm{SNR}^2 = V \int_0^{k_i<k_{\max}} \prod_{i=1}^{3} \frac{\mathrm{d}^3 k_i}{(2\pi)^3} \frac{B_{\mathrm{DM}}(k_1,k_2,k_3)^2}{6P(k_1)P(k_2)P(k_3)} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

- V: survey volume $\sim 1(h^{-1}\mathrm{Gpc})^3$
- $k_{\max}$: trusted largest wavenumber $\sim 0.1h\mathrm{Mpc}^{-1}$
- $P(\mathrm{k})$: adiabatic power spectrum
- Set SNR=2

$$f_{\mathrm{ULDM},0} \leq 3.21\times10^{-9} C^{-\frac{1}{3}} \left(\frac{k_*}{1h\mathrm{Mpc}^{-1}}\right)^2 \times \left[\frac{V}{1(h^{-1}\mathrm{Gpc})^3}\right]^{-1/6} I(k_{\max})^{-1/6}$$

Curvature Perturbation Evolution

$$\dot{\zeta} = -\frac{H}{\rho_{\rm tot} + p_{\rm tot}} \delta p_{\rm nad} \,, \tag{17}$$

where the dot is physical time derivative $\frac{d}{dt}$. The isocurvature source term

$$\delta p_{\rm nad} = \delta p_{\rm tot} - \frac{\dot{p}_{\rm tot}}{\dot{\rho}_{\rm tot}} \delta \rho_{\rm tot} \,. \tag{18}$$

In our setup,

$$\rho_{\rm tot} = \rho_a(a) + \rho_{\rm SM}(a) + \rho_{\rm DM-ALP}(a) \,, \quad p_{\rm tot} = \frac{1}{3}\rho_{\rm SM}(a) \,, \tag{19}$$

and the time derivatives

$$\dot{\rho}_{\rm tot} = -H\left[4\rho_{\rm SM}(a) + 3\rho_{\rm DM}(a)\right] \,, \quad \dot{p}_{\rm tot} = -\frac{4}{3} H \rho_{\rm SM}(a) \,, \tag{20}$$

Together with the total energy density perturbation

$$\delta\rho_{\rm tot} = \delta\rho_a \equiv \rho_a \delta_a \,, \quad \delta p_{\rm tot} = 0 \,, \tag{21}$$

we have

$$\delta p_{\rm nad} = -\frac{\rho_a(a)}{3\left[1 + \frac{3}{4}\frac{\rho_{\rm DM}(a)}{\rho_{\rm SM}(a)}\right]} \delta_a \,. \tag{22}$$

Curvature Perturbation Evolution

Define $f_A(a) = \rho_a(a)/\rho_{\rm SM}(a) = f_{A,\rm osc}(a/a_{\rm osc})$, and $f_{\rm DM}(a) = \rho_{\rm DM}(a)/\rho_{\rm SM}(a)$. The evolution of the curvature perturbation is therefore

$$\dot{\zeta} = aH\frac{d\zeta}{da} = \frac{H\frac{\rho_a(a)}{3\left(1+\frac{3}{4}f_{\rm DM}(a)\right)}}{\frac{4}{3}\left(1+\frac{3}{4}f_{\rm DM}(a)\right)\rho_{\rm SM}(a)}\,\delta_a = \frac{Hf_A(a)}{4\left(1+\frac{3}{4}f_{\rm DM}(a)\right)^2}\,\delta_a\,. \tag{23}$$

If $f_{\rm DM} \ll 1$ when the perturbation modes we're interested in enter the horizon,

$$\frac{d\zeta}{da} \approx \frac{1}{4}f_{A,\rm osc}a_{\rm osc}^{-1}\,\delta_a \Rightarrow \zeta(a) \approx \frac{f_{A,\rm osc}}{4}\left(\frac{a}{a_{\rm osc}}\right)\delta_a = \frac{f_{A,\rm osc}}{4}\left(\frac{\tau}{\tau_{\rm osc}}\right)\delta_a\,. \tag{24}$$

For the initial perturbation of a k-mode at horizon reentry, $k\tau \approx 1$, and

$$\zeta(k) \approx \frac{f_{A,\rm osc}}{4}(k\tau_{\rm osc})^{-1}\,\delta_a(k)\,. \tag{25}$$

Analytical Bounds

	Gaussian distribution (bound on $f_{\mathrm{ULDM},0}$)	Cosmic string (bound on f_{a} in [GeV])
CMB	$1.52\left(\frac{k_{\mathrm{osc}}}{1\mathrm{Mpc}^{-1}}\right)^{2}$	$5.49\times10^{15}\left(\frac{m}{10^{-24}\mathrm{eV}}\right)^{\frac{1}{4}}$
LSS	$2.34\times10^{-8}C^{-\frac{1}{3}}\left(\frac{k_{\mathrm{osc}}}{1h\mathrm{Mpc}^{-1}}\right)^{2}\times\left[\frac{V}{1(h^{-1}\mathrm{Gpc})^{3}}\right]^{-1/6}I(k_{\mathrm{max}})^{-1/6}$	$6.99\times10^{11}C^{-\frac{1}{6}}h^{-1}\left(\frac{m}{10^{-24}\mathrm{eV}}\right)^{1/4}\times\left[\frac{V}{1(h^{-1}\mathrm{Gpc})^{3}}\right]^{-1/12}I(k_{\mathrm{max}})^{-1/12}$

Nonlinearity in LSS

