Absence of linear power corrections in heavy-quark pair production at NNLO in a non-Abelian gauge theory



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- In an era of ultraprecise measurements, **non-perturbative corrections** need to be addressed

$$d\sigma_x = \sum_{ij} \int dx_i\, dx_j\, f_i(x_i)\, f_j(x_j)\, d\sigma^{\text{part}}_{ij\to x} \times \Big[1 + \mathcal{O}(\lambda^p)\Big], \quad \text{with} \quad \lambda \equiv \frac{\Lambda_{\text{QCD}}}{Q}$$

- Given $\lambda \ll 1$, power corrections will matter only if

  a) the observable is measured in an ultraprecise way, *b)* $\lambda^p = \lambda^1$: LINEAR corrections

- Then, do precisely measured observables receive linear power corrections?

- Such corrections are currently a source of debate in the interpretation of some observables...

  ...and are expected to be a recurrent issue as the experimental precision keeps increasing

- Studying these corrections from first principles is difficult. The renormalon model was often used [Beneke, 9807443]
  - One considers a certain class of diagrams, with the dominant contribution
  - The sensitivity to linear corrections can be probed by computing perturbative QCD corrections with a fake gluon mass
  - Yet, such class of diagrams is only known whenever there are no non-Abelian interactions

How to probe linear power corrections for general processes involving non-Abelian interactions ?

- This question may be related to KLN theorem [Kinoshita, 1963] [Lee, Nauenberg, 1964]
  - While in dim reg, KLN means $\frac{1}{\epsilon_{\rm IR}}$ cancel, with a cutoff λ it means $\log\lambda$ cancel

    To see this in more detail, let us write $\sigma[O] = \int d\Phi \, \frac{d\sigma}{d\Phi} \, O(\Phi)$. Then,
  - **Logarithmic** accuracy: $$\sigma[O]\big|_{\log\lambda} = \int d\Phi \left[\frac{d\sigma}{d\Phi}\Big|_{\log\lambda} O(\Phi)\big|_{\lambda^0} + \frac{d\sigma}{d\Phi}\Big|_{\lambda^0} O(\Phi)\big|_{\log\lambda} \right]$$

    At this accuracy, the usual KLN theorem says $\int d\Phi \, \frac{d\sigma}{d\Phi}\Big|_{\log\lambda} O(\Phi)\big|_{\lambda^0} = 0$
  - **Linear accuracy:** $$\sigma[O]\big|_{\lambda^1} = \int d\Phi \left[\frac{d\sigma}{d\Phi}\Big|_{\lambda^1} O(\Phi)\big|_{\lambda^0} + \frac{d\sigma}{d\Phi}\Big|_{\lambda^0} O(\Phi)\big|_{\lambda^1} \right]$$

    What happens at this accuracy? Could it be that $\int d\Phi \, \frac{d\sigma}{d\Phi}\Big|_{\lambda^1} O(\Phi)\big|_{\lambda^0} = 0$?
- This argument was developed several years ago, but we lack a clear proof in the non-Abelian case [Akhoury, Zakharov, 9512433]
- We need a framework to probe IR sensitivity — and, in particular, linear IR sensitivity — in general processes governed by non-Abelian interactions

- In this talk I present one such framework
  - Inspired by the renormalon model, we promote the gluon mass to a parameter of a proper QFT
  - Such QFT is a non-Abelian gauge theory, where the gluon gets its mass via the Higgs mechanism
  - The gluon mass m_g , assumed to be the smallest scale, plays the role of the IR scale
    - In particular, a linear dependence of an observable on m_g is a sign of linear power corrections
- As a first non-trivial application, I calculate heavy-quark pair production at NNLO, the first order in which non-Abelian effects first contribute
- I will show that, up to that order, $\left.\frac{d\sigma_{t\bar{t}}}{d\Phi}\right|_{m_g} = 0$, in agreement with the generalized KLN
- I will show this cancellation without really solving any integral

- The model is a renormalizable gauge theory, where SU(2) is spontaneously broken
- In analogy with QCD,
  - I refer to the gauge fields G as gluons
  - I refer to the massive fermion ψ_{t} as top quark
  - I consider n_{f} flavors of massless fermions ψ_k
- Defining $D_\mu = \partial_\mu + ig\hat{T}^a G^a_\mu$, the Lagrangian is

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \overline{\psi}_{\mathrm{t}}(\mathrm{i}\not{D} - m_{\mathrm{t}})\psi_{\mathrm{t}} + \sum_{k=1}^{n_{\mathrm{f}}} \overline{\psi}_k \mathrm{i}\not{D}\psi_k + (D_\mu\Phi)^\dagger(D^\mu\Phi) + \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{Ghost}}$$

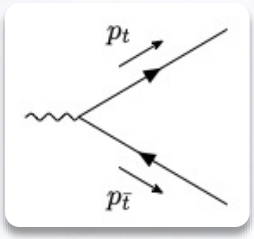
- With $\mu^2 > 0,\ \lambda > 0,$ the Higgs doublet Φ acquires a vacuum expectation value v
- The gluon and the Higgs boson acquire masses

$$m_{\mathrm{g}} = \frac{gv}{2}\,, \qquad m_{\mathrm{H}} = \sqrt{2\lambda}\,v\,, \quad \text{with} \quad v = \sqrt{\frac{\mu^2}{\lambda}}$$

  → probes IR sensitivity

- We take $m_{\mathrm{g}} \ll m_{\mathrm{t}} \sim m_{\mathrm{H}}$

- LO
  - I consider heavy-quark pair production by a neutral vector current

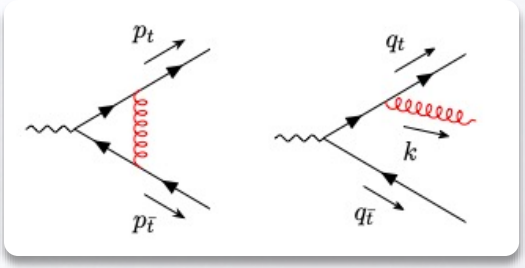


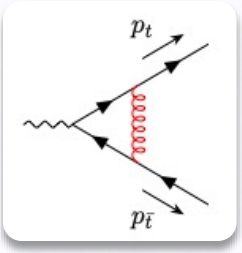
$$\mathcal{M}^{(0)} = \bar{u}(p_t)\gamma^\mu v(p_{\bar{t}})\,\epsilon_\mu \quad \Longrightarrow \quad \frac{\mathrm{d}\sigma^{(0)}_{t\bar{t}}}{\mathrm{d}\Omega_t} = \sigma_0 \beta^{1-2\epsilon}\left(3 - 2\epsilon - \beta^2\right)$$

$$\beta = \sqrt{1 - 4m_t^2/s}$$

Clearly, $\left.\dfrac{\mathrm{d}\sigma^{(0)}_{t\bar{t}}}{\mathrm{d}\Omega_t}\right|_{m_g} = 0$

- NLO
  - Here, we have to consider both virtual and real contributions
  - Individual pieces contain terms linear in m_g
  - To isolate them, we use the method of regions [Beneke, Smirnov, 9711391]
  - There are two regions for a loop momentum k: hard, $k \sim m_t$ soft, $k \sim m_g$
  - For our purposes, only the soft region matters





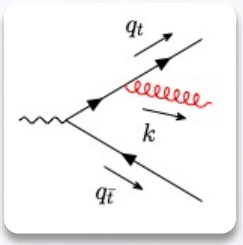
- Virtual contribution
  - Using Integration-By-Parts identities, we obtain just one master integral,

$$I_{\mathrm{V}} \equiv \int \frac{\mathrm{d}^d k}{i(2\pi)^d} \frac{1}{(k^2 - m_{\mathrm{g}}^2 + i0)} \frac{1}{(2k \cdot p_t + i0)}$$

  - Trick: solve the k^0 integral using residues. Result:

$$I_{\mathrm{V}} = \int \frac{\mathrm{d}^d k}{(2\pi)^{d-1}} \theta(k^0) \delta_+(k^2 - m_{\mathrm{g}}^2) \frac{1}{2k \cdot p_t} \equiv I_{\mathrm{R}}$$

  - This is a *real-emission integral*: it represents the emission of an on-shell gluon



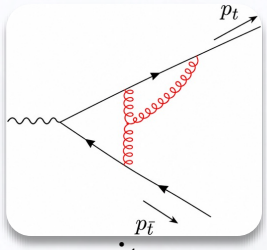
- Real-emission contribution
  - The phase space of final state is different from that of the Born process
  - Yet, one can map them with $\mathcal{O}(m_{\mathrm{g}})$ accuracy, e.g. $q_t = p_t - \frac{1+\chi}{2} k + A\, p_t + B\, p_{\bar{t}}$. We find:

$$\mathrm{dLips}(P; q_t, q_{\bar{t}}, k) = \mathrm{dLips}_{\mathrm{LO}}(P; p_t, p_{\bar{t}}) \frac{\mathrm{d}^d k}{(2\pi)^{d-1}} \delta_+(k^2 - m_{\mathrm{g}}^2) \Bigg\{ 1 + \frac{1}{(p_t \cdot p_{\bar{t}})^2 - m_{\mathrm{t}}^4} \Bigg[-(1-\epsilon) m_{\mathrm{t}}^2$$
$$\times \Big[(1+\chi)(p_t \cdot k) + (1-\chi)(p_{\bar{t}} \cdot k) \Big] + (p_t \cdot p_{\bar{t}}) \Big[[\epsilon - \chi(1-\epsilon)] (p_t \cdot k) + [\epsilon + \chi(1-\epsilon)] (p_{\bar{t}} \cdot k) \Big] \Bigg] + \mathcal{O}(k^2) \Bigg\}$$

  - The final result for the real-emission contribution is also written in terms of I_{R}

- Other contributions and total result
  - Besides renormalization, we have to use a short-distance mass for the heavy quark
  - This leads to extra contributions, which are all written in terms of I_{R}
  - All the terms are written in terms of I_{R} , and they *cancel* each other
  - So, we obtain $\left.\dfrac{\mathrm{d}\sigma^{(1)}_{t\bar{t}}}{\mathrm{d}\Omega_t}\right|_{m_g} = 0$ *without really having solved the master integral*

- NNLO



  - Many contributions: double-virtual, virtual-real, double-real, etc.
  - Still, we use the same strategy as at NLO. In particular, we use residue theorem to rewrite virtual integrals as real-emission ones
  - In this case, however, this rewriting is much more involved, and instead of just one master integral, there are many. Example, $\mathrm{Re}\, I^{(1)}_{\mathrm{VV}} = \mathrm{Re}\, I^{(1)}_{\mathrm{RV}} + \mathrm{Re}\, I^{(2)}_{\mathrm{RV}} - I^{(1)}_{\mathrm{RR}} - I^{(2)}_{\mathrm{RR}}$
  - The master integrals that depend on the scalar product $p_t \cdot p_{\bar{t}}$ are notoriously challenging
  - Putting everything together, we find $\left.\dfrac{\mathrm{d}\sigma^{(2)}_{t\bar{t}}}{\mathrm{d}\Omega_t}\right|_{m_g} = 0$
  - As at NLO, we obtained this cancellation without having solved any master integral

- With increasing experimental precision, we need to consider **non-perturbative corrections**
- I proposed a framework to investigate such corrections from a perturbative perspective
  - It is a renormalizable toy model based on an SU(2) gauge theory whose symmetry is spontaneously broken through the Higgs mechanism
  - Gluons acquire a small mass m_{g} in a theoretically consistent manner. A linear dependence on m_{g} probes a linear IR sensitivity of observables
- As a first non-trivial application, I calculated heavy-quark pair production through NNLO
- Using modern loop techniques, I was able to show $\left.\dfrac{\mathrm{d}\sigma_{t\bar{t}}}{\mathrm{d}\Omega_t}\right|_{m_g} = 0$ to this order
- This cancellation was obtained without really having solved any integral
- It is a concrete example of the extension of the KLN theorem to linear sensitivity
- Next step: heavy-quark pair production in actual QCD