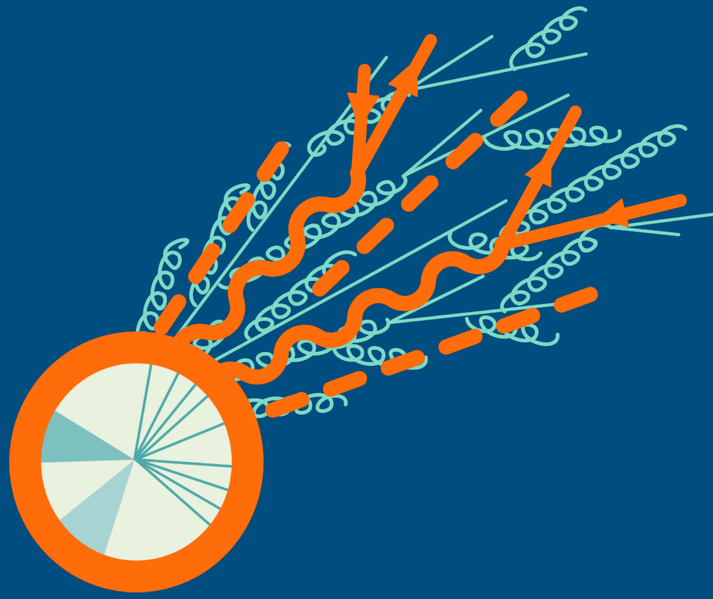


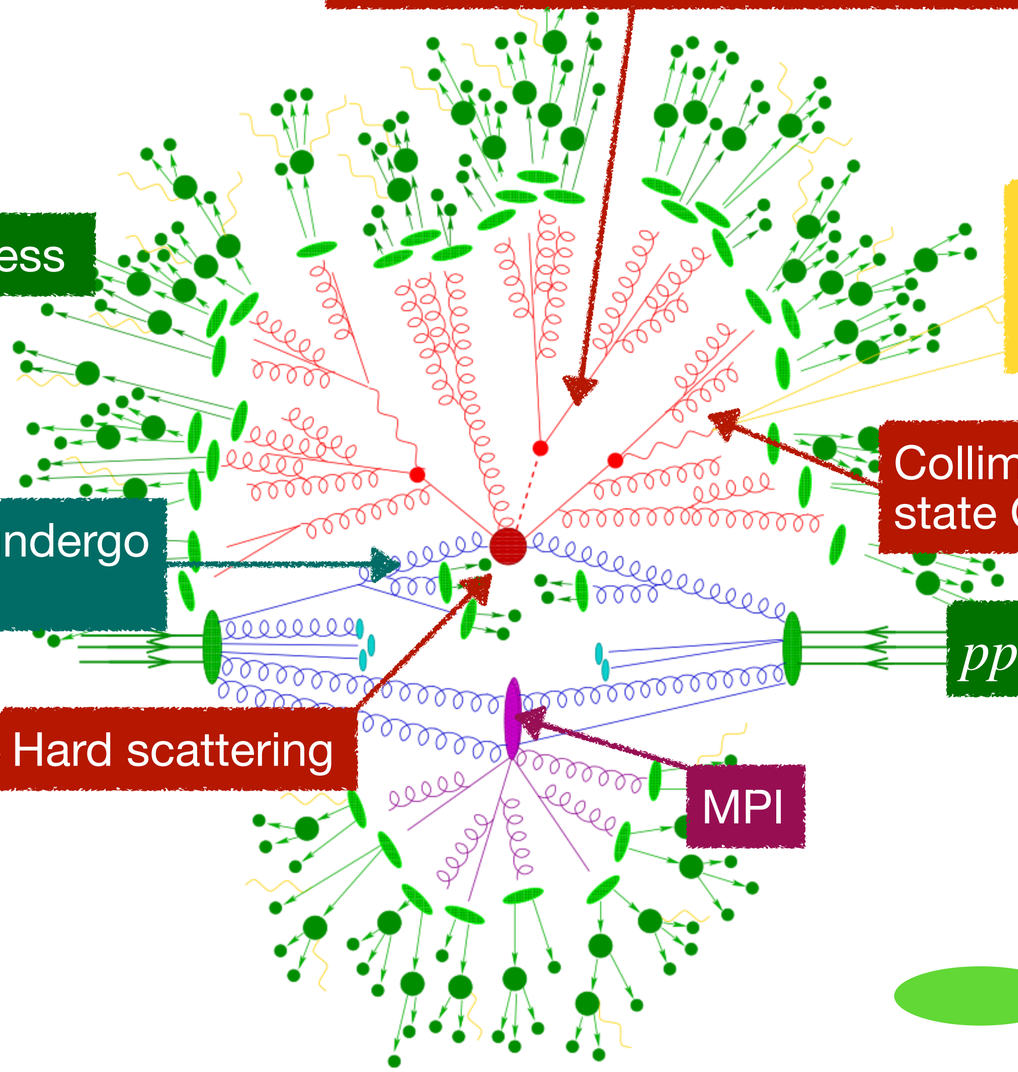
Towards Higher Accuracy Resummation in Heavy flavour processes

Based on results of arxiv:2606.20798 in collaboration with L. Mai, S. Marzani, D. Reichelt, S. Schumann, L. Stoecker, and on work in progress with L. Bason, S. Devoto, S. Marzani, P.F. Monni and G. Ridolfi.

University of Torino

Andrea Ghira, 15/10/2026

Introduction



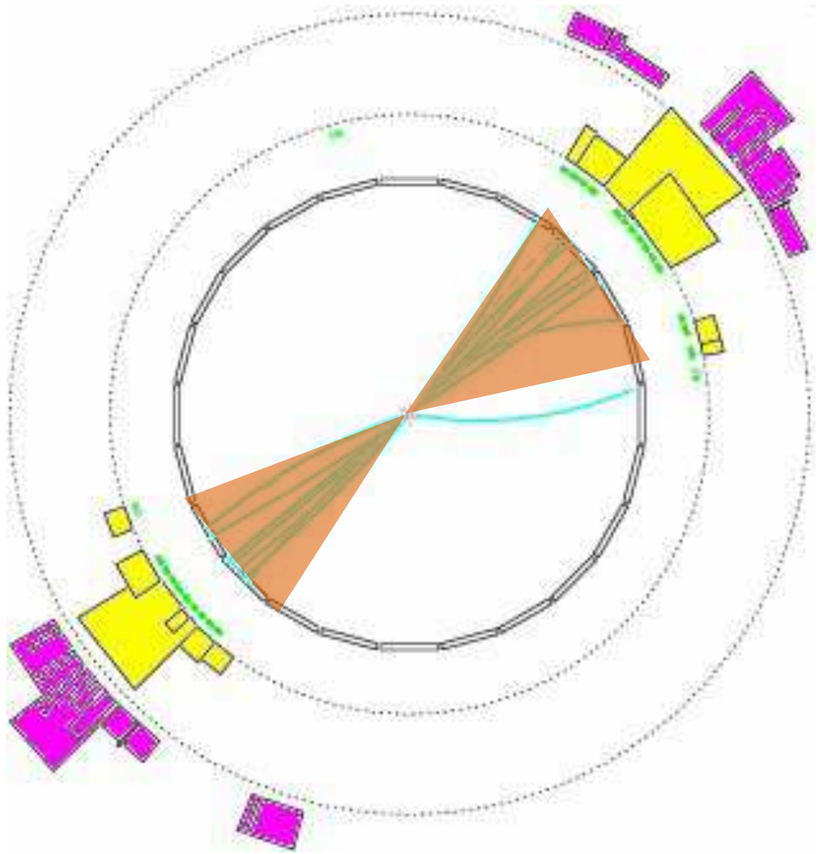
Quarks and gluon produced in the final state split into quarks and gluon at smaller and smaller angles until they reach a non perturbative scale $\Lambda_{\mathrm{QCD}} \sim 1\mathrm{GeV}$

= non perturbative scale

Cartoon for $t\bar{t}H$ production (T. Gleisberg et al.)

What is a jet?

- Overall, high energy produced in the collision appear in the final state as collimated bunch of hadrons: jets!



Jet definition is ambiguous, it depends on our definition of collimated. We need a defined procedure that tells us how to recombine together final state particles:

- Jet algorithms: how do we cluster particles together?
- Jet radius: how do we say that two particles belong to the same jet?
- Recombination scheme: what is the jet kinematics?

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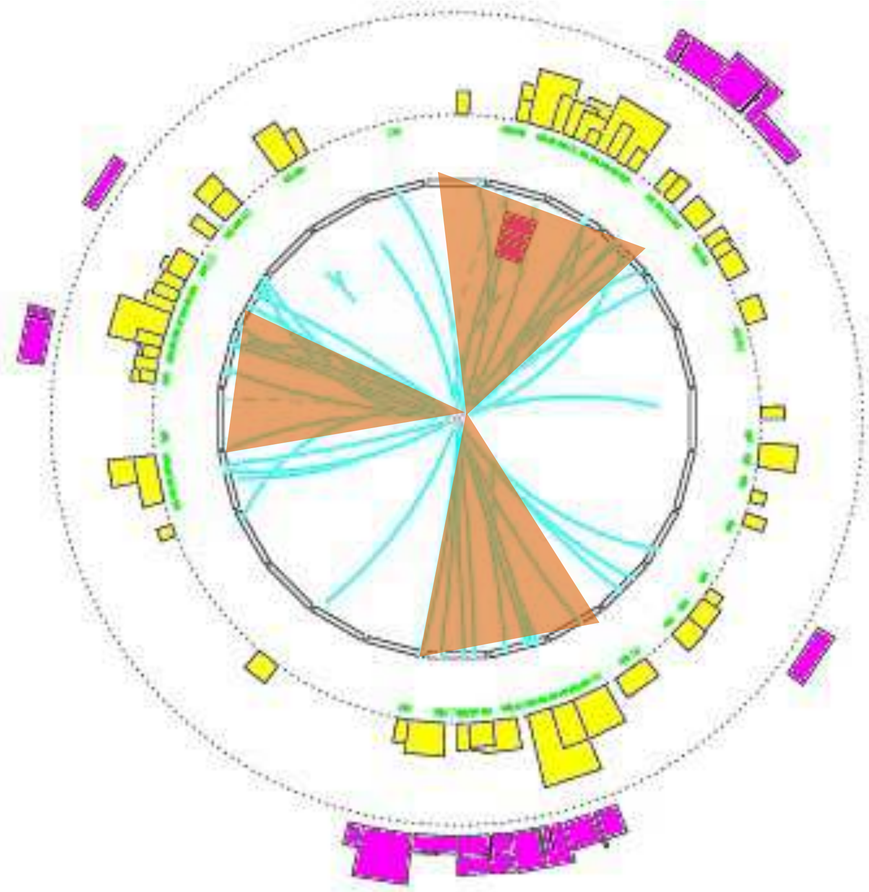

3 or 4 jets event?

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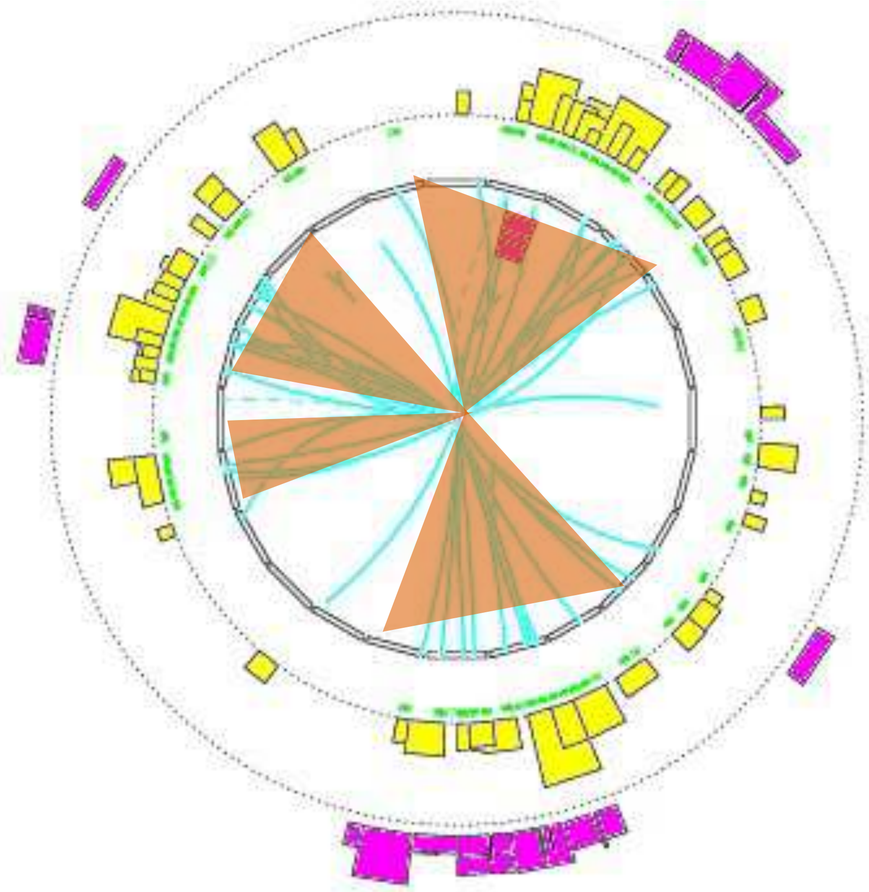


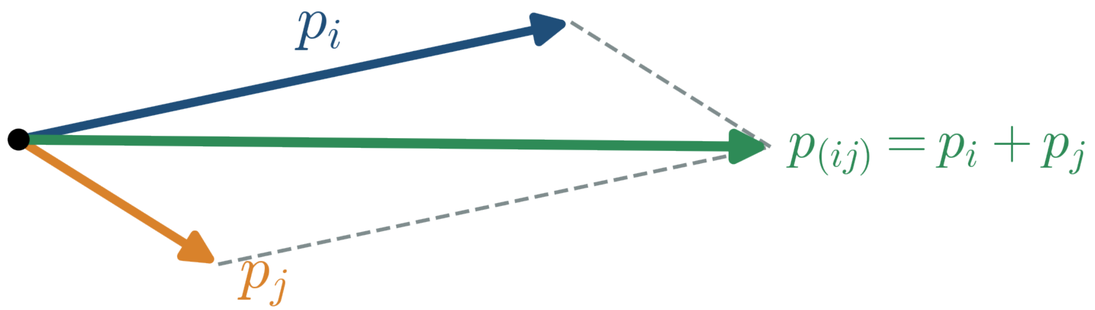
3 or 4 jets event?

Recombination scheme

These are two of the mostly used recombination algorithms:

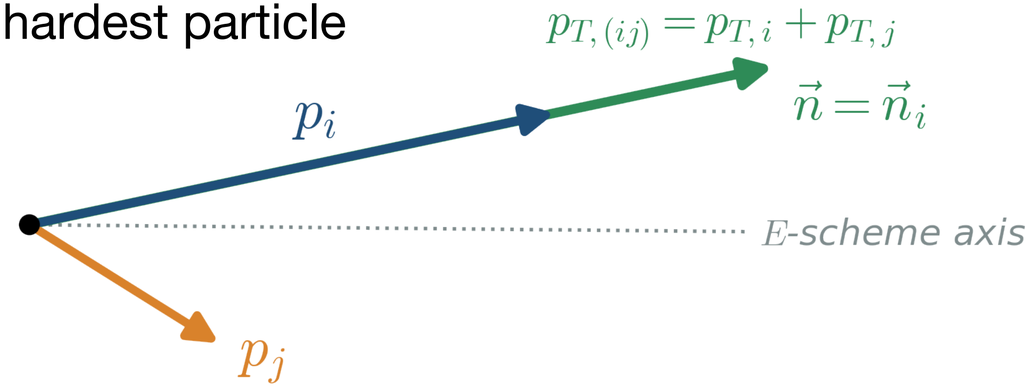
E-scheme: the 4-momentum of the recombined particle is the sum of its constituents

- $p_{(ij)} = p_i + p_j$, with (ij) recombined particle according to some metric, i, j original particles.



WTA-scheme (A.Larkoski, D. Neill, J. Thaler): the p_T of the recombined particle is the sum of its constituents. The direction is determined by the hardest particle

- $p_{T_{(ij)}} = p_{T_i} + p_{T_j}$ and $\vec{n} = \begin{cases} \vec{n}_i & \text{if i harder than j} \\ \vec{n}_j & \text{otherwise} \end{cases}$



Gen-k_t algorithms

- Take all the particles in one event as the initial list of object
- Build two set of distances: $d_{ij} = \min\left(p_{T_i}^{2p}, p_{T_j}^{2p}\right)\Delta R_{ij}^2$ Distance between final state particles

$d_{i\mathrm{B}} = p_{T_i}^{2p} R_0^2$ Distance between particles and beams

Jet radius

- Find the minimum of all of all $d_{ij}, d_{i\mathrm{B}}$.
- If the smallest is one of the d_{ij} cluster (ij) into a single pseudo jet, which is added to the list of objects
- If the smallest is one of the $d_{i\mathrm{B}}$, then i is a jet and it is removed from the list
- Return to step 2 until the list is empty

$$\Delta R_{ij} = \sqrt{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}$$

Gen-k_t algorithms

- $p = 1$: k_t **algorithm** (Catani et al.): motivated by the fact that in the soft and collinear limit QCD matrix factorizes as:

$$|\mathscr{M}|^2 d\phi \sim |\mathscr{M}_0|^2 d\phi_0 \frac{dk_t^2}{k_t^2} \frac{dz}{z}$$

$$d_{ij} = \min(p_{T_i}^2, p_{T_j}^2)\Delta R_{ij}^2, \quad d_{i\mathrm{B}} = p_{T_i}^2 R_0^2$$

- $p = 0$: **Cambridge/Aachen algorithm** (Dokshitzer et al.): purely geometrical algorithm. Particles close in angle are recombined first (collinear enhancement of QCD)

$$d_{ij} = \Delta R_{ij}^2, \quad d_{i\mathrm{B}} = R_0^2$$

- $p = -1$: **anti-k_t algorithm** (M. Cacciari, G. Salam and G. Soyez): soft particles cluster first around a hard core until they reach a size R_0. Less sensitive to soft radiation.

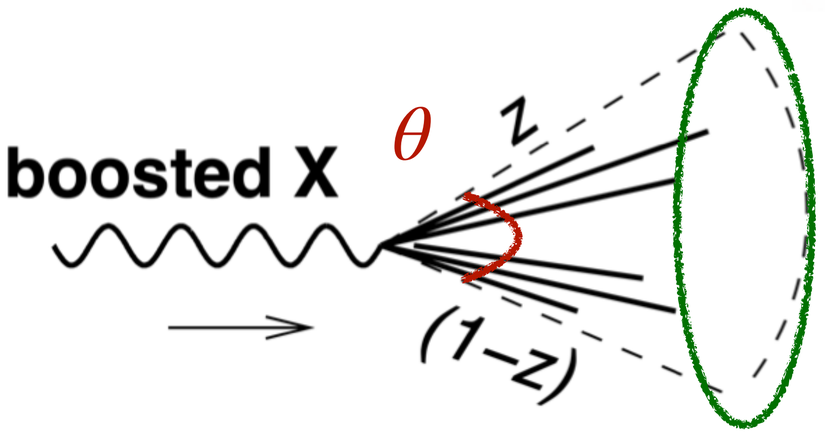
$$d_{ij} = \min\left(\frac{1}{p_{T_i}^2}, \frac{1}{p_{T_j}^2}\right)\Delta R_{ij}^2, \quad d_{i\mathrm{B}} = \frac{1}{p_{T_i}^2} R_0^2$$

Looking inside jets



- What is the interaction between jets constituents? We study jets internal structure: it may have multiple cores in the same jet. E.g. we want to distinguish QCD from a E-W boson jet

- In the boosted regime, different decay product may end up in a single jet. What size are we talking about?



$R_0 \sim 0.4$

$$M_X^2 = 2E_X^2 z(1-z)\cos\theta \sim E_X^2 z(1-z)\theta^2 \implies \theta \sim \underbrace{\frac{2M_X}{E_X}} \sim 0.15$$

$$M_X \sim 80\,\text{GeV}, E_X \sim 1\,\text{TeV}$$

Decay product of the EW boson are likely to end up in a single jet

How do we distinguish the signal from the background? Look at a substructure observable

Sudakov resummation

For substructure observable we mean a function of external momenta that need to be IRC safe.

- There are kinematical regions in which the convergence of the series is spoiled due to a non complete cancellation between real and virtual corrections:

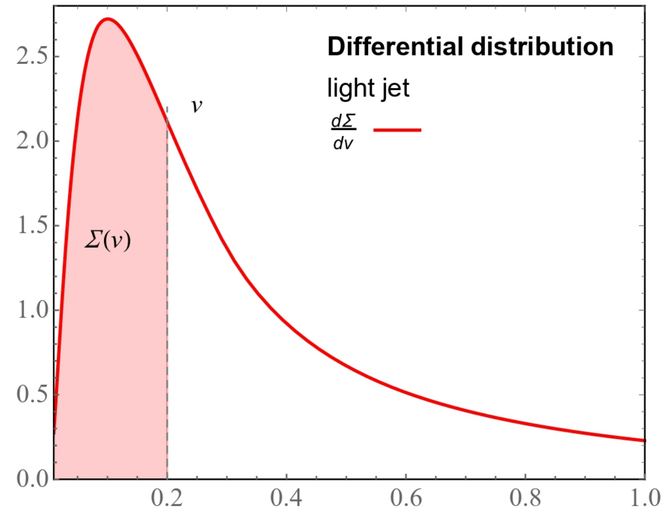
$$\frac{1}{\sigma_0}\frac{d\sigma}{dv} \overset{v\to 0}{\longrightarrow} \sum_n^{\infty} \sum_k^{2n-1} \alpha_S^n \, c_k \frac{1}{v} \log^k v$$

v function of the external momenta that tends to 0 in the Born configuration, i.e. soft and collinear limit

Cumulative distribution:

$$\Sigma(v) = \frac{1}{\sigma_0}\int_0^v dv' \frac{d\sigma}{dv'} \overset{v\to 0}{\longrightarrow} \sum_n^{\infty} \alpha_S^n \sum_k^{2n} c_k \log^k v$$

Need resummation to get reliable predictions!



Exponentiation of additive observables

Assume that in the case of n emissions, our observable behaves as: (case of jet angularities, thrust, jet mass, etc..)

$$\mathscr{V} = \sum_{i \in J}^{n} \left(\frac{k_{t_i}}{Q} \right) e^{-b\eta_i} = \sum_{i \in J}^{n} \mathscr{V}_i$$

At leading log accuracy therefore write the all-order cumulative distribution as a sum over uncorrelated emission:

$$\Sigma(v) = \sum_{n} \frac{1}{n!} \prod_{i}^{n} \int_{0}^{Q^2} \frac{dk_{t_i}^2}{k_{t_i}^2} \int_{0}^{1} dz_i \frac{\alpha_S}{2\pi} P_{qg} \left(1 - \Theta\left(v - \mathscr{V} \right) \right) \Theta_{i \in J}$$

→Only emissions inside the jet contribute to the jet mass. Virtual corrections always allowed

The kinematics factorizes in Laplace space where

$$\Theta(v - \mathscr{V}) = \frac{1}{2\pi i} \int_{\gamma} \frac{dN}{N} e^{Nv} \prod_{i}^{n} e^{-N\mathscr{V}_i}$$

Laplace inversion

- The overall cumulative distribution can be therefore be written as:

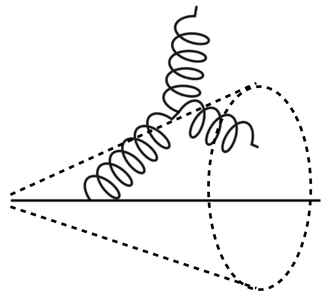
$$\Sigma(v) = \frac{1}{2\pi i}\int_\gamma \frac{dN}{N} e^{Nv} e^{-R\left(\frac{1}{N}\right)}$$

We expand around $N = 1/v$ (Catani, et al.)

At LL. one finds: $\Sigma(v) \simeq e^{-R(v)}$, $R(v) = \int_0^{Q^2} \frac{dk_t^2}{k_t^2}\int_0^1 dz P_{qg}\frac{\alpha_S(k_t^2)}{2\pi}\Theta\left[\left(\frac{k_t}{Q}\right)^a e^{-b\eta} - v\right]$

- Within this approximation, the all order cumulative distribution is given by the exponentiation of the single gluon emission as in QED
- Gluon emissions are strongly ordered in energy and in angle: $\Theta(\sum_{i\in J}\mathscr{V}_i - v) \simeq \Theta\left(\max \mathscr{V}_i - v\right)$: single gluon emission dominates the observable

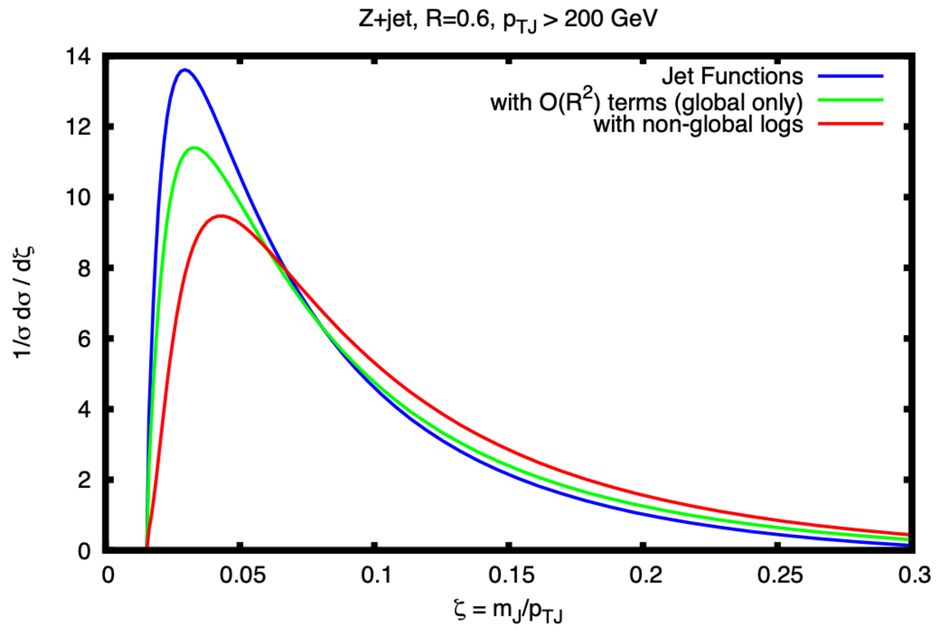
Towards higher accuracy



NGL configuration

We have simplified a lot the derivation of the all order calculation: many missing ingredient

- Clustering effects (clustering condition is more complicated to deal with already at NLL)
- CMW effective coupling ($\alpha_S(k_t) \to \alpha_S^{\mathrm{CMW}}(k_t)$)
- Non global logs (correlated gluon emissions at the border of the jet)
- Multiple emission corrections to the constraint $\Theta(\sum_{i \in J} \mathscr{V}_i - v)$
- Power corrections in the jet radius

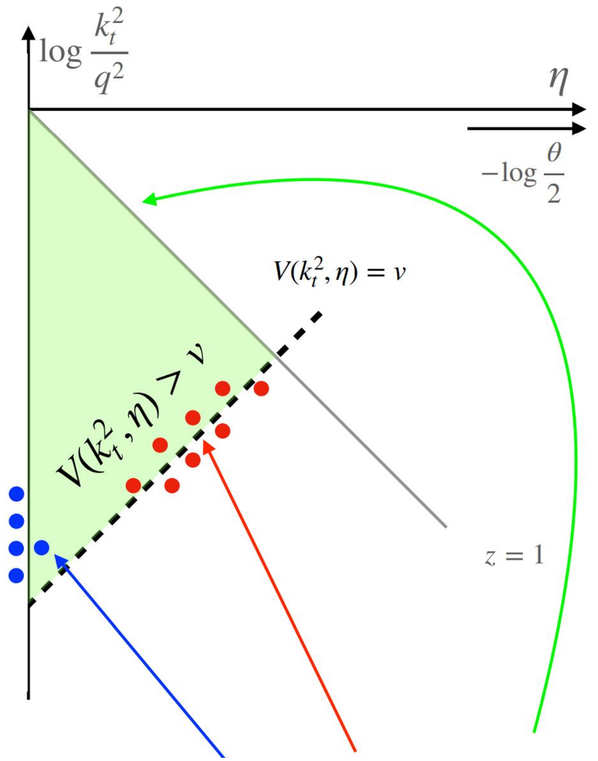


Dasgupta et al.

Graphical interpretation: Lund Plane

- We are vetoing all the emissions that would lead a value of the observable grater than v.
- The value of the cumulative distribution is related to the green area: $\Sigma(v) \simeq e^{-\frac{\alpha_S C_F}{2\pi}\log^2 v}$
- At NLL, the simple exponentiation receives corrections from non-global logs and multiple emissions.
  - Multiple emission: @NLL the cumulative is corrected by the fact that the observable depends on more than one emission
  - Non global configuration: gluon emission at commensurate angles in the vicinity of the jet boundary

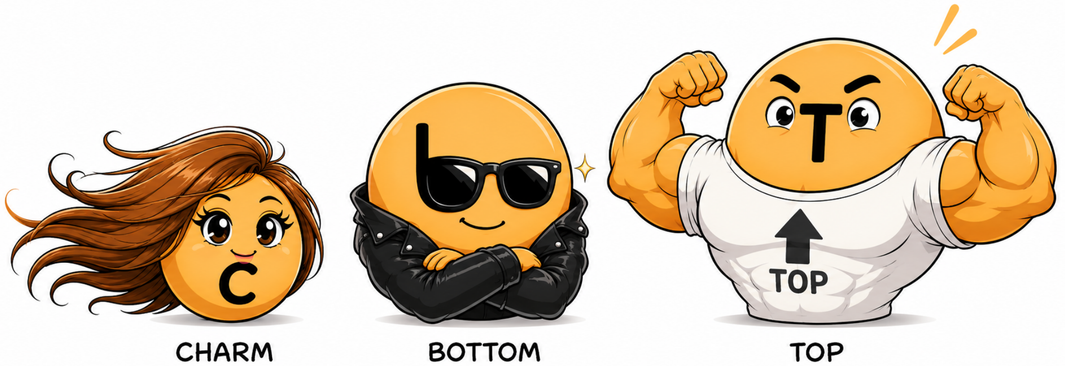
Lund plane: available phase space of soft and collinear emission



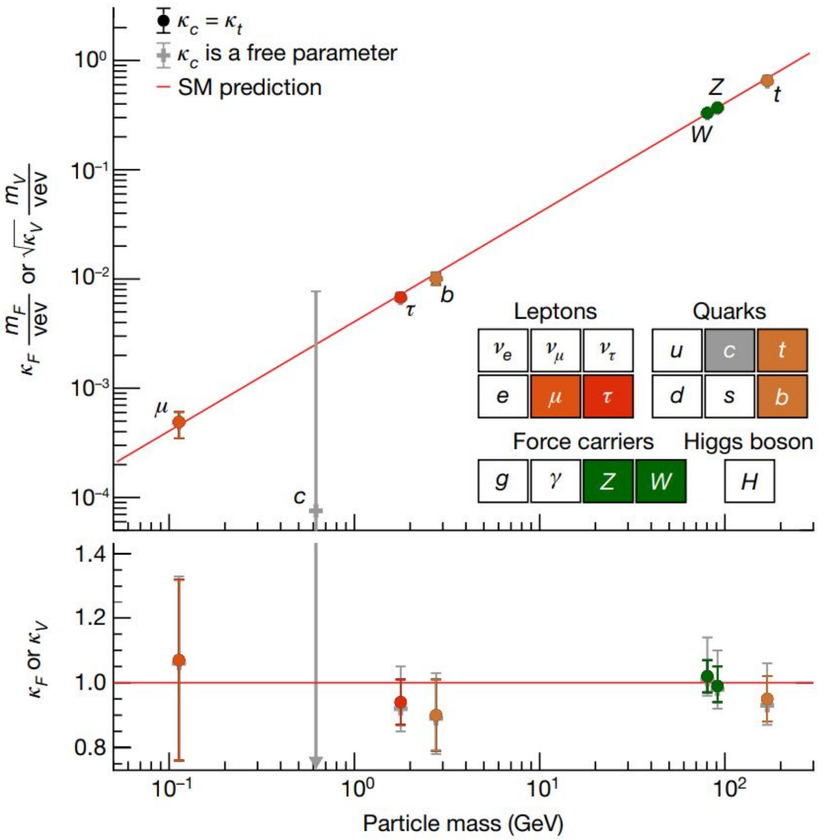
$$\Sigma(v) = S \times \mathcal{F} \times e^{-R}$$

Heavy flavour jet substructure

Motivation



- All the fundamental particles acquire mass through the Higgs mechanism: SM prediction $Y_f \propto m_f$
- Heavy flavor processes offer a more robust test of pQCD
- $b - c$ quarks constitute portal to non perturbative physics
- Recent measurements on jss observable for H-F jets:
  - Lund b jet plane by LHCb collaboration,
  - θ_g, z_g observable for charm jets by ALICE,
  - Upcoming measurements at ATLAS and CMS for b-jet substructure to expose dead-cone effect



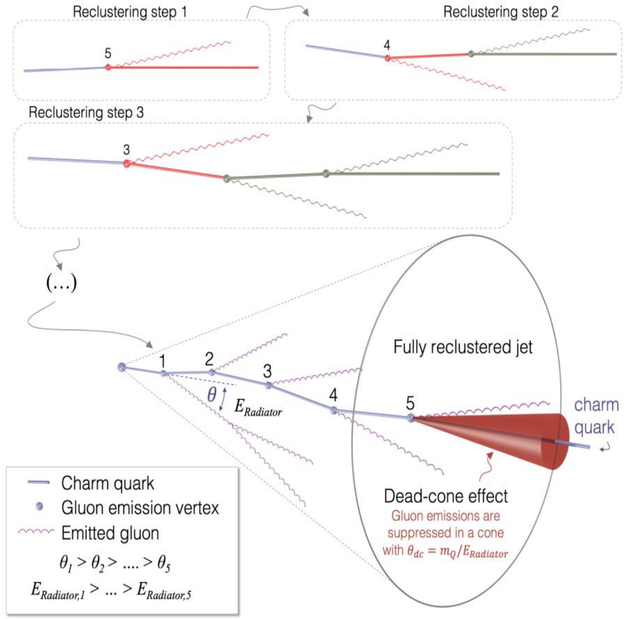
Heavy quarks jets

When jets are initiated by a heavy flavor, the quark mass shields the collinear singularity

$$d\sigma \simeq \frac{\alpha_S C_F}{2\pi} \frac{dz}{z} \frac{d\theta^2}{\theta^2 + \frac{m^2}{E^2}}$$

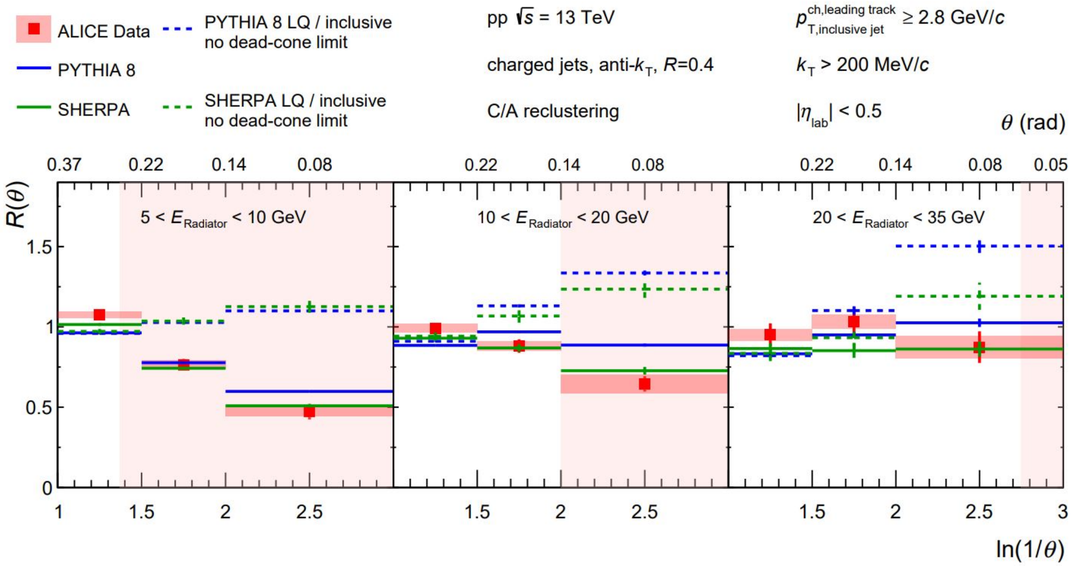
$m =$ heavy quark mass

$E =$ heavy quark energy



Dead cone effect

- the radiation emitted off a heavy flavor is suppressed inside a cone of opening angle $\theta_D \sim m/E$ (ALICE)
- Many recent and ongoing measurements on H-F jets by CMS, ATLAS, ALICE, LHCb

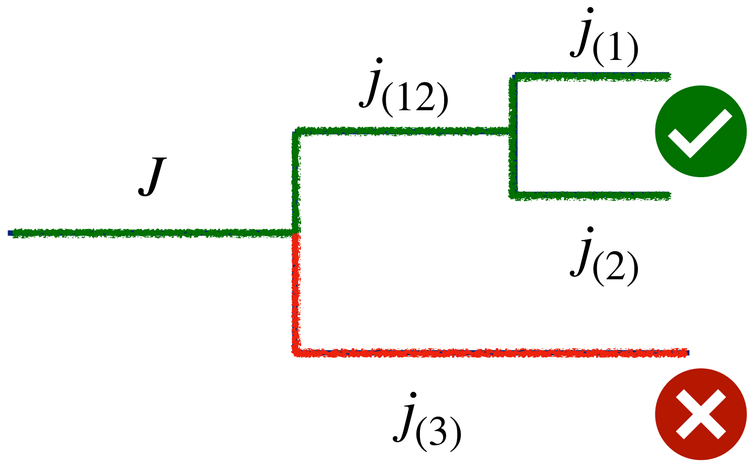


Soft Drop algorithm to expose the dead-cone

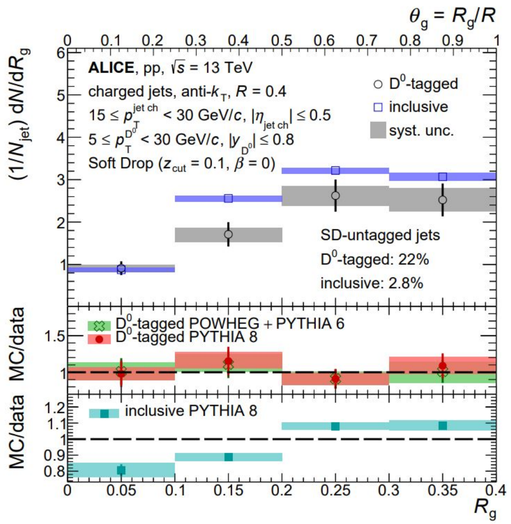
The Soft- Drop algorithm is a widely used grooming technique (removing gluon radiation at large)

- Recluster your anti-k_t jet with C/A
- Undo the the last step of C/A and look if the following condition is satisfied:
- $$\frac{\min(p_{T_{(12)}}, p_{T_3})}{p_{T_{(12)}} + p_{T_3}} > z_{\text{cut}} \left(\frac{\Delta R_{(12),3}}{R_0} \right)^{\beta},$$
- If the criterion is satisfied J is the Soft-Drop jet. If not the softest branch is descarded and we iterate following the hardest branch

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$



A. Larkoski, S. Marzani, G. Soyez and J. Thaler



ALICE collaboration

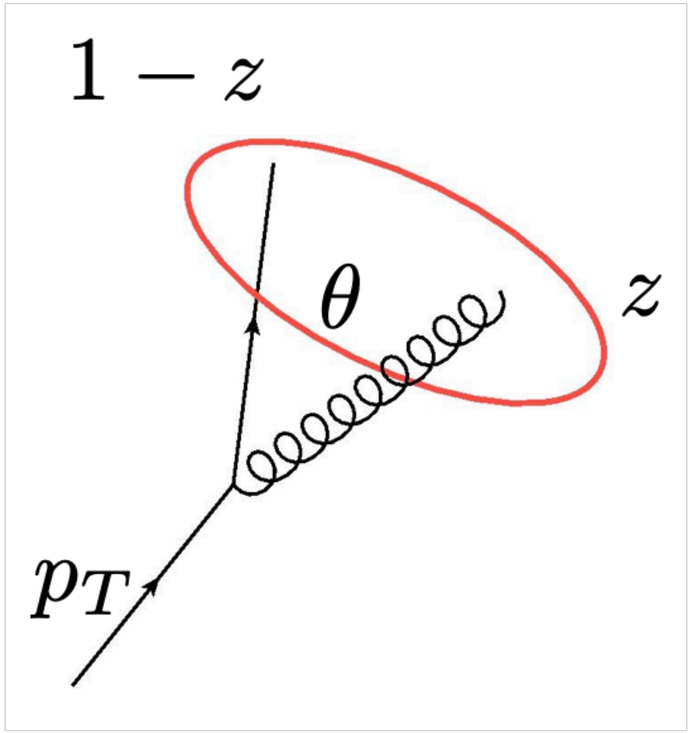
Theoretical computation performed by S. Caletti, A. G., S. Marzani

Jet angularities at hadron colliders

- We want to study jet angularities (sensitive to radiation pattern, i.e. to HF jet substructure).
- In case of massless theory with just one emission $\lambda_\alpha \simeq z\theta^\alpha$
- Many more possible definitions when massive particles are involved:

$$\lambda_\alpha = \sum_i \frac{p_{ti}}{p_T}\left(\frac{\Delta R_i}{R_0}\right)^\alpha, \quad \mathring{\lambda}_\alpha = \sum_i \frac{p_{ti}}{p_T}\left(\frac{2p_i\cdot n}{p_{ti}R_0}\right)^{\frac{\alpha}{2}}, \quad \mathring{\lambda}_{\alpha,0} = \sum_i \frac{p_{ti}}{p_T}\left(\frac{2p_i\cdot n_0}{p_{ti}R_0}\right)^{\frac{\alpha}{2}}$$

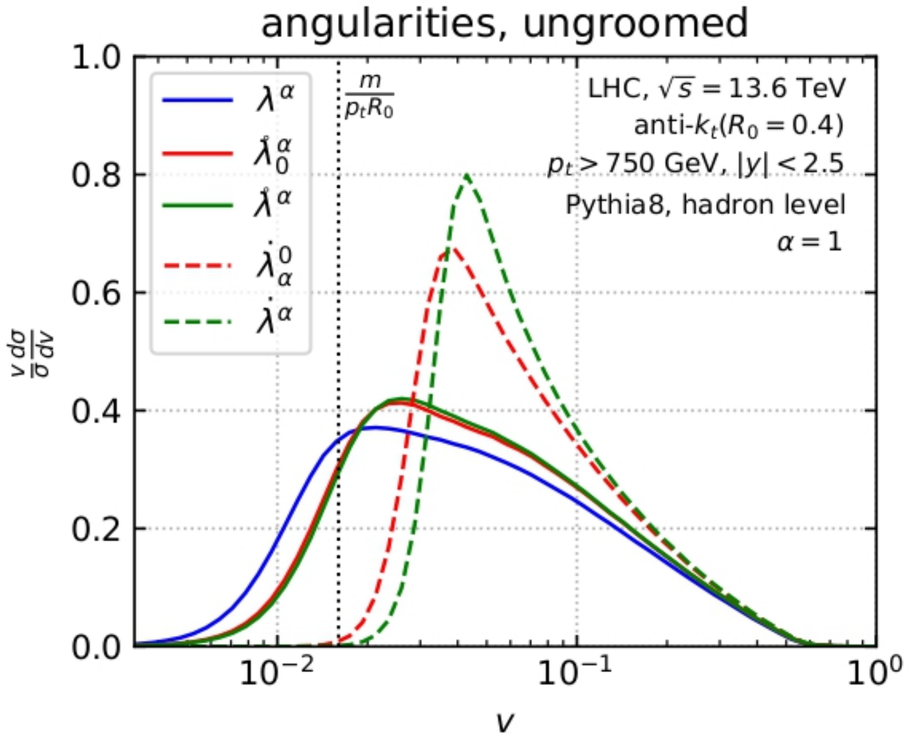
- All these variables coincide in the massless case.
- All these variables do not coincide when one take the quasi-collinear limit ($\theta_{k,\text{jet}} \sim m/p_T$)



$n(n_0)$ is a massive (massless) 4 vector aligned with the WTA axis

See also (C. Lee, P. Shrivastava, V. Vaidya), for SCET analyses of ECFs.

Jet angularities definitions: Monte Carlo study



$$\lambda_\alpha = \sum_i \frac{p_{ti}}{p_T}\left(\frac{\Delta R_i}{R_0}\right)^\alpha$$

P.K. Dhani et al.

$$\mathring{\lambda}_\alpha \simeq \sum_i \frac{p_{ti}}{p_T}\left(\frac{m_i^2}{p_{ti}^2 R_0^2} + \frac{m_n^2}{p_T^2 R_0^2} + \frac{\Delta R_i^2}{R_0^2}\right)^{\frac{\alpha}{2}}$$

$$\mathring{\lambda}_{\alpha,0} \simeq \sum_i \frac{p_{ti}}{p_T}\left(\frac{m_i^2}{p_{ti}^2 R_0^2} + \frac{\Delta R_i^2}{R_0^2}\right)^{\frac{\alpha}{2}}$$

- The circled observables exhibits a larger peak than λ_α : more sensitive to mass effects as mass enter both at amplitude and at substructure observable level

- The mass contribution in on λ_α distribution is only “dynamical” as mass effects enter only at amplitude level. More sensitive to suppression of collinear radiation

Resummation in the CAESAR formalism

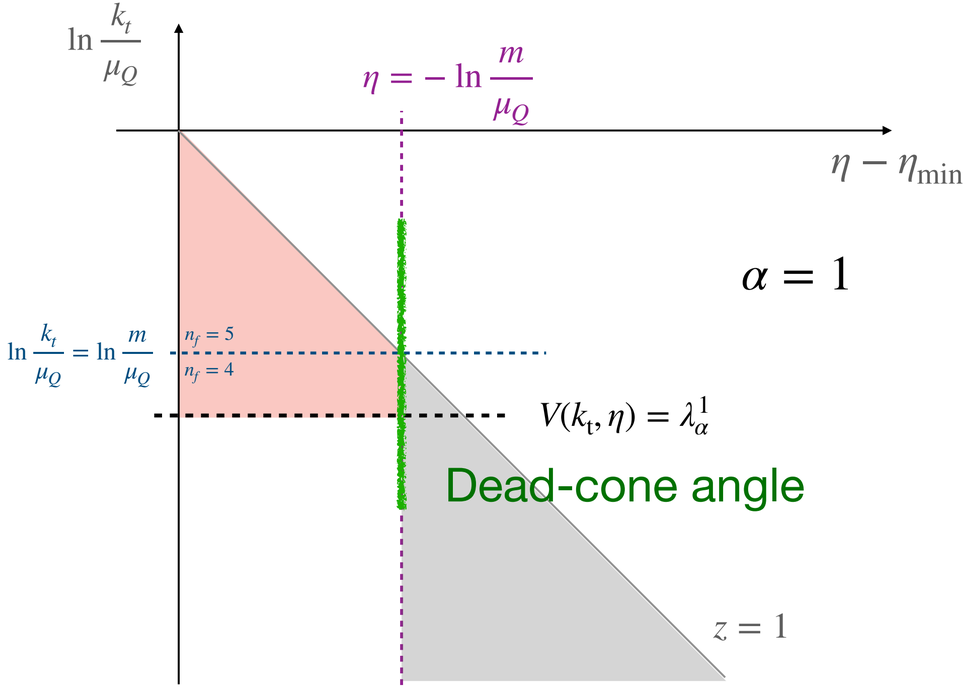
Resummation performed using the CAESAR formalism (A. Banfi, G. Salam, G. Zanderighi)

$$\Sigma_{\rm res}(v) = \sum_{\delta} \int d\mathcal{B}_\delta \frac{d\sigma_\delta}{d\mathcal{B}_\delta} \exp\left[-\sum_{l\in\delta} R_l^{\mathcal{B}_\delta}(v)\right] \mathcal{S}^{\mathcal{B}_\delta}(v)\, \mathcal{P}^{\mathcal{B}_\delta}(v)\, \mathcal{F}^{\mathcal{B}_\delta}(v)\, \mathcal{H}^{\delta}(\mathcal{B}_\delta),$$

D. Reichelt

Here, R denotes the massive radiator:

$$R = \int_0^1 dz \int_{z^2m^2}^{\mu_Q^2} \frac{dk_t^2}{k_t^2} \frac{\alpha_S^{\rm CMW}(k_t^2)}{2\pi} P_{gb}(z, k_t^2 - z^2m^2) \times \Theta_\alpha \times \Theta(\eta - \eta_{\rm min})$$



Θ_α = observable constraint

$\eta_{\rm min}$= lower bound in rapidity

P_{gb} = massive splitting function

Resummation in the CAESAR formalism for groomed jets

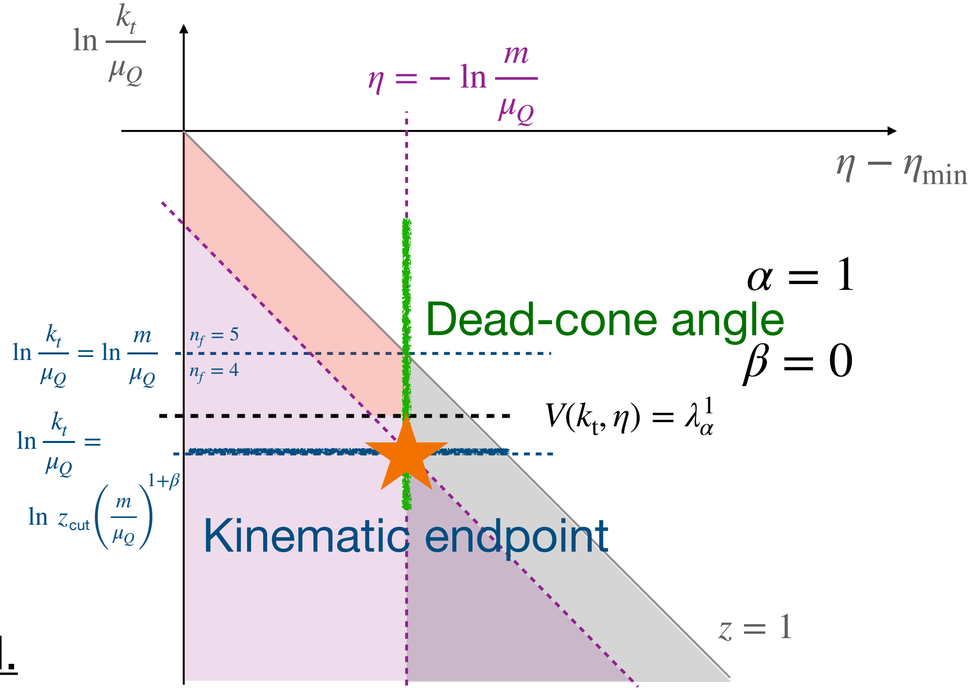
Resummation performed using the CAESAR formalism (A. Banfi, G. Salam, G. Zanderighi)

$$\Sigma_{\rm res}(v) = \sum_{\delta} \int d\mathcal{B}_\delta \frac{d\sigma_\delta}{d\mathcal{B}_\delta} \exp\left[-\sum_{l\in\delta} R_l^{\mathcal{B}_\delta}(v)\right] \mathcal{S}^{\mathcal{B}_\delta}(v)\, \mathcal{P}^{\mathcal{B}_\delta}(v)\, \mathcal{F}^{\mathcal{B}_\delta}(v)\, \mathcal{H}^{\delta}(\mathcal{B}_\delta),$$

D. Reichelt, J. Baron et al.

Here, R denotes the massive radiator:

$$R = \int_0^1 dz \int_{z^2m^2}^{\mu_Q^2} \frac{dk_t^2}{k_t^2} \frac{\alpha_S^{\rm CMW}(k_t^2)}{2\pi} P_{gb}(z, k_t^2 - z^2m^2) \times \Theta_\alpha \times \Theta(\eta - \eta_{\rm min}) \times \Theta_{\rm SD}$$



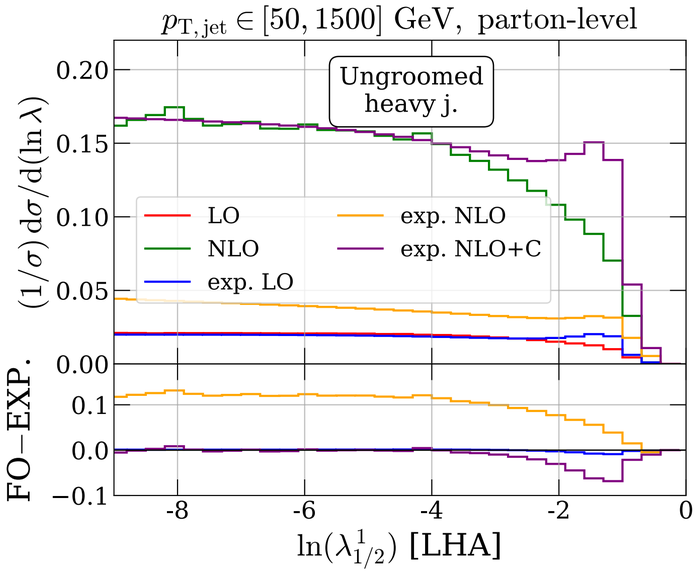
Θ_α = observable constraint

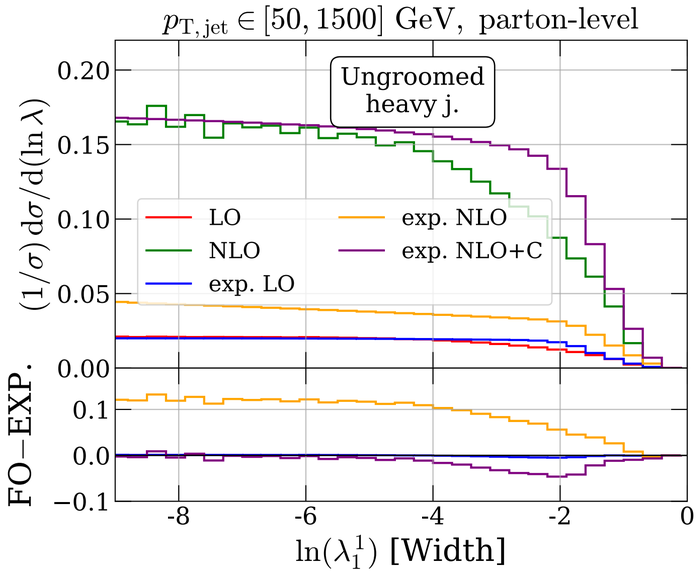
$\Theta_{\rm SD}$ = soft drop constraint

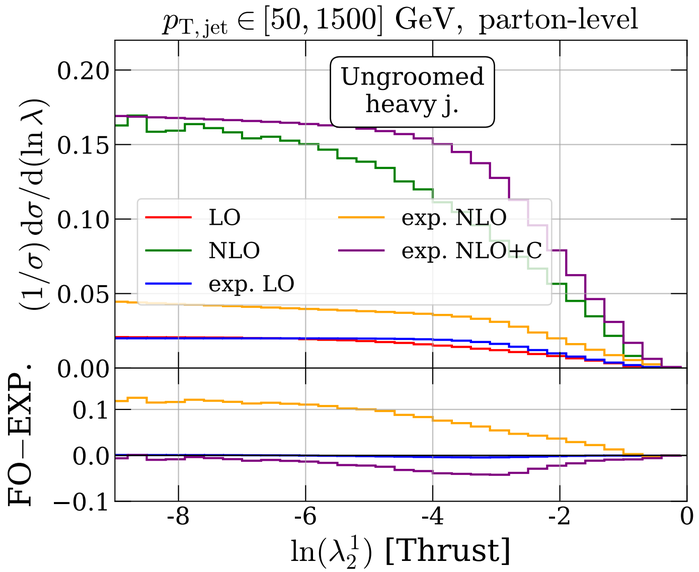
$\eta_{\rm min}$= lower bound in rapidity

P_{gb} = massive splitting kernel

Validation against fixed order







$pp \to Z + b$

Comparison between expansion of the resummation at NLO against fixed order

- Distribution overlapped at leading (the extremely small offset is driven by power corrections in the mass and in the jet radius)
- At NLO the difference is given by a constant off set C obtain through matching.

$$\Sigma(\lambda^1_\alpha) \sim e^{-\alpha_S L}$$

The distribution is single logarithmic for $\lambda^1_\alpha \ll (m/\mu_Q)^\alpha$

Effects of leading power corrections

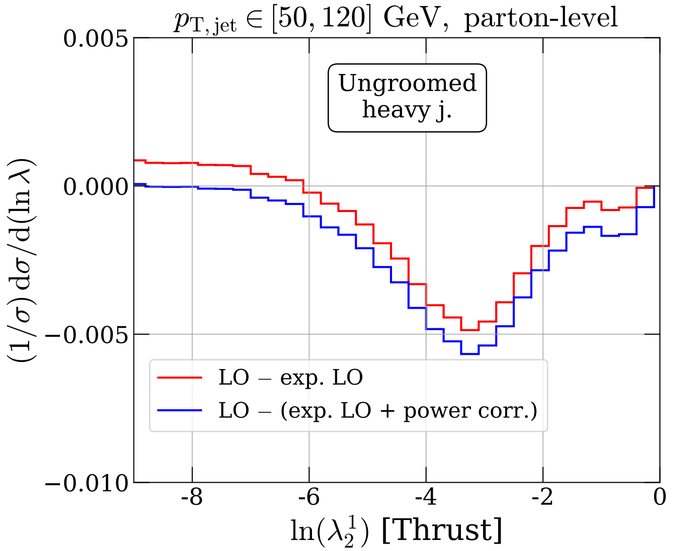
- In the small observable limit at order α_S, the leading power corrections arise only from the calculation of soft function
- Jet radius power corrections computed in M. Dasgupta et al.

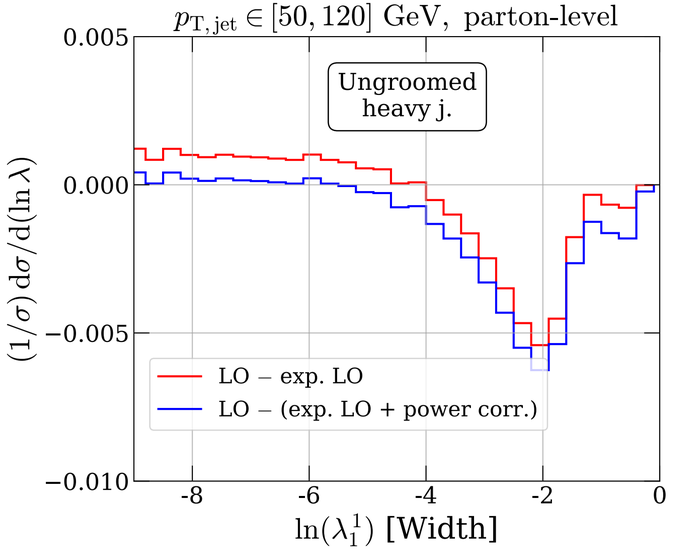
$$\frac{d\sigma}{d\ln\lambda_\alpha^1} = \lambda_\alpha^1 \sum_{ij} \mathbf{T}_i \cdot \mathbf{T}_j \int d^3k w_{ij} \delta\left(\mathscr{V}(k_t,\eta) - \lambda_\alpha^1\right) \Theta(\eta - \eta_{\min})$$

$$w_{ij} = \frac{(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)} - \frac{p_i^2}{2(p_i \cdot k)^2} - \frac{p_j^2}{2(p_j \cdot k)^2}$$

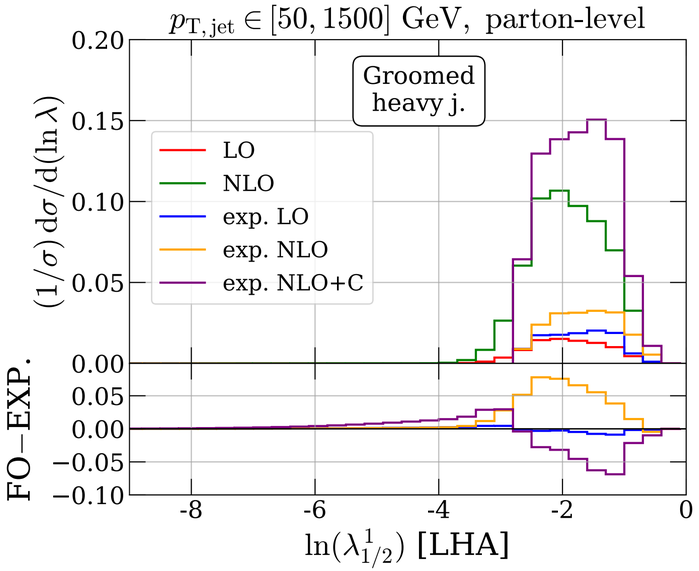
Massive Eikonal

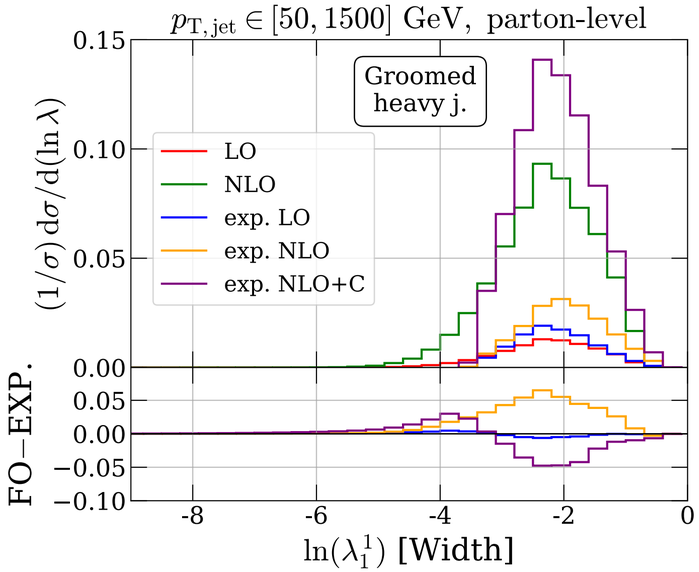
Power corrections computed in the limit where $m/p_T \sim \Delta R_k$

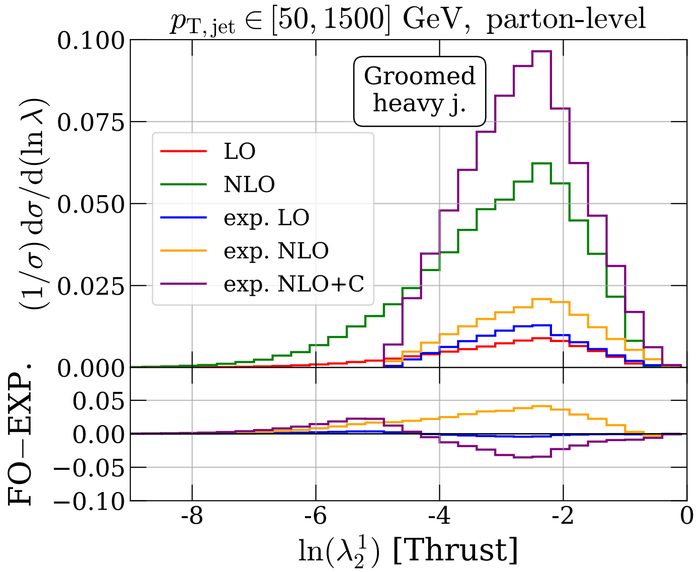




Validation against fixed order: groomed case







$$pp \to Z + b$$

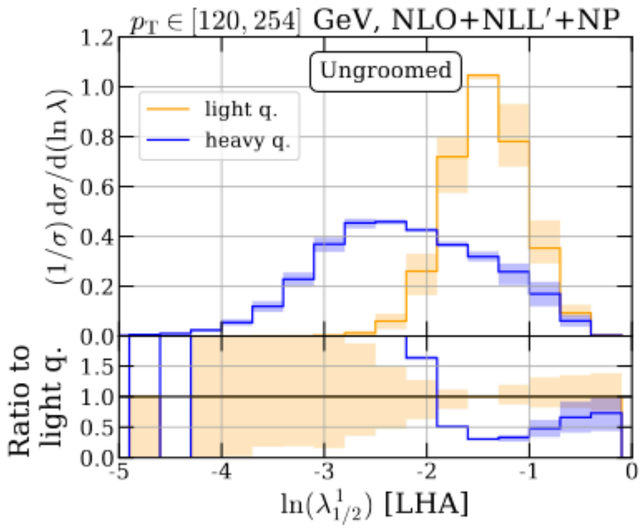
Comparison between expansion of the resummation at NLO against fixed order for $\beta = 0$

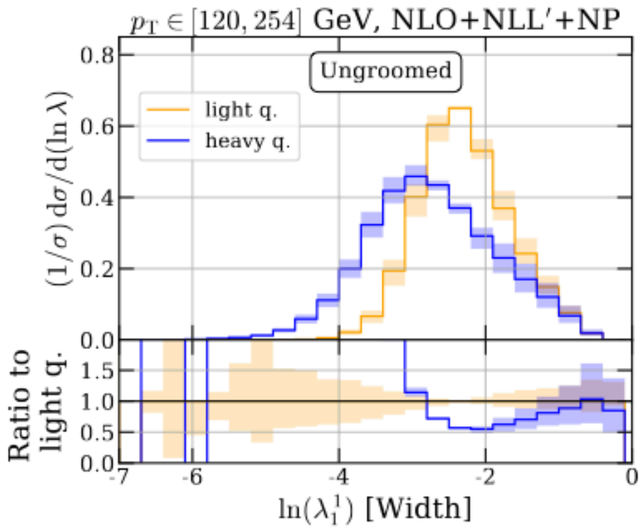
- In principle $\mathcal{O}(z_{\mathrm{cut}})$ effects, but very small, not observed numerically.

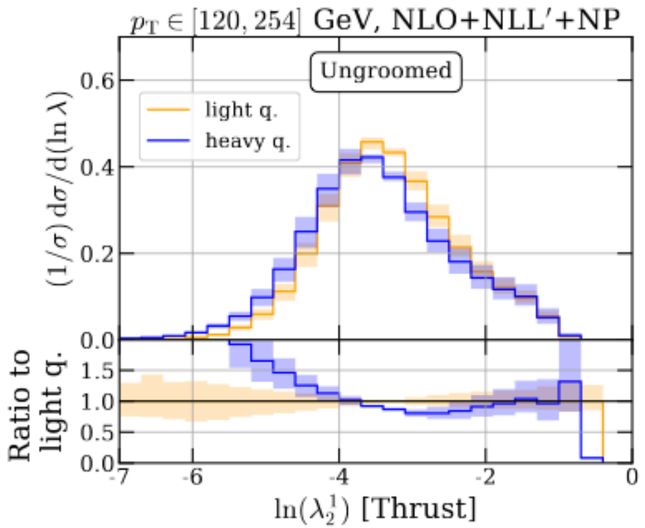
$$\Sigma(\lambda^1_\alpha) \sim e^{-\alpha_S L_\mu}, L_\mu = \log\frac{\mu_Q}{m}$$

- The distribution tend to 0 in the asymptotic limit: the cumulative becomes constant.
- Negative derivative of the radiator over a part of the spectrum.

$b-$jet vs light quark jet: ungroomed case



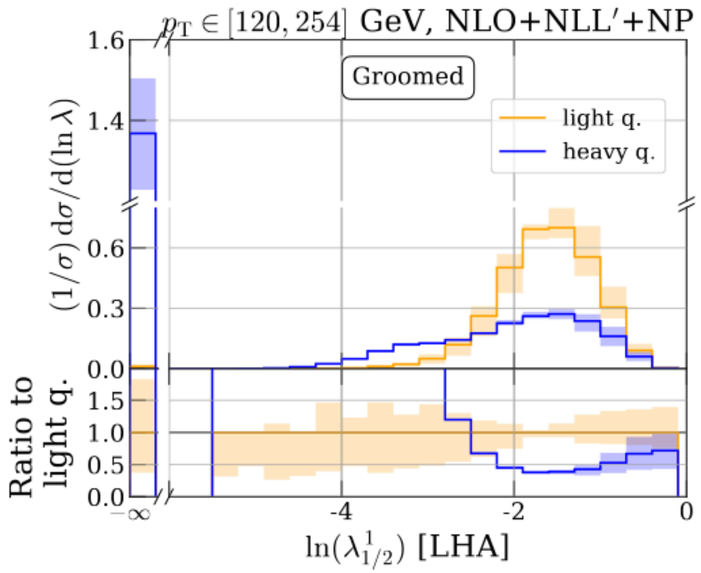


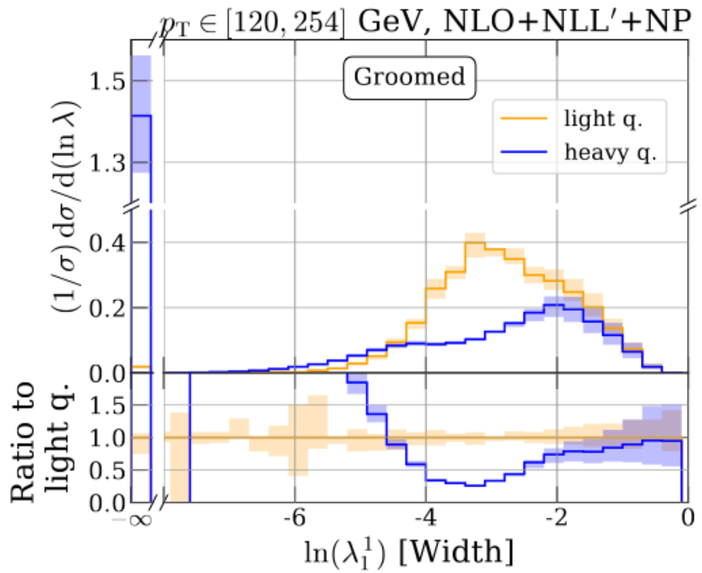


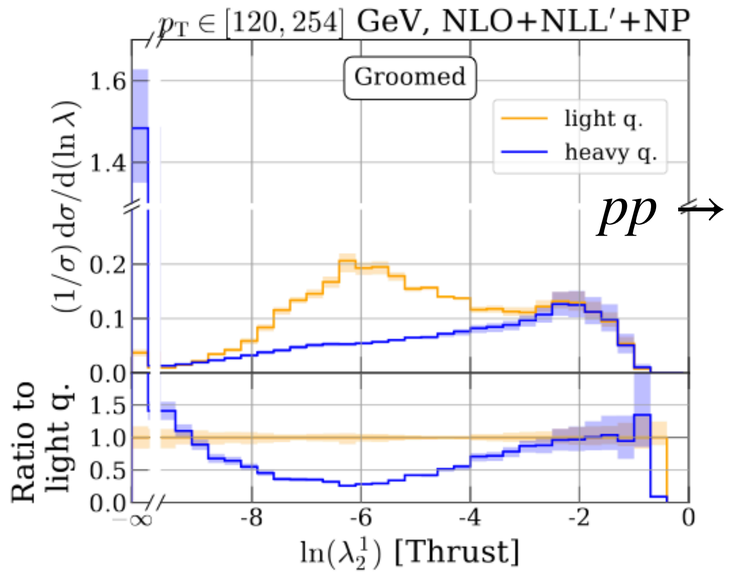
$pp \to Z + b$

- Mass effects scale as $\sim m^{\alpha}$: H-F distribution exhibits a larger deviation from the light quark distribution.
- Longer tail: reduced Sudakov suppression compared to the massless case $\Sigma_b \sim e^{-\alpha_S L}, \quad \Sigma_l \sim e^{-\alpha_S L^2}$.
- Comparison performed with stable B hadrons (P.K. Dhani et al.)
- Non perturbative corrections implemented with the transfer matrix approach (Reichelt et al.)

$b-$jet vs light quark jet: groomed case







$pp \leftrightarrow Z + b$

- Huge peak at the end of the distribution, already present at parton level: $\delta(\lambda^1_\alpha)$ contribution to the resummation (jets composed of a single parton)
- Non perturbative effects smears out this effect, but configurations remain in which the jet has only the B hadron
- This effect can be removed by using Soft Drop in tagging mode, as suggested in S. Caletti, A.G., S. Marzani. In this case it is natural to consider ratio b/l distributions

Towards higher accuracy: CMW effective coupling

Towards higher accuracy: scale of the coupling and CMW scheme

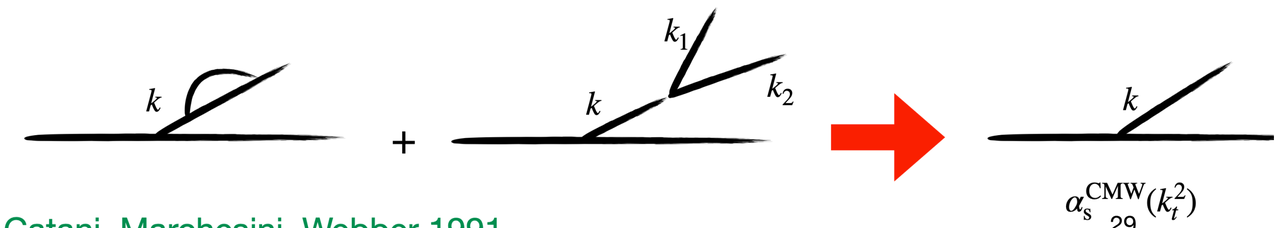
In order to improve the accuracy of our calculation one the missing ingredients is the running coupling:

$$\alpha_s^2(\mu^2)[dk]\,|M_{1-\text{loop,N.A.}}(k) + \alpha_s^2(\mu^2)\int [dk_1][dk_2]\,|M_{N.A.}(k_1,k_2)\delta^3(k-k_1-k_2)$$

$$= \alpha_s^2(\mu^2)[dk]\,|M(k,\mu)|^2\left(\beta_0 \ln\frac{k_t^2}{\mu^2} + \frac{K^{\text{CMW}}}{2\pi}\right)$$

$$\alpha_s(k_t^2) = \alpha_s(\mu^2)\left(1 + \alpha_s\beta_0 \ln\frac{k_t^2}{\mu^2}\right)$$

$$\alpha_s^{\text{CMW}} = \alpha_s^{\overline{\text{MS}}}\left(1 + K^{\text{CMW}}\frac{\alpha_s^{\overline{\text{MS}}}}{2\pi}\right)$$



Catani, Marchesini, Webber 1991

- This prescription is perfectly fine even if the emitter is massive at NLL accuracy as shown in for fragmentation (A.G., S. Marzani and G. Ridolfi)
- Used for recent developments for NLL shower with massive particles (M. Van Beekved et al)
- It does not work at NNLL

Slide adapted from G. Zanderighi

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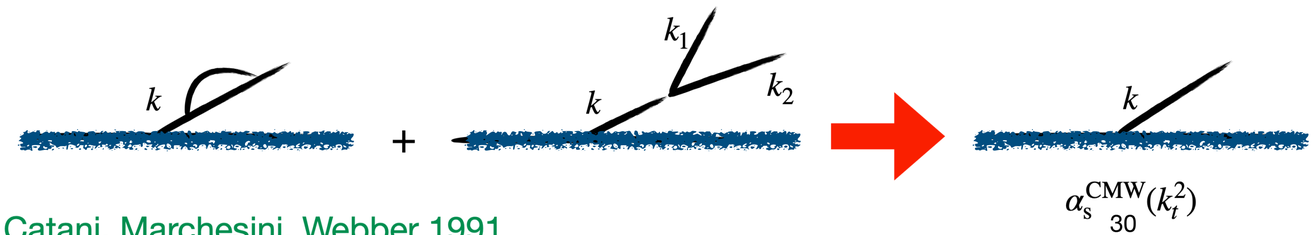

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Repeat the calculation for massive emitter

Web function

We start from eikonal factorization at leading power :

$$|\mathscr{M}(p_1, p_2, k_1, \cdots, k_N)|^2 \to |\mathscr{M}(p_1, p_2)|^2 \times |J_i(k_1, \cdots, k_N)|^2$$

Reduced amplitude Eikonal current

From the eikonal factorization one can derive the all order web function (A. Banfi, B. El-Menoufi, P.F. Monni)

$$w_i(k;\epsilon) = \mathscr{N}_\epsilon(k_T)\left\{ \frac{\delta_+(k^2)}{(2\pi)^{d-1}} |J_i(k)|^2 + \sum_{N=2}^{\infty} \sum_{\{a_1,\ldots,a_N\}} \sigma_{a_1\ldots a_N} \int \left[\prod_{\ell=1}^{N} [dk_\ell]\right] \times W_i^{a_1\ldots a_N}(k_1, \ldots, k_N)\delta^{(d)}\left(k - \sum_{\ell=1}^{N} k_\ell\right)\right\}$$

- $|J_i(k)|^2$ loop correction to single emission
- $W^{a_1,\ldots,a_N}$ eikonal current for the emission of N partons

Real and virtual singularities cancel at every perturbative order, rendering w_i IRC safe to any order in perturbation theory

The CMW effective coupling

Since the massless web depend only on two scales ($k_{\mathrm{T}}, m_{\mathrm{T}}$), the k_{T} effective CMW coupling depend only after one specify the scale that is held fixed: (Catani et al.)

$$\mathscr{A}_{\mathrm{T},i}\left(\alpha_s\right) = \frac{1}{2}\mu^2 \int_0^\infty dm_{\mathrm{T}}^2\, dk_{\mathrm{T}}^2 \delta\left(\mu^2 - k_{\mathrm{T}}^2\right) w_i(k;0) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \mathscr{A}_{\mathrm{T},i}^{(n)} \qquad \mathscr{A}_{\mathrm{T},i}^{(2)} = \frac{C_F K^{\mathrm{CMW}}}{2}$$

In the massive case, the web function has a non trivial dependence on the longitudinal dof of the off-shell emission. Thus we need to specify which longitudinal variable we fix. For this reason we introduce:

$$\frac{d^2\mathcal{S}_{\mathcal{Q}}}{dk_{\mathrm{T}}^2 d\zeta} = \frac{1}{2}\int dk^2 w_{\mathcal{Q}}(k(k^2, k_{\mathrm{T}}, \zeta); \epsilon)\,. \qquad \zeta = \begin{cases} \ln z = & \ln \frac{k_+}{p_{1+}} \\ y = & \frac{1}{2}\ln \frac{k_+}{k_-} \end{cases}$$

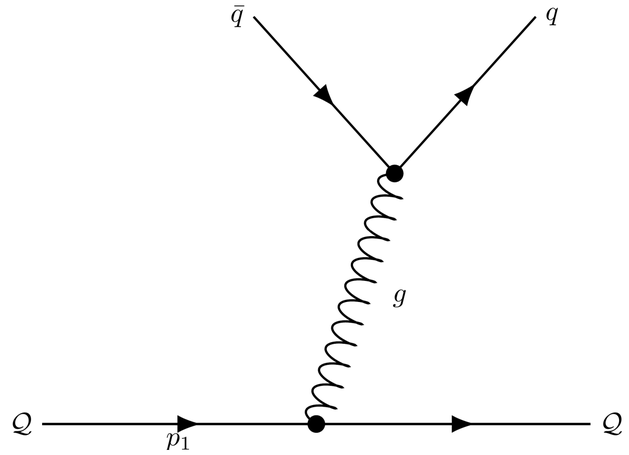
The definition of a CMW coupling entails the definition of a mapping: $\mathbb{P} : k, k^2 \neq 0 \to p_k, p_k^2 = 0$

A. Banfi, F. Dreyer, P.F. Monni

Sketch of the calculation: $n_f T_{\mathrm{R}}$ contribution

We sketch the calculation of the $n_f T_{\mathrm{R}}$ contribution at NLO:

$$\frac{d^2\mathcal{S}_{\mathcal{Q},n_f}}{dk_{\mathrm{T}}^2 dy} = C_F n_f T_{\mathrm{R}} \mathrm{Eik}(p_k) \left(\frac{1}{3} \log \frac{k_{\mathrm{T}}^2}{\mu^2} - \frac{5}{9} \right)$$



- Here the massive eikonal $\mathrm{Eik}(p_k)$ factorizes completely

- This calculation indicates that the scale for the running of the coupling is k_{T}^2

$$\frac{d^2\mathcal{S}^{\mathrm{q.c.1}}{}_{\mathcal{Q},n_f}}{dk_{\mathrm{T}}^2 d\ln z} = C_F n_f T_{\mathrm{R}} \left[\left(\frac{2p_1\cdot p_2}{(p_1\cdot p_k)(p_2\cdot p_k)} - \frac{m_1^2}{(p_1\cdot p_k)^2} \right) \left(\frac{1}{3}\log\frac{k_{\mathrm{T}}^2 + z^2 m_1^2}{\mu^2} - \frac{5}{9} \right) - \frac{1}{3}\frac{m_1^2}{(p_1\cdot p_k)} \right]$$

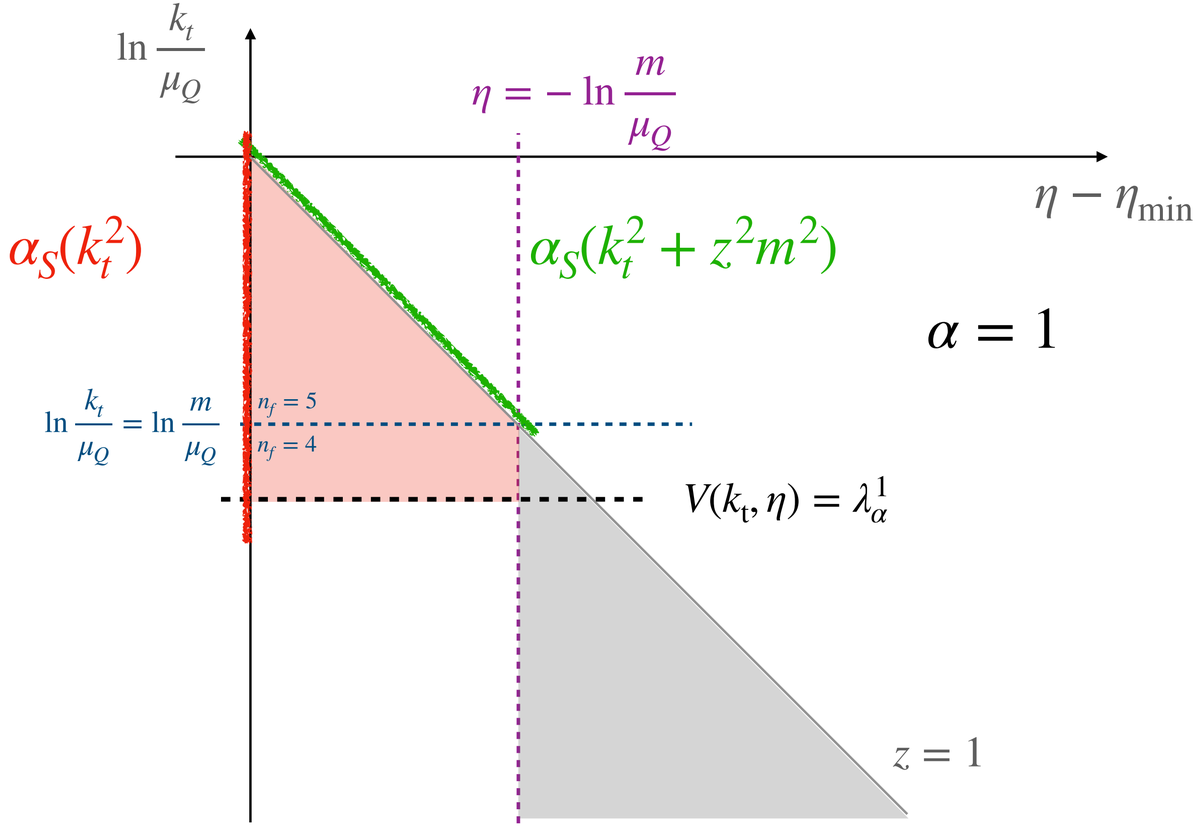
Scale that appear in quasi collinear factorization (Catani et al.)

- Here the eikonal does not factorize even in the quasi collinear limit $k_{\mathrm{T}}^2 \sim m_1^2 \ll (p_1+p_2)^2$

- This calculation suggest that the natural scale for the running of the coupling is $k_{\mathrm{T}}^2 + z^2 m_1^2$

$\ln z$ vs y prescription

- The previous expression agrees in the massless limit, but there are different finite-mass contributions
- This happens because y, z refers to the kinematics of the off-shell gluon and they are not related by on-shell relations between rapidity and longitudinal momentum fraction.
- Overall, the rapidity prescription seems more natural, as the integral over the invariant mass can be carried out analytically.



- Other possible choice of the prescription:
  - at large angle $k_+ \sim k_- \implies \mu^2 = k_t^2$
  - In the collinear region $k_- \ll k_+ \sim p_{1+} \implies \mu^2 = k_t^2 + z^2 m_1^2$

Structure of the result for the rapidity prescription

The rapidity spectrum reads:

$$\frac{d^2\mathcal{S}_{\mathbb{Q}}}{d\ln k_{\mathrm{T}}^2 dy} = \frac{\alpha_S(\mu^2)C_F}{\pi}\mathcal{B}(y)\left[1+\frac{\alpha_S(\mu^2)}{2\pi}\left(-2\pi\beta_0\ln\frac{k_{\mathrm{T}}^2}{\mu^2}+\frac{67}{18}C_A-\frac{10}{9}T_{\mathrm{R}}n_f\right)\right]+\left(\frac{\alpha_S(\mu^2)}{\pi}\right)^2\frac{C_F C_A}{2}F(y,\varphi)$$

$$\mathcal{B}(y) = \frac{\sinh^2\varphi}{(\cosh\varphi+\cosh 2y)^2},$$

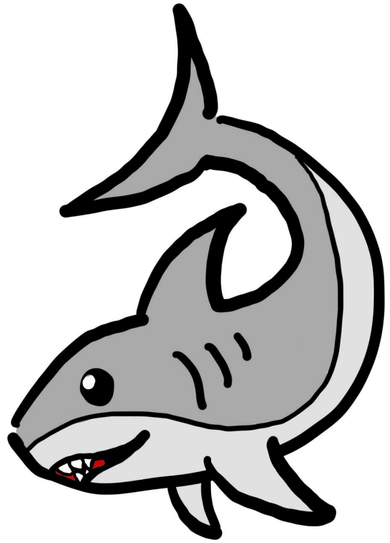
- The calculation can be also carried out using the results of Ze Long Liu and P.F. Monni

$\varphi = \tanh^{-1} v$, the cusp angle ,$v = \sqrt{1-\frac{m_1^2 m_2^2}{(p_1\cdot p_2)^2}}$ the relative velocity

where by construction: $\lim_{\varphi\to\infty} F(y,\varphi) = -\zeta_2$. This contribution explicitly violates the factorization of the LO result.

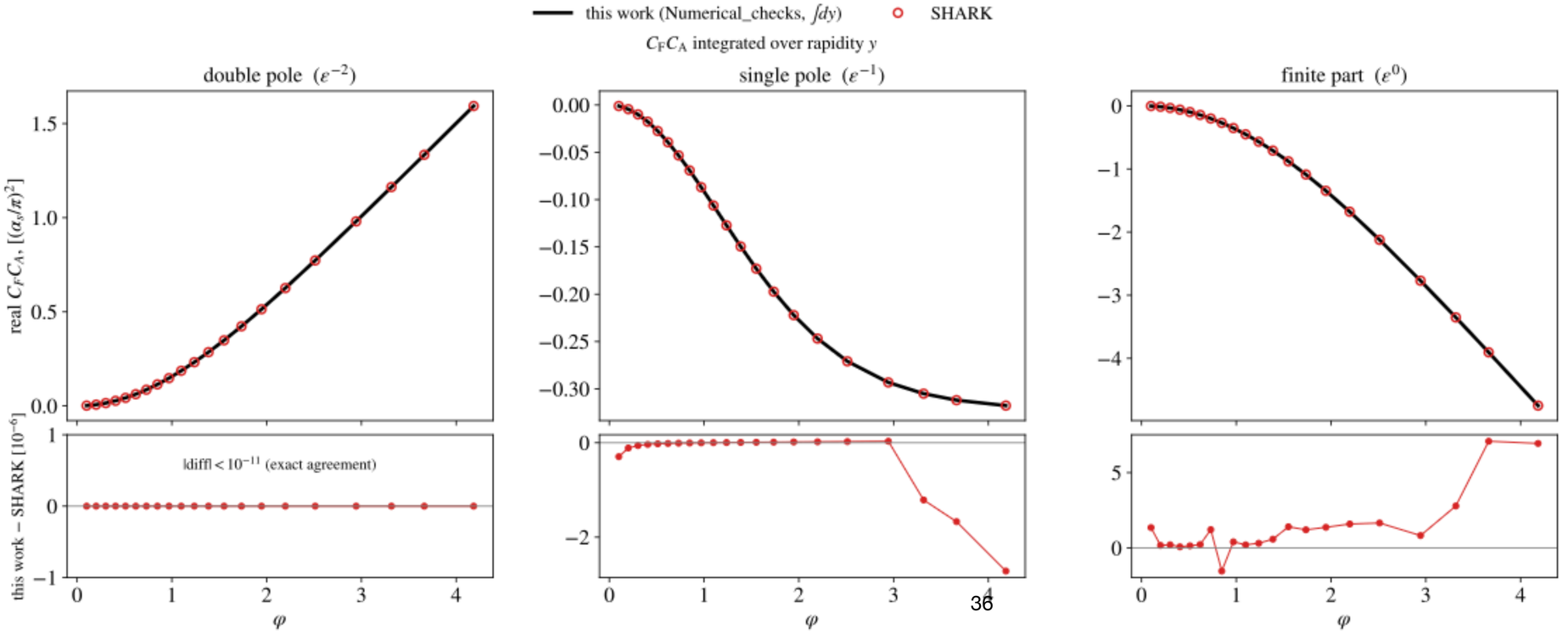
Moreover in the threshold limit $\varphi \to 0$: $F(y,\varphi) \sim -\frac{\pi^2}{\varphi}\mathcal{B}(y)$, Coulomb enhancement

Connection with q_T soft function



The q_T soft function was computed by S. Catani et al. for back to back and by S. Devoto and J. Mazzitelli for general kinematics (SHARK)

Performed cross check for every color channel against the SHARK. In particular we can look at the non trivial structure proportional to $C_F C_A$.



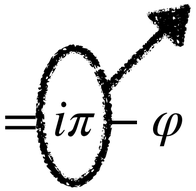
Numerical integration starts to become unstable in the large φ limit

Connection with two loop soft anomalous dimension

For a color singlet dipole, the soft anomalous dimension computed by T. Becher and M. Neubert, (see also A. Von Manteufell, R. Schabinger and H.X. Zhu) becomes:

Coulomb phase (spacelike to timelike)

$$\mathbf{\Gamma}_{\mathcal{Q}\bar{\mathcal{Q}}} = C_F \gamma_{\text{cusp}}(\beta, \alpha_S) + 2\gamma^{\mathcal{Q}}(\alpha_S), \quad \beta = i\pi - \varphi$$



$$\gamma_{\text{cusp}}(\beta, \alpha_s) = \gamma_{\text{cusp}}(\alpha_s)\,\beta\coth\beta + \frac{C_A}{2}\left(\frac{\alpha_s}{\pi}\right)^2\left\{\coth^2\beta\left[\text{Li}_3(e^{-2\beta}) + \beta\,\text{Li}_2(e^{-2\beta}) - \zeta_3 + \frac{\pi^2}{6}\beta + \frac{\beta^3}{3}\right]\right.$$
$$\left. + \coth\beta\left[\text{Li}_2(e^{-2\beta}) - 2\beta\,\ln(1-e^{-2\beta}) - \frac{\pi^2}{6}(1+\beta) - \beta^2 - \frac{\beta^3}{3}\right] + \frac{\pi^2}{6} + \zeta_3 + \beta^2\right\} + \dots .$$

$$\gamma_{\text{cusp}}(\alpha_S) = \frac{\alpha_S}{4\pi}\gamma_0 + \left(\frac{\alpha_S}{4\pi}\right)^2\gamma_1 + \mathcal{O}\left(\alpha_S^2\right)$$

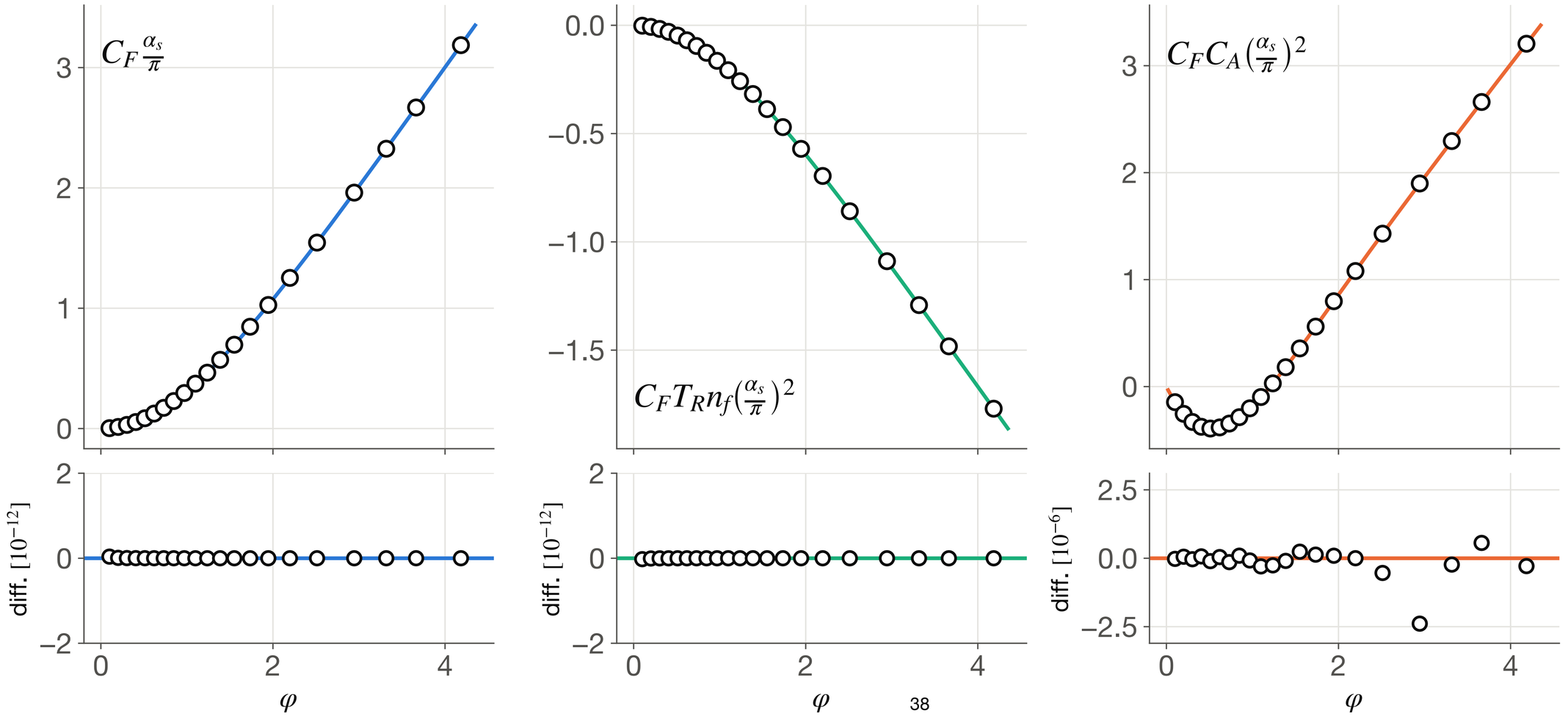
$$\gamma^{\mathcal{Q}}(\alpha_S) = \frac{\alpha_S}{4\pi}\gamma_0^{\mathcal{Q}} + \left(\frac{\alpha_S}{4\pi}\right)^2\gamma_1^{\mathcal{Q}} + \mathcal{O}\left(\alpha_S^2\right)$$

Presence of the masses breaks the factorization of the Born level anomalous dimension

We check that holds the identity: $\int dy \frac{d^2\mathcal{S}_{\mathcal{Q}}}{d\ln k_{\text{T}}^2 dy} = \mathfrak{R}\left(\mathbf{\Gamma}_{\mathcal{Q}\bar{\mathcal{Q}}}\right)$ for every color structure: $C_F, C_F n_f T_{\text{R}}, C_F C_A$

Cross-checks: two loop soft anomalous dimension

Becher–Neubert: $\mathrm{Re}\,[\,C_F\gamma_{\rm cusp}(\beta)+2\gamma^Q\,],\ \ \beta=i\pi-\varphi$ ○ this work: $\int dy\,\frac{d^2S_Q(k_T,y)}{dy\,d\ln k_T^2}$ at $\mu^2=k_T^2$



Definition of the massive CMW coupling

The definition of a massive CMW scheme is not unique. One possible definition is suggested by the fact that:

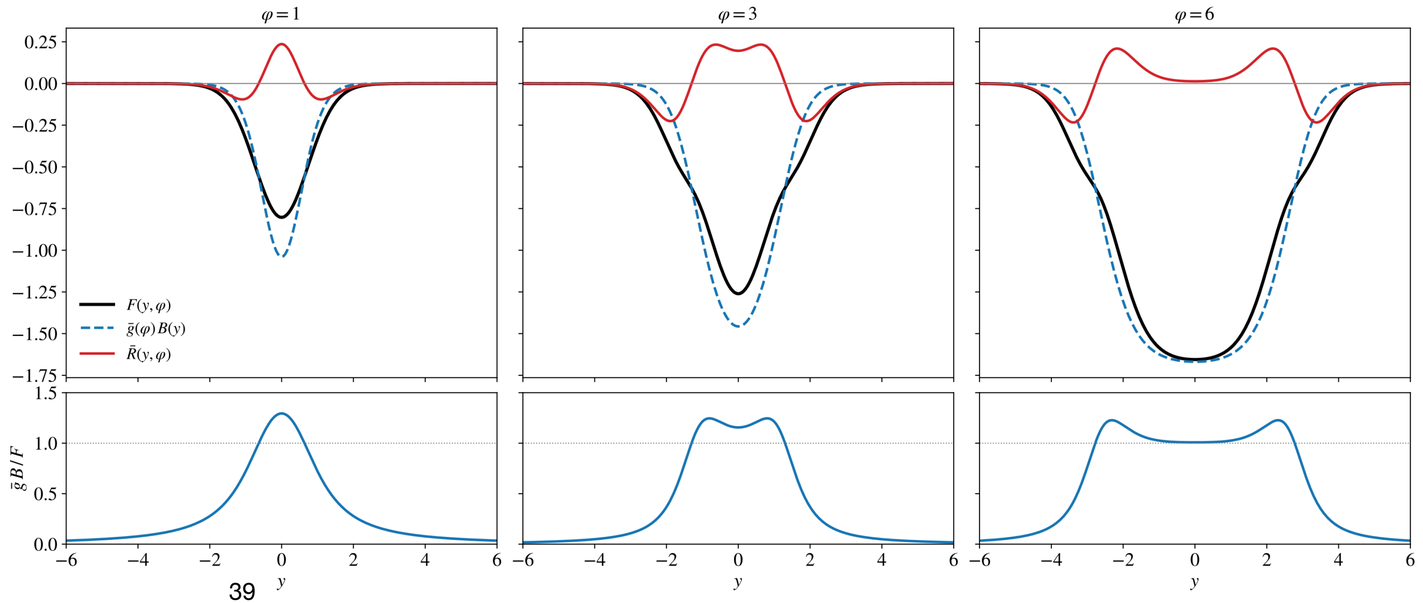
$$\int dy \frac{d^2\mathcal{S}_{\mathcal{Q}}}{d\ln k_{\mathrm{T}}^2 dy}\Big|_{\mu^2=k_{\mathrm{T}}^2} = \Re\left(\mathbf{\Gamma}_{\mathcal{Q}\bar{\mathcal{Q}}}\right)$$

$$F(y,\varphi) = \bar{g}(\varphi)\mathcal{B}(y) + \bar{R}(y,\varphi)$$

Born factorization and $\bar{g}$ depends only on the kinematics of the dipole

Integrate to 0 over rapidity, enhanced in the dead-cone region

- At large φ (boosted region), the contribution of $\bar{R}$ is negligible: recover factorization.
- Enhancement at small φ: Coulomb effect



Definition of the massive CMW coupling

The above considerations lead us to define the massive CMW coupling as

$$\mathscr{A}_{\mathcal{Q}}^{\mathrm{CMW}}(\alpha_S) \equiv C_F \frac{\alpha_S}{\pi}\left(1+\frac{\alpha_S}{2\pi}K_{\mathcal{Q}}\right)+\mathcal{O}\left(\alpha_S^3\right), \quad K_{\mathcal{Q}}=\frac{67}{18}C_A-\frac{10}{9}T_{\mathrm{R}}n_f+\bar{g}(\varphi)\,.$$

- In the massless limit ($m_i \to 0, i=1,2$), $\varphi \to \infty$, we recover the standard CMW coupling: $K_{\mathcal{Q}} \to K$.
- In the opposite (threshold limit) the cusp angle φ and, hence the velocity v, go to zero and we have $K_{\mathcal{Q}} \to -C_A\frac{\pi^2}{v}$. Therefore, the effective coupling also accounts for the Coulomb enhancement at small velociti

With the above prescription, the scale of running coupling remains k_{T} as in the massless case: $\alpha_S = \alpha_S(k_{\mathrm{T}}^2)$

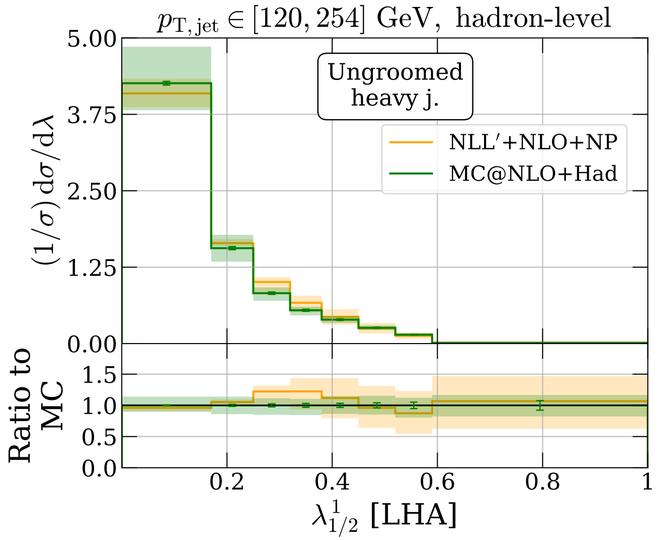
Conclusion and outlook

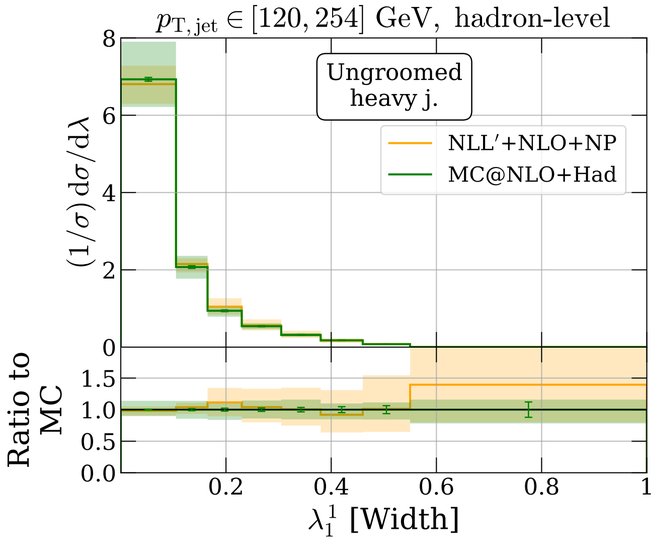
- Examined recent results on jet angularities for H.F. jets.
- Extension to higher accuracy: examined choice of the resummation scale in the case of massive emitters and definition of the CMW coupling.
- Results obtained for general masses in the dipole. Interesting cases: $m_1 = m_2 = m;$ $m_1 = m, m_2 = 0.$
- Outlook: test our prescription for substructure observables to gauge the impact of the CMW scheme.

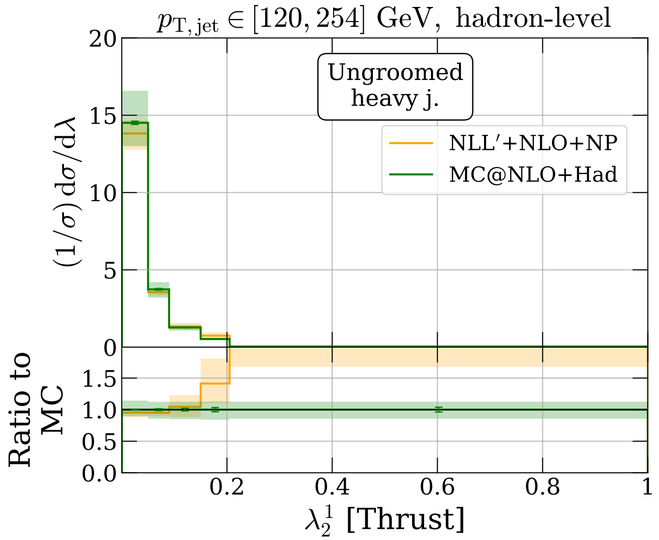
Thanks for your attention!!!

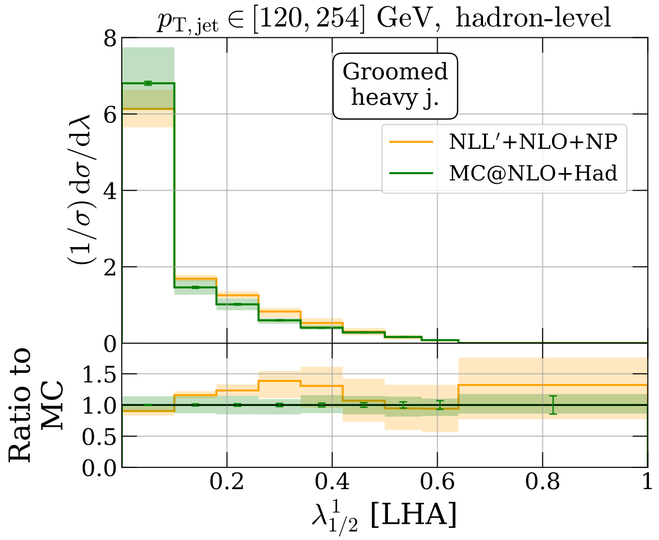
Backup slides

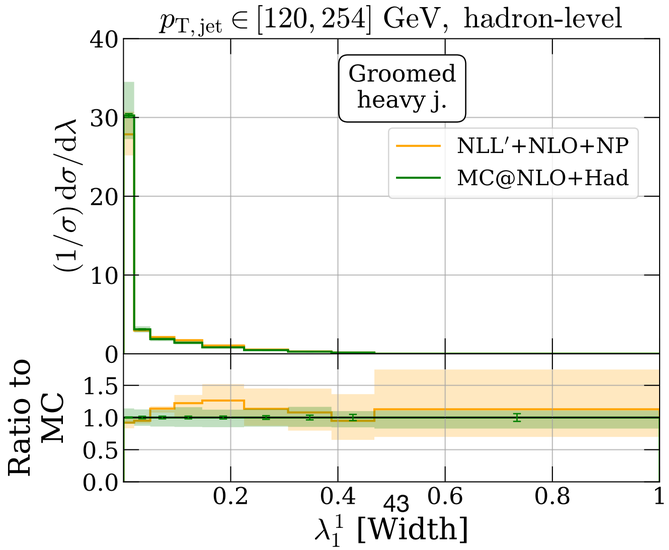
Comparison against SHERPA MC@NLO

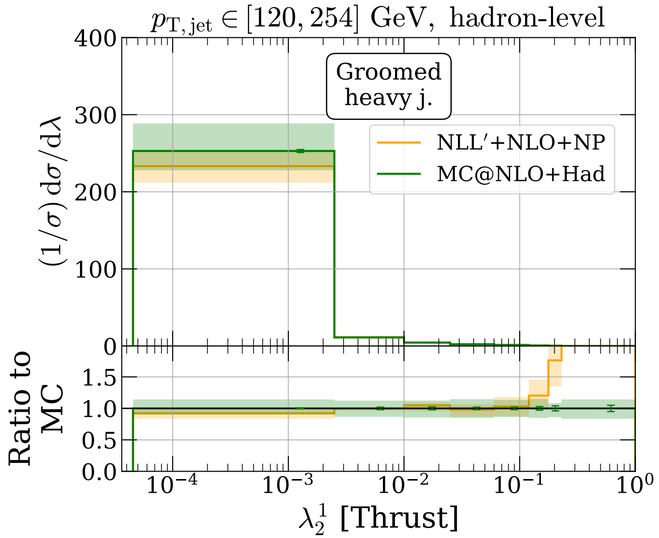










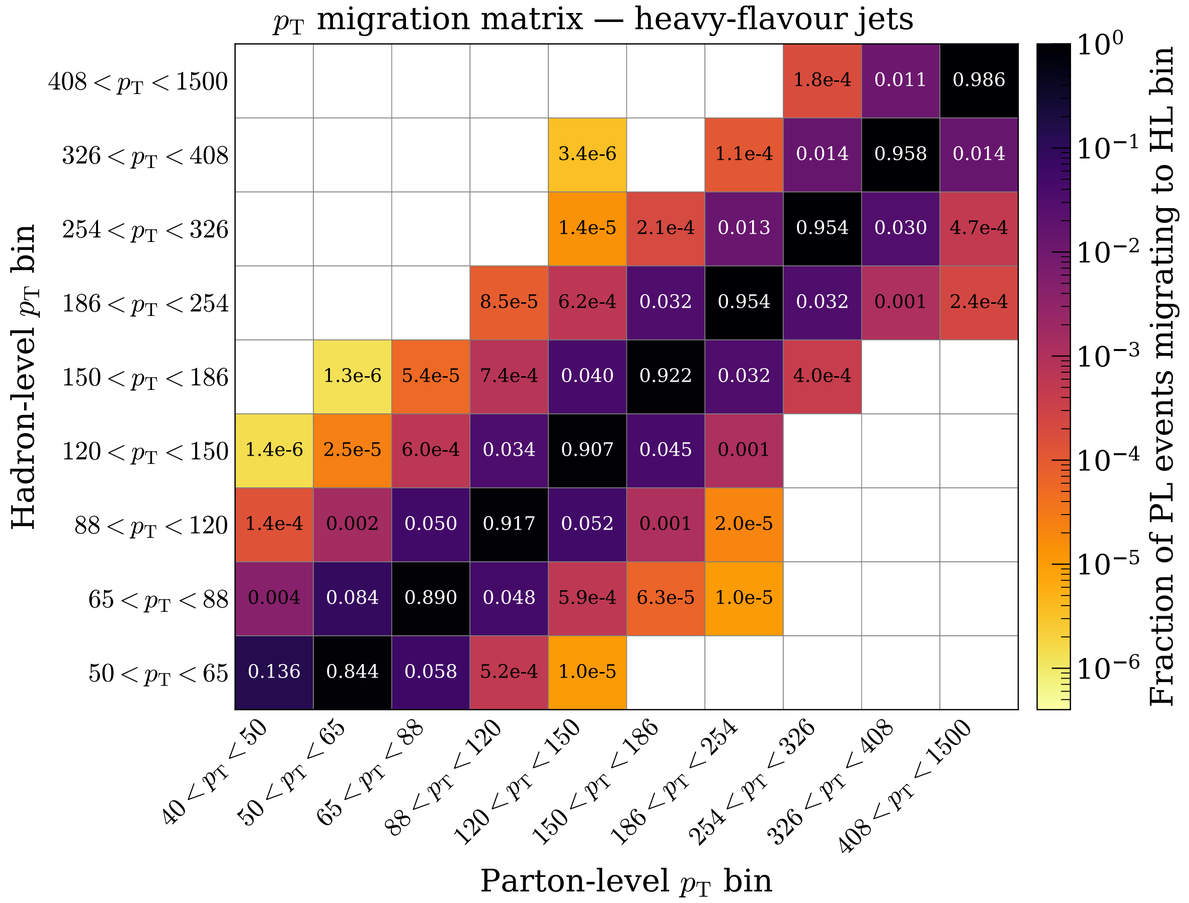


Transfer matrix approach

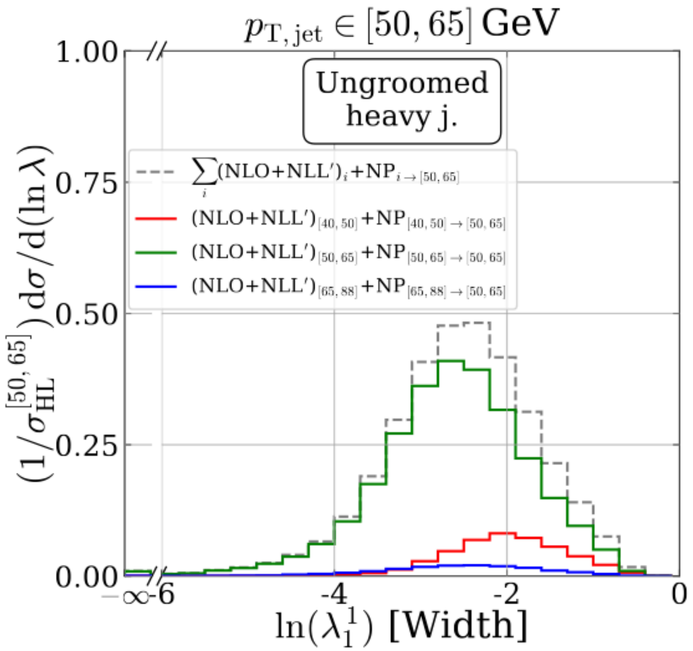
Hadronisation correction included with the transfer matrix approach (Reichelt et al.)

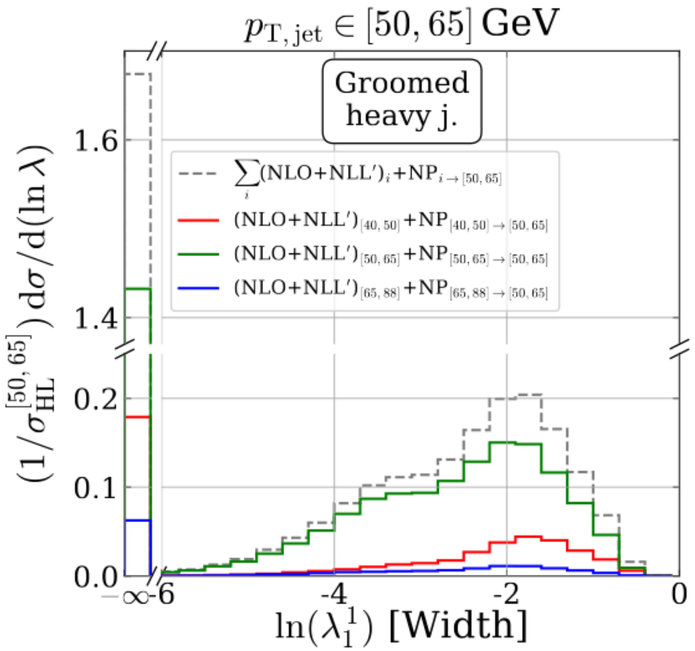
$$\mathcal{T}\left(\vec{v}^{h}|\vec{v}^{p}\right) = \frac{\int \mathrm{d}\mathcal{P}\frac{\mathrm{d}\sigma^{p}}{\mathrm{d}\mathcal{P}}\delta^{(m)}\left(\vec{v}^{p}-\vec{V}(\mathcal{P})\right)\delta^{(m)}\left(\vec{v}^{h}-\vec{V}(\mathcal{H}(\mathcal{P}))\right)}{\int \mathrm{d}\mathcal{P}\frac{\mathrm{d}\sigma^{p}}{\mathrm{d}\mathcal{P}}\delta^{(m)}\left(\vec{v}^{p}-\vec{V}(\mathcal{P})\right)}.$$

- We consider distribution of kinematic variables and observables to determine the transfer operator
- Migration of the cross section in different p_T bins from parton to hadron level due to UE and hadronization corrections
- Transfer matrix almost diagonal in high p_T bins, due to the fact that we keep the B hadron stable (less fragmentation product).



Hadronisation corrections via transfer matrix



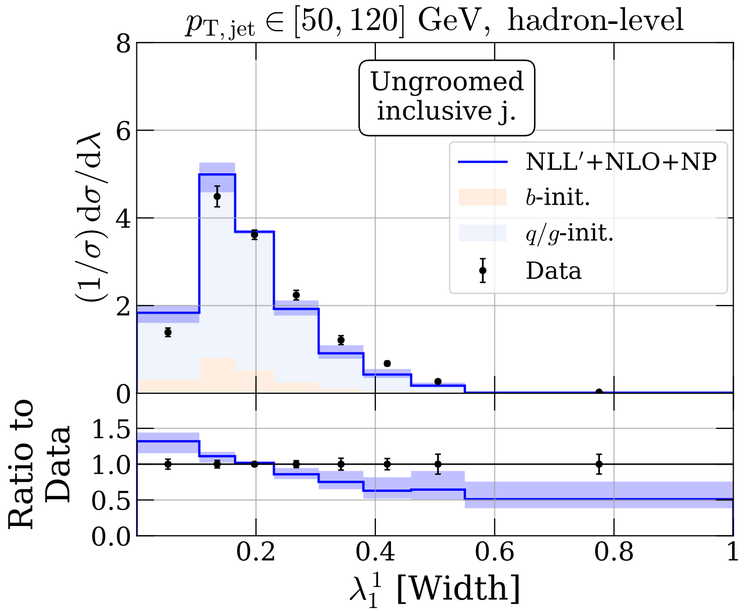


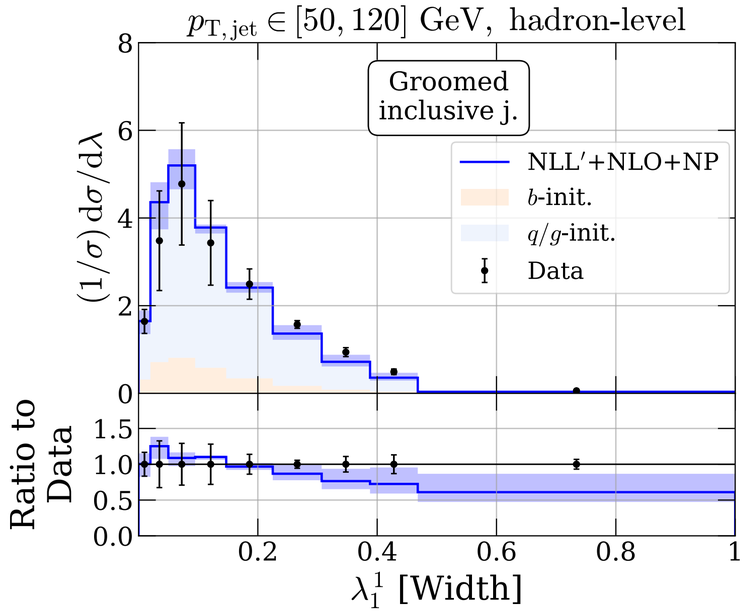
$\mathscr{P}([65,88] \to [50,65]) = 12.8\,\%$

$\mathscr{P}([40,50] \to [50,65]) = 7.6\,\%$

- For both cases, at small p_T the transfer matrices have rather large off-diagonal elements, populating significantly larger observable values at hadron than at parton level.
- In the groomed case at hadron level we have a more pronounced peak at $\lambda_\alpha^1 = 0$

Comparison to CMS data for untagged jets





Comparison against CMS data for the inclusive case for NLL+NL+NP angularity distributions

- We refine the theoretical framework of S. Caletti et al. by incorporating contributions from b-initiated jets and by including running coupling effects in the light flavour case

- Blue band: light quarks + gluon, Orange band: $b-$jets. Good agreement with data, confirming the results of S. Caletti et al.

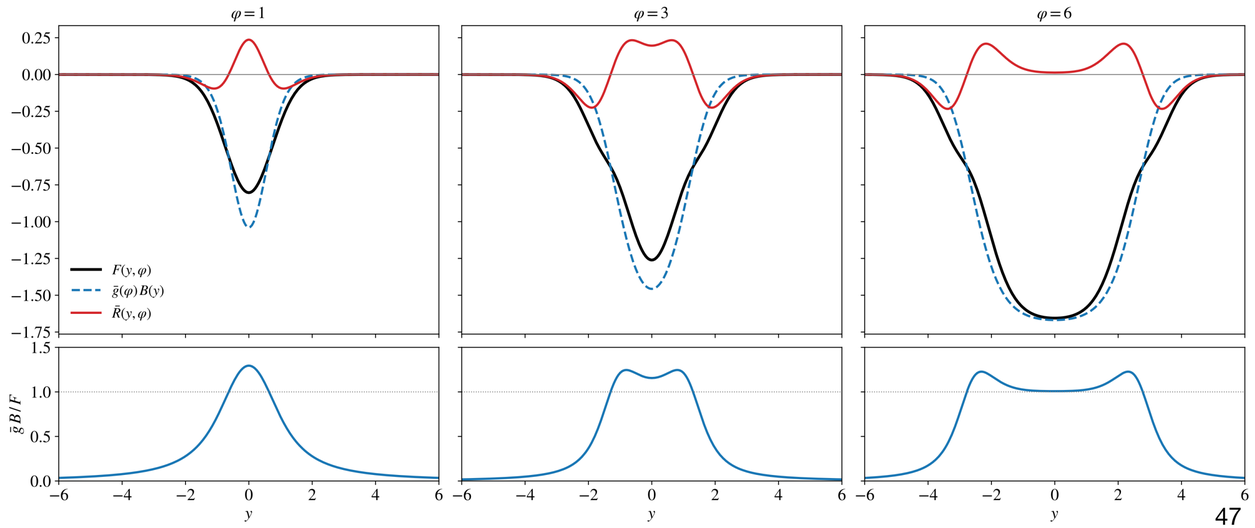
Definition of the massive CMW coupling

The rapidity spectrum reads:

$$\mathscr{B}(y) = \frac{\sinh^2\varphi}{(\cosh\varphi + \cosh 2y)^2}, \varphi \text{ the cusp angle}$$

$$\frac{d^2\mathcal{S}_{\mathbb{Q}}}{d\ln k_{\mathrm{T}}^2 dy} = \frac{\alpha_S(\mu^2)C_F}{\pi}\mathscr{B}(y)\left[1 + \frac{\alpha_S(\mu^2)}{2\pi}\left(-2\pi\beta_0 \ln\frac{k_{\mathrm{T}}^2}{\mu^2} + \frac{67}{18}C_A - \frac{10}{9}T_{\mathrm{R}}n_f\right)\right] + \left(\frac{\alpha_S(\mu^2)}{\pi}\right)^2 \frac{C_F C_A}{2}F(y,\varphi)$$

where by construction: $\lim_{\varphi\to\infty} F(y,\varphi) = -\zeta_2$. This contribution explicitly violates the factorization of the LO result.



$$F(y,\varphi) = \bar{g}(\varphi)\mathscr{B}(y) + \bar{R}(y,\varphi)$$

At large φ (boosted region), the contribution of $\bar{R}$ is negligible: recover factorization