Global Structure, Non-invertible PQ, and DFSZ Domain Wall Problem

Sungwoo Hong
KAIST

Based on:
2309.05636: with Clay Cordova and Lian-Tao Wang
2510.18404: with Hyungyu Kim, Sung Mook Lee, and Dongmin Seo
2603.26867: with Gongjun Choi and Seth Koren,
* with Youngho Go, Sung Mook Lee, and Lian-Tao Wang
* with T. Daniel Brennan

Sydney-CPPC Seminar

(Theoretical) Particle Physics

Why is the Universe so big?

Why are elementary particles so small?



Credit: https://caldwellop.org/where-we-stand/new-cosmology/

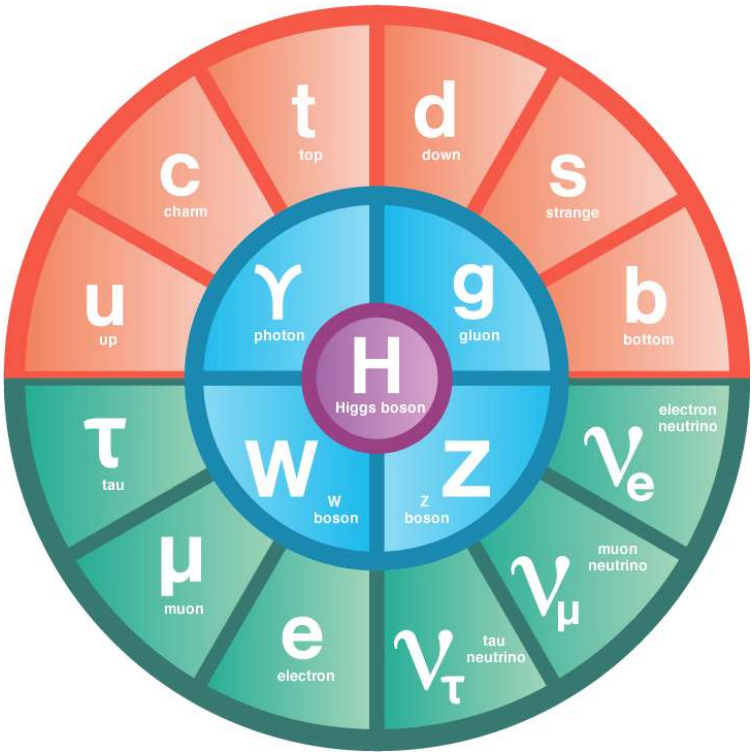
(Theoretical) Particle Physics

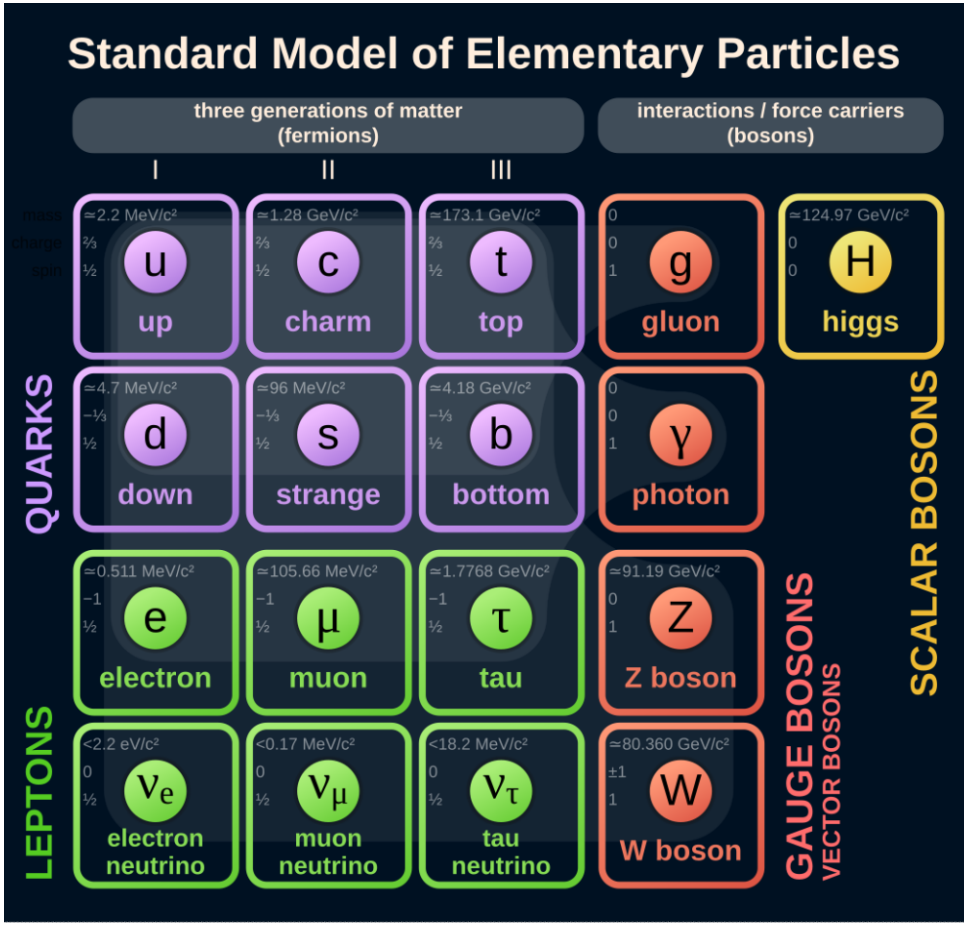
Many Important and unsolved **Puzzles**:

1. Dark Matter
2. Generation of Baryon-anti-baryon : Baryogenesis
3. Origin of tiny neutrino masses
4. Large hierarchy between weak force and gravity :
   Electroweak Hierarchy Problem
5. Unnaturally small CP-violating phase in QCD :
   Strong-CP Problem
6. Phases of strongly interacting QFT :
   Confinement and mass gap generation of QCD
7. Global Structure of the Standard Model
8. On-shell scattering amplitudes
9. Why the Universe is so big when particles are very small?
   : Cosmological Constant Problem
10. And many many more

Great Success of Particle Physics

And yet many interesting puzzles to resolve!

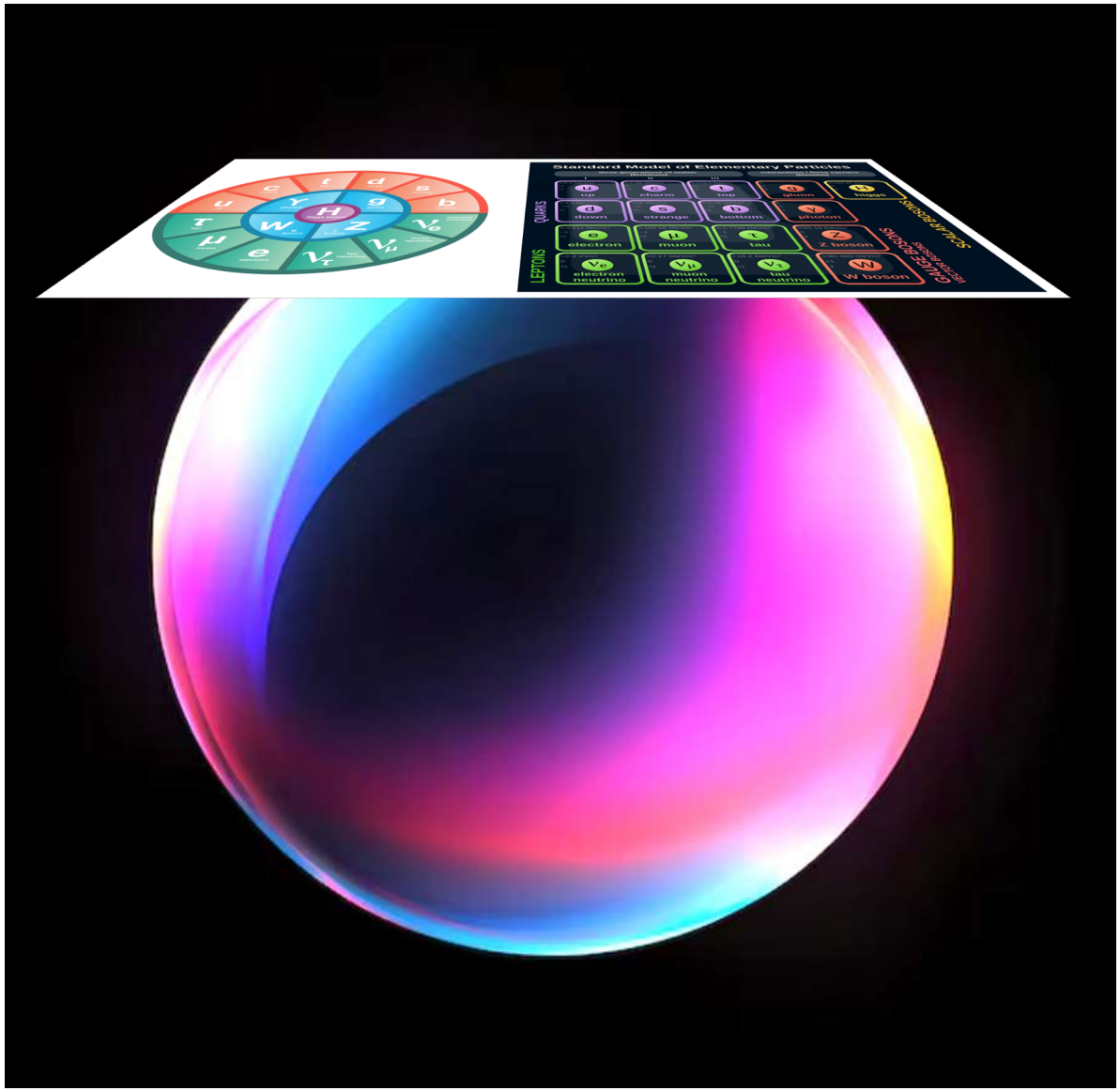




https://commons.wikimedia.org/wiki/File:Standard_Model_of_Elementary_Particles_dark.svg

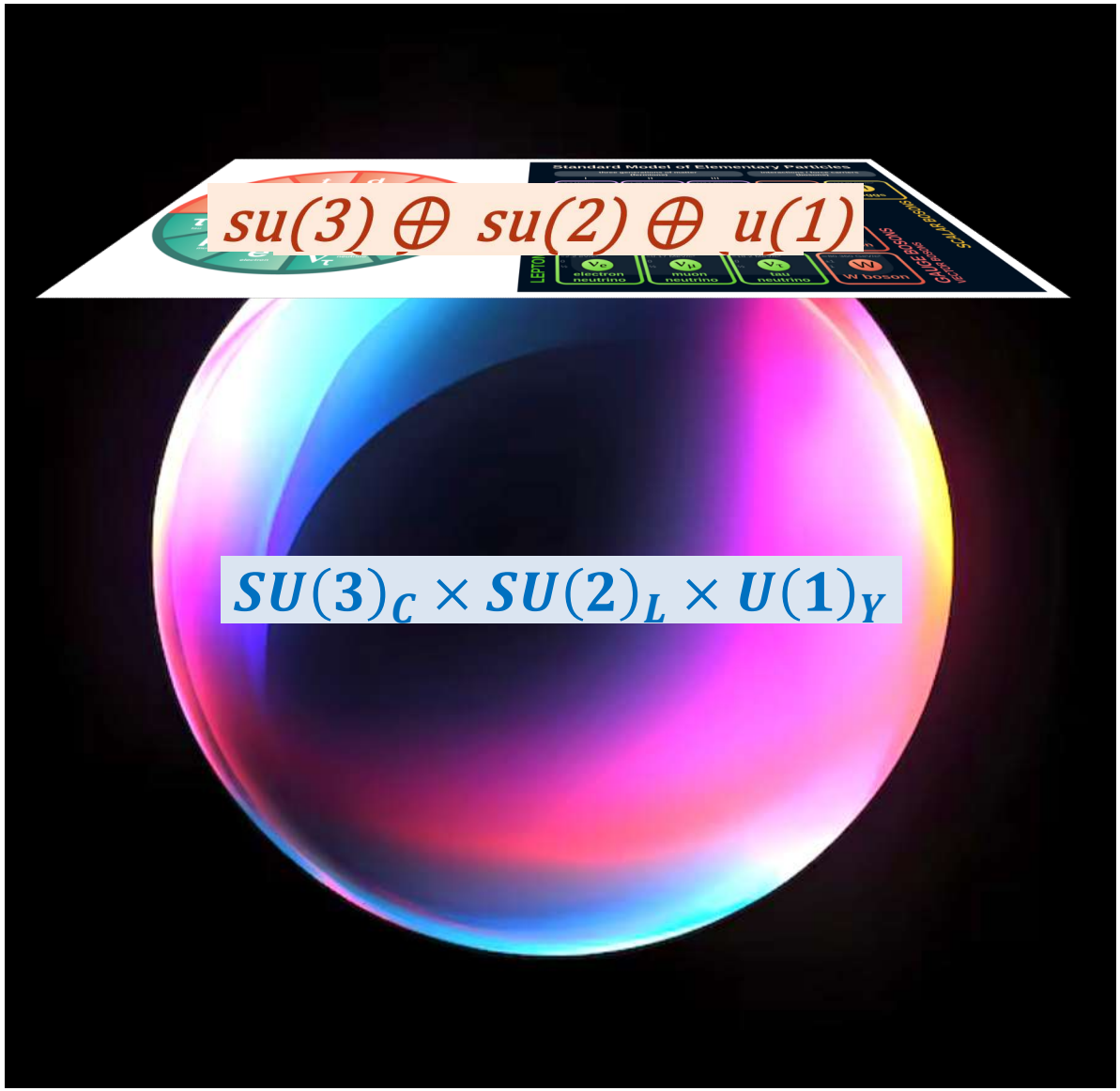
Main focus of Particle Physics so far

Lie algebra data: representations, charges, etc



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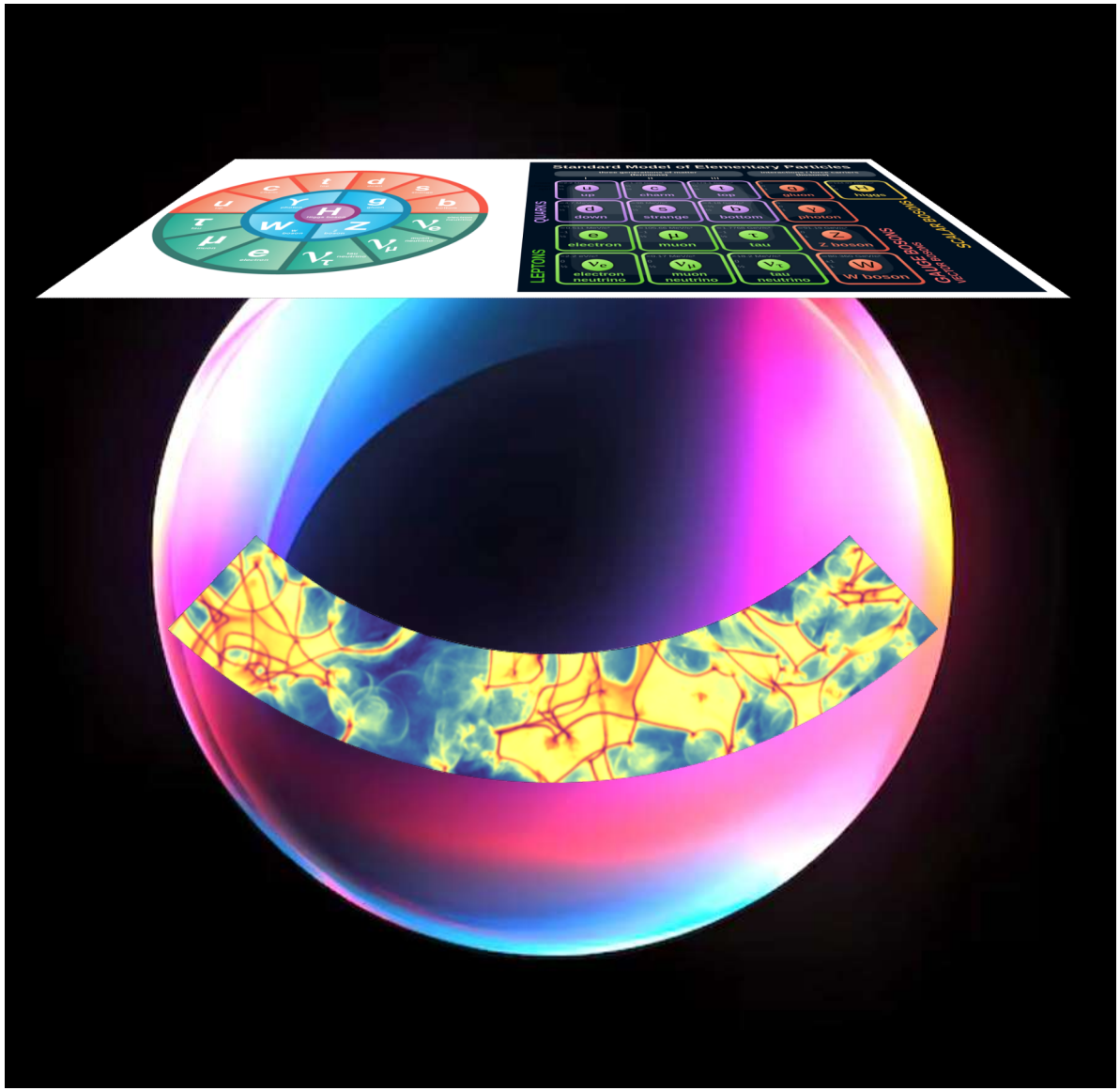


e.g.

	Q
$su(3)$	3
$su(2)$	2
$u(1)$	1

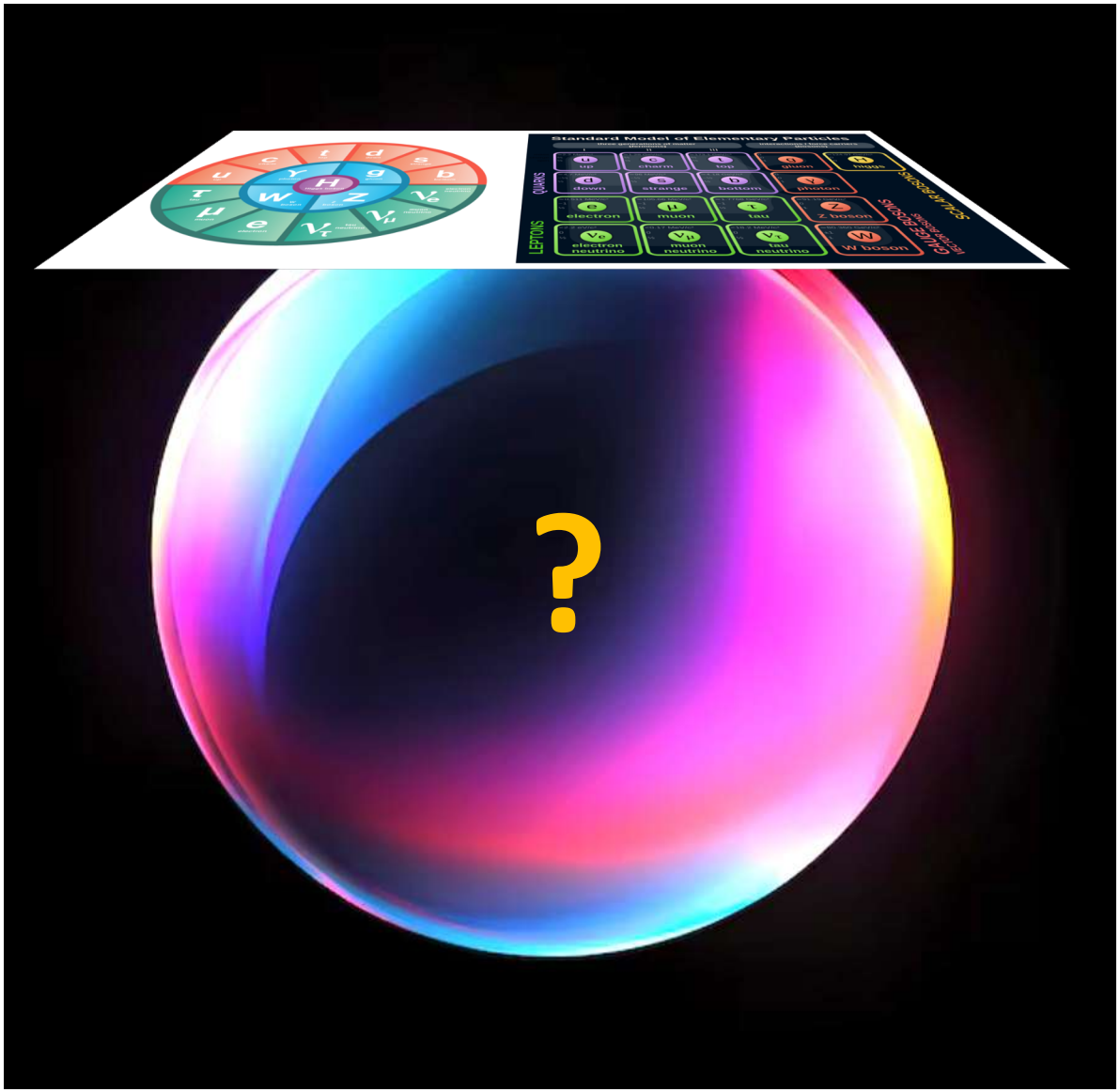
Main focus of Particle Physics so far

Topological Defects: cosmic strings, domain walls, etc



Important to think about

"Global Aspects" of **Particle** and **Defect** Physics



Recall

Global Aspects of Current Algebra

Nuclear Physics B223 (1983) 422–432
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GLOBAL ASPECTS OF CURRENT ALGEBRA

Edward WITTEN*

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA

Received 4 March 1983

A new mathematical framework for the Wess-Zumino chiral effective action is described. It is shown that this action obeys an a priori quantization law, analogous to Dirac's quantization of magnetic change. It incorporates in current algebra both perturbative and non-perturbative anomalies.

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Volume 37B, number 1 PHYSICS LETTERS 1 November 1971

CONSEQUENCES OF ANOMALOUS WARD IDENTITIES

J. WESS
CERN, Geneva, Switzerland
and
University of Karlsruhe, Karlsruhe, Germany

and

B. ZUMINO
CERN, Geneva, Switzerland

Received 7 September 1971

The anomalies of Ward identities are shown to satisfy consistency or integrability relations, which restrict their possible form. For the case of SU(3) × SU(3) we verify that the anomalies given by Bardeen satisfy the consistency relations. A solution of the anomalous Ward identities is also given which describes concisely all anomalous contributions to low energy theorems. The contributions to strong five pseudoscalar interactions, to K_{l4}, to one- and two-photon interactions with three pseudoscalars are explicitly exhibited.

Recall

Global Aspects of Current Algebra

$$\mathcal{L} = \frac{f_\pi^2}{16} Tr\left[\partial_\mu U^\dagger \partial^\mu U\right] + O(\partial^4)$$

(Q) Chiral anomalies in QCD (UV)

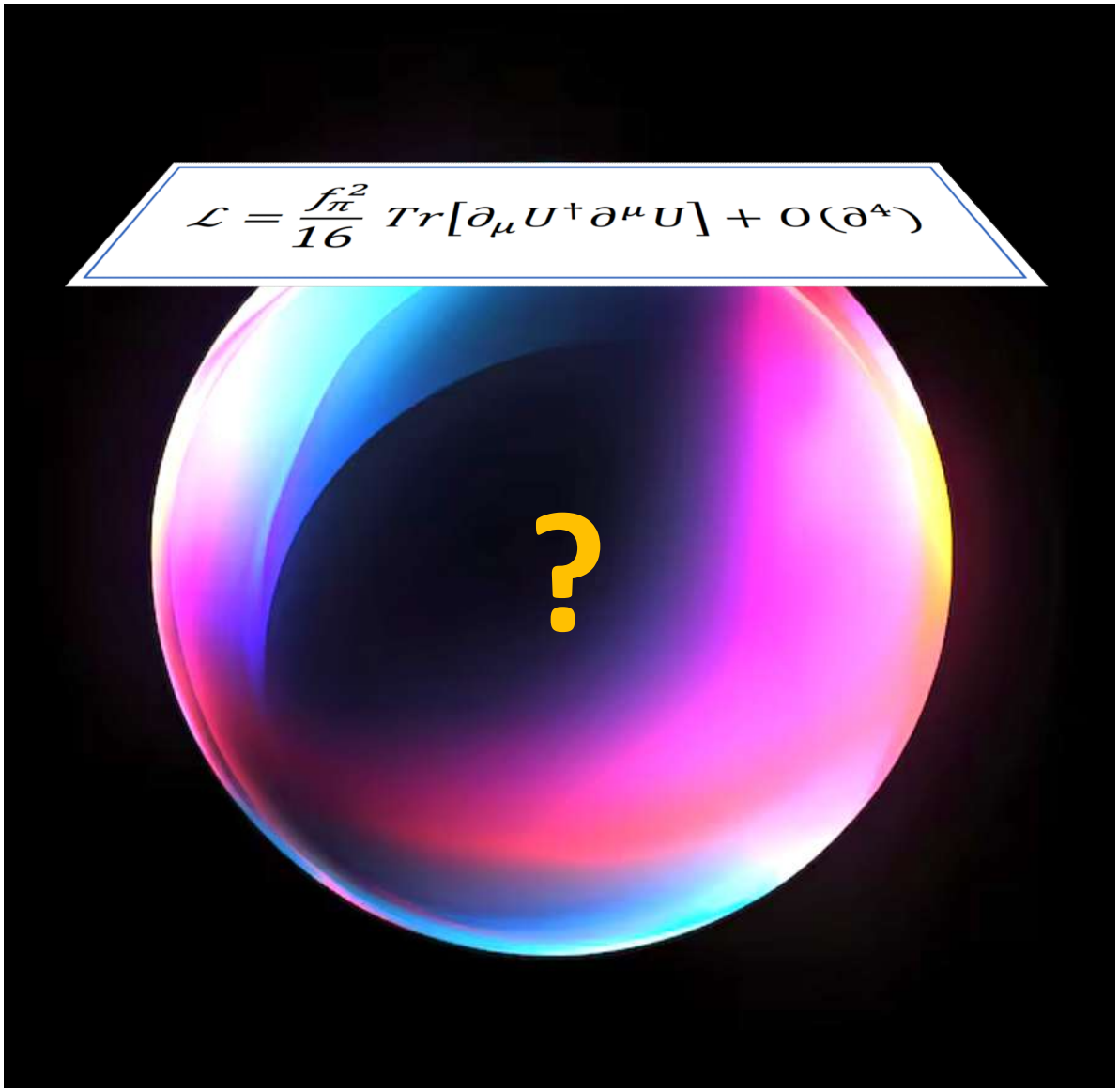
$\Downarrow$??

Same anomalies in ChPT (IR)

(Q) $K^+ K^- \to \pi^+ \pi^0 \pi^-$ [CP-violation] ?

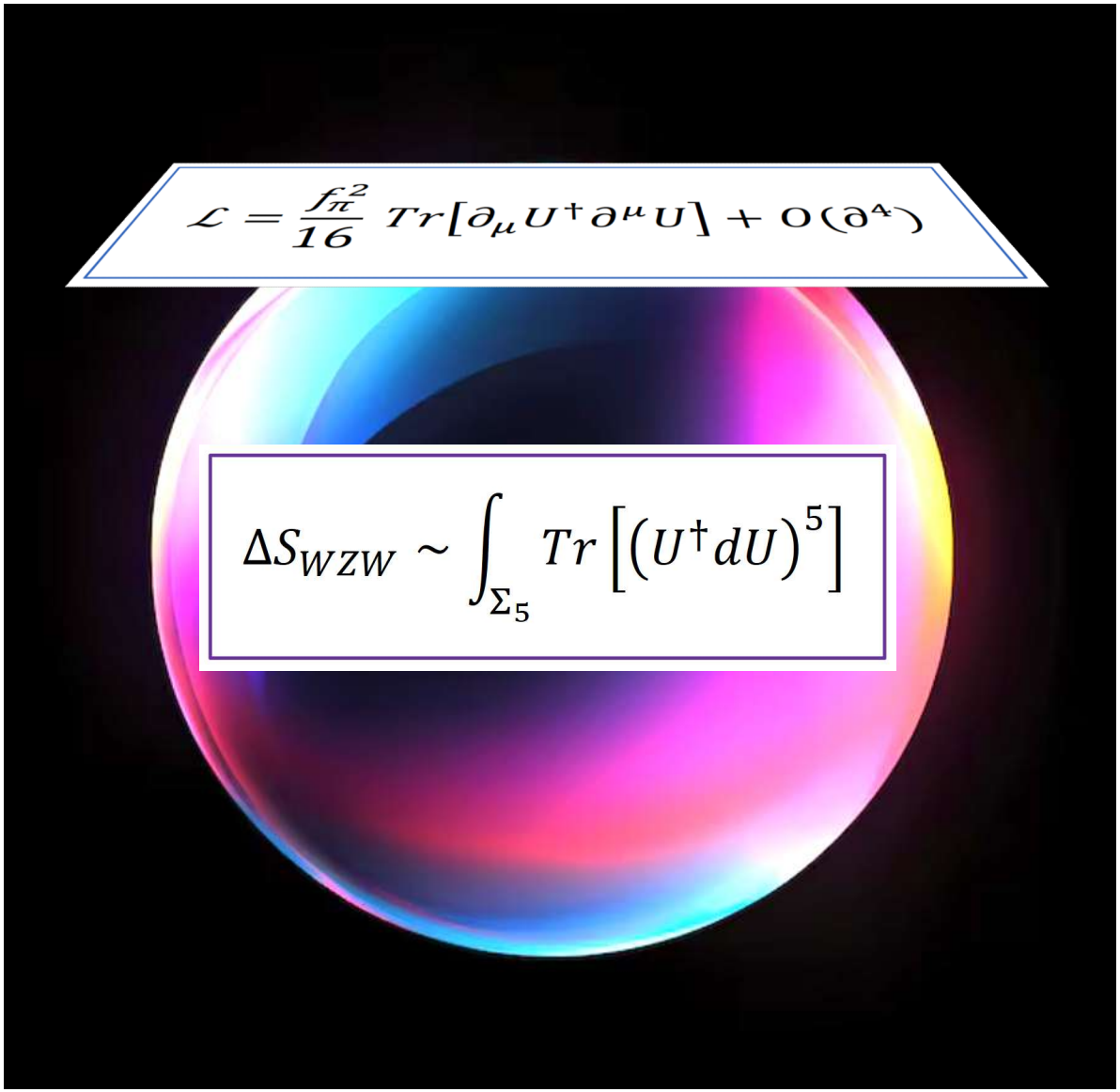
Recall

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Recall

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What are Global Aspects ?

I will mostly discuss one kind : "**Global Structure**"

$$G_{\text{sym}} = \frac{\tilde{G}}{\Gamma} \qquad \text{with } \tilde{G} = \text{simply-connected } \left(\pi_1\left(\tilde{G}\right) = 0\right)$$
$$\Gamma = \text{(discrete) center}$$

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e.g. (i) $SU(N)$ vs $PSU(N) = SU(N)/\mathbb{Z}_N$

(ii) $\frac{SU(N)\times U(1)}{\mathbb{Z}_k}$

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(ii) $\frac{SU(N)\times U(1)}{\mathbb{Z}_k}$

Possibilities: $\frac{G_c\times G_c'}{\Gamma}$, $\frac{G_c\times G_g}{\Gamma}$, $\frac{G_g\times G_g'}{\Gamma}$ (effects on particles / defects)

Why care Global Aspects ?

Point 1: Question of full/correct (generalized) symmetries of QFT

Point 2: ? UV sensitivity

Point 3: selection rules on particle and topological defect spectrum

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Why care Global Aspects ?

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And …..

To be honest, I don't have full answer yet and this is precisely **why**!

Only deeply understanding methods to determine global aspects and comprehensive physics implications will answer why.

Axion String-Domain Wall Network and Global Structure

$$\left(\frac{G_C \times G_g}{\Gamma} \text{ and } \frac{G_C \times G_C'}{\Gamma}\right)$$

2603.26867: with Gongjun Choi and Seth Koren
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DFSZ Axon Model

$$\mathcal{L} \supset y_u H_u Q \bar{u} + y_d H_d Q \bar{d} + \lambda_3 H_u H_d \phi + h.c.$$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-N_cL}$	$U(1)_{\mathrm{PQ}_3}$
Q	3	2	$+1$	$+1$	$+1$
$\bar{u}$	$\bar{3}$	–	-4	-1	0
$\bar{d}$	$\bar{3}$	–	$+2$	-1	0
H_u	–	2	$+3$	0	-1
H_d	–	2	-3	0	-1
ϕ	–	–	0	0	$+2$

Counting = 6 (fields) - 3 (terms in $\mathcal{L}$) - 1 (Y gauged) = 2 ($[B - N_cL]$ and **PQ**)

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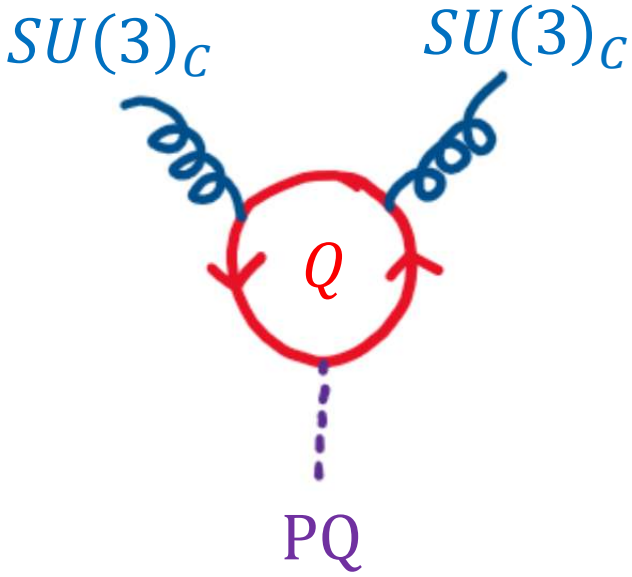
Axion = linear combination of phases of H_u , H_d, and ϕ

DFSZ Axon Model

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$$\partial_\mu J^\mu_{PQ} = \frac{A}{16\pi^2} \mathrm{Tr}\left(G\tilde{G}\right), \quad A = 2N_g$$

$$U(1)_{PQ} \rightarrow \mathbb{Z}_{2N_g}$$



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$\mathrm{V}(\theta) \sim \Lambda^4_{\mathrm{QCD}} \cos\left(2N_g\theta\right)$ from QCD instanton

$\Rightarrow$ when $H \sim m_a \approx \Lambda^2_{\mathrm{QCD}}/f_a$: formations of $\mathbb{Z}_{2\mathrm{N_g}}$ domain walls

$$\left(H \approx \frac{T^2}{M_{pl}}\right)$$

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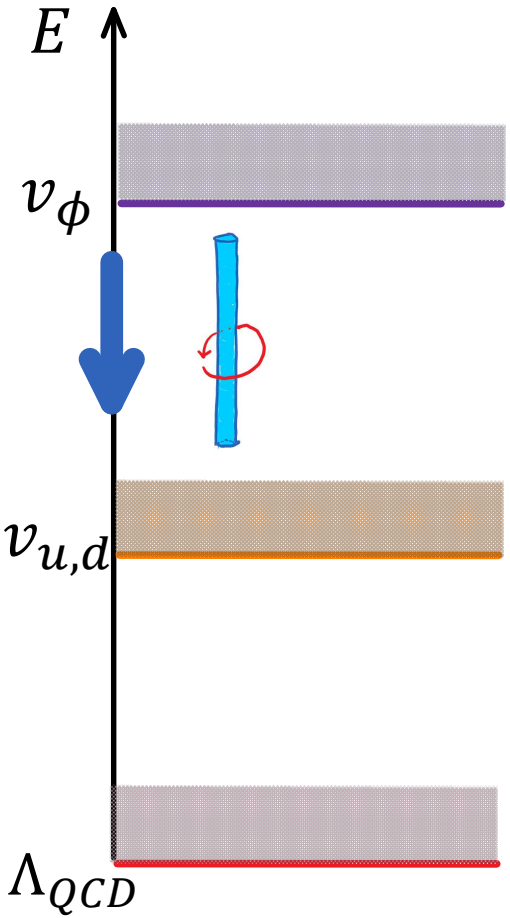
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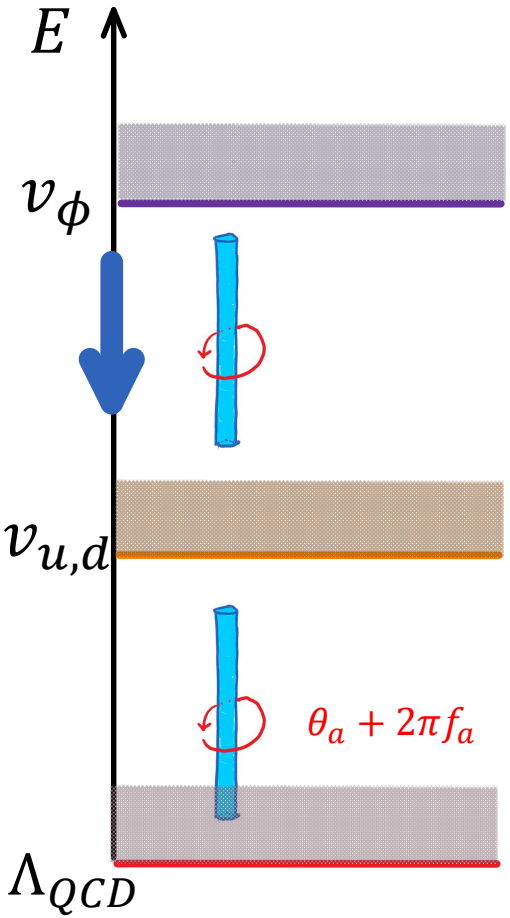
1. $T \sim \langle \phi \rangle = v_\phi$

SSB of $U(1)_{PQ}$: $\pi_1\big(U(1)\big) = \mathbb{Z} \rightarrow$ axion string

DFSZ Axon Model

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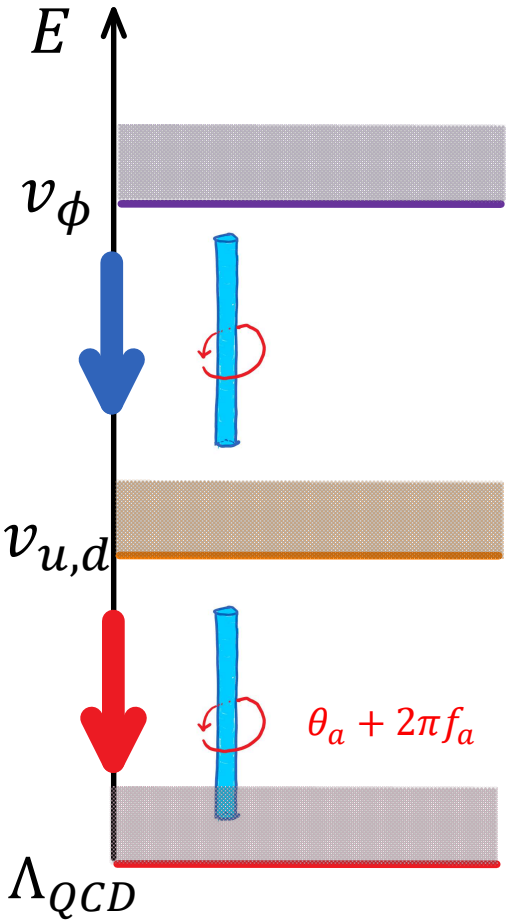
2. $T \sim v_{EW}$

EWSB, vacuum realignment for PQ breaking

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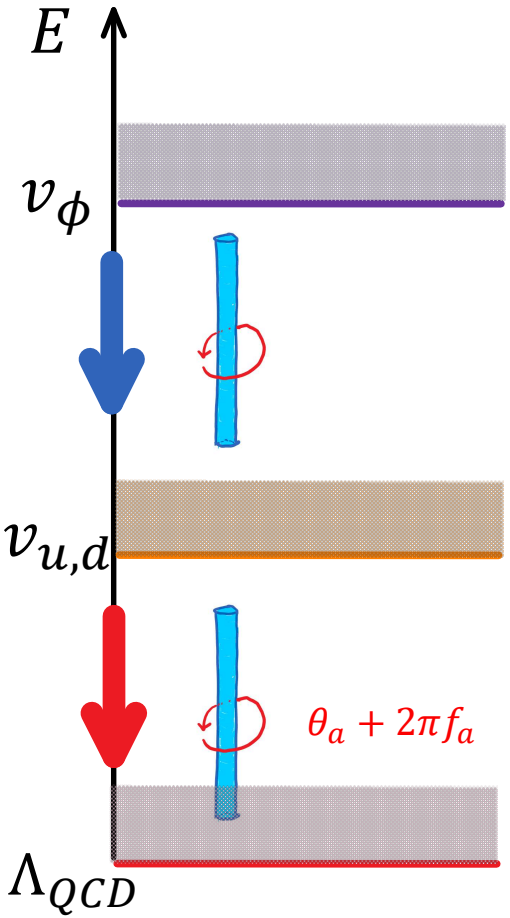
3. $T \sim \Lambda_{QCD}$

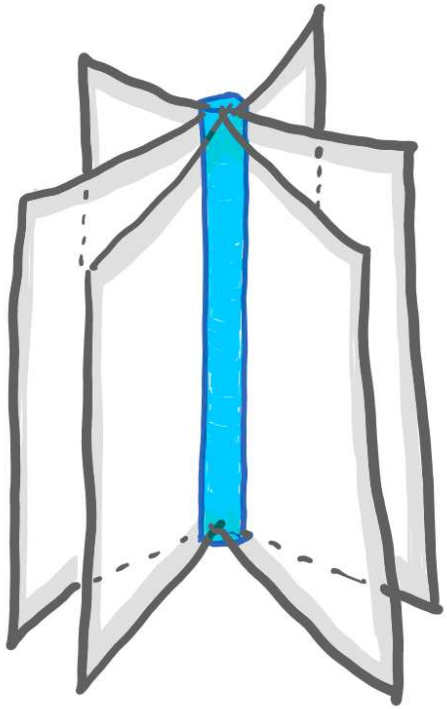
   QCD confinement $\rightarrow$ $V(\theta) \rightarrow$ SSB of $\mathbb{Z}_{N_{DW}}$
   $\rightarrow \pi_0\big(\mathbb{Z}_{N_{DW}}\big) = \mathbb{Z}_{N_{DW}} \rightarrow$ axion DW

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$$\mathcal{L} \supset y_u H_u Q \bar{u} + y_d H_d Q \bar{d} + \lambda_3 H_u H_d \phi + h.c.$$

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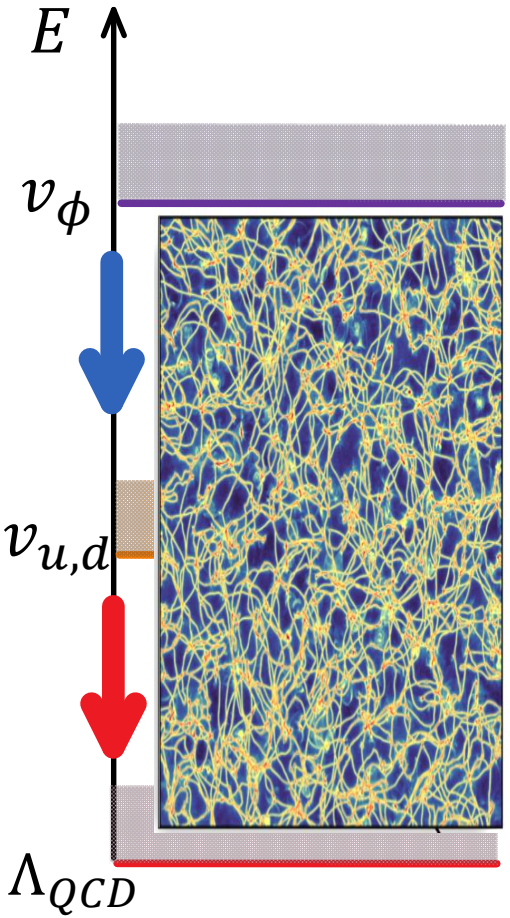




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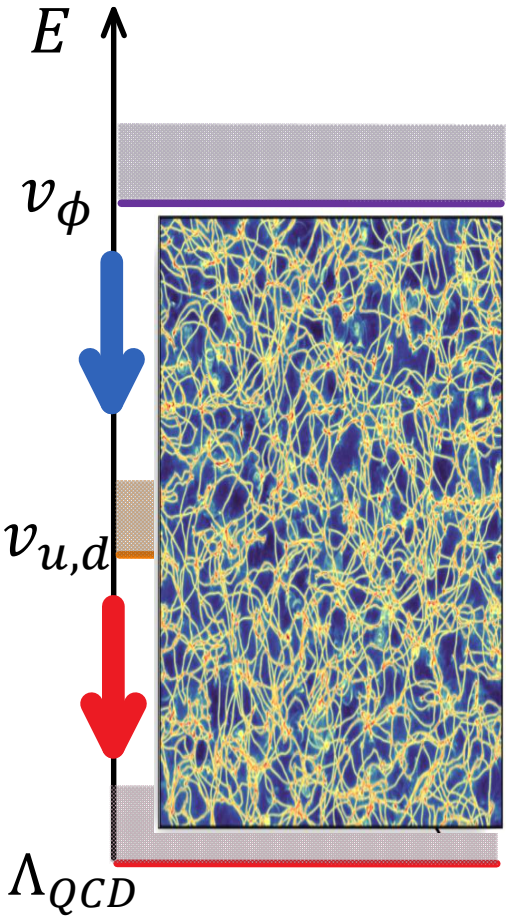


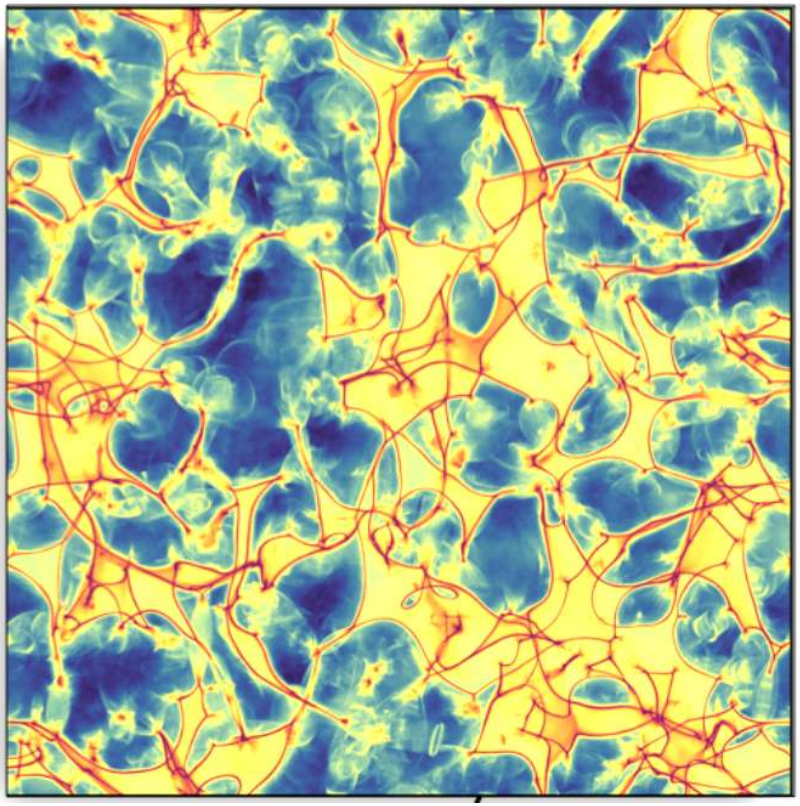
Credit: https://arxiv.org/pdf/2112.05117

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Credit: https://arxiv.org/pdf/2112.05117

Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

$$\mathcal{L} \supset y_u H_u Q \bar{u} + y_d H_d Q \bar{d} + \lambda_3 H_u H_d \phi + h.c.$$

axion mode

$$H_u = \begin{pmatrix} 0 \\ \frac{v_u}{\sqrt{2}} \end{pmatrix} e^{i\theta_u}, \qquad H_d = \begin{pmatrix} 0 \\ \frac{v_d}{\sqrt{2}} \end{pmatrix} e^{i\theta_d}, \qquad \phi = \frac{v_\phi}{\sqrt{2}} e^{i\theta_\phi}$$

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Useful to introduce:

$$\vec{\vartheta} = (v_u \theta_u, v_d \theta_d, v_\phi \theta_\phi)$$

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$$\vec{\vartheta} = (v_u\theta_u, v_d\theta_d, v_\phi\theta_\phi)$$

$$\vec{Y} = (v_u y_u, v_d y_d, v_\phi y_\phi) = (3v_u, -3v_d, 0) \quad : \quad \hat{Y} = \frac{\vec{Y}}{|\vec{Y}|} = \frac{\vec{Y}}{3v_{EW}}$$

$$\vec{P} = (v_u p_u, v_d p_d, v_\phi p_\phi) = (-v_u, -v_d, 2v_\phi) : \quad \hat{P} = \frac{\vec{P}}{|\vec{P}|}$$

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$$\vec{\vartheta} = (v_u\theta_u, v_d\theta_d, v_\phi\theta_\phi) \; : \quad \vec{\vartheta} \mapsto \vec{\vartheta} + \alpha_Y \vec{Y} + \alpha_{PQ}\vec{P}$$

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axion mode

$$\hat{a} = \frac{1}{f_a}\left(\vec{P} - \left(\vec{P}\cdot\vec{Y}\right)\hat{Y}\right), \qquad f_a = \left|\vec{P} - \left(\vec{P}\cdot\vec{Y}\right)\hat{Y}\right|$$

$$\theta_a = \vec{\vartheta}\cdot\hat{a} = \frac{2}{f_a}\left(v_\phi^2\theta_\phi - \frac{v_u^2 v_d^2}{v_{EW}^2}(\theta_u + \theta_d)\right)$$

Note: axion direction is $U(1)_Y$ - invariant.

$$\vec{\vartheta} = (v_u\theta_u, v_d\theta_d, v_\phi\theta_\phi)$$
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H_d	–	2	-3	0	-1
ϕ	–	–	0	0	$+2$

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H_u	–	2	$+3$	0	-1
H_d	–	2	-3	0	-1
ϕ	–	–	0	0	$+2$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$

Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

$$\mathcal{L} \supset y_u H_u Q \bar{u} + y_d H_d Q \bar{d} + \lambda_3 H_u H_d \phi + h.c.$$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-N_cL}$	$U(1)_{\mathrm{PQ}_3}$
Q	3	2	$+1$	$+1$	$+1$
$\bar{u}$	$\bar{3}$	–	-4	-1	0
$\bar{d}$	$\bar{3}$	–	$+2$	-1	0
H_u	–	2	$+3$	0	-1
H_d	–	2	-3	0	-1
ϕ	–	–	0	0	$+2$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$

Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

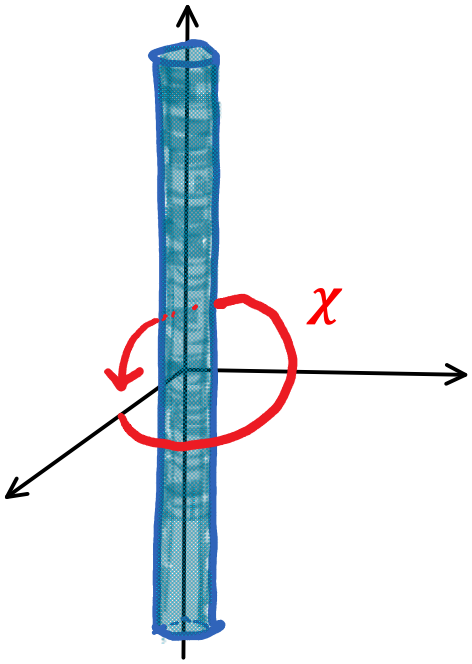
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	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-N_cL}$	$U(1)_{\mathrm{PQ}_3}$
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H_u	–	2	$+3$	0	-1
H_d	–	2	-3	0	-1
ϕ	–	–	0	0	$+2$

$$\frac{\boldsymbol{SU(2)_L \times U(1)_Y \times U(1)_{PQ}}}{\boldsymbol{\Gamma \times \mathbb{Z}_2}}$$

Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$

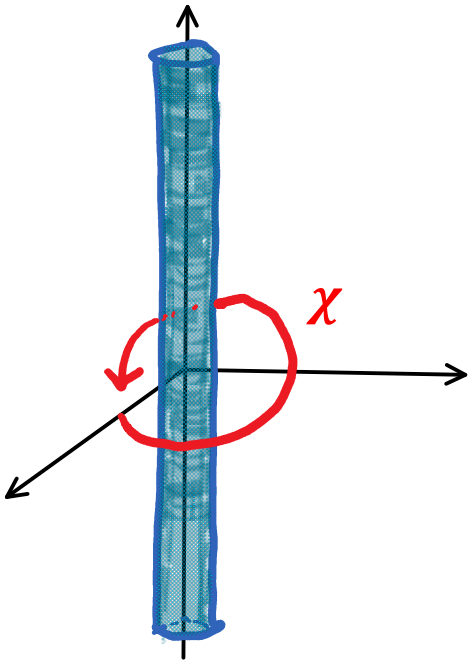


$$\vec{\vartheta}(\chi = 0) \;\mapsto\; \vec{\vartheta}(\chi = 2\pi) = \vec{\vartheta}(0) + \frac{2\pi}{2}\left(\vec{Y} + \vec{P}\right)$$

$$= \begin{pmatrix} v_u(\theta_u(0) + 2\pi) \\ v_d(\theta_d(0) - 4\pi) \\ v_\phi(\theta_\phi(0) + 2\pi) \end{pmatrix}$$

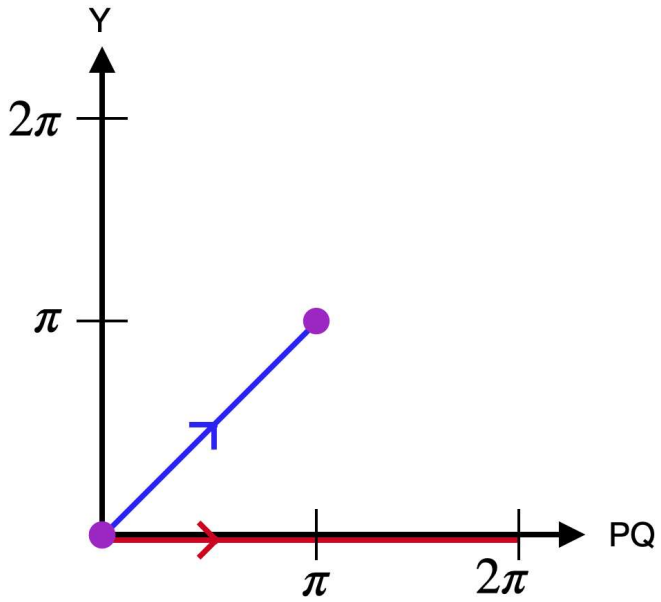
Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$



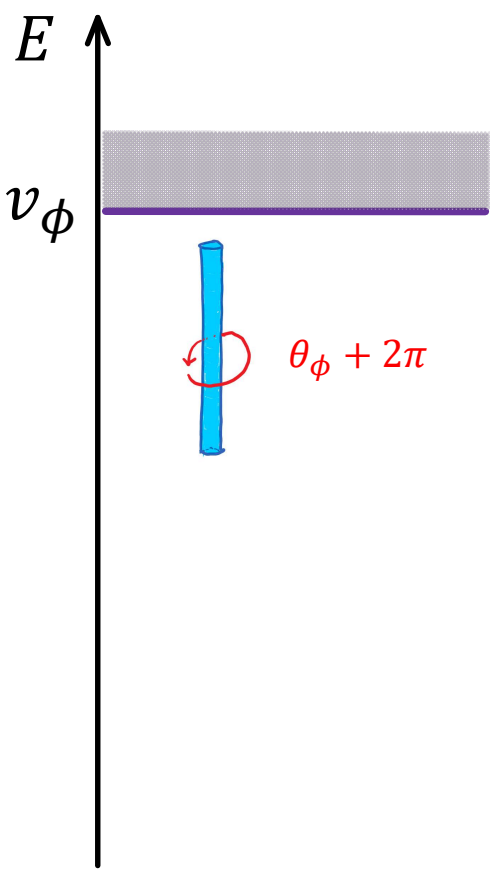
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Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$



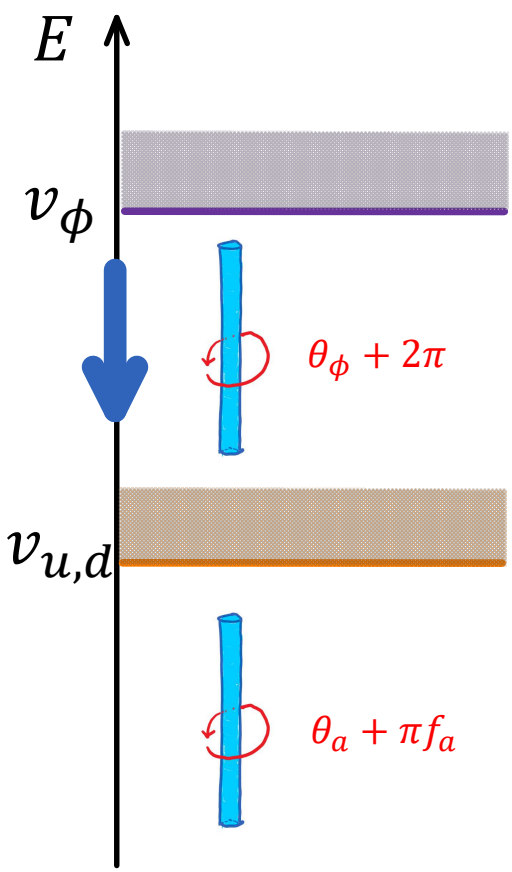
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Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$



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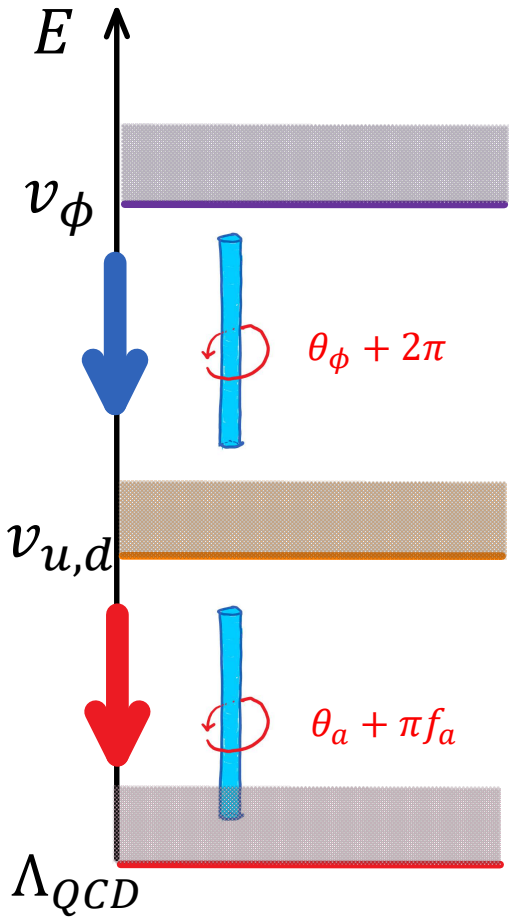
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Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

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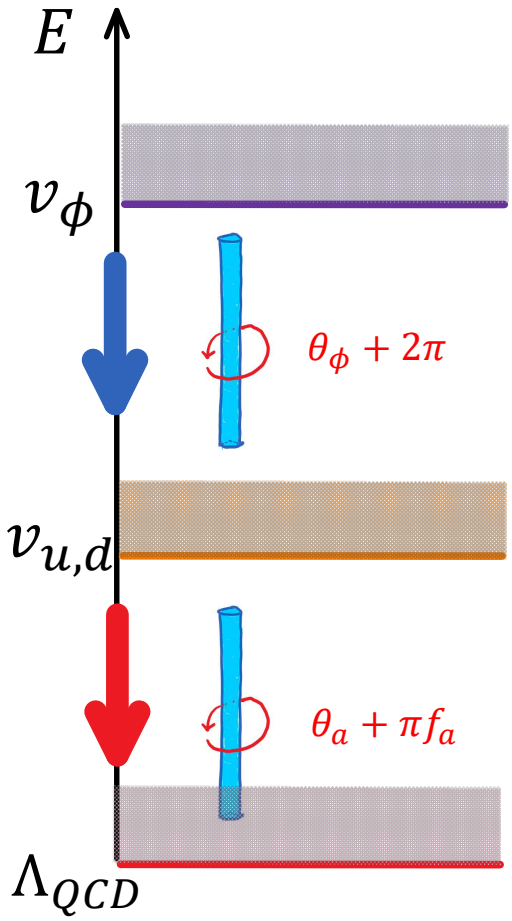
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Recall: $\partial_\mu J^\mu_{PQ} = \frac{A}{16\pi^2} \mathrm{Tr}\left(G\tilde{G}\right), \quad A = 2N_g$

$$U(1)_{PQ} \rightarrow \mathbb{Z}_{2N_g} : \theta_a \mapsto \theta_a + \frac{2\pi}{2N_g} f_a$$

Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

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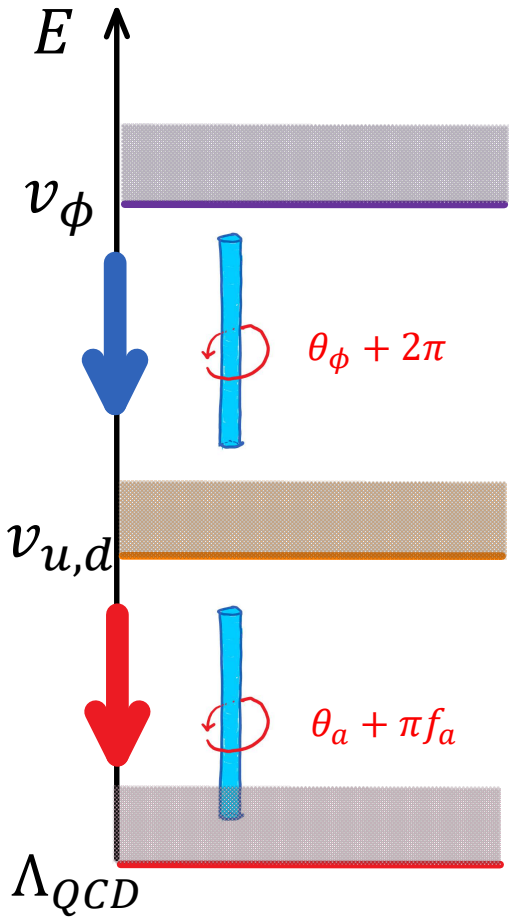
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Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$



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$\Rightarrow$ N_g **domain walls** are attached to a string

Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

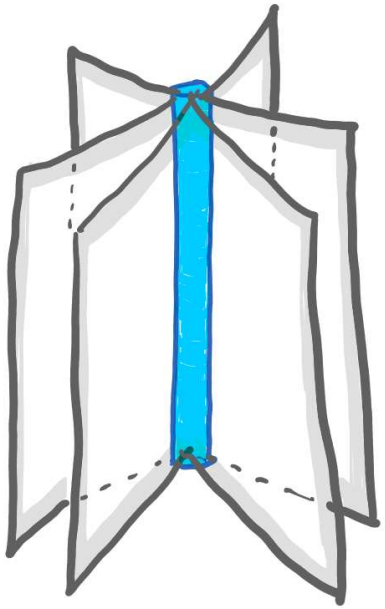
$U(1)_Y \times U(1)_{PQ}$ vs $\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$

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Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

$U(1)_Y \times U(1)_{PQ}$ vs $\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$

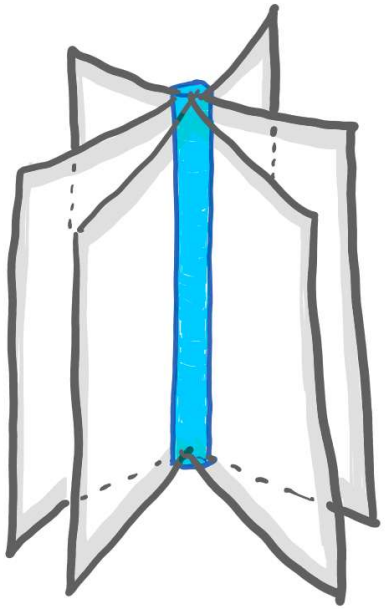
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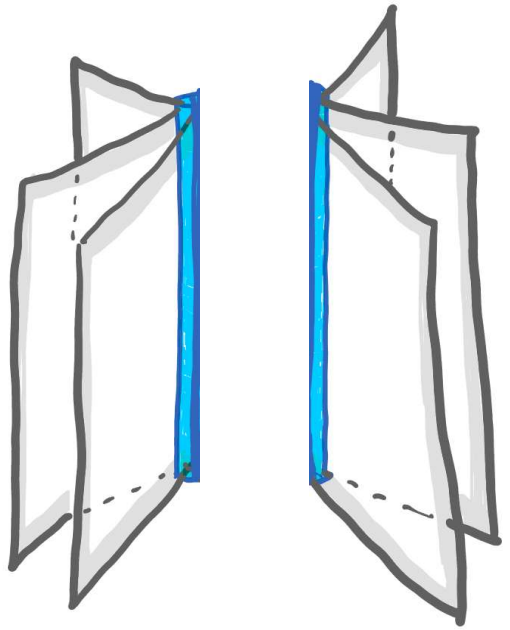


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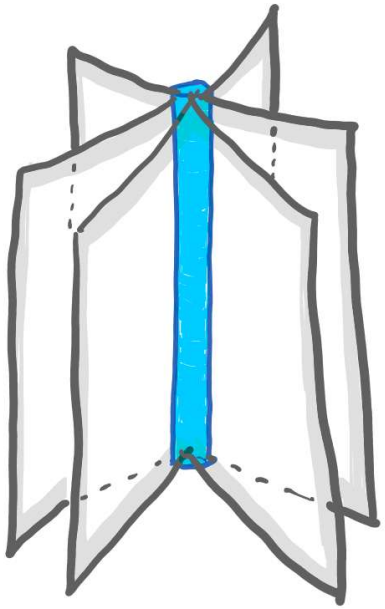


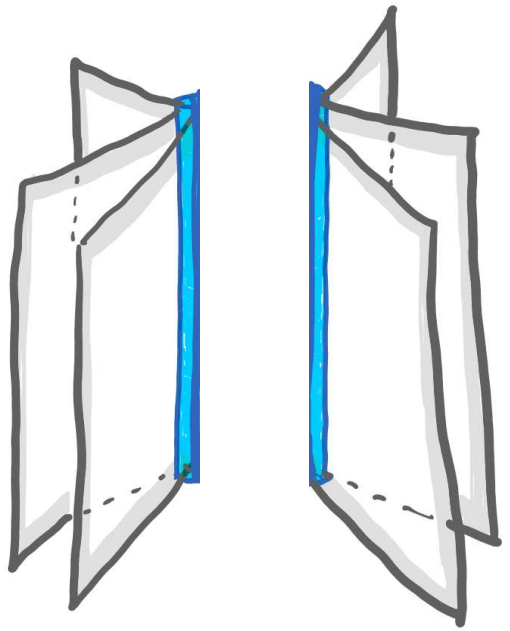


Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

$U(1)_Y \times U(1)_{PQ}$ vs $\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$

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$\mathbb{Z}_{2N_g}$ Domain Wall Problem

$\mathbb{Z}_{N_g}$ Domain Wall Problem

Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

Comment 1: In a slightly different language, this feature was observed by Lazarides and Shafi (1982)

$$\frac{Spin(10) \times U(1)_{PQ}}{\mathbb{Z}_4} \quad \text{with} \quad A = 4N_g$$

Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

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$$\frac{Spin(10)\times U(1)_{PQ}}{\mathbb{Z}_4} \quad \text{with} \quad A = 4N_g$$

Comment 2: Lazarides-Shafi mechanism does not always work (precise string-wall network is UV-sensitive)

e.g. [Q. Lu, M. Reece, Z. Sun '23]

Gauge unbroken (anom-free) $\mathbb{Z}_5 \subset U(1)_{PQ}$ with $\left[\phi = v_\phi e^{i\theta_\phi}\right]_{PQ} = 2$

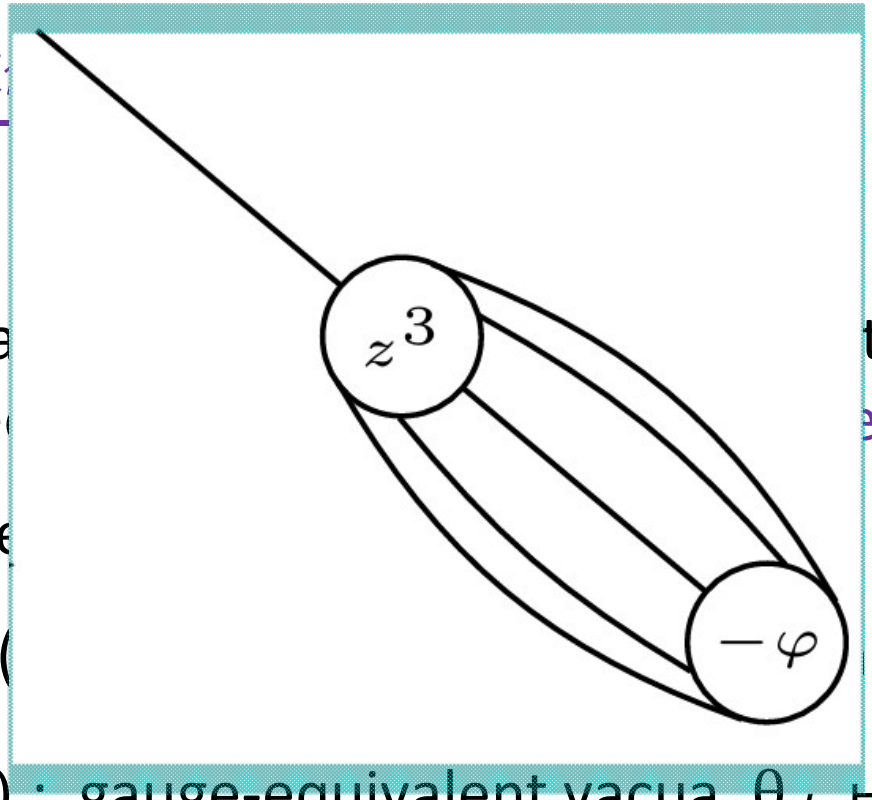
SSB of $\mathbb{Z}_5$ by $V(\theta)$: gauge-equivalent vacua $\theta_\phi \mapsto \theta_\phi + 2\pi/5$

$\mathbb{Z}_5$ string: $\oint_{z^k} \theta_\phi = 2\pi\left(\frac{2k}{5} + n\right) = 2\pi\left(\frac{2\cdot 3}{5} - 1\right)$

Correct Domain Wall Spectrum from $\frac{G_c \times G_g}{\Gamma}$

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Spi



Comment 2: Laza t always work (pre ensitive)

e.g. [Q. Lu, M. Ree

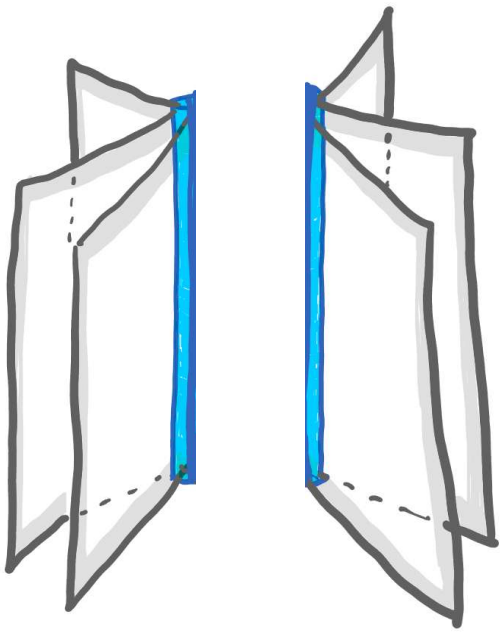
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Solving $\mathbb{Z}_{N_g}$ Domain Wall Problem from $\frac{G_c \times G_c'}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$

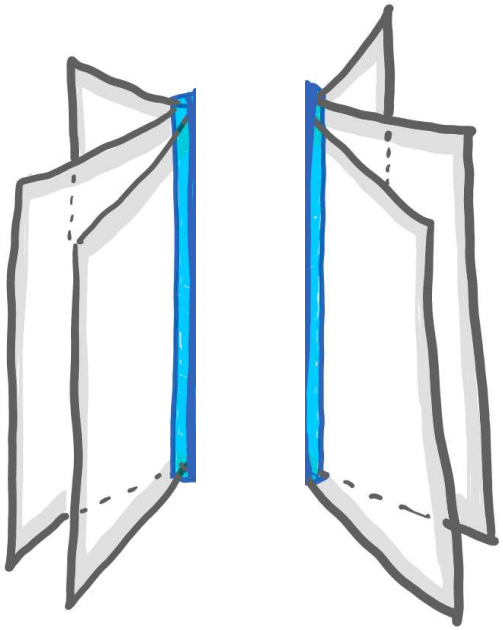


$\mathbb{Z}_{N_g}$ Domain Wall Problem

Solving $\mathbb{Z}_{N_g}$ Domain Wall Problem from $\frac{G_C \times G_C'}{\Gamma}$

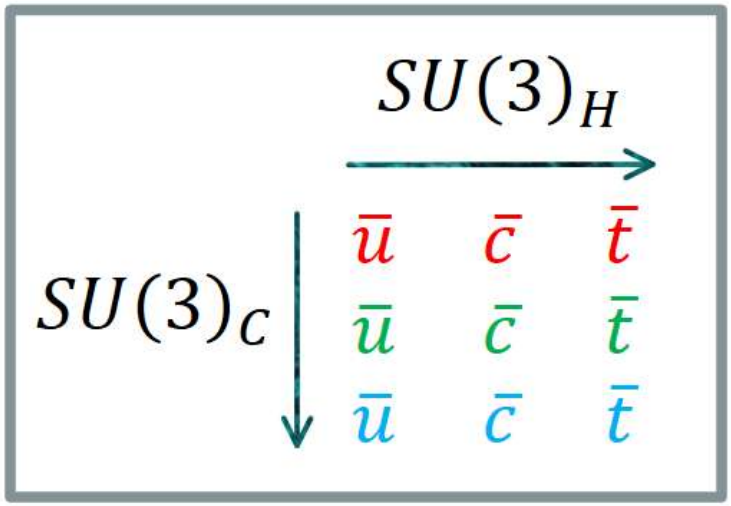
$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$

$$\frac{SU(3)_C \times SU(3)_H}{\mathbb{Z}_3}$$



$\mathbb{Z}_{N_g}$ Domain Wall Problem

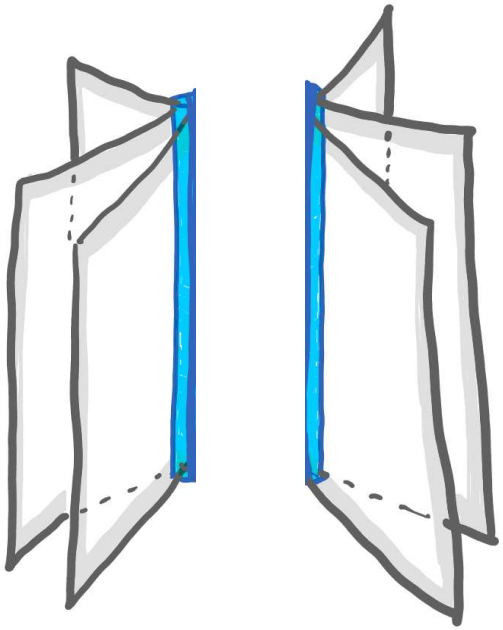
$$N_C = N_H \bmod 1 = \frac{3-1}{3}\int \frac{w_2 \wedge w_2}{2} \bmod 1$$



Solving $\mathbb{Z}_{N_g}$ Domain Wall Problem from $\frac{G_c \times G_c'}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$

$$\frac{SU(3)_C \times SU(3)_H}{\mathbb{Z}_3}$$



$$N_C = N_H \bmod 1 = \frac{3-1}{3}\int \frac{w_2 \wedge w_2}{2} \bmod 1$$

$$A_C = 2N_g, \qquad A_H = 2N_c, \qquad A_{CH} = 4$$

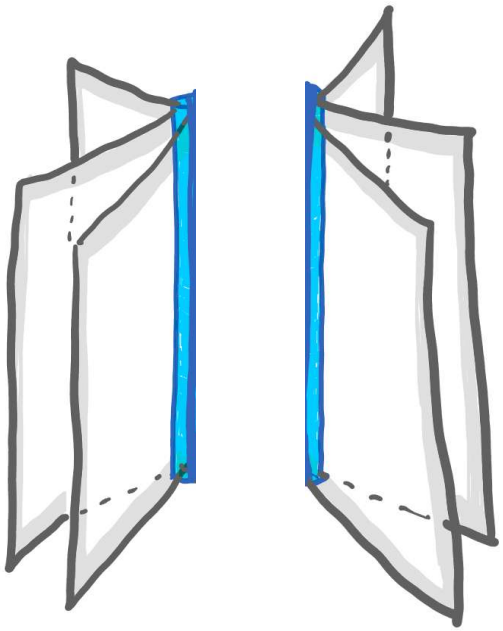
Naively, $U(1)_{PQ} \to \mathbb{Z}_{\gcd(A_c, A_H, A_{CH})} = \mathbb{Z}_2$

$\mathbb{Z}_{N_g}$ Domain Wall Problem

Solving $\mathbb{Z}_{N_g}$ Domain Wall Problem from $\frac{G_c \times G_c'}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$

$$\frac{SU(3)_C \times SU(3)_H}{\mathbb{Z}_3}$$



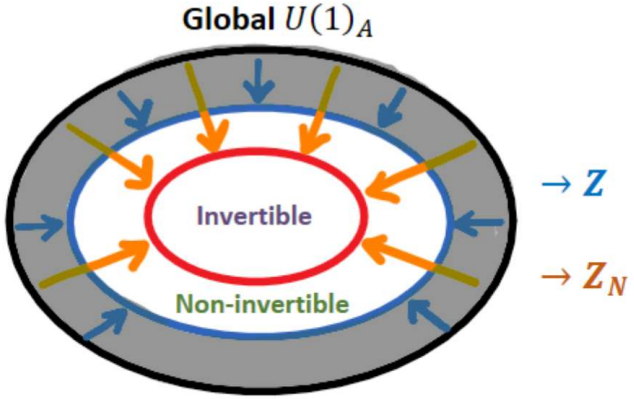
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Naively, $U(1)_{PQ} \to \mathbb{Z}_{\gcd(A_C, A_H, A_{CH})} = \mathbb{Z}_2$

Correct answer: $U(1)_{PQ} \to \mathbb{Z}_2^I \times \mathbb{Z}_3^{NI}$



[Digression]

Non-Invertible Symmetry

Non-Invertible Symmetry

1. From $U(1)$ Instanton

Consider $U(1)_{EM}$ with $\Psi_+,\ \Psi_-$

EoM: $d * F_2 = j_\Psi \quad \left(d * F_2 = 0 \ \Rightarrow \ U(1)^{(1)}(e)\right)$

charged op: Wilson $W_1 = e^{i\oint A_1}$, SDO $U(\Sigma_2) = e^{i\oint * F_2}$

Bianchi id: $dF_2 = 0 \ \Rightarrow \ U(1)^{(1)}(m)$

charged op: 't Hooft $T_1 = e^{i\oint \tilde{A}_1}$, SDO $U(\Sigma_2) = e^{i\oint F_2}$

$U(1)^{(0)}{}_A$: $\Psi_+ \to e^{i\alpha}\,\Psi_+$, $\Psi_- \to e^{i\alpha}\,\Psi_- \ \Rightarrow \ d * j_A = \frac{K}{8\pi^2} F_2 \wedge F_2$

$\Rightarrow \ \Pi_3\big(U(1)\big) = \emptyset$ and $U(1)_A$ turns into non-invertible symm

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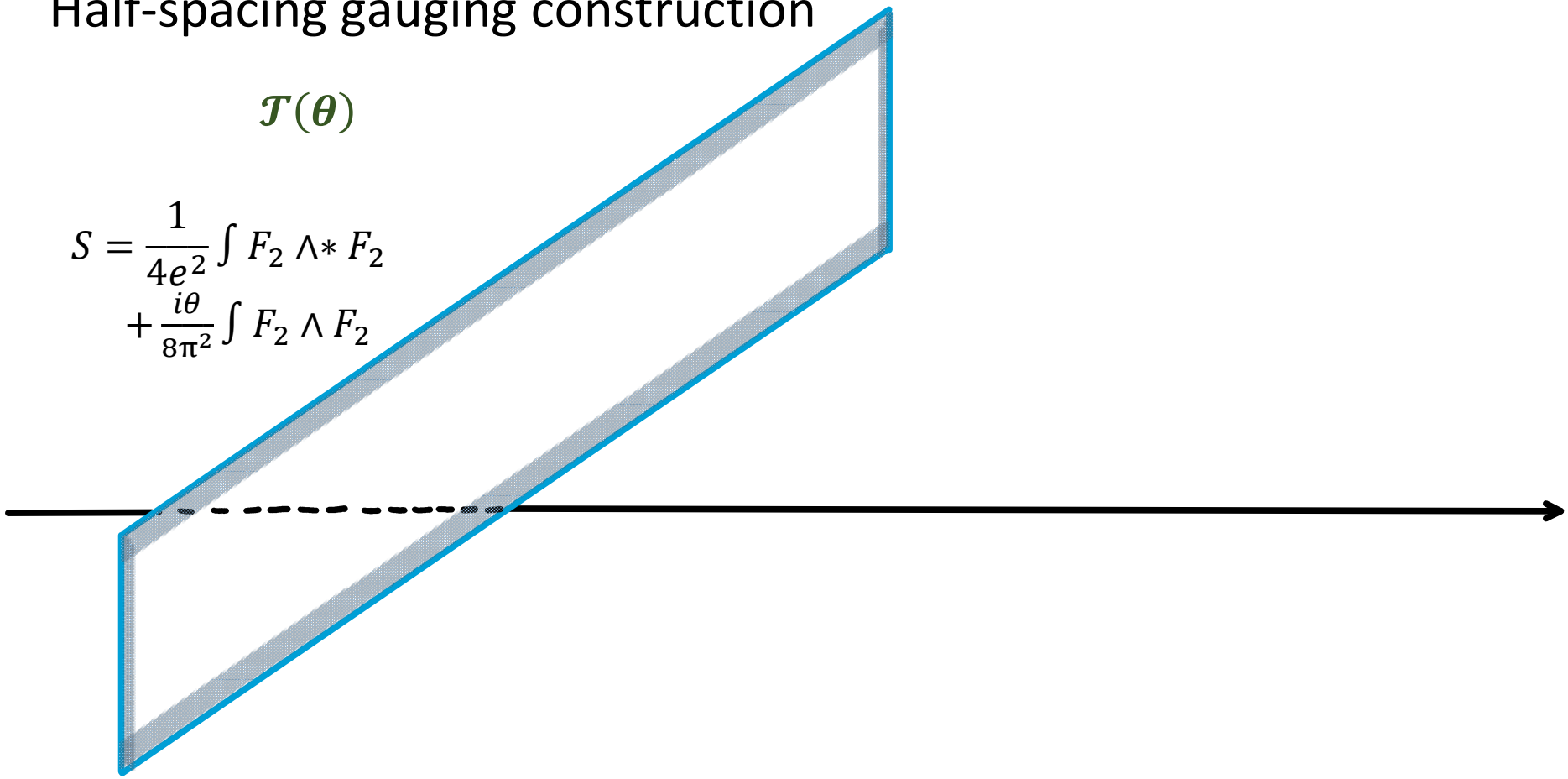
Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction

$$\mathcal{T}(\theta)$$

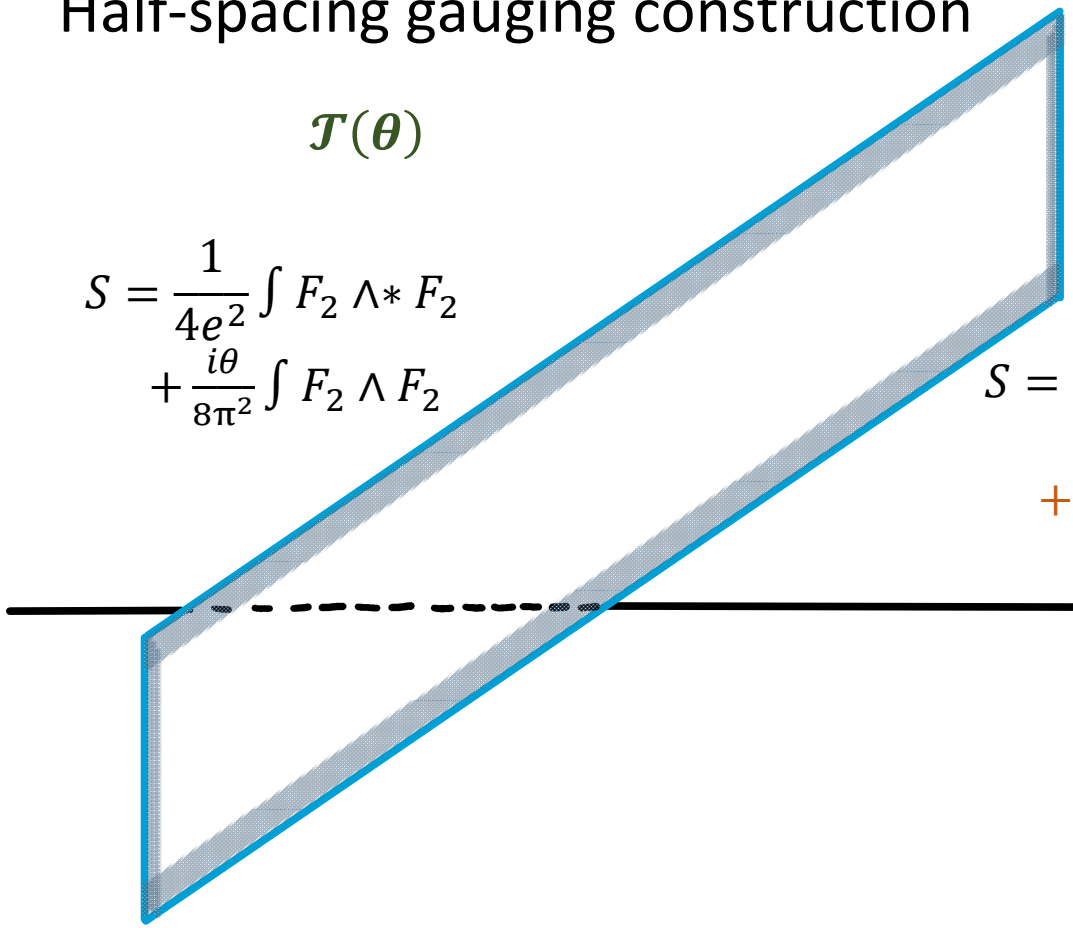
$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$



Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction



$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$

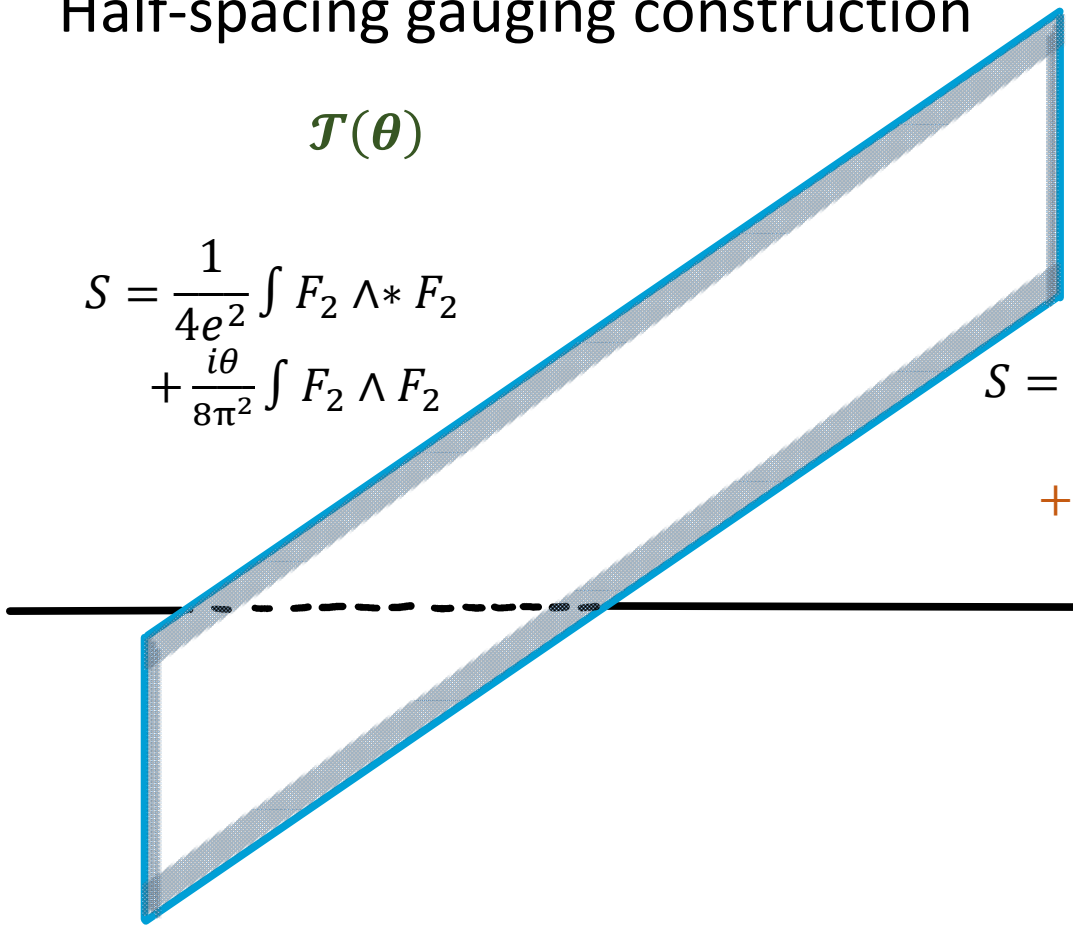
[Gauge $Z_N^{(1)}(m) \subset U(1)^{(1)}(m)$]

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2 + \frac{i}{2\pi}\int B_2 \wedge F_2 + \frac{iN}{2\pi}\int B_2 \wedge dC_1 + \frac{ikN}{4\pi}\int B_2 \wedge B_2$$

Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction



$\mathcal{T}(\theta)$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$

[Gauge $Z_N^{(1)}(m) \subset U(1)^{(1)}(m)$]

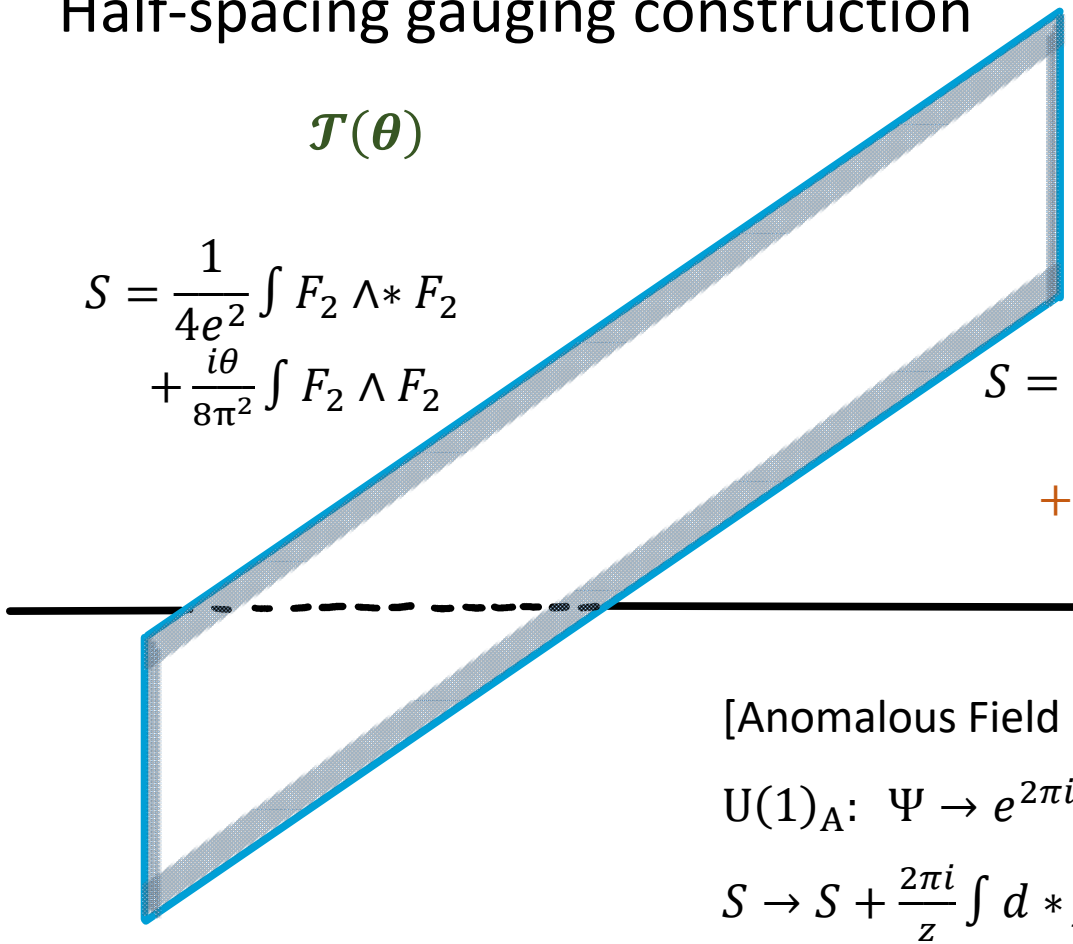
$$\mathcal{T}\left(\theta - \frac{2\pi p}{N}\right) \quad (kp=1 \bmod N)$$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2 + \frac{i}{2\pi}\int B_2 \wedge F_2 + \frac{iN}{2\pi}\int B_2 \wedge dC_1 + \frac{ikN}{4\pi}\int B_2 \wedge B_2$$

Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction



$\mathcal{T}(\theta)$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$

[Gauge $Z_N^{(1)}(m) \subset U(1)^{(1)}(m)$]

$$\mathcal{T}\left(\theta - \frac{2\pi p}{N}\right) \quad (kp=1 \bmod N)$$

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[Anomalous Field Redefinition]

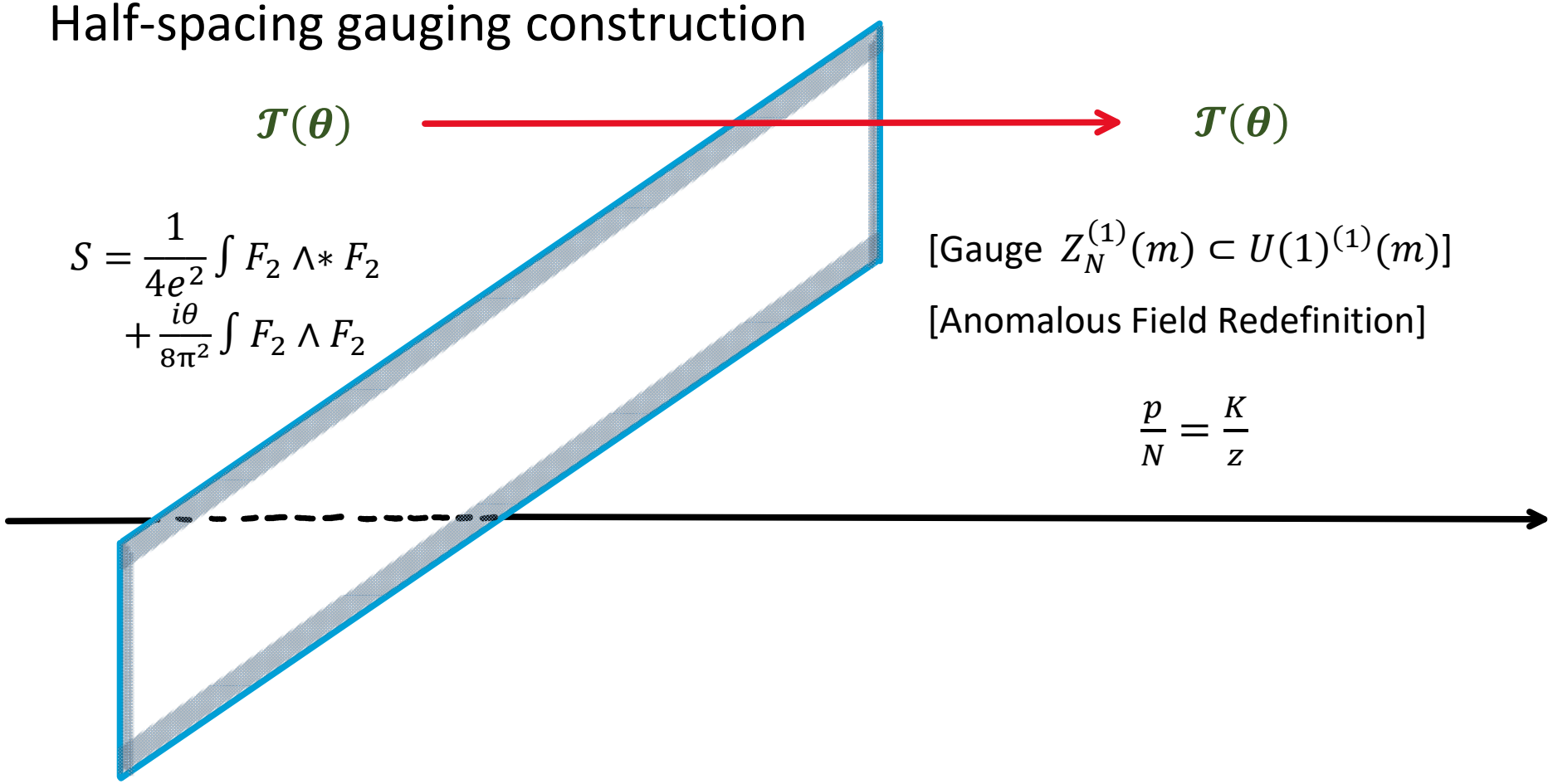
$\mathrm{U(1)_A}$: $\Psi \to e^{2\pi i/z}\,\Psi$

$$S \to S + \frac{2\pi i}{z}\int d * j_1 + \frac{2\pi i K}{z}\int \frac{F\wedge F}{8\pi^2} \quad \leftrightarrow \quad \left[\mathcal{T}(\theta) \to \mathcal{T}\left(\theta + \frac{2\pi K}{z}\right)\right]$$

Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction



$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$

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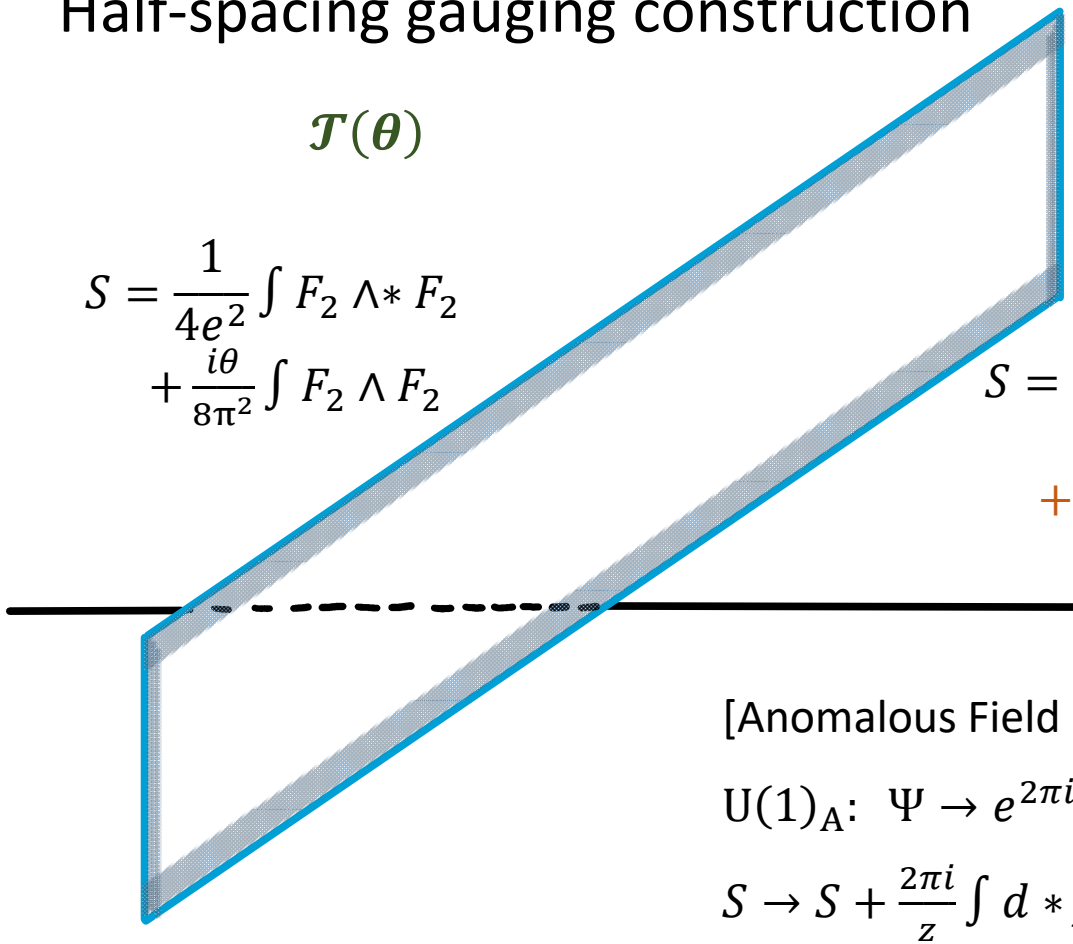
[Anomalous Field Redefinition]

$$\frac{p}{N} = \frac{K}{z}$$

Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction



$\mathcal{T}(\theta)$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$

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[Anomalous Field Redefinition]

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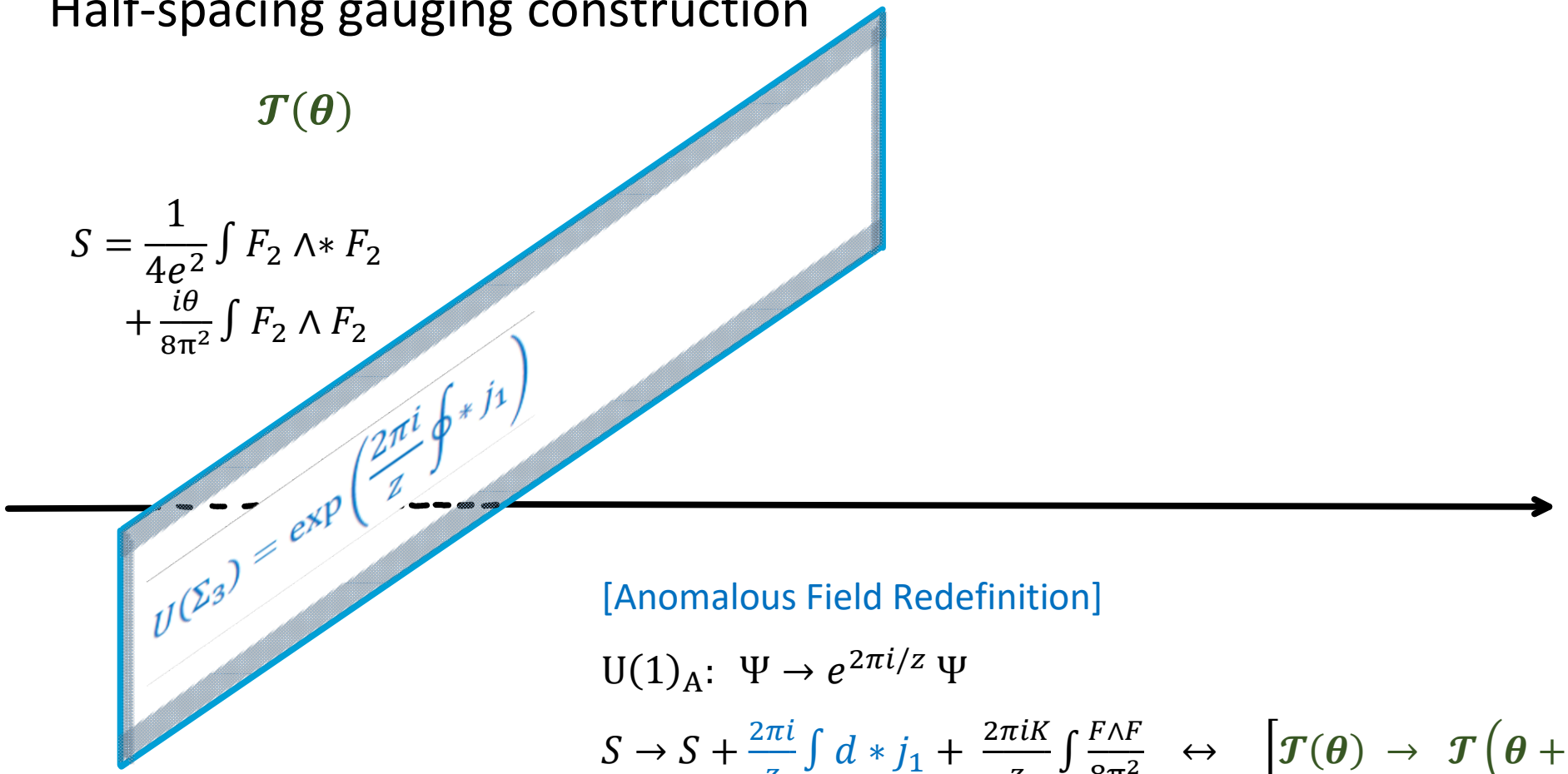
Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction

$$\mathcal{T}(\theta)$$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$



[Anomalous Field Redefinition]

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Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction

$\mathcal{T}(\theta)$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$

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$U(\Sigma_3) = \exp\left(\frac{2\pi i}{z}\oint * j_1\right)$ $\mathcal{A}^{N,p}(D_2 = F_2/N)$

[Anomalous Field Redefinition]

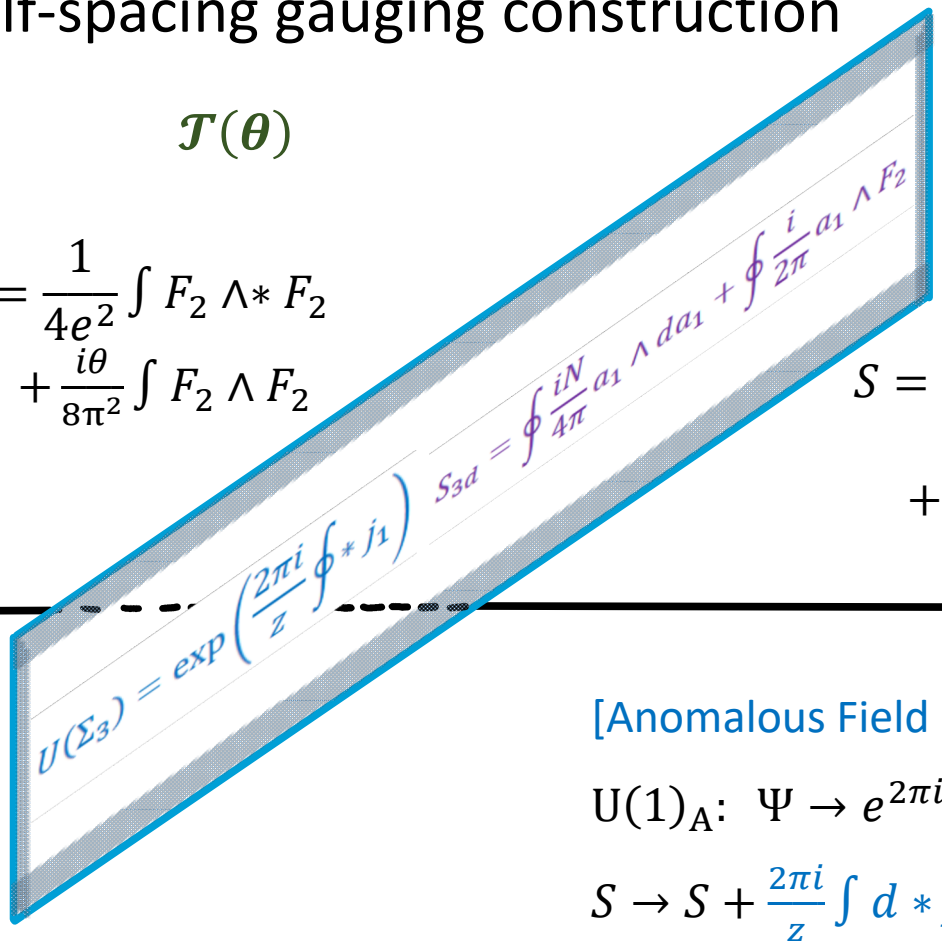
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Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction



$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$

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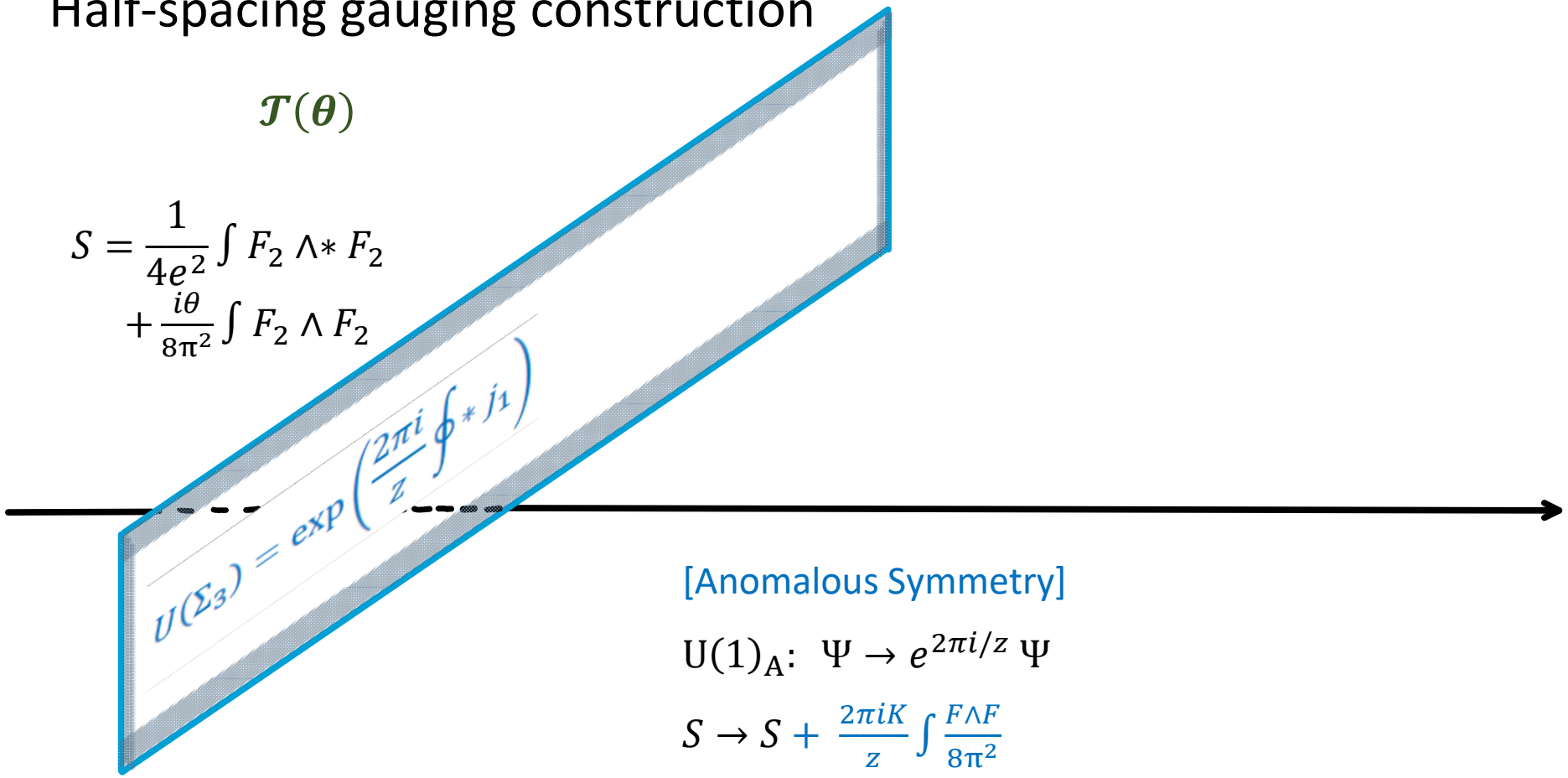
Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction

$$\mathcal{T}(\theta)$$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$



[Anomalous Symmetry]

$\mathrm{U}(1)_{\mathrm{A}}$: $\Psi \to e^{2\pi i/z}\,\Psi$

$S \to S + \frac{2\pi i K}{z}\int \frac{F\wedge F}{8\pi^2}$

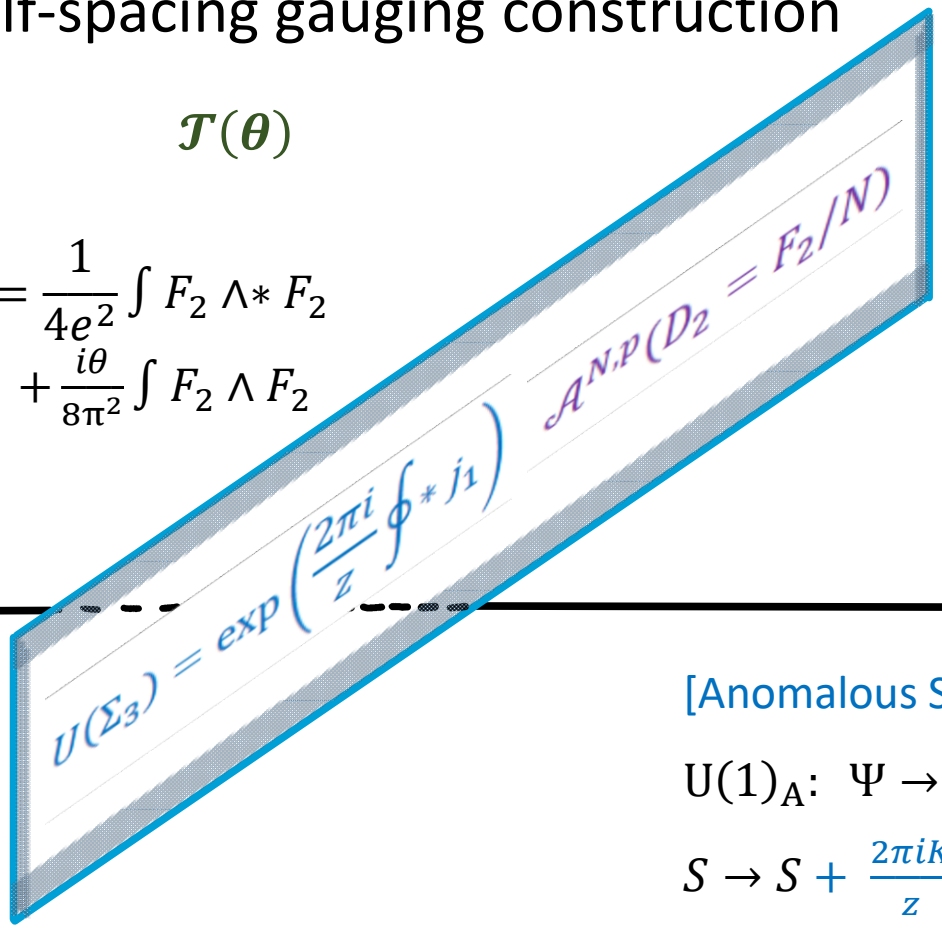
Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction

$$\mathcal{T}(\theta)$$

$$S = \frac{1}{4e^2}\int F_2 \wedge * F_2 + \frac{i\theta}{8\pi^2}\int F_2 \wedge F_2$$



[Anomalous Symmetry] × [$\mathcal{A}^{N,p}(F_2/N)$]

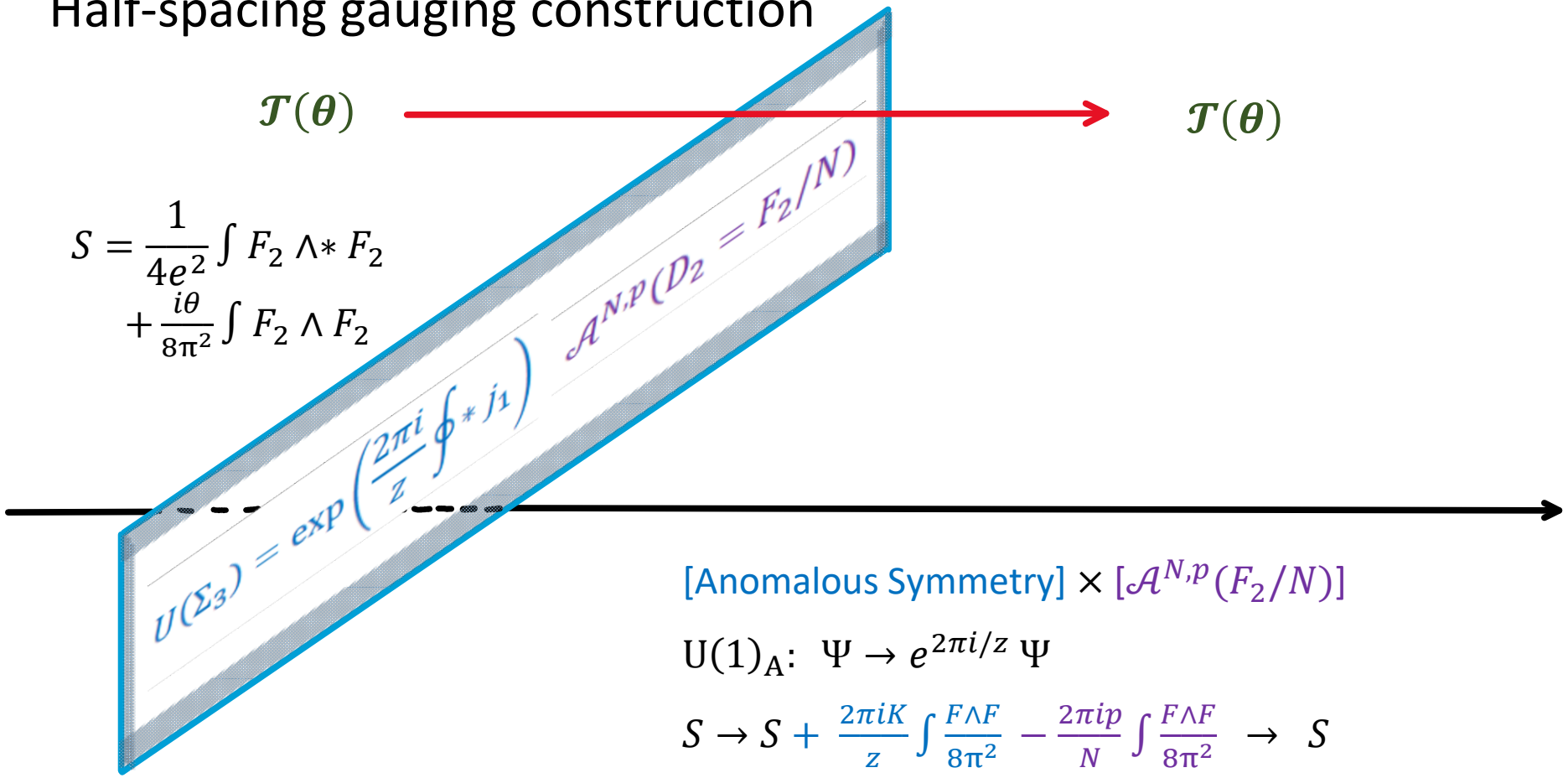
$\mathrm{U}(1)_\mathrm{A}$: $\Psi \to e^{2\pi i/z}\,\Psi$

$$S \to S + \frac{2\pi i K}{z}\int \frac{F\wedge F}{8\pi^2} - \frac{2\pi i p}{N}\int \frac{F\wedge F}{8\pi^2} \to S$$

Non-Invertible Symmetry

1. From $U(1)$ Instanton

Half-spacing gauging construction



[Anomalous Symmetry] × $[\mathcal{A}^{N,p}(F_2/N)]$

$\mathrm{U}(1)_{\mathrm{A}}$: $\Psi \to e^{2\pi i/z}\,\Psi$

$S \to S + \frac{2\pi i K}{z}\int \frac{F\wedge F}{8\pi^2} - \frac{2\pi i p}{N}\int \frac{F\wedge F}{8\pi^2} \to S$

Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)$

electric 1-form: Z_N

magnetic 1-form: none

Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

Non-Invertible Symmetry

2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$ electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $S \to S + \frac{2\pi K i}{z}\underbrace{\int_{M_4} \frac{G\wedge G}{8\pi^2}}_{\in Z} + \frac{2\pi K i}{z}\underbrace{\left(\frac{L-1}{L}\right)\int_{M_4} \frac{w_2 \wedge w_2}{2}}_{\in Z_L}$

Non-Invertible Symmetry

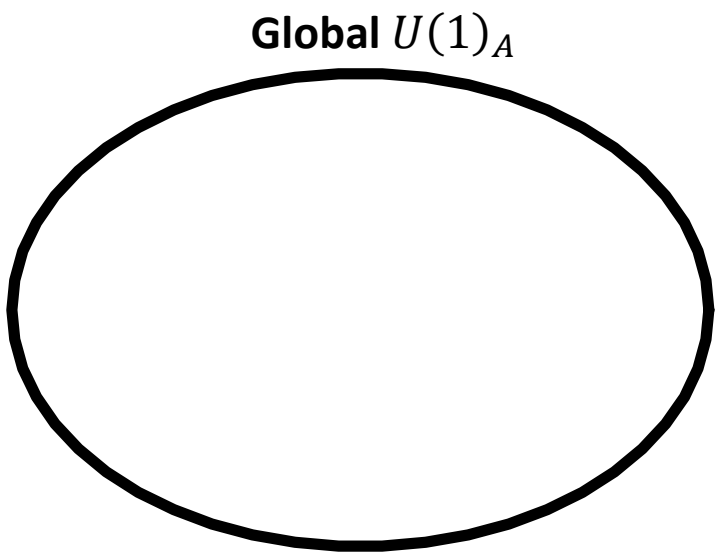
2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$ electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $S \to S + \frac{2\pi K i}{z}\int_{M_4}\frac{G\wedge G}{8\pi^2} + \frac{2\pi K i}{z}\left(\frac{L-1}{L}\right)\int_{M_4}\frac{w_2\wedge w_2}{2}$

$\in Z$ $\in Z_L$



Non-Invertible Symmetry

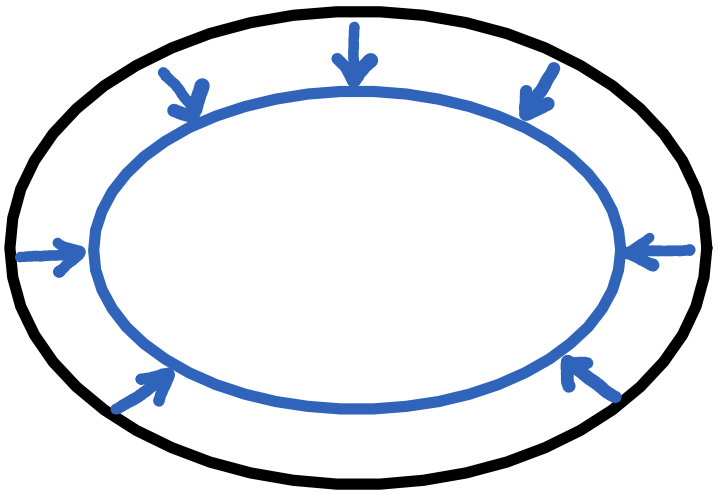
2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$ electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $S \to S + \frac{2\pi K i}{z}\underbrace{\int_{M_4} \frac{G\wedge G}{8\pi^2}}_{\in Z} + \frac{2\pi K i}{z}\underbrace{\left(\frac{L-1}{L}\right)\int_{M_4} \frac{w_2 \wedge w_2}{2}}_{\in Z_L}$

Global $U(1)_A$



→ **Z Instanton**

Non-Invertible Symmetry

2. From Fractional Instanton

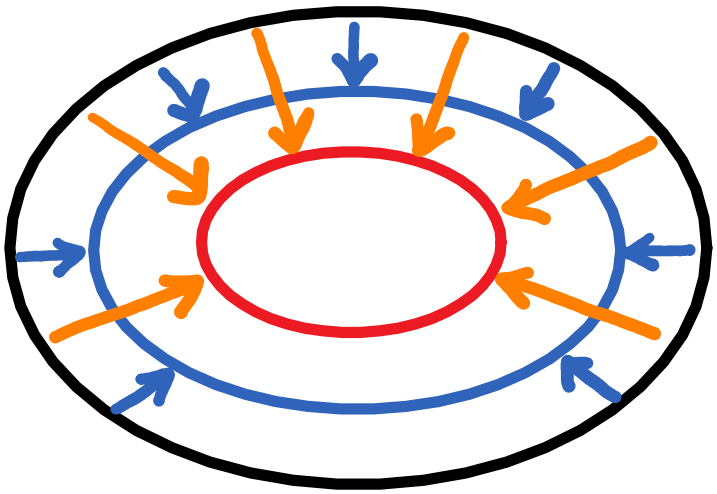
e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $\quad S \to S + \frac{2\pi K i}{z}\underbrace{\int_{M_4} \frac{G\wedge G}{8\pi^2}}_{\in Z} + \frac{2\pi K i}{z}\underbrace{\left(\frac{L-1}{L}\right)\int_{M_4} \frac{w_2 \wedge w_2}{2}}_{\in Z_L}$

Global $U(1)_A$



→ **Z Instanton**

→ **Z_L (fractional) Instanton**

Non-Invertible Symmetry

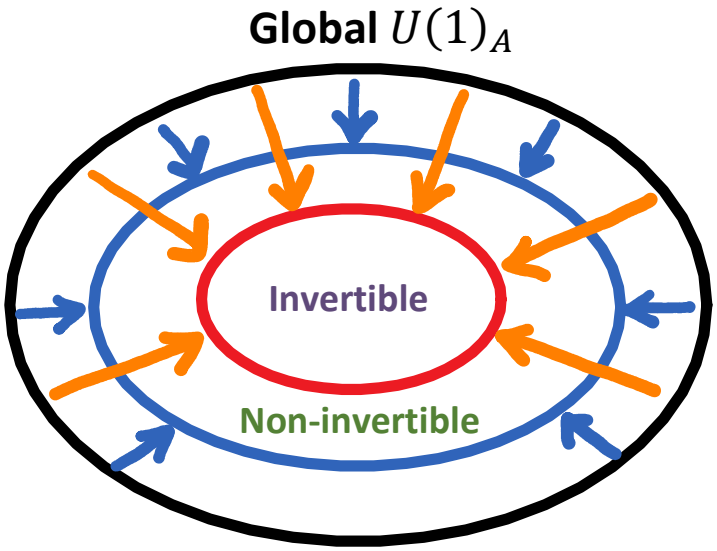
2. From Fractional Instanton

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{z}$, $\quad S \to S + \frac{2\pi K i}{z}\underbrace{\int_{M_4} \frac{G\wedge G}{8\pi^2}}_{\in Z} + \frac{2\pi K i}{z}\underbrace{\left(\frac{L-1}{L}\right)\int_{M_4} \frac{w_2 \wedge w_2}{2}}_{\in Z_L}$



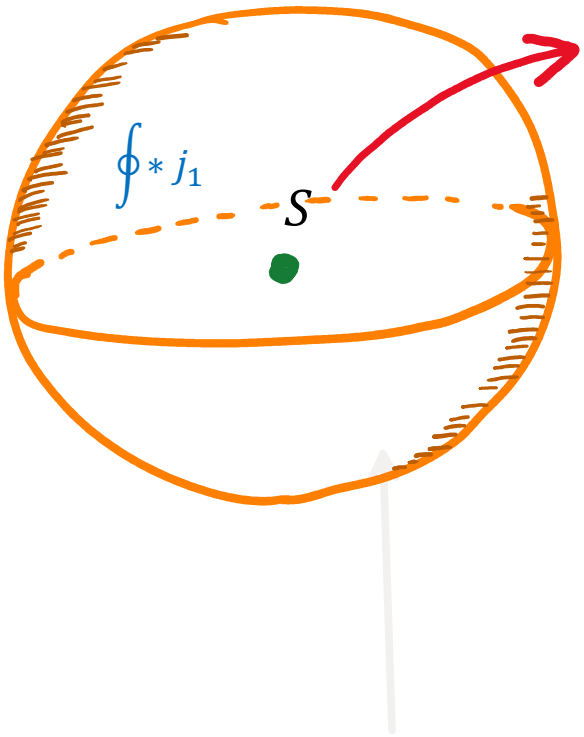
→ Z Instanton

→ Z_L (fractional) Instanton

Non-Invertible Symmetry

2. From Fractional Instanton

$$U(1)^{(0)}_A : \quad \Psi_+ \to e^{i\alpha}\,\Psi_+ \ , \quad \Psi_- \to e^{i\alpha}\,\Psi_- \quad \Rightarrow \quad d * j_A \propto w_2 \wedge w_2$$



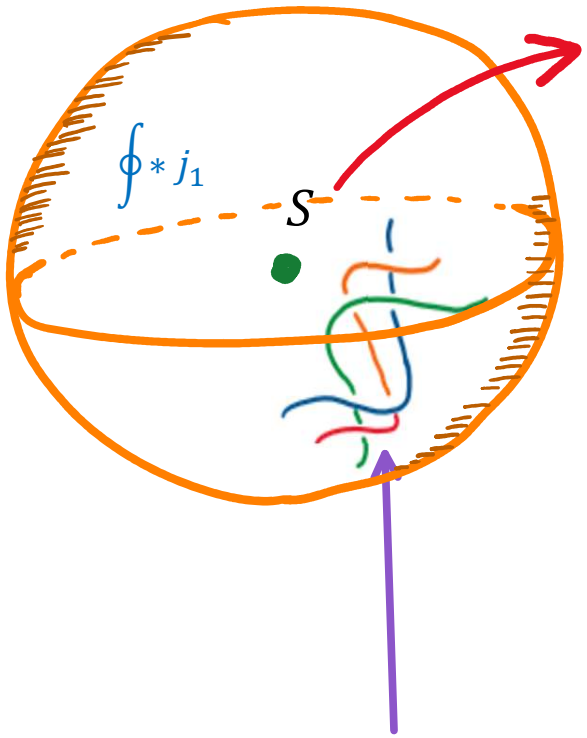
$$S \ \to \ S + \frac{2\pi i K\ell}{K}\left(\frac{L-1}{L}\right)\int \frac{w_2 \wedge w_2}{2}$$

$$\underbrace{\exp\left(\frac{2\pi\ell i}{K}\oint * j_1\right)}_{U\left(\frac{2\pi\ell}{K}, \Sigma_3\right)}$$

Non-Invertible Symmetry

2. From Fractional Instanton

$$U(1)^{(0)}_A : \quad \Psi_+ \to e^{i\alpha}\,\Psi_+ \ , \quad \Psi_- \to e^{i\alpha}\,\Psi_- \quad \Rightarrow \quad d * j_A \propto w_2 \wedge w_2$$



$$S \ \to \ S + \frac{2\pi i K \ell}{K}\left(\frac{L-1}{L}\right)\int \frac{w_2 \wedge w_2}{2} - \frac{2\pi i p}{L}\int \frac{w_2 \wedge w_2}{2} \ \to \ S$$

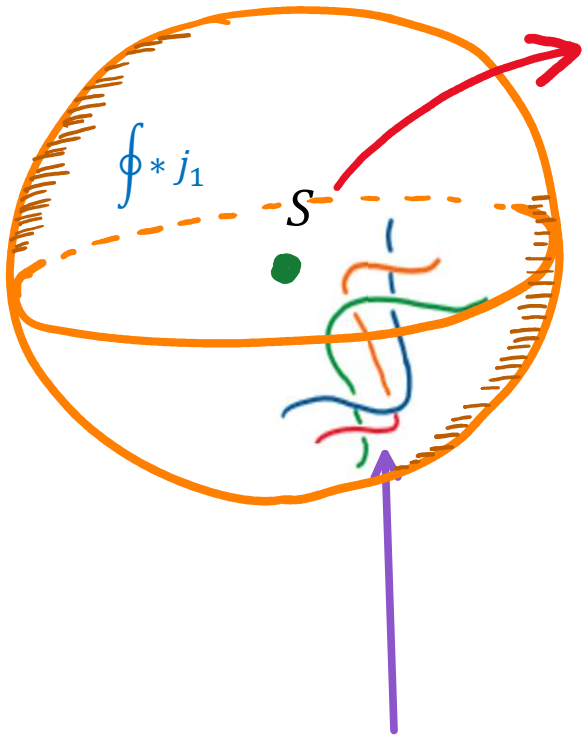
$$\underbrace{\exp\left(\frac{2\pi \ell i}{K}\oint * j_1\right)}_{U\left(\frac{2\pi\ell}{K}, \Sigma_3\right)} \times \mathcal{A}^{\mathrm{L,p}}(\mathrm{w}_2)$$

$$S_{3d} = \frac{iLp}{4\pi}\int_{\Sigma_3} a_1 \wedge da_2 + ip\int_{\Sigma_3} a_1 \wedge w_2$$

Non-Invertible Symmetry

2. From Fractional Instanton

$$U(1)^{(0)}_A : \quad \Psi_+ \to e^{i\alpha}\,\Psi_+ \ , \quad \Psi_- \to e^{i\alpha}\,\Psi_- \quad \Rightarrow \quad d * j_A \propto w_2 \wedge w_2$$



$$S \ \to \ S + \frac{2\pi i K \ell}{K}\left(\frac{L-1}{L}\right)\int \frac{w_2 \wedge w_2}{2} - \frac{2\pi i p}{L}\int \frac{w_2 \wedge w_2}{2} \ \to \ S$$

$$\boldsymbol{\mathcal{D}_\ell(\Sigma_3)} = \underbrace{\exp\left(\frac{2\pi \ell i}{K}\oint * j_1\right)}_{U\left(\frac{2\pi\ell}{K}, \Sigma_3\right)} \times \mathcal{A}^{\mathrm{L,p}}(\mathrm{w_2})$$

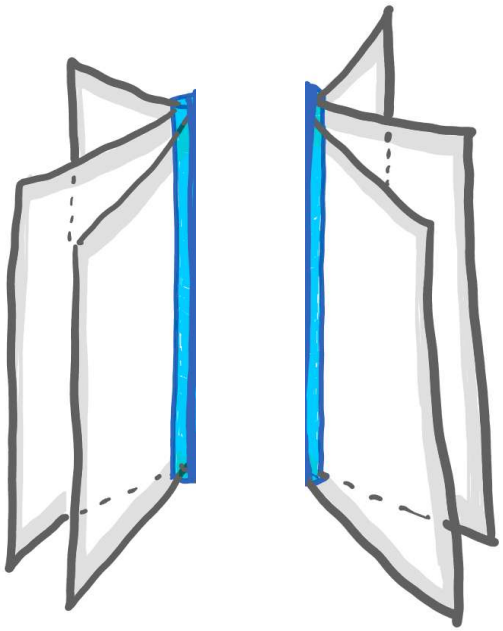
$$\mathcal{D}_\ell(\Sigma_3) \times \overline{\mathcal{D}}_\ell(\Sigma_3) \sim \sum_S \xi(S) \exp\left(\frac{i}{2\pi L}\int_s w_2\right) \neq 1$$

$$S_{3d} = \frac{iLp}{4\pi}\int_{\Sigma_3} a_1 \wedge da_2 + ip\int_{\Sigma_3} a_1 \wedge w_2$$

Solving $\mathbb{Z}_{N_g}$ Domain Wall Problem from $\frac{G_c \times G_c'}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$

$$\frac{SU(3)_C \times SU(3)_H}{\mathbb{Z}_3}$$



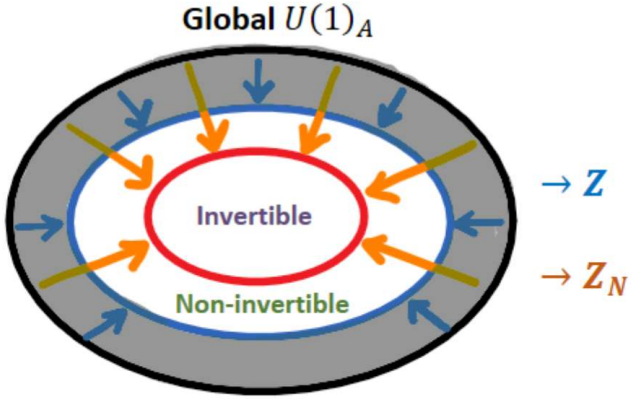
$\mathbb{Z}_{N_g}$ Domain Wall Problem

$$N_C = N_H \bmod 1 = \frac{3-1}{3}\int \frac{w_2 \wedge w_2}{2} \bmod 1$$

$$A_C = 2N_g, \qquad A_H = 2N_c, \qquad A_{CH} = 4$$

Naively, $U(1)_{PQ} \to \mathbb{Z}_{\gcd(A_C, A_H, A_{CH})} = \mathbb{Z}_2$

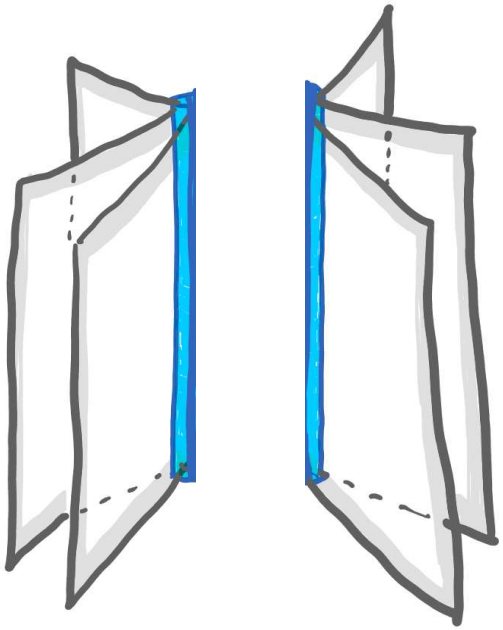
Correct answer: $U(1)_{PQ} \to \mathbb{Z}_2^I \times \mathbb{Z}_3^{NI}$



Solving $\mathbb{Z}_{N_g}$ Domain Wall Problem from $\frac{G_c \times G_c'}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$

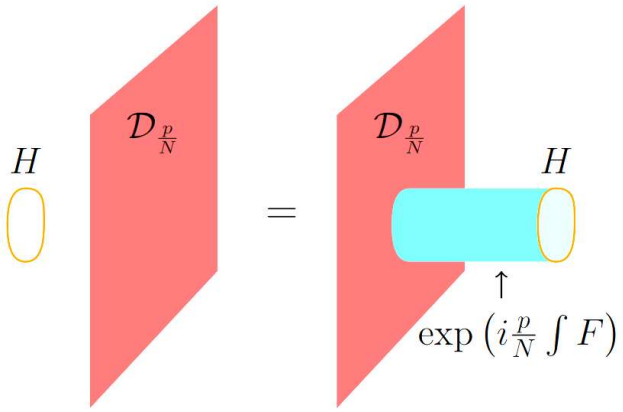
$$\frac{SU(3)_C \times SU(3)_H}{\mathbb{Z}_3}$$



$\mathbb{Z}_{N_g}$ Domain Wall Problem

$$U(1)_{PQ} \to \mathbb{Z}_2^I \times \mathbb{Z}_3^{NI}$$

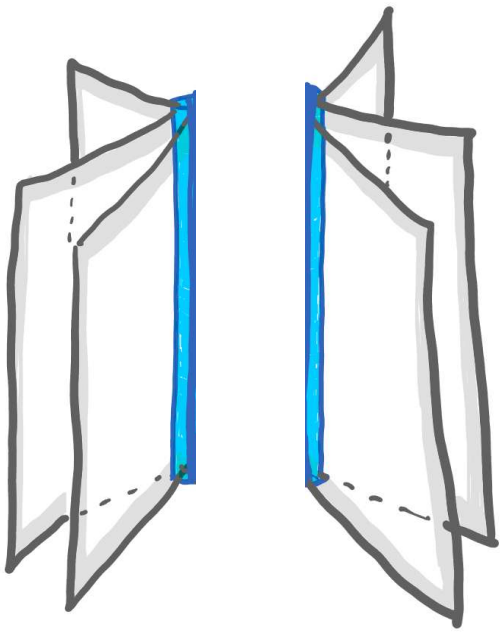
$$D_z \times D_z^\dagger \neq 1 \quad \text{(non-invertible)}$$



https://arxiv.org/pdf/2205.05086

Solving $\mathbb{Z}_{N_g}$ Domain Wall Problem from $\frac{G_c \times G_c'}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$



$\mathbb{Z}_{N_g}$ Domain Wall Problem

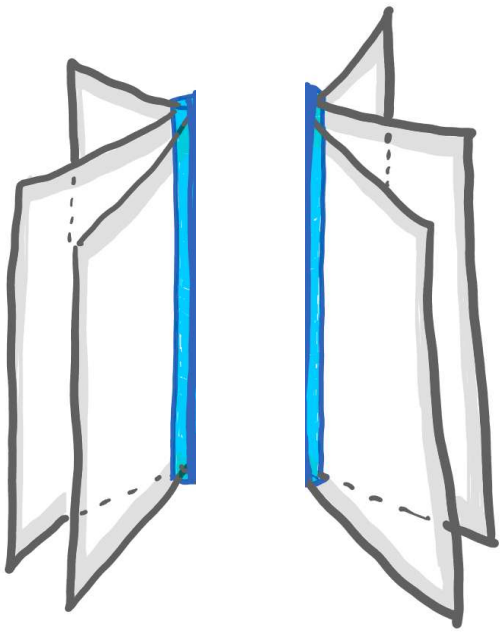
$$\frac{SU(3)_C \times SU(3)_H}{\mathbb{Z}_3}$$

$\mathbb{Z}_{N_g}^{NI}$ Domain Wall Problem

1. need (small) explicit breaking of $\mathbb{Z}_{N_g}^{NI}$
2. Non-invertible symmetry breaking by IR $\hookrightarrow$ UV s.t. $\exists$ dynamical monopoles (small instantons)

Solving $\mathbb{Z}_{N_g}$ Domain Wall Problem from $\frac{G_c \times G_c'}{\Gamma}$

$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$



$\mathbb{Z}_{N_g}$ Domain Wall Problem

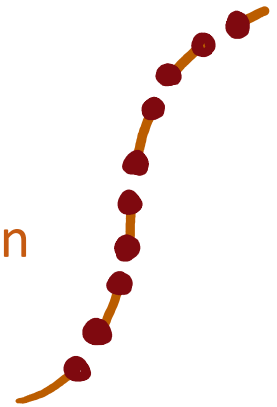
$$\frac{SU(3)_C \times SU(3)_H}{\mathbb{Z}_3}$$

$\mathbb{Z}_{N_g}^{NI}$ Domain Wall Problem

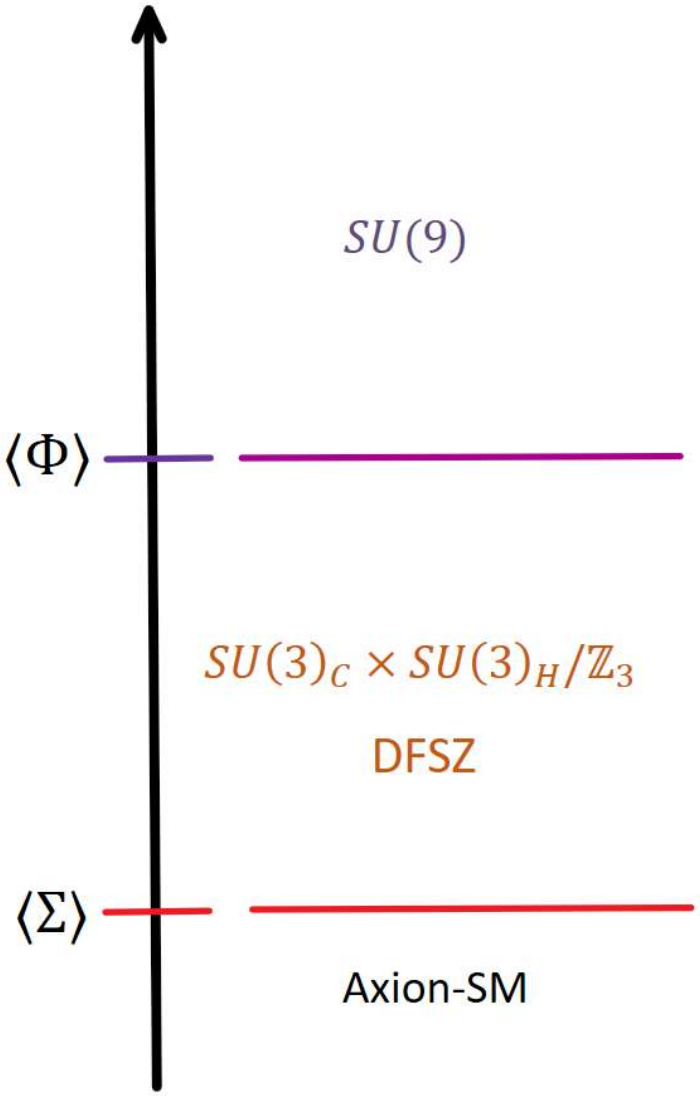
1. need (small) explicit breaking of $\mathbb{Z}_{N_g}^{NI}$

2. Non-invertible symmetry breaking by IR $\hookrightarrow$ UV
   s.t. $\exists$ dynamical monopoles (small instantons)

Monopoles in UV
$\rightarrow$ 1-form magnetic symm broken
$\rightarrow$ non-invertible symm in IR broken



Solving $\mathbb{Z}_{N_g}$ Domain Wall Problem from $\frac{G_c \times G_c'}{\Gamma}$

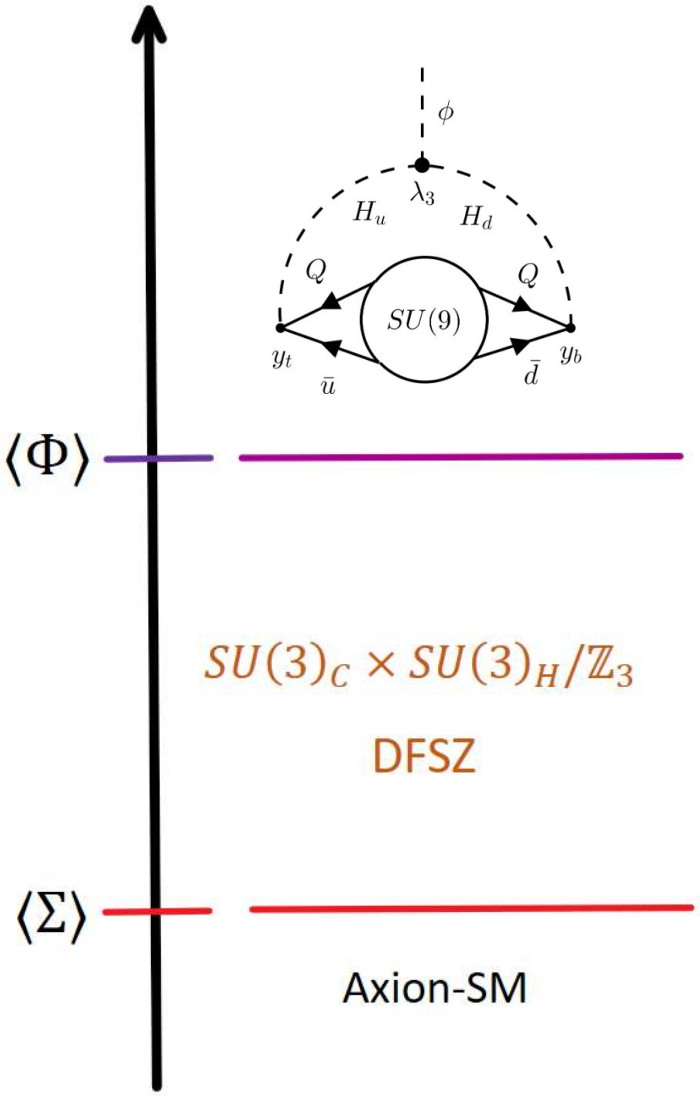


$$\frac{SU(3)_C \times SU(3)_H}{\mathbb{Z}_3}$$

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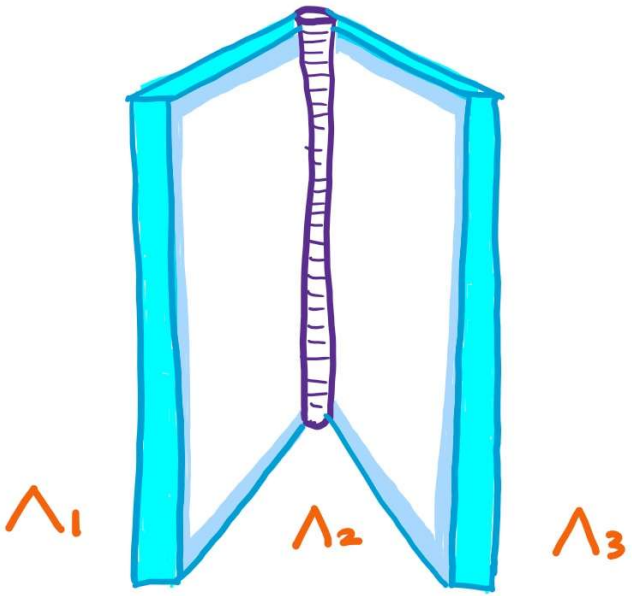
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Solving $\mathbb{Z}_{N_g}$ Domain Wall Problem from $\frac{G_c \times G_c'}{\Gamma}$

$$V(a) = V(a) + \Delta V(a)$$

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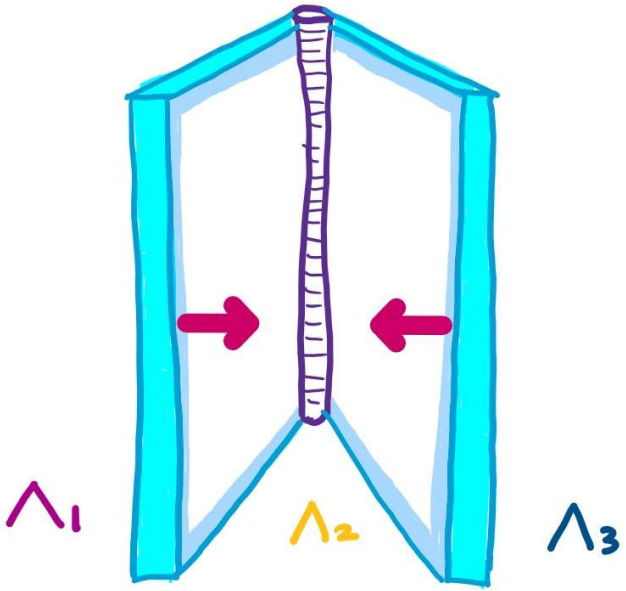
$$\Lambda_1 = \Lambda_2 = \Lambda_3$$

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($\widehat{G_{IR}}$ - instantons) (G_{UV}/G_{IR} - instantons)

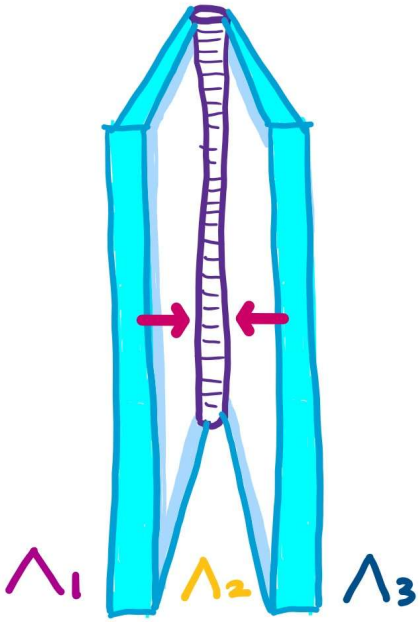


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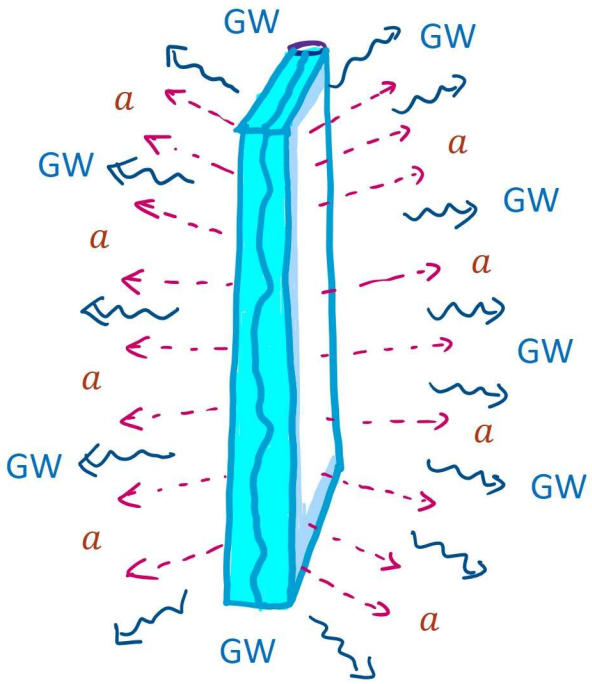


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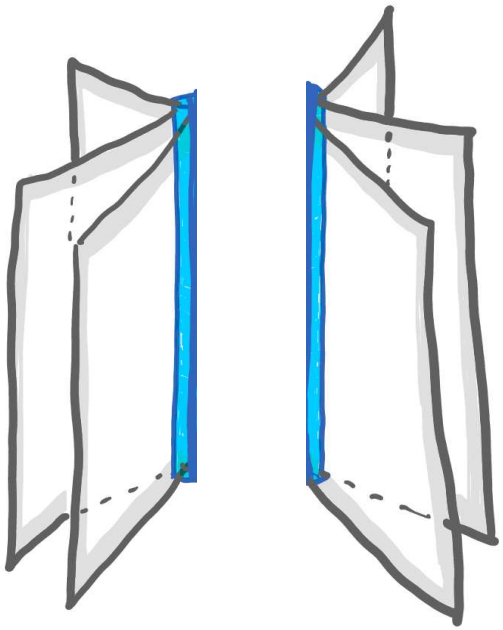
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$$\frac{G_c \times G_g}{\Gamma} \Leftarrow \textbf{[Domain Wall Problem]} \Rightarrow \frac{G_c \times G_c'}{\Gamma}$$

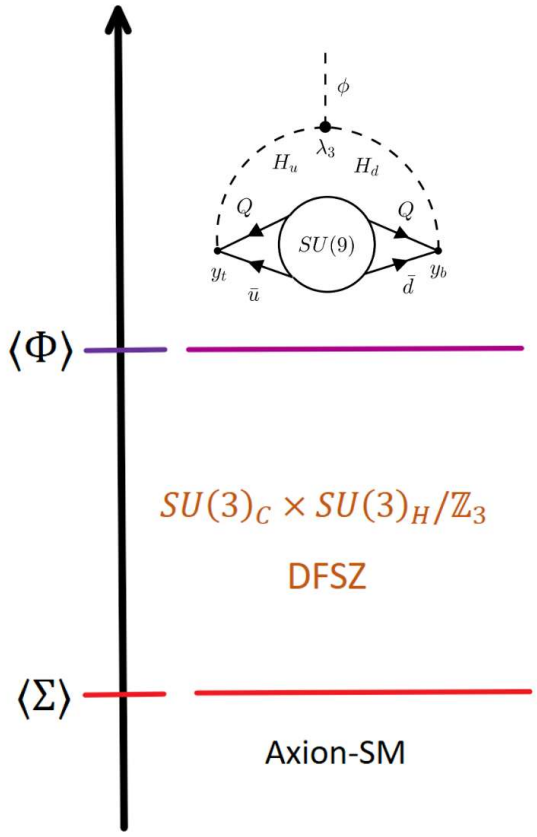
$$\frac{U(1)_Y \times U(1)_{PQ}}{\mathbb{Z}_2}$$

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$\mathbb{Z}_{N_g}$ Domain Wall Problem

$$U(1)_{PQ} \to \mathbb{Z}_2^I \times \mathbb{Z}_3^{NI}$$



Conclusion = things to think about

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[c.f. "Global Aspects of Current Algebra" - E. Witten '83]

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[WIP: with Youngho Go, Sung Mook Lee, and Lian-Tao Wang]

Global Structures in Spontaneously Broken Gauge Theories

Youngho Go,[1] **Sungwoo Hong,**[1] **Sung Mook Lee,**[2] **Lian-Tao Wang**[3,4]

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[2] *Theoretical Physics Department, CERN, CH-1211 Geneva 23, Switzerland*

[3] *Department of Physics, Kadanoff Center for Theoretical Physics & Enrico Fermi Institute, University of Chicago*

[4] *Kavli Institute for Cosmological Physics, University of Chicago, Chicago IL 60637, USA*

E-mail: edwardgo@kaist.ac.kr, sungwooh@kaist.ac.kr, sungmook.lee@cern.ch, liantaow@uchicago.edu

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(vi) Much more to think about!

Thank You
For
Your Attention!

Back-up-1

Global Form of SM Gauge Group

What is, Why $G_{SM} = \widetilde{\mathrm{G}}_{\mathrm{SM}}/\Gamma$?

$$G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}, \quad \Gamma = 1, Z_2, Z_3, Z_6$$

Point 1: Question of 1-form symmetries of SM

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Point 1: Question of 1-form symmetries of SM

i. The entire SM matter fields are neutral under Z_6 transformation generated by

$$e^{\frac{2\pi i}{3}\lambda_8} = e^{\frac{2\pi i}{3}} I_3 \in SU(3)_C, \quad e^{\frac{2\pi i}{2}T_3} = -I_2 \in SU(2)_L, \quad e^{\frac{2\pi i}{6}Q_Y} \in U(1)_Y$$

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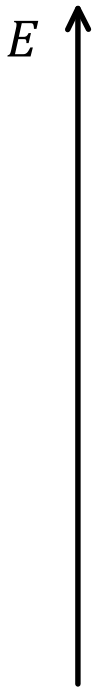
iii. Gauging $\Gamma = Z_p^{(1)} \subset Z_6^{(1)} \Rightarrow \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $Z_{6/p}^{(1)}(e) \times Z_p^{(1)}(m)$

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$SU(5)\,,\ SO(10)\,,\ E_6$
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~~$\Gamma = Z_6$: $SU(5)$, $SO(10)$, E_6~~

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Point 3: Selection rules on **particle** and defect spectrum ?

if $\boldsymbol{\Gamma = \mathbb{Z}_3}$ $\Rightarrow$ SM: $\mathrm{Q} = (3,2)_1\,,\ \bar{u} = \left(\bar{3},1\right)_{-4},\ \bar{d} = \left(\bar{3},1\right)_2,$

$$L = (1,2)_{-3}\,,\ \bar{e} = (1,1)_6,\ H = (1,2)_3$$

$\Rightarrow$ **BSM:** no new particles with $e^{\frac{2\pi i}{3}\lambda_8} e^{-\frac{2\pi i}{6}Q_Y}\,\Psi \neq \Psi$

(see for e.g. 2406.17850, 2507.16900)

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Point 1: Question of 1-form symmetries of SM

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Point 3: Selection rules on particle and **defect spectrum** ?

for given $\boldsymbol{\Gamma = \mathbb{Z}_2}$ $\Rightarrow$ Any EW (non) topological defects sensitive to Γ ?

if yes, super exciting and important!
if not, can we understand why (underlying structure) ?

(WIP with T. Daniel Brennan)