ÖAW Austrian Academy of Sciences

Thermal Effects of Standard-Model Leptons in the Early Universe

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Based on: S. Kanemura & **Lisp**, PRL 134 (2025) 21

Lisp & A. Pilaftsis, arXiv: 2601.15921; 2604.06493

Outline

- The Standard Model, matter-antimatter asymmetry & seesaw models
- Power counting of couplings in QFT, thermal resummation of leptons
- Three kinds of low-scale type-I leptogenesis
- Conclusions

The Standard Model (SM)

$$-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}$$

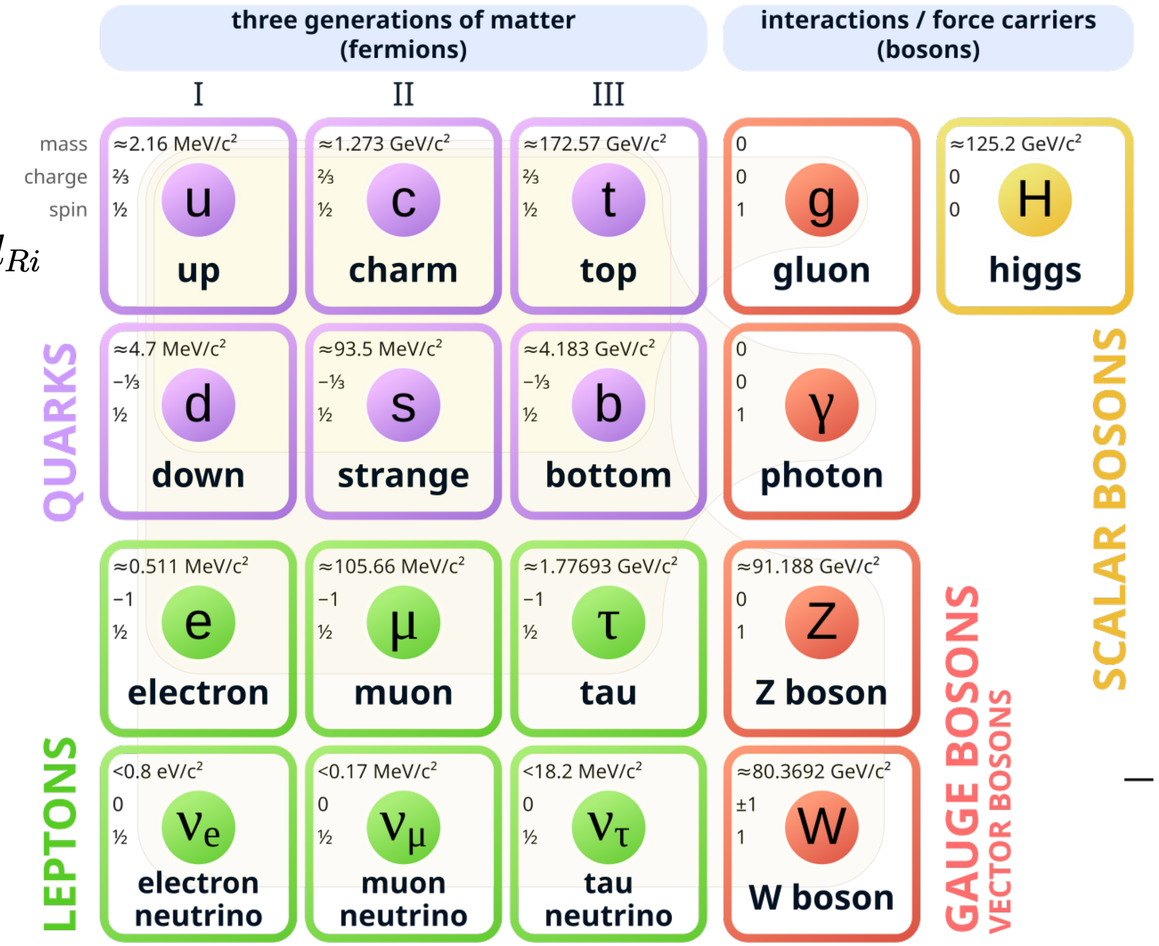
$$\overline{Q_i}\,i\not{D}\,Q_i + \overline{u_{Ri}}\,i\not{D}\,u_{Ri} + \overline{d_{Ri}}\,i\not{D}\,d_{Ri}$$

$$- Y^d_{ij}\left(\overline{Q_i}\,d_{Rj}\,H + H^\dagger\,\overline{d_{Rj}}\,Q_i\right)$$

$$- Y^u_{ij}\left(\overline{Q_i}\,u_{Rj}\,\tilde{H} + \tilde{H}^\dagger\,\overline{u_{Rj}}\,Q_i\right)$$

$$+ \overline{L_i}\,i\not{D}\,L_i + \overline{e_{Ri}}\,i\not{D}\,e_{Ri}$$

$$- Y^e_{ij}(\overline{L_i}\,e_{Rj}\,H + H^\dagger\overline{e_{Rj}}\,L_i)$$



$$(D_\mu H)^\dagger(D^\mu H) + \mu^2 H^\dagger H - \lambda(H^\dagger H)^2$$

$$-\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

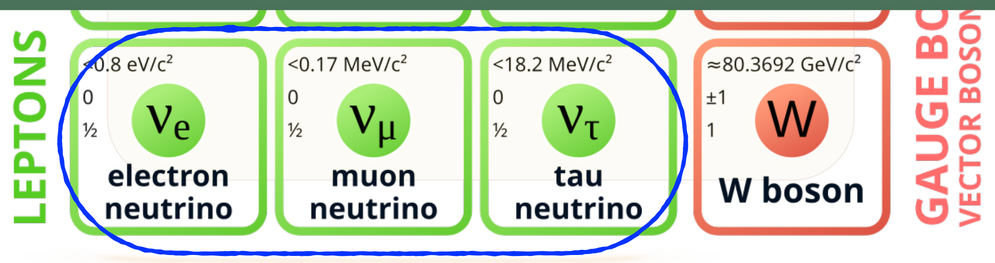
The Standard Model (SM)

$$-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}$$

Standard Model of Elementary Particles

These Lagrangian int. describe ALL the particle phenomenology at colliders, astrophysics, and cosmology, except for

- Unknown Lagrangian for massive neutrino oscillations
- Unknown Lagrangian interactions for dark matter & dark energy
- Unknown Lagrangian interactions for the matter-antimatter asymmetry observed in the Universe
- First-order phase transitions, primordial black holes, stochastic gravitational waves, …

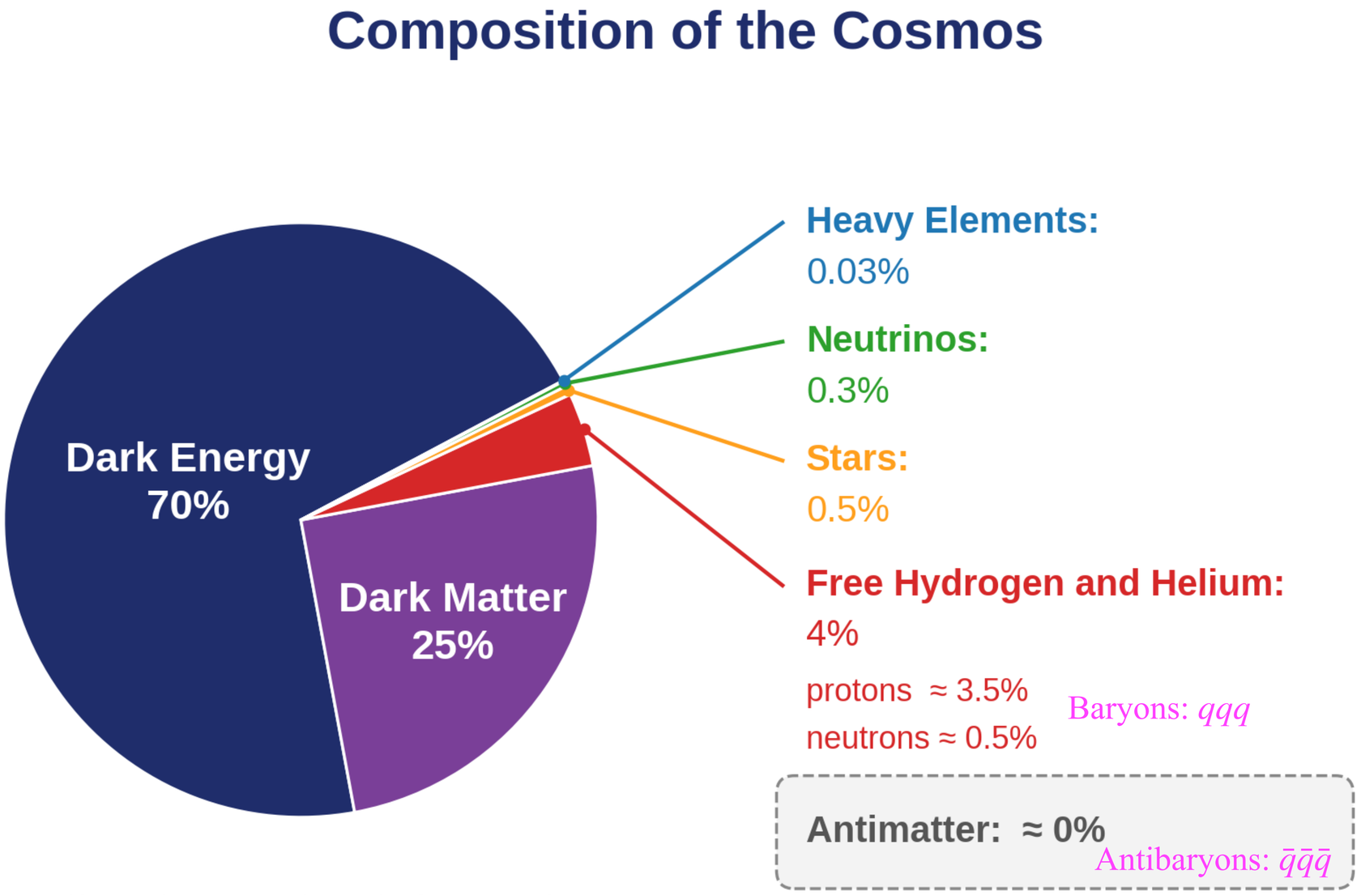


$$-\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

one of the most exciting directions for going beyond the SM to address the open questions

—The Nobel Prize in 2026 (1988, 1995, 2002, 2015) for neutrinos!

Matter-antimatter asymmetry in our Universe



Matter-antimatter asymmetry in our Universe (BAU)

- Observations from Cosmic Microwave Background (CMB)

[PLANCK Collaboration, N. Aghanim et al., Astron. Astrophys. 641 (2020) A6]

$$\eta_B^{\rm CMB} \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} = (6.104 \pm 0.058) \times 10^{-10}$$

- Neutrino energy fraction reaches 40 % at 3 minutes after Big Bang, but reduces to 0.3 % after 14 billion years. So, is BAU also a numerical effect of cosmic expansion? NO

$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} = (8.67 \pm 0.082) \times 10^{-11}$$

For baryon number already becoming conserved at a very high temperature in the early Universe, $n_B - n_{\bar{B}}$ & entropy density s scaling with the cosmic expansion scale factor a are the same:

$(n_B - n_{\bar{B}})a^3 = \text{constant}$ (baryon-number conservation), $sa^3 = \text{constant}$ (entropy conservation)



Y_B should have been $\mathcal{O}(1)$ constant if $n_B, n_{\bar{B}}, s \propto T^3$—strong hint at some dynamical & non-thermal reason to create a small difference between baryons and anti-baryons at the very early Universe

Matter-antimatter asymmetry in our Universe (BAU)

- Three conditions to generate the BAU (Sakharov's condition) [A.D. Sakharov, JETP Lett. 5 (1967) 24]
  - Baryon-number (B) violating interactions (I)
  - C and CP violation in a CPT-invariant theory (II)
  - Out-of-equilibrium dynamics (III)

SM: (I)—$B + L$ violating sphaleron; (II)—CKM phase in quark sector; (III)—Too small to be relevant

- One of the leading mechanisms for the BAU problem: leptogenesis [M. Fukugita & T. Yanagida, PLB 174 (1986) 45; M. A. Luty, PRD 45 (1992) 455]

Leptogenesis in type-I seesaw, in type-II seesaw, in Dirac seesaw, ….



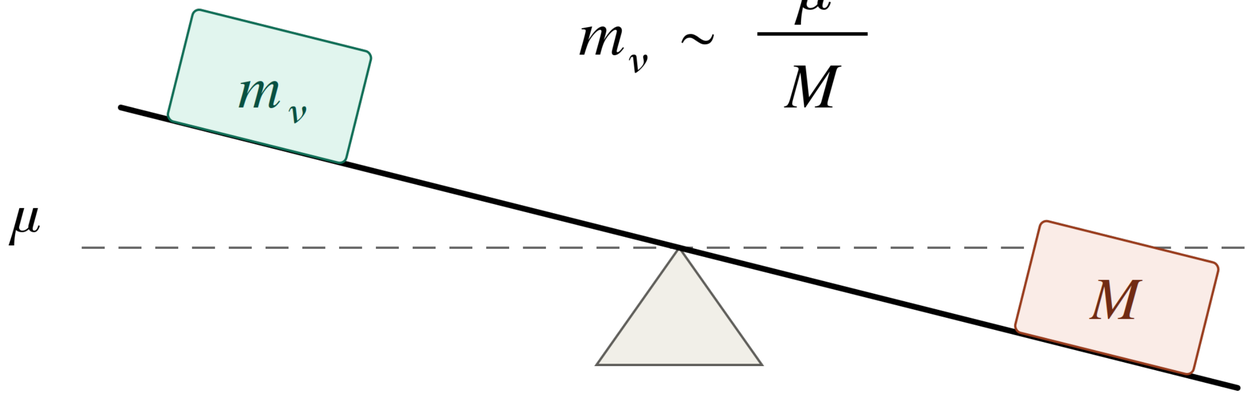
A big lesson: physics responsible for massive neutrinos is also capable of explaining the BAU!

The seesaw models

- A generic seesaw mechanism: new heavy particles beyond the SM coupling to neutrinos

m_ν: SM neutrino mass

$$m_\nu \sim \frac{\mu^2}{M}$$



M: heavy particle mass

μ: a balanced point/scale/level

1. no higher than the electroweak scale
2. No first-principles to fix

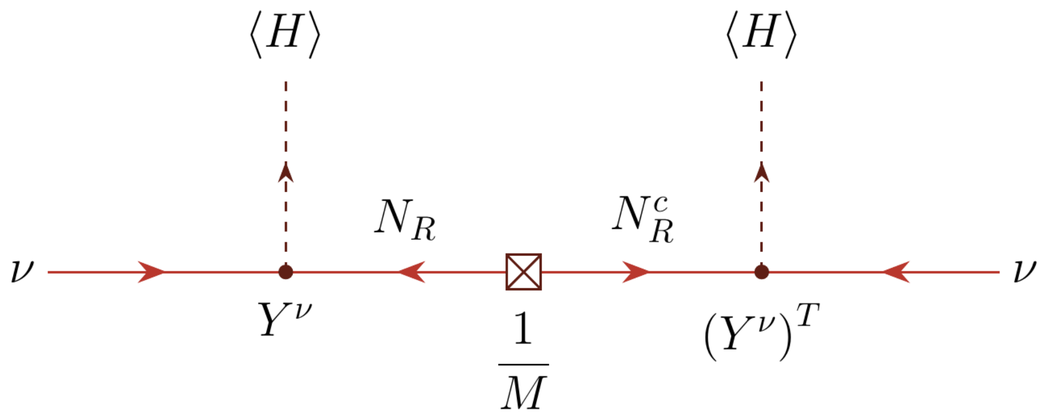
Seesaw mechanism from hierarchical mass: $m_\nu \ll \mu \ll M$

The seesaw models

[P. Minkowski, PLB 67 (1977) 421; T. Yanagida, PTP, 64 (1980) 1103; R. N. Mohapatra and G. Senjanovic, PRL. 44 (1980) 912; J. Schechter and J. W. F. Valle, PRD 22 (1980) 2227; …]

- Type-I seesaw: introduce gauge fermion singlet (right-handed Majorana neutrinos/sterile neutrinos/heavy neutral leptons)

$$\mathcal{L}_{\text{type-I}} = -Y^{\nu}_{ij}\bar{L}_i\tilde{H}N_R - \frac{1}{2}M_{ij}\overline{N^c_R}N_R + \text{h.c.}$$



$m_\nu = -(Y^\nu v_{\text{EW}})M^{-1}(Y^\nu v_{\text{EW}})^T, \mu \sim Y^\nu v_{\text{EW}}, v_{\text{EW}} = 246\ \text{GeV}$

Traditional high-scale type-I seesaw: $\mu \sim Y^\nu v_{\text{EW}} \sim 10^2\ \text{GeV}, m_\nu \sim 10^{-2}\ \text{eV}$ ⟶ $M \sim M_{\text{GUT}} \sim 10^{15}\ \text{GeV}$

Low-scale type-I seesaw: $M \lesssim 1\ \text{TeV}, m_\nu \sim 10^{-2}\ \text{eV}$ ⟶ $\mu \sim Y^\nu v_{\text{EW}} \lesssim 10^2\ \text{keV}$

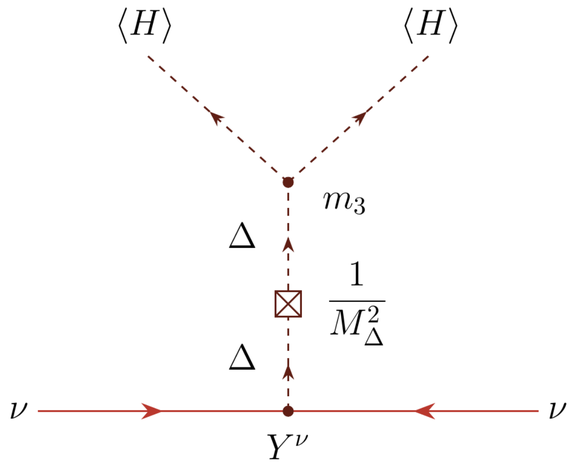
The seesaw models

[M. Magg & C. Wetterich, PLB 94, 61 (1980); J. Schechter & J. W. F. Valle, PRD 22, 2227 (1980); T. P. Cheng & L.-F. Li, PRD 22, 2860 (1980); C. Wetterich, NPB 87, 343 (1981); …]

- Type-II seesaw: introduce a gauge scalar triplet

$$\mathcal{L}_{\text{type-II}} = \text{Tr}\left[(D_\mu \Delta)^\dagger (D^\mu \Delta)\right] - M_\Delta^2 \,\text{Tr}(\Delta^\dagger \Delta) - \left[Y^\nu_{ij}\, \overline{L_i^c}\, i\sigma_2 \Delta\, L_j + m_3\, H^T i\sigma_2 \Delta^\dagger H + \text{h.c.}\right]$$

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$



$$m_\nu = Y^\nu v_\Delta = \frac{Y^\nu m_3 v_{\text{EW}}^2}{M_\Delta^2}, \mu^2 \sim \frac{Y^\nu m_3 v_{\text{EW}}^2}{M_\Delta}$$

High-scale type-II seesaw: $M_\Delta \gg v_{\text{EW}}$

Low-scale type-II seesaw: $M_\Delta \lesssim 10$ TeV

- Other seesaw: type-III, radiative seesaw, scotogenic, Dirac seesaw,…

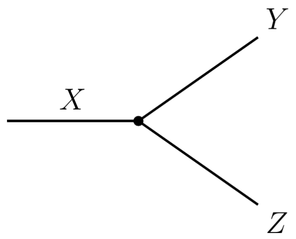
Power counting of couplings in vacuum & at finite temperatures

Perturbative Feynman diagram expansion in vacuum

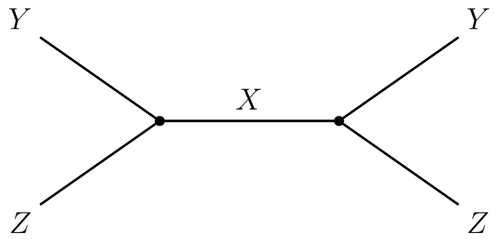
- The (zero-temperature) QFT treats interactions as perturbations and expands the interactions by powers of the weak coupling constants

$\mathscr{L} \supset gXYZ$

Tree level process



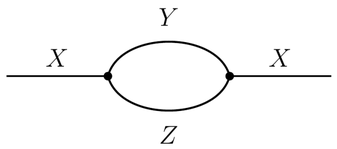
$\mathcal{M} \sim g$



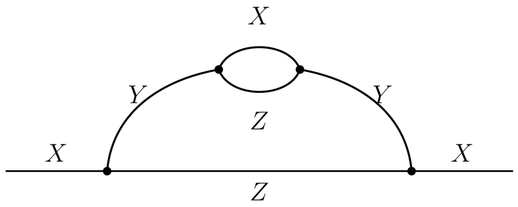
$\mathcal{M} \sim g^2$

For most of the time, decay is more efficient than scattering

Loop level process



$\mathcal{M} \sim g^2 \times \frac{1}{(4\pi)^2}$



$\mathcal{M} \sim g^4 \times \frac{1}{(4\pi)^4}$

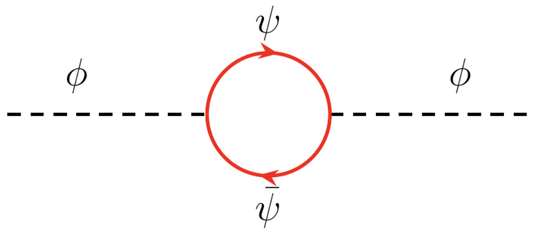
For most of the time, one loop suffices to be leading corrections we need

Perturbative Feynman diagram expansion at finite T

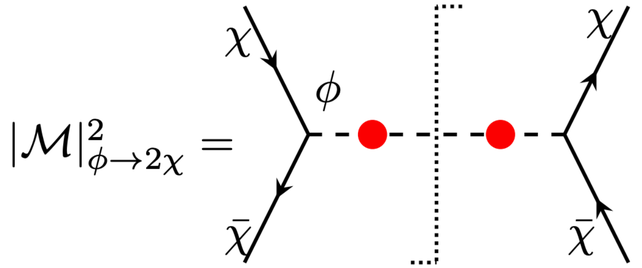
- The finite-T QFT also treats interactions as perturbations and expands the interactions by powers of the weak coupling constants, but finite-temperature effects can break the naive coupling counting even perturbatively

- Example: *Forbidden decay* $\mathscr{L} = y_\chi \bar{\chi}\chi\phi + y_\psi \bar{\psi}\psi\phi$, $m_\phi < 2m_\chi$, $\phi \to \bar{\chi} + \chi$ is kinematically forbidden in vacuum but opens at finite T, due to thermal corrections from ψ

[**Lisp**, PRD 108 (2023) 1, 015014]



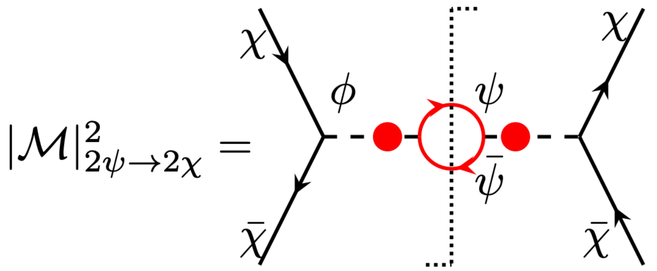
Thermal mass: $m_\phi^2(T) \sim y_\psi^2 T^2$



Dimension argument

$\propto y_\chi^2 m_\phi^2(T) \propto y_\chi^2 y_\psi^2$

Power counting



$\propto y_\chi^2 y_\psi^2$

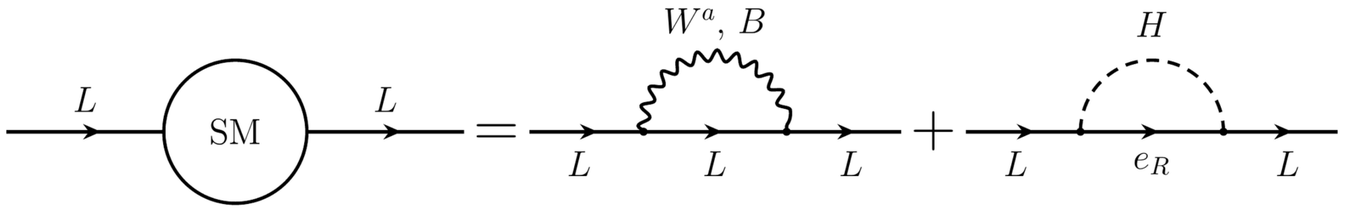
Both decay and scattering have the same power of weak couplings!

Thermal resummation of leptons

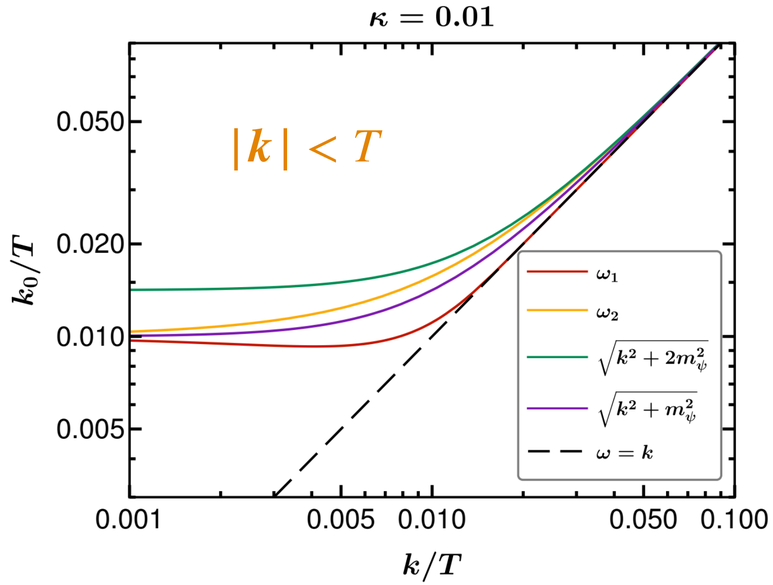
[H. A. Weldon, PRD 26 (1982) 2789; M. Drewes & J. U. Kang, NPB 875 (2013) 315; **Lisp,** JCAP 05 (2023) 008]

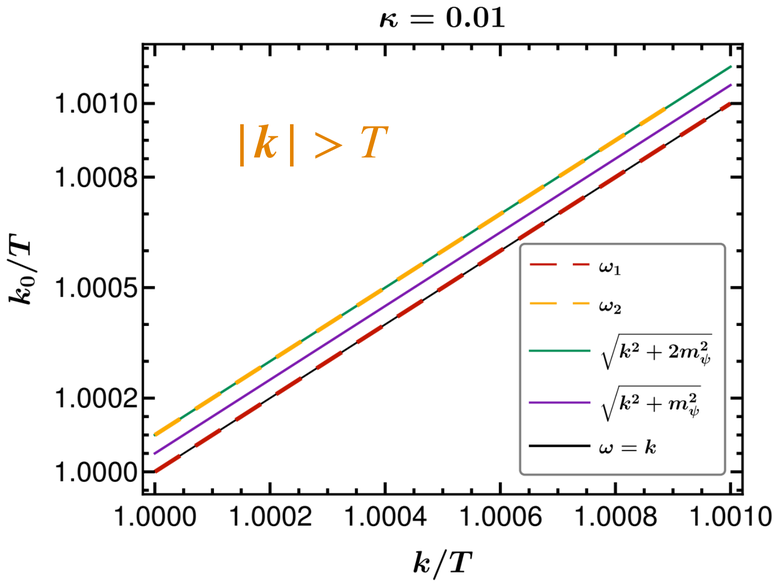
- Before gauge symmetry breaking, SM leptons are massless: $k^2 = 0$ in vacuum, but finite-T corrections give two asymptotic quasiparticle modes

$k^2 \approx 0\,, \quad k^2 \approx 2m^2(T),$ or $\omega_1 = |\boldsymbol{k}|\,, \quad \omega_2 = \sqrt{|\boldsymbol{k}|^2 + 2m^2(T)}, \quad \sqrt{2}m(T)$: the asymptotic ($|\boldsymbol{k}| \gtrsim T$) thermal mass



$$m^2(T > v_{\rm EW}) \equiv \kappa^2 T^2 = \left(\frac{3}{32}g_2^2 + \frac{1}{32}g_1^2 + \frac{1}{16}y_\alpha^2\right) T^2$$





Low-scale type-I leptogenesis

Low-scale leptogenesis in type-I seesaw

- Low-scale leptogenesis in type-I seesaw: the introduced right-handed Majorana neutrino mass is at GeV-TeV scale

- Two leading low-scale leptogenesis in type-I seesaw:

  - Resonant leptogenesis: A. Pilaftsis, PRD 56 (1997) 5431; A. Pilaftsis & T. E. J. Underwood, NPB 692 (2004) 303; PRD 72 (2005) 113001,…

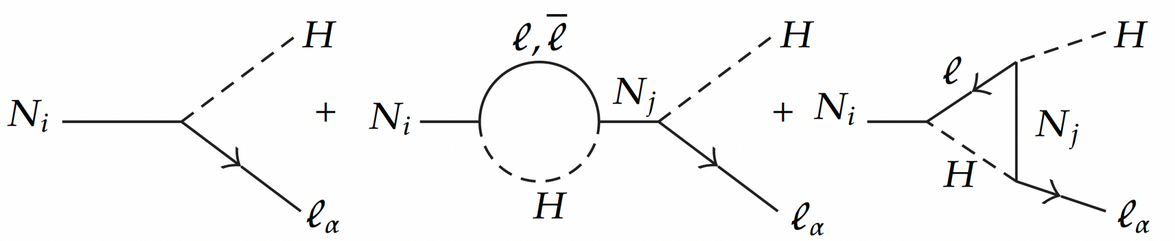
  - Sterile neutrino oscillations (ARS mechanism): Akhmedov, Rubakov & Smirnov: PRL 81 (1998) 1359; T. Asaka & M. Shaposhnikov, PLB 620 (2005) 17,…

Low-scale leptogenesis in type-I seesaw

- Resonant leptogenesis from neutrino decay

A. Pilaftsis, PRD 56 (1997) 5431; A. Pilaftsis & T. E. J. Underwood, NPB 692 (2004) 303; PRD 72 (2005) 113001

$$\mathcal{L}_{\rm type-I} = -Y^{\nu}_{ij}\bar{L}_i\tilde{H}N_R - \frac{1}{2}M_{ij}\overline{N^c_R}N_R + \text{h.c.}$$



Decay rate in Boltzmann kinetic equations (dim-4)

$$\gamma_{N\to HL} - \gamma^{\rm CP}_{N\to HL} \propto \text{Im}\left[(Y^{\nu})^4\right] \times \left(\frac{M^2}{M_i^2 - M_j^2}\right) \times M^4$$

Dim-4 dominated by heavy neutrino mass

Yukawa phase (CP-odd phase): complex phases from Yukawa couplings

Resonant enhancement from sterile-neutrino mass difference

Kinetic phase/CP-even phase/strong phase: imaginary contribution from on-shell loop particles, if $m_{\rm loop} < m_{\rm external}$

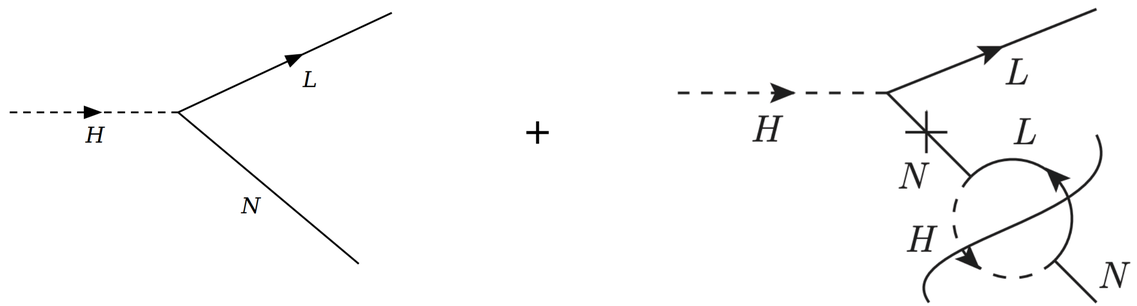
RL: $\mathcal{O}[(Y^{\nu})^4]$, lepton-number violating effect

Low-scale leptogenesis in type-I seesaw

- Resonant leptogenesis from Higgs decay

T. Hambye & D. Teresi, PRL, 117 (2016), no. 9 091801; PRD 96 (2017), no. 1 015031

$$\mathcal{L}_{\text{type-I}} = -Y^{\nu}_{ij}\bar{L}_i\tilde{H}N_R - \frac{1}{2}M_{ij}\overline{N^c_R}N_R + \text{h.c.}$$



$$\gamma_{H\to N\bar{L}} - \gamma^{\text{CP}}_{H\to N\bar{L}} \propto \text{Im}\left[(Y^{\nu})^4\right] \times \left(\frac{M^2}{M_i^2 - M_j^2}\right)\left(\frac{M}{m_H}\right) \times m_H^4$$

Dim-4 dominated by Higgs thermal+vacuum mass

CP violation source

Resonant enhancement from sterile-neutrino mass difference

Neutrino chirality flip—lepton number violating

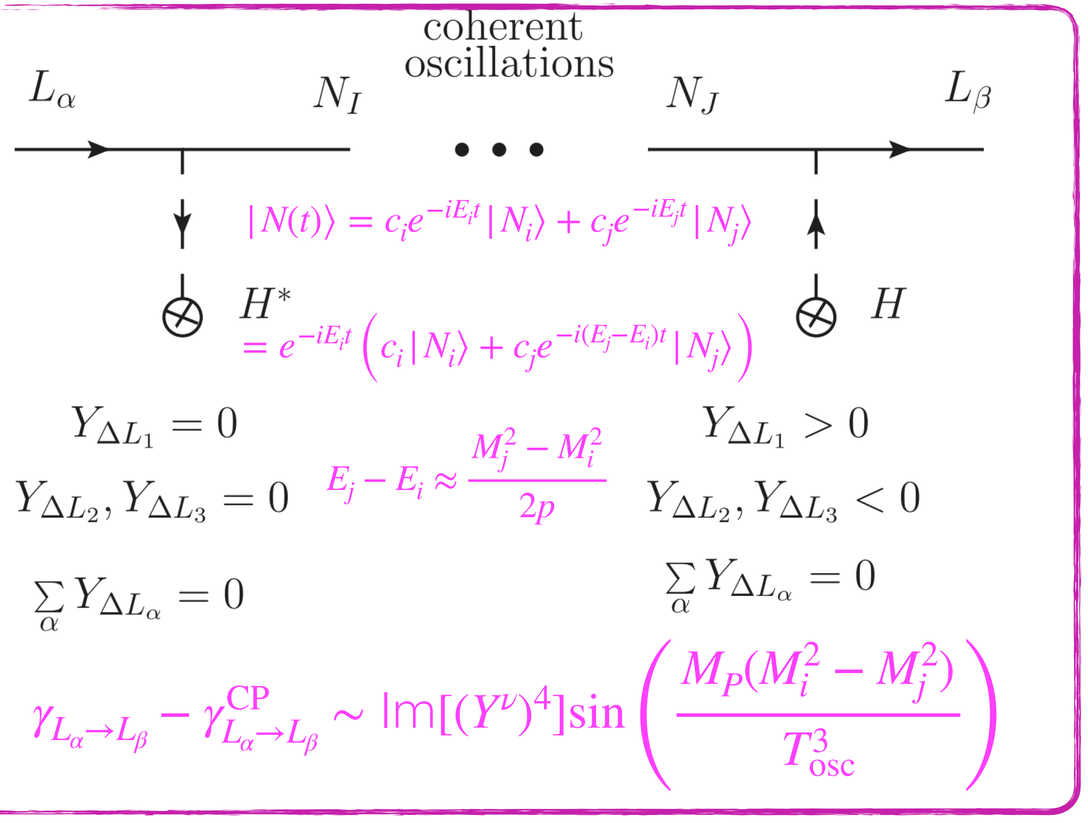
RL: $\mathcal{O}[(Y^{\nu})^4]$, lepton-number violating effect

Low-scale leptogenesis in type-I seesaw

Akhmedov, Rubakov & Smirnov: PRL 81 (1998) 1359; T. Asaka & M. Shaposhnikov, PLB 620 (2005) 17; B. Shuve & I. Yavin, PRD 89 (2014) 7, 075014

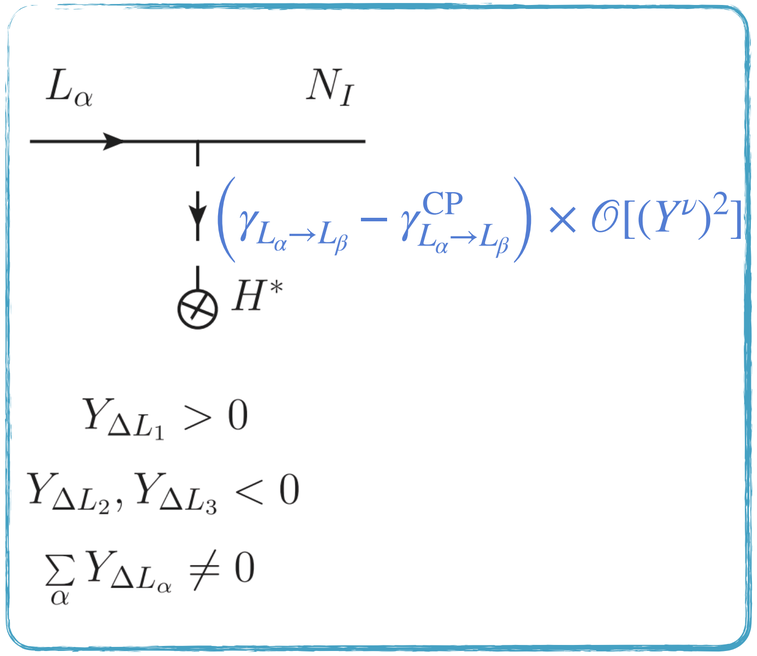
- ARS leptogenesis (freeze-in leptogenesis)

ARS: $\mathcal{O}[(Y^\nu)^6]$, lepton-number conserving effect



Lepton-flavour asymmetry most active at $M_P(M_i^2 - M_j^2) \sim T_{\rm osc}^3$

$\gg$ Smaller small splitting, later oscillation $T_{\rm osc}$



Lepton-flavour asymmetry partly converted and stored in N-sector, total SM lepton number not conserved.

$\gg$ Later oscillation $T_{\rm osc}$, smaller Hubble expansion ($\sim T^2/M_{\rm P}$), more efficient conversion

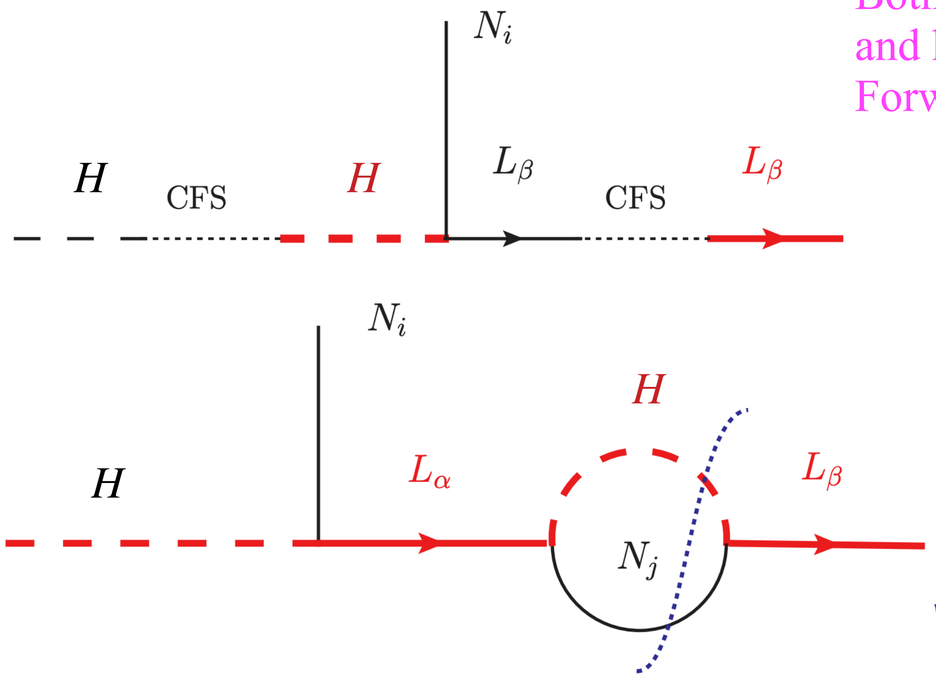
$\sum_\alpha L_{L_\alpha} + \sum_i L_{N_i} = 0$ throughout

SM $B+L$ violating sphaleron

A third low-scale type-I leptogenesis-TIRL

Lisp & A. Pilaftsis, arXiv: 2601.15921; 2604.06493

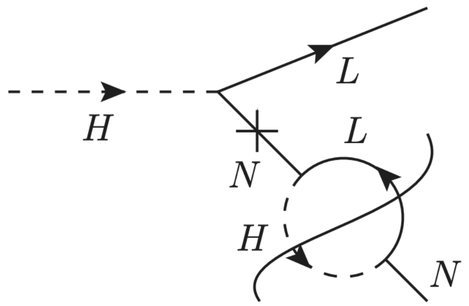
- Thermally induced resonant leptogenesis (TIRL) from Higgs decay



Both Higgs and leptons are in (quasi) thermal equilibrium, and have significant thermal mass corrections (Coherent Forward Scattering) from the plasma

$$\gamma_{H\to N\bar{L}} - \gamma^{\rm CP}_{H\to N\bar{L}} \propto {\rm Im}\left[(Y^\nu)^4\right] \times \left(\frac{T^2}{\tilde{m}_\alpha^2 - \tilde{m}_\beta^2}\right) \times m_H^4$$

Resonant enhancement from lepton thermal mass difference: $\tilde{m}_\alpha^2 - \tilde{m}_\beta^2 \propto y_\alpha^2 - y_\beta^2$

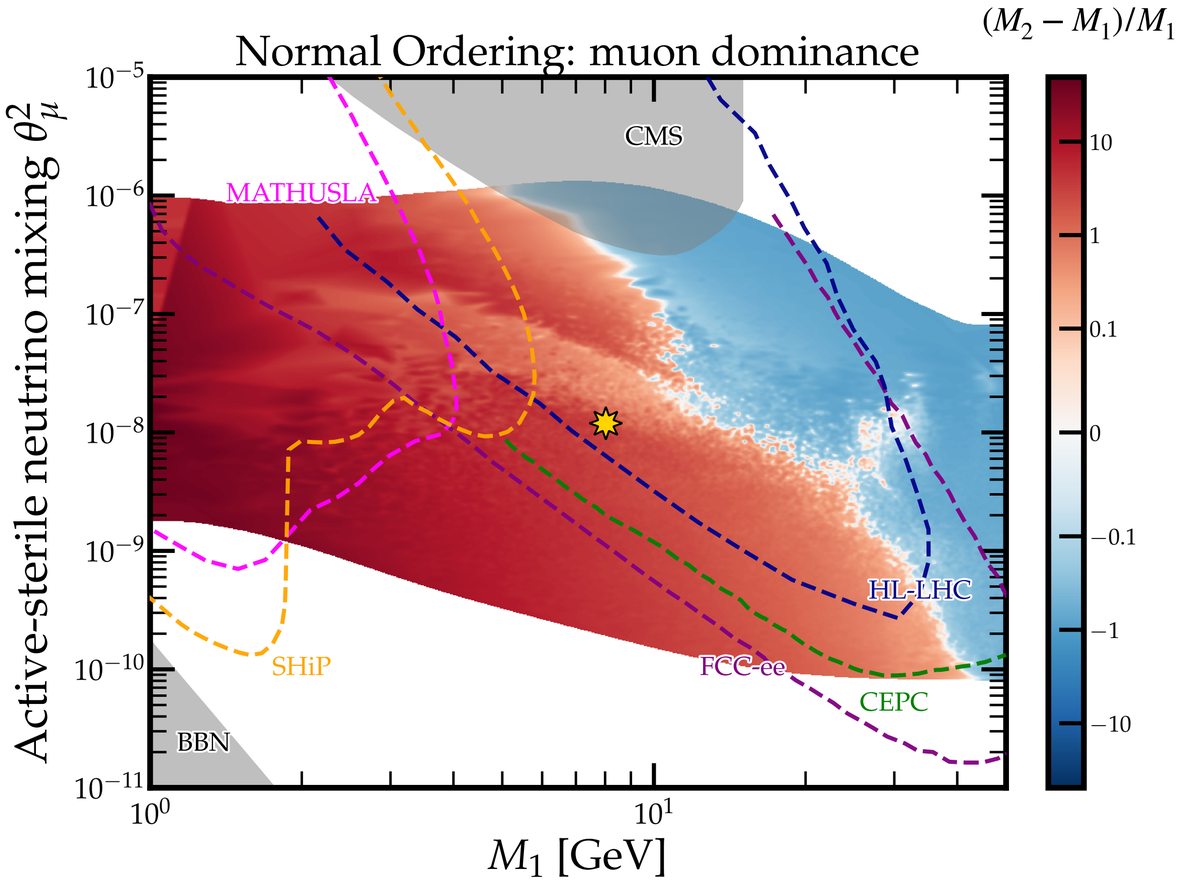


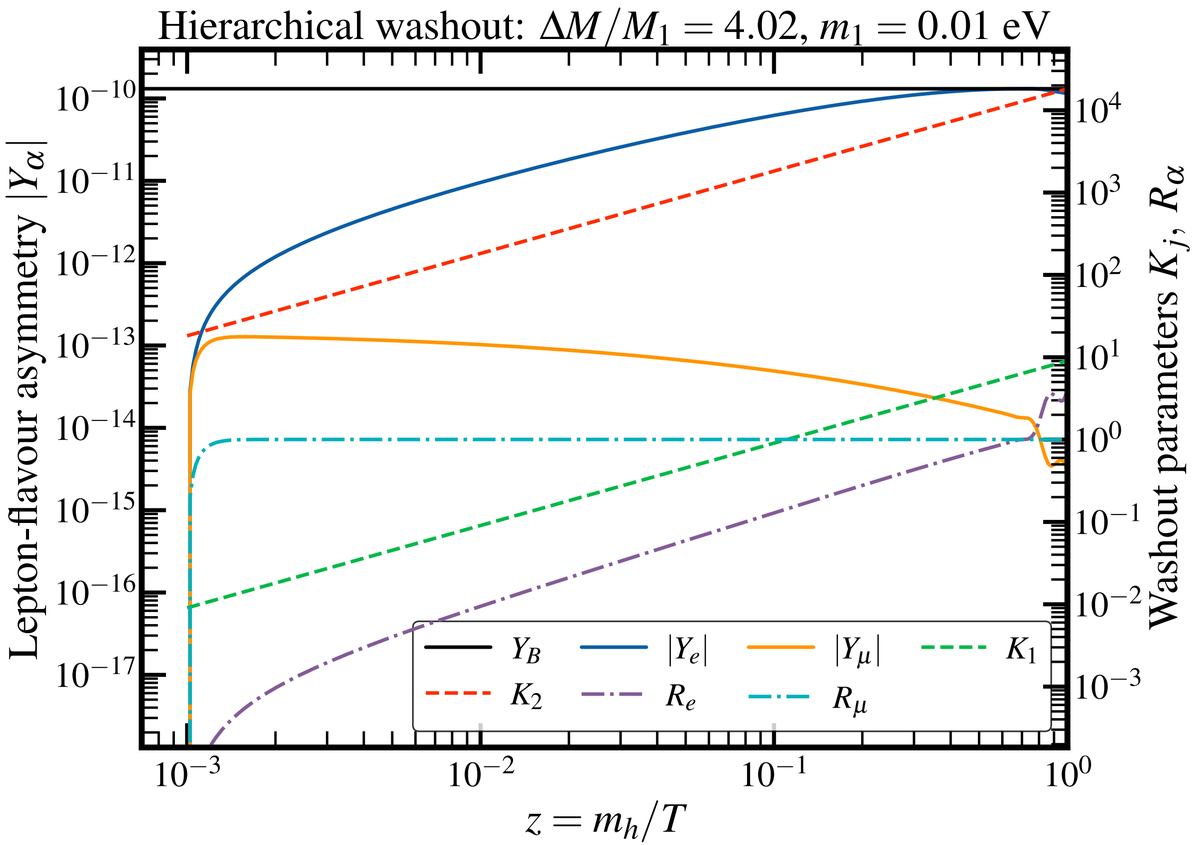
TIRL: $\mathcal{O}[(Y^\nu)^4]$, lepton-number conserving effect

TIRL		RL
$\frac{T^2}{\tilde{m}_\alpha^2 - \tilde{m}_\beta^2}$	V.S.	$\frac{M^2}{M_i^2 - M_j^2}$

Parameter space of TIRL

- Detection via active-sterile neutrino mixing angle $\theta_\alpha^2 = \sum_i \frac{v_{\rm EW}^2}{M_i^2} |Y_{\alpha i}^\nu|^2$





Produce $Y_B = [10^{-11}, 10^{-10}]$, where $Y_B^{\rm obs} \approx 8.75 \times 10^{-11}$

Experimental probe

3*N*-based leptogenesis does not require small mass splittings

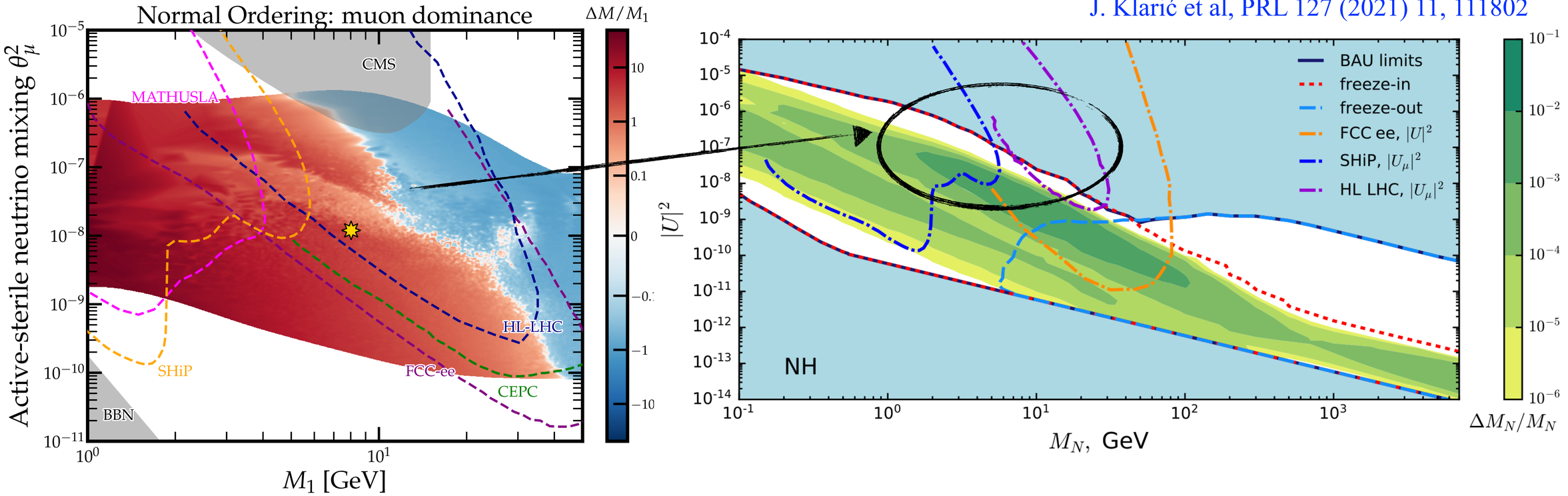
M. Drewes & B. Garbrecht, JHEP 03 63 (2013) 096;

A. Abada *et al,* JHEP 01 (2019) 164

- One of the key differences among three low-scale type-I leptogenesis (RL, ARS, TIRL): sterile-neutrino mass splitting

Lisp & A. Pilaftsis, arXiv: 2601.15921

J. Klarić et al, PRL 127 (2021) 11, 111802



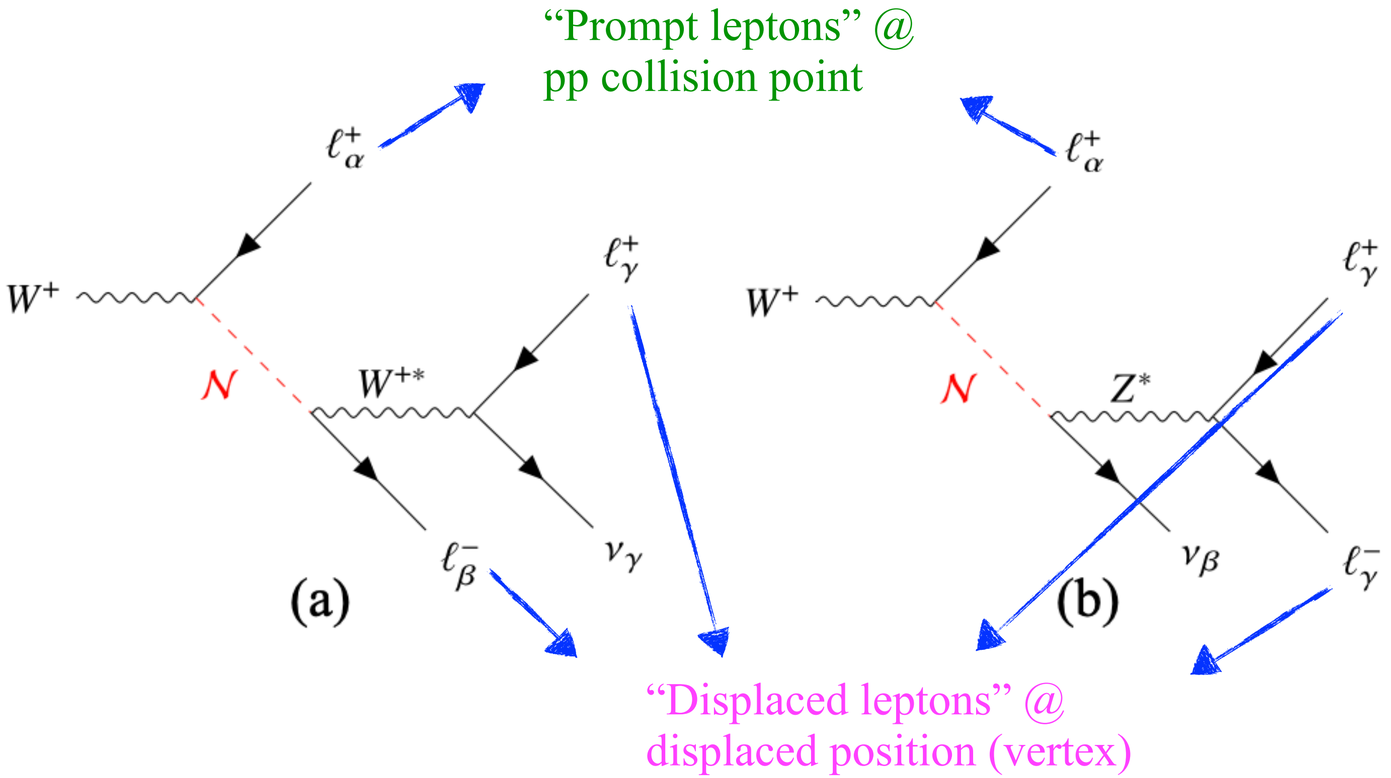
For two sterile neutrinos, ARS/RL:$\Delta M/M \ll 1$; TIRL: mass degeneracy not required when $\Delta M/M \ll 10^{-6}$, RL dominates TIRL

Lisp & A. Pilaftsis, arXiv:2604.06493

Reconstruction of two invariant-mass peaks?

- Production and decay of sterile neutrinos in displaced-vertex searches

CMS Collaboration, JHEP 07 (2022) 081; ATLAS Collaboration, PRL 131 (2023), no. 6 061803;…



- With energy conservation in W and $\mathcal{N}$ decays, 4-momentum of $\mathcal{N}$ is obtained by W-boson mass, three-charged-lepton kinematics, and hence the reconstruction of $\mathcal{N}$-mass

Reconstruction of two invariant-mass peaks?

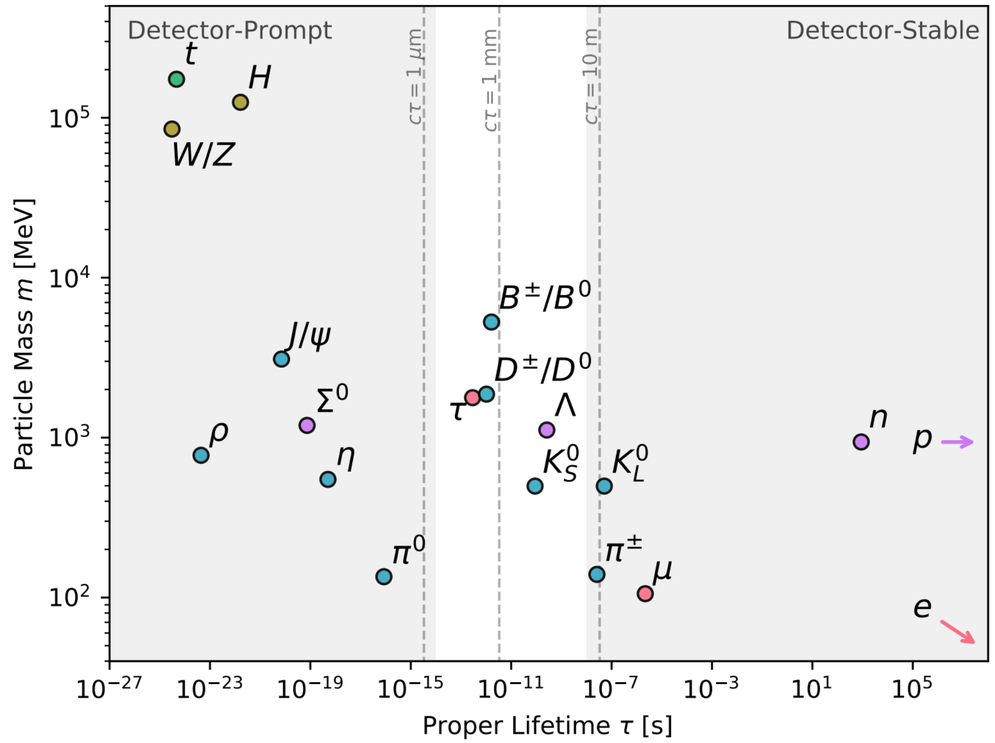
R. Aleksan *et al*, EPJC 85 (2025) 1, 14;

- Can we see two resonant peaks?

For two sterile neutrinos, quasi-degenerate ARS/RL gives one unresolved peak when $\Delta M/M \ll 1$ far below the resolution

Two resolved peaks in TIRL with ΔM of order GeV may point away from a degenerate pair as the origin of the BAU

L. Lee *et al*, Prog.Part.Nucl.Phys. 106 (2019) 210



Lifetimes: M. Gronau, C. N. Leung & J. L. Rosner, PRD 29, 2539 (1984).

$$\tau_i = 5.4 \times 10^2\ \text{m} \times \left(\frac{10^{-8}}{\theta_{\alpha i}^2}\right)\left(\frac{M_i}{3\ \text{GeV}}\right)^{-5} = 1.8 \times 10^{-6}\ \text{s}\left(\frac{10^{-8}}{\theta_{\alpha i}^2}\right)\left(\frac{M_i}{3\ \text{GeV}}\right)^{-5}$$

Muon dominance: $\theta_{\mu 1}^2 = \dfrac{v_{\text{EW}}^2 |\, Y_{\mu 1}^{\nu}\,|^2}{M_1^2}$, $\theta_{\mu 2}^2 = \dfrac{v_{\text{EW}}^2 |\, Y_{\mu 2}^{\nu}\,|^2}{M_2^2}$,

≫ Both separate displaced vertices & two resonance are required, depending on mass splitting & Yukawa couplings

≫ Challenges?

Summary

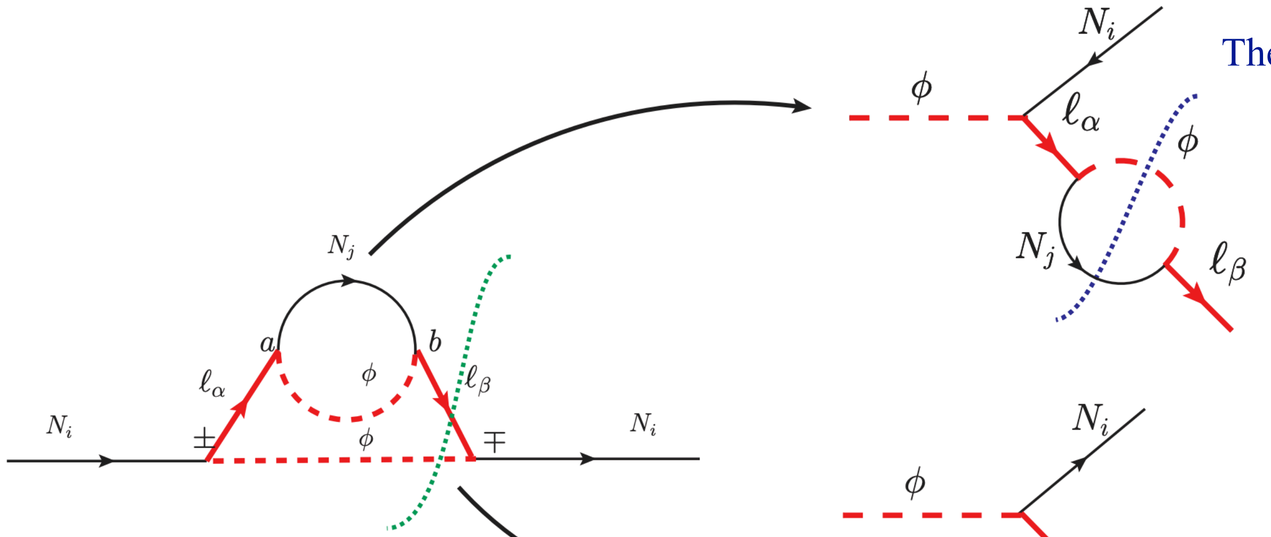
- SM leaves several open questions in the theoretical front: neutrino masses, matter-antimatter asymmetry, dark matter,…
- Seesaw models are promising beyond-the-SM candidates
- Higher-order thermal effects are important: resummation, modified dispersion relation,…
- Resummation of thermal leptons in the early Universe can yield significant contribution to a third low-scale leptogenesis—TIRL
- Collider probes are important to see sterile neutrinos & tell whether ARS, RL or TIRL dominates to explain the BAU problem: reconstruction of resonance peaks,…

Thank you!

Technical aspects of TIRL

S. Kanemura & **Lisp**, PRL 134 (2025) 21

- A two-loop diagrammatic way to see the resonant process



Thermal cut: on-shell quasi-particle ϕ, N_j

When either ℓ_α or ℓ_β goes on-shell:

$$\propto \frac{1}{p_\beta^2 - \tilde{m}_\alpha^2} \to \frac{1}{\tilde{m}_\beta^2 - \tilde{m}_\alpha^2}$$

Technical aspects of TIRL

Lisp & A. Pilaftsis, arXiv: 2601.15921; 2604.06493

- Quantum kinetic approach—flavour-covariant Kadanoff-Byam formulation

$$\frac{\mathrm{d}\Delta' n_{\alpha\beta}}{\mathrm{d}t} + i\Delta\omega_{\alpha\beta}\,\Sigma' n_{\alpha\beta} \;=\; -(y_\alpha^2 + y_\beta^2) C_\ell\,\Delta' n_{\alpha\beta}\,,$$

$$\frac{\mathrm{d}\Sigma' n_{\alpha\beta}}{\mathrm{d}t} + i\Delta\omega_{\alpha\beta}\,\Delta' n_{\alpha\beta} \;=\; 2y'^{*}_{\alpha j} y'_{\beta j}\, C_{N_j} - (y_\alpha^2 + y_\beta^2) C_\ell\,\Sigma' n_{\alpha\beta} - 2g^4 C_V\,\Sigma' n_{\alpha\beta}\,,$$

$\Delta' n_{\alpha\beta} \equiv n_{\alpha\beta} - \bar{n}_{\alpha\beta}$ at $\alpha \neq \beta$: NO one-loop (leading order) source rate

$\Sigma' n_{\alpha\beta} \equiv n_{\alpha\beta} + \bar{n}_{\alpha\beta}$ at $\alpha \neq \beta$: One-loop source rate + strong gauge damping rate.

- An approximation for lepton-flavour coherences: $\mathrm{d}\Sigma' n_{\alpha\beta}/\mathrm{d}t \approx 0\,, \Sigma' n_{\alpha\beta} \approx 0$

$$\Delta' n_{\alpha\beta} \approx \frac{-2i y'_{\beta j} y'^{\dagger}_{j\alpha} \Delta m_j^2}{\tilde{b}_\alpha - \tilde{b}_\beta} \qquad \tilde{b}_\alpha - \tilde{b}_\beta \propto \tilde{m}_\alpha^2 - \tilde{m}_\beta^2$$

$$\times \int \mathrm{d}\Pi_\ell \mathrm{d}\Pi_\phi \mathrm{d}\Pi_N (2\pi)^4 \tilde{\delta}^{(4)}(k_\ell + k_\phi - k_N)\,\delta f_{N_j}(E_N)\,\left(f_\phi^{\mathrm{eq}}(E_\phi) + f_\ell^{\mathrm{eq}}(E_\ell)\right)$$

Technical aspects of TIRL

Lisp & A. Pilaftsis, arXiv: 2601.15921; 2604.06493

- Lepton-flavour off-diagonal coherences $\Delta' n_{\alpha\beta}$ enters the diagonal lepton asymmetry

$$\frac{\mathrm{d}\Delta n_{\alpha\beta}}{\mathrm{d}t} - i\left(\tilde{b}_\alpha - \tilde{b}_\beta\right)\langle \Sigma n_{\alpha\beta}\rangle = \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_0}{2\pi}\mathrm{Tr}\left(\mathcal{C}_{\alpha\beta} + \mathcal{C}^\dagger_{\alpha\beta}\right)$$

$$\mathcal{C}_{\alpha\beta} \equiv [i\Sigma_>]_{\alpha\gamma}[iS_<]_{\gamma\beta} - [i\Sigma_<]_{\alpha\gamma}[iS_>]_{\gamma\beta}$$

$\Delta' n_{\alpha\beta} \equiv n_{\alpha\beta} - \bar{n}_{\alpha\beta}$: appear through $[iS_<]_{\gamma\beta}$, $[iS_>]_{\gamma\beta}$, at $\gamma \neq \beta$

$$\Delta n_\ell \equiv \mathrm{Tr}(\Delta n_{\alpha\beta}) = \sum_{\alpha=1}^{3}\Delta n_{\alpha\alpha}$$

A two-loop CP-violating source enhanced by charged-lepton Yukawa couplings

$$\frac{\mathrm{d}\Delta n_\ell}{\mathrm{d}t} = \sum_{\substack{\alpha,\gamma\\ \alpha\neq\gamma}}\sum_{i,j}\frac{\Delta m_i^2\Delta m_j^2\,\mathrm{Im}(y'_{\gamma i}y'^*_{\alpha i}y'_{\alpha j}y'^*_{\gamma j})}{256\pi^4(\tilde{m}_\gamma^2 - \tilde{m}_\alpha^2)}\int\frac{dE_\ell}{E_\ell}\int_{E_{\rm low}}^{\infty}dE_N\int_{E_{\rm low}}^{\infty}dE'_N\mathcal{F}_{ij}$$

$$\mathcal{F}_{ij} \equiv \left(f_\phi^{\rm eq}(E_\ell + E_N) + f_{N_i}(E_N)\right)\left(f_\ell^{\rm eq}(E_\ell) + f_\phi^{\rm eq}(E_\ell + E'_N)\right)\delta f_{N_j}(E'_N)$$

Technical aspects of TIRL

- Stability against finite thermal width by expanding lepton thermal masses

$$\frac{\mathrm{d}\Delta n_\ell}{\mathrm{d}t} = \sum_{\substack{\alpha,\gamma \\ \alpha\neq\gamma}} \sum_{i,j} \frac{\Delta m_i^2 \Delta m_j^2 \,\mathrm{Im}(y'_{\gamma i} y'^{*}_{\alpha i} y'_{\alpha j} y'^{*}_{\gamma j})}{256\pi^4(\tilde{m}_\gamma^2 - \tilde{m}_\alpha^2)} \int \frac{dE_\ell}{E_\ell} \int_{E_{\rm low}}^{\infty} dE_N \int_{E_{\rm low}}^{\infty} dE'_N \mathcal{F}_{ij}$$



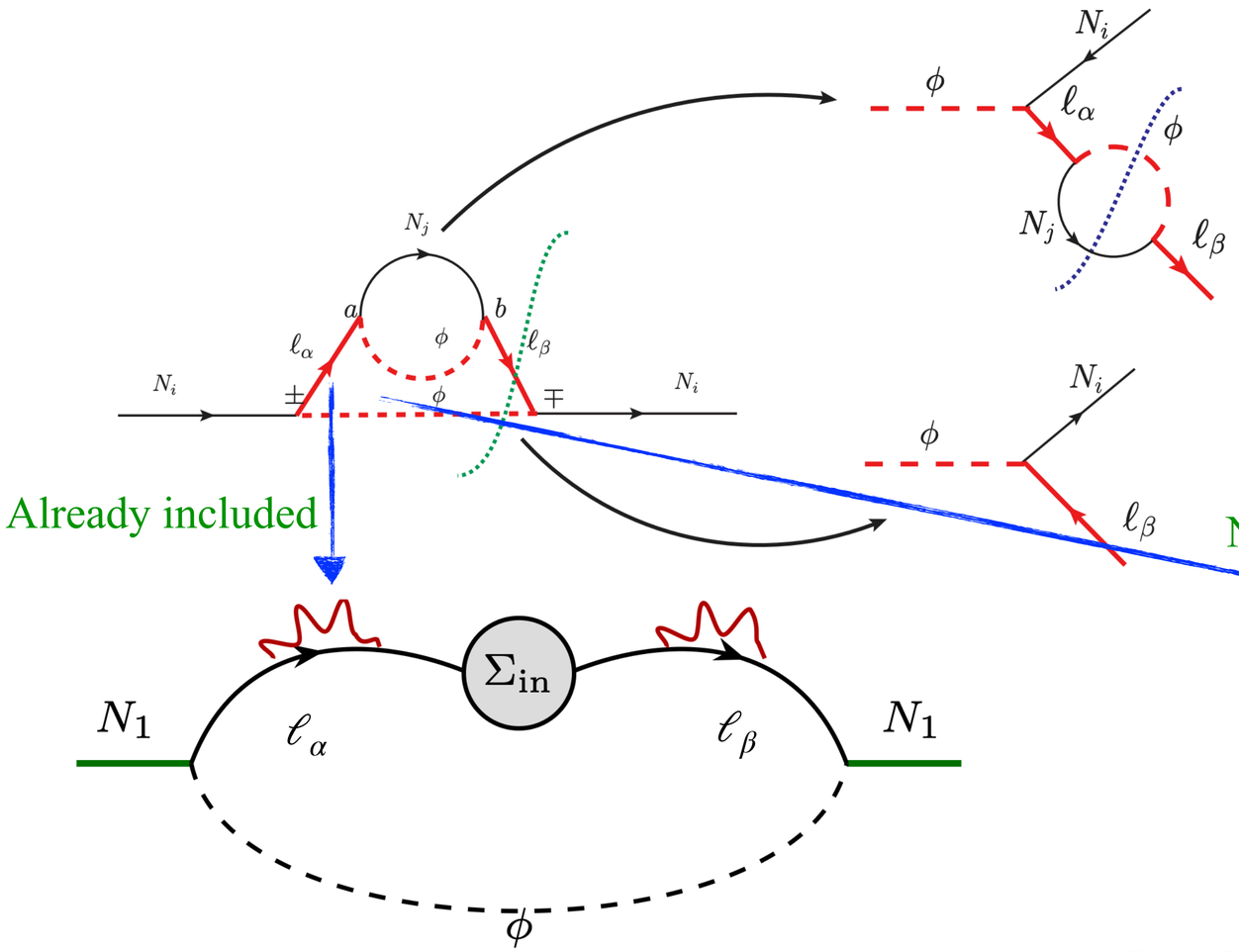
modified dispersion relation + finite thermal width

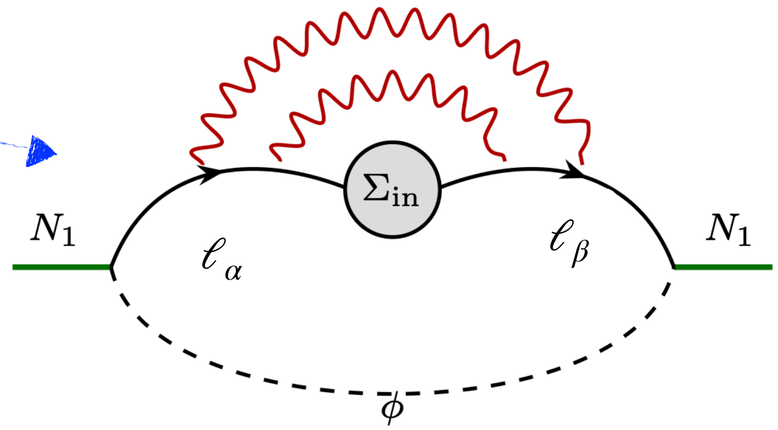
$$p^2 \approx 2m_\alpha^2(T) + \mathcal{O}(\tilde{m}^4/|\boldsymbol{p}|^2) \qquad \Gamma_\alpha(T) \sim g^4 T + g^2 y_\alpha^2 T + \mathcal{O}(y_\alpha^4) T$$

$$\sim m_\alpha^2 - m_\gamma^2 + i\,\kappa\, g^2\,(m_\alpha^2 - m_\gamma^2), \quad \kappa < 1$$

Open questions

- Stability against finite thermal width by including ladder resummation?





S. Jeon, PRD 52, (1995) 3591; S. Jeon & L. G. Yaffe, PRD 53 (1996) 5799

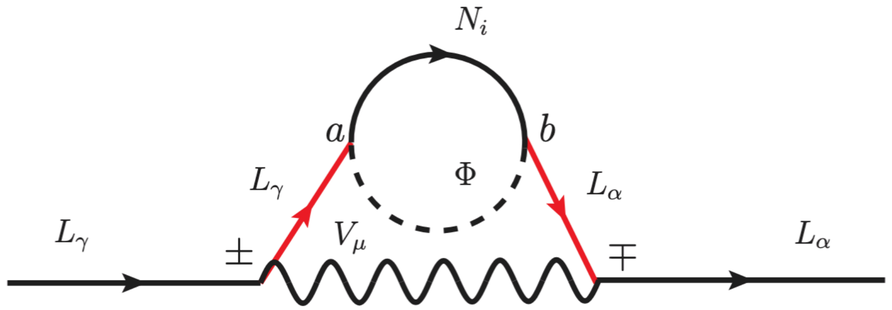
Open questions

- Thermal lepton mass difference originates from a CP-even phase, like in ARS?

$$|N(t)\rangle = c_i e^{-iE_i t}|N_i\rangle + c_j e^{-iE_j t}|N_j\rangle$$

$$= e^{-iE_i t}\left(c_i|N_i\rangle + c_j e^{-i(E_j-E_i)t}|N_j\rangle\right) \qquad E_j - E_i \approx \frac{M_j^2 - M_i^2}{2p}$$

$$\gamma_{L_\alpha\to L_\beta} - \gamma^{\rm CP}_{L_\alpha\to L_\beta} \sim {\rm Im}[(Y^\nu)^4]\sin\left(\frac{M_P(M_i^2 - M_j^2)}{T^3_{\rm osc}}\right)$$

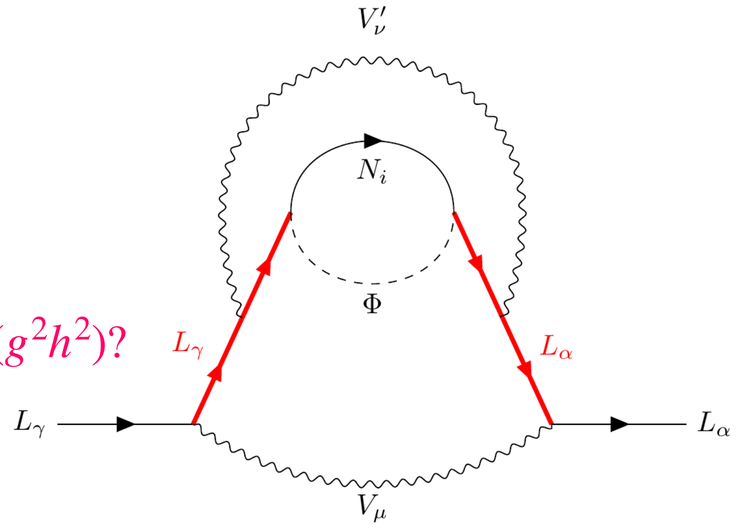


M. Beneke *et al,* NPB 843 (2011) 177; B. Garbrecht, NPB 868 (2013) 557

$$\frac{{\rm d}\Delta' n_{\alpha\beta}}{{\rm d}t} + i\Delta\omega_{\alpha\beta}\,\Sigma' n_{\alpha\beta} = -(y_\alpha^2 + y_\beta^2)C_\ell\,\Delta' n_{\alpha\beta}\,,$$

$$\frac{{\rm d}\Sigma' n_{\alpha\beta}}{{\rm d}t} + i\Delta\omega_{\alpha\beta}\,\Delta' n_{\alpha\beta} = 2y'^*_{\alpha j}y'_{\beta j}\,C_{N_j} - (y_\alpha^2 + y_\beta^2)C_\ell\,\Sigma' n_{\alpha\beta} - 2g^4 C_V\,\Sigma' n_{\alpha\beta}$$

+ Higher-order source rate $\mathcal{O}(g^2h^2)$?



Quasi-steady state regime implies $\Delta' n_{\alpha\beta} = \mathcal{O}\left(\frac{g^2h^2}{y^2}\right)$

Backup

- Why the ARS oscillation peaks at $M_P(M_i^2 - M_j^2) \sim T_{\rm osc}^3$?

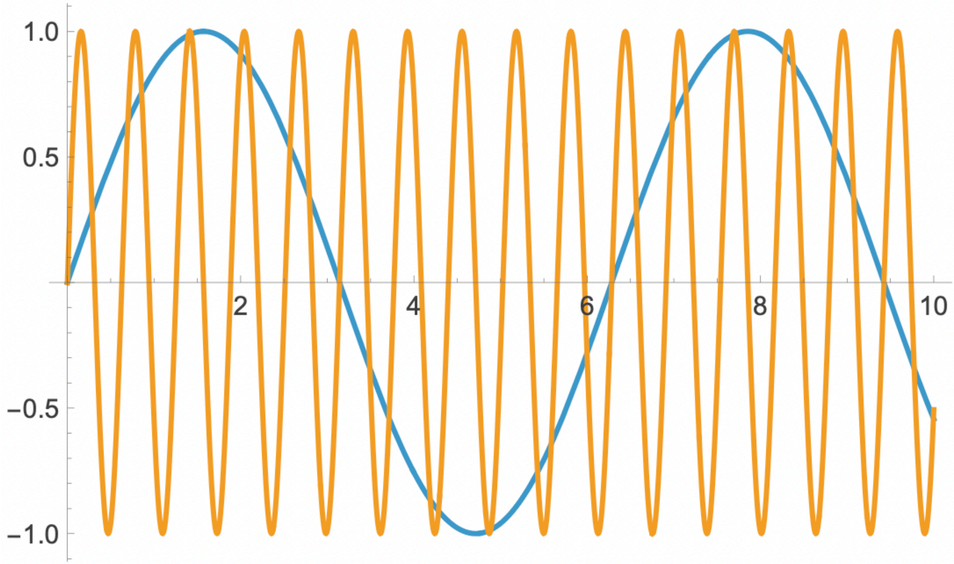
$$|N(t)\rangle = c_i e^{-iE_i t}|N_i\rangle + c_j e^{-iE_j t}|N_j\rangle$$

$$= e^{-iE_i t}\left(c_i|N_i\rangle + c_j e^{-i(E_j - E_i)t}|N_j\rangle\right)$$

$$E_j - E_i \approx \frac{M_j^2 - M_i^2}{2p}$$

$$\gamma_{L_\alpha \to L_\beta} - \gamma^{\rm CP}_{L_\alpha \to L_\beta} \sim {\rm Im}[(Y^\nu)^4]\sin\left(\frac{M_P(M_i^2 - M_j^2)}{T_{\rm osc}^3}\right)$$

Plot[{Sin[x], Sin[10 x]}, {x, 0, 10}]



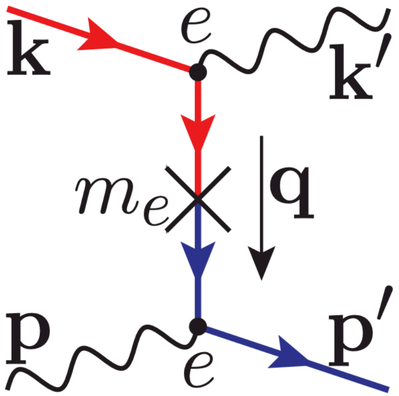
{Integrate[Sin[0.01 x], {x, 0, 10.}], Integrate[Sin[x], {x, 0, 10.}], Integrate[Sin[100 x], {x, 0, 10.}]}

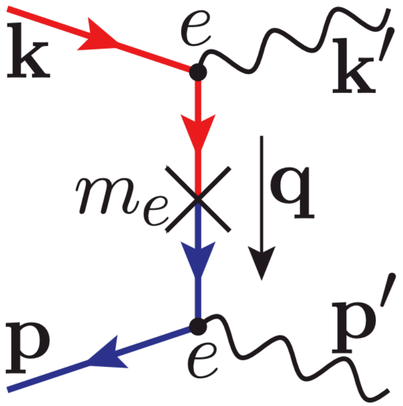
{0.499583, 1.83907, 0.00437621}

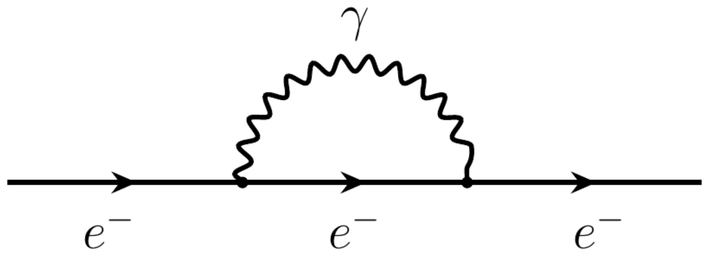
Perturbative Feynman diagram expansion at finite T

[A. Boyarsky *et al*, PRL 126 (2021) 2, 021801; PRD 103 (2021) 1, 013003]

- Example 2: *Electron chirality flip rate in the early Universe*







Self-energy correction at finite T:

$$\Sigma(q, T \gg m_e) \sim \sqrt{\alpha} T$$

Chirality-flip picks up the mass

Naive estimate by vacuum QFT

$$S_e = \frac{\not{q} + m_e}{q^2 - m_e^2} \to \frac{m_e}{E_k E_{k'} - |\boldsymbol{k}||\boldsymbol{k'}| \cos\theta_{kk'}}$$

$$\Gamma_{\text{flip}}^{\text{naive}} \propto \left(\frac{m_e}{T}\right)^2 \alpha^2 T$$

Finite-T QFT

$$S_e = \frac{\not{q} + m_e}{(\not{q} - \Sigma(q,T))^2 - m_e^2} \xrightarrow{q^2 \sim \alpha T^2 \gg m_e} \frac{m_e}{\alpha T^2}$$

$$\Gamma_{\text{flip}} \propto \alpha^2 \times \frac{m_e}{\alpha T^2} \propto \left(\frac{m_e}{T}\right)^2 \alpha T$$

A factor of $1/\alpha \sim 100$ enhanced!