POLOCALC drone calibrator: Attitude reconstruction

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End-of-the-year 2 - seminar
September 18rd 2026

Flying artificial calibrator







- ❏ POLOCALC project
- ❏ Drone based calibration
- ❏ mm wave source
- ❏ SO site (5200m asl)
- ❏ Photogrammetry

Cosmic Microwave Background (CMB)



Inflation
Accelerated expansion of the Universe

Formation of light and matter

Light and matter are coupled
Dark matter evolves independently: it starts clumping and forming a web of structures

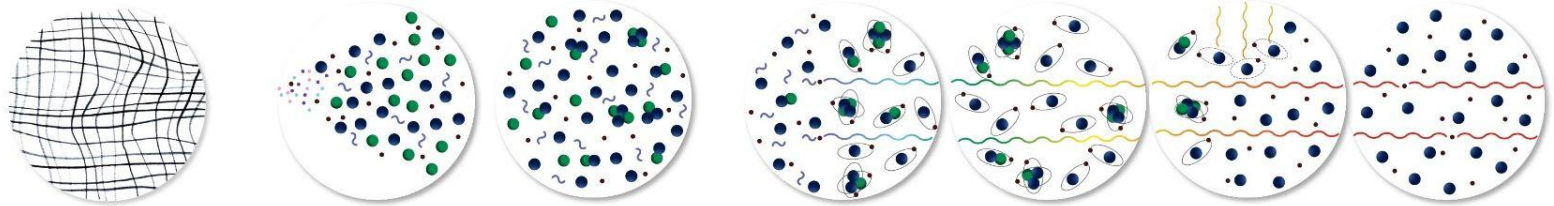
Light and matter separate
- Protons and electrons form atoms
- Light starts travelling freely: it will become the Cosmic Microwave Background (CMB)

Dark ages
Atoms start feeling the gravity of the cosmic web of dark matter

First stars
The first stars and galaxies form in the densest knots of the cosmic web

Galaxy evolution

The present Universe



- *Tiny fluctuations: the seeds of future structures*
- *Gravitational waves?*

Frequent collisions between normal matter and light

As the Universe expands, particles collide less frequently

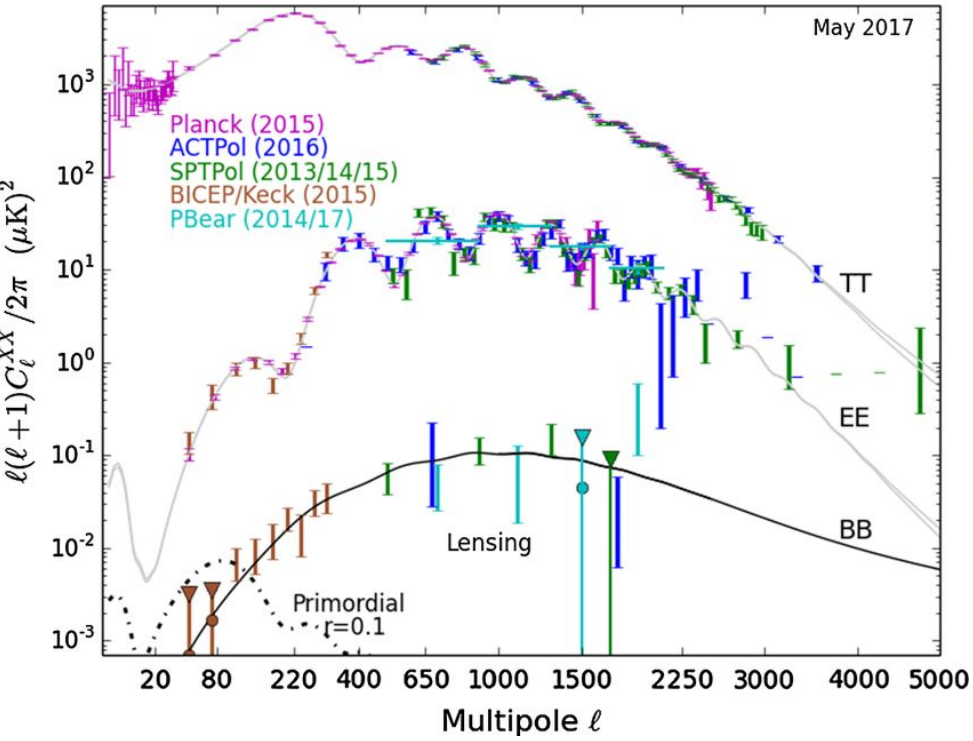
Last scattering of light off electrons
→ ***Polarisation***

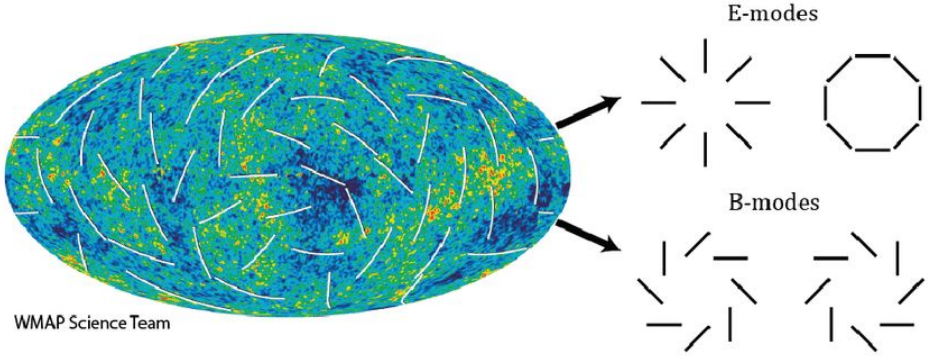
The Universe is dark as stars and galaxies are yet to form

Light from first stars and galaxies breaks atoms apart and "reionises" the Universe

Light can interact again with electrons
→ ***Polarisation***

CMB anisotropies





Scalar perturbation (E-mode): sourced by Thomson scattering

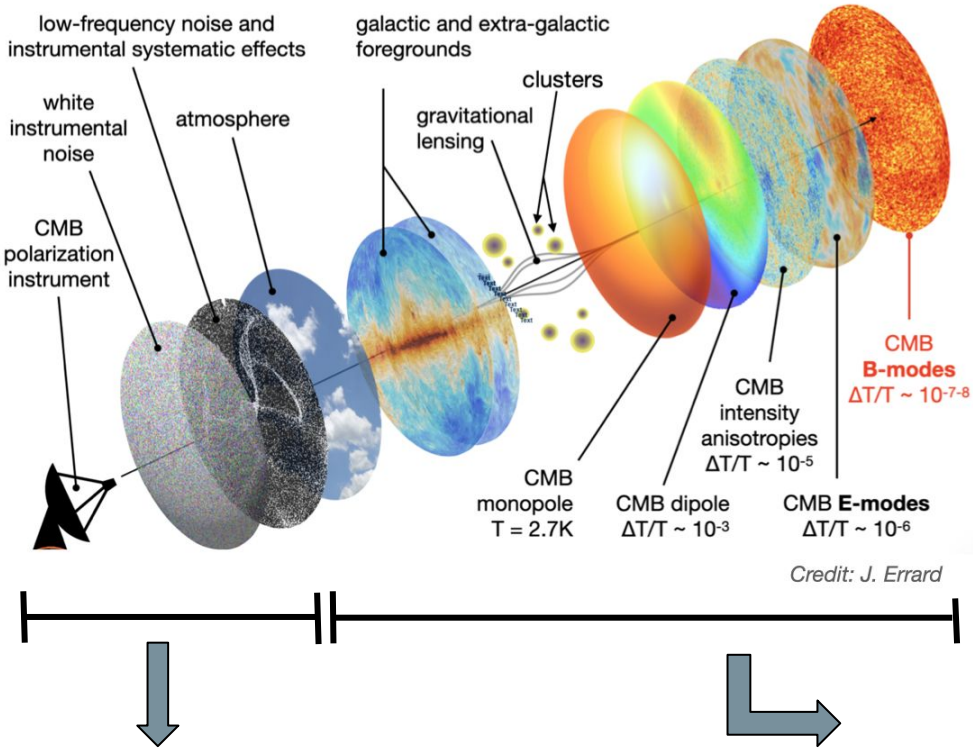
Tensor perturbation (B-mode): sourced by Primordial Gravitational waves

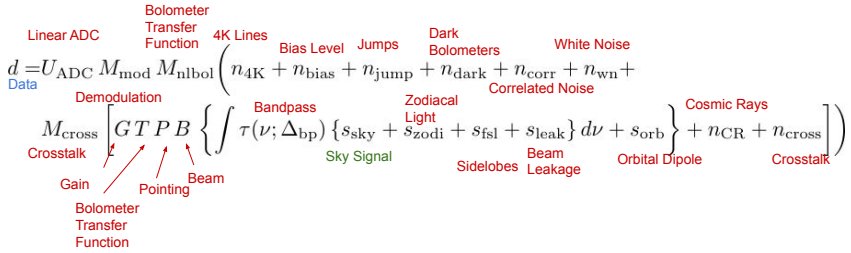
tensor-to-scalar ratio :

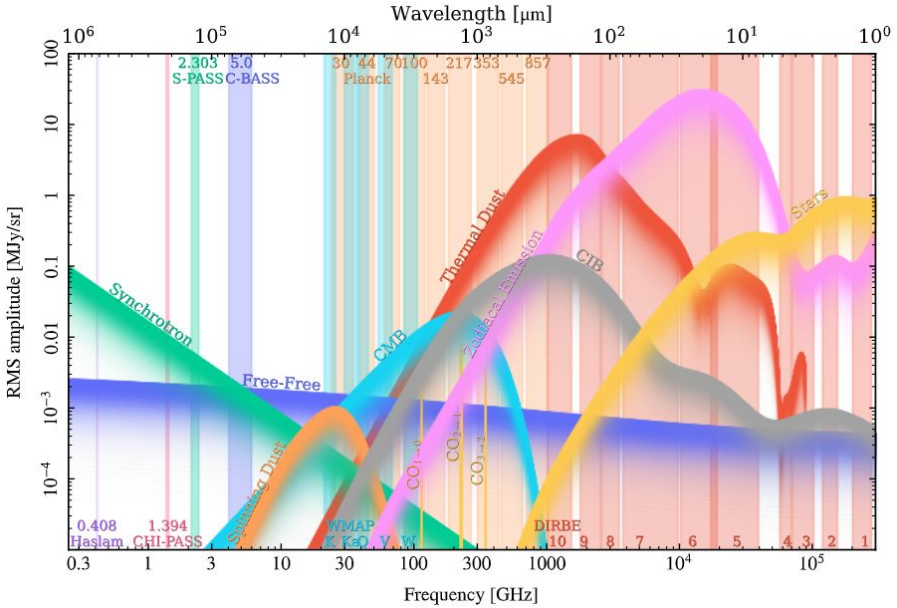
$$r = \frac{P_t(k)}{P_s(k)}$$

$$\Delta T(\theta,\phi) = \sum_{lm} a_{lm} Y_l^m(\theta,\phi)$$

Systematics & Foreground





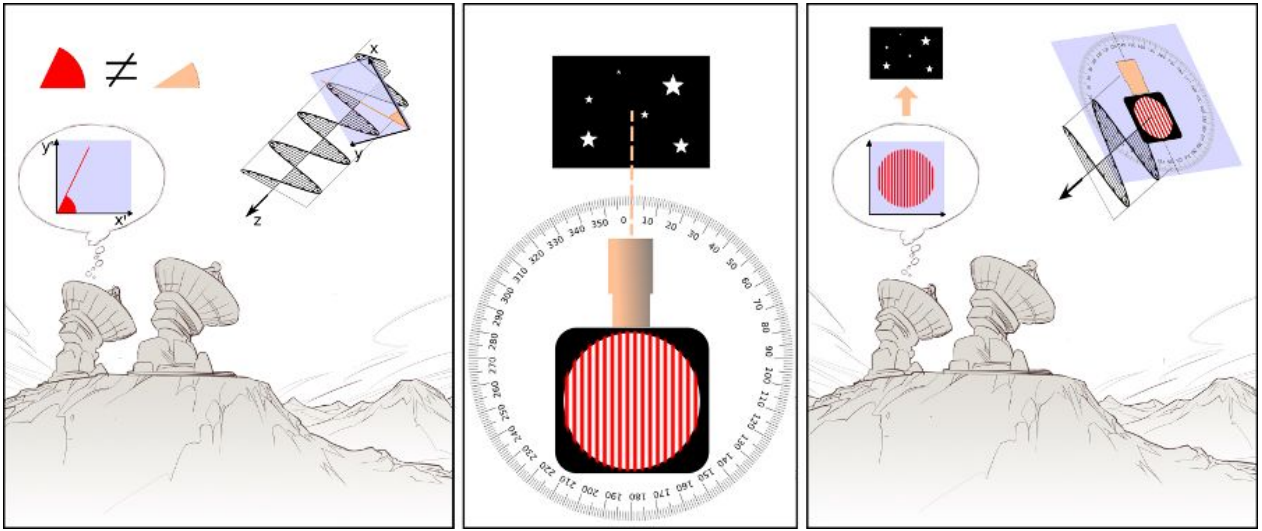


POLOCALC*

POLOCALC & Absolute polarization angle

The problem

The absolute polarization angle is the inclination of the incoming photon's electric field vector (or CMB photon direction) relative to the telescope's detector plane.



The solution

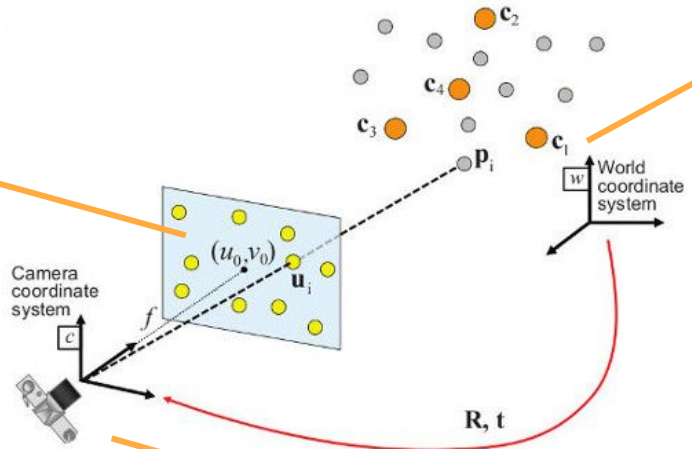
Well know attitude of the calibration source

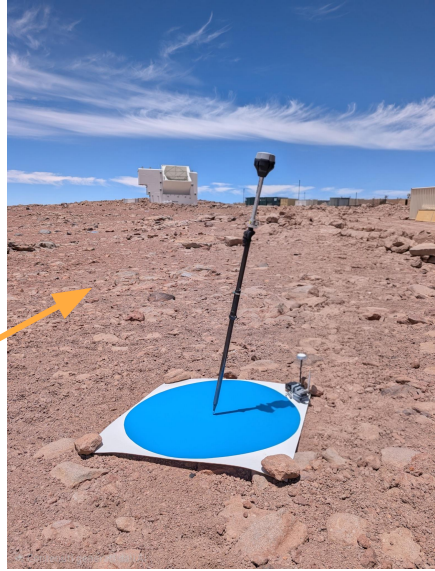


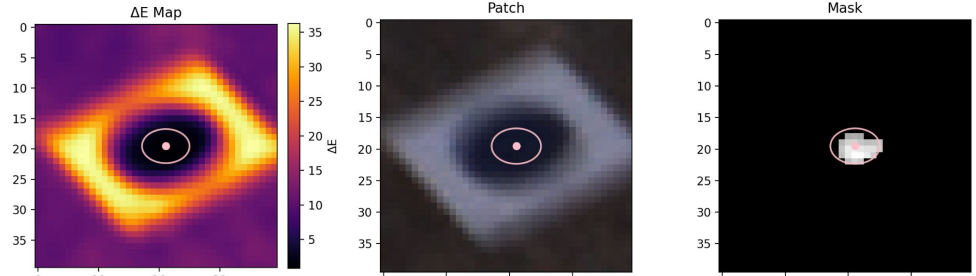
PHOTOGRAMMETRY



$$\min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{n} \| \mathbf{p}_i - \pi(\mathbf{K}, \mathbf{R}, \mathbf{t}, \mathbf{P}_i) \|^2$$







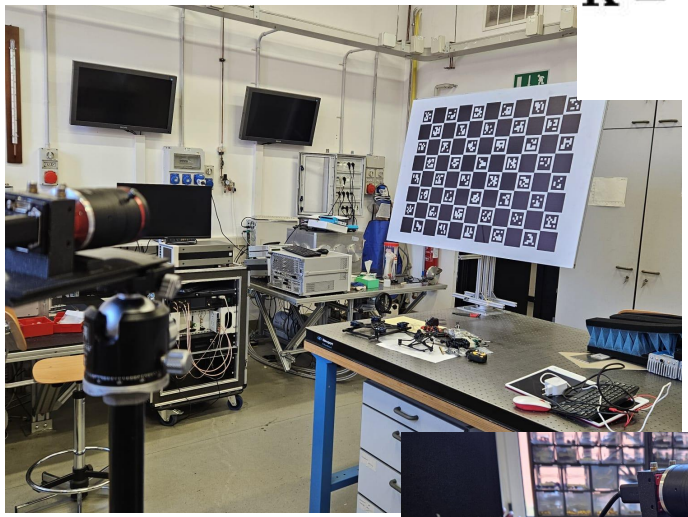
$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \mathbf{K} \, [\mathbf{R} \mid \mathbf{t}] \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

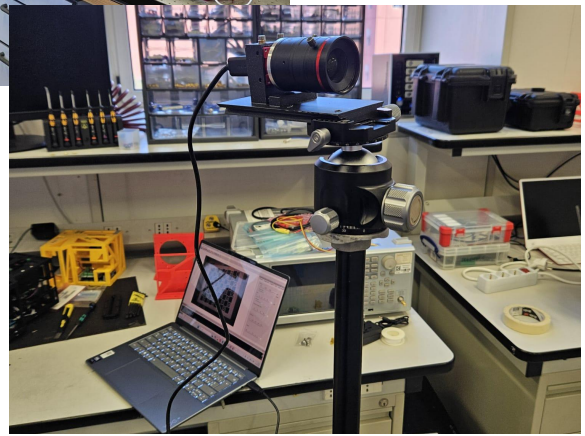


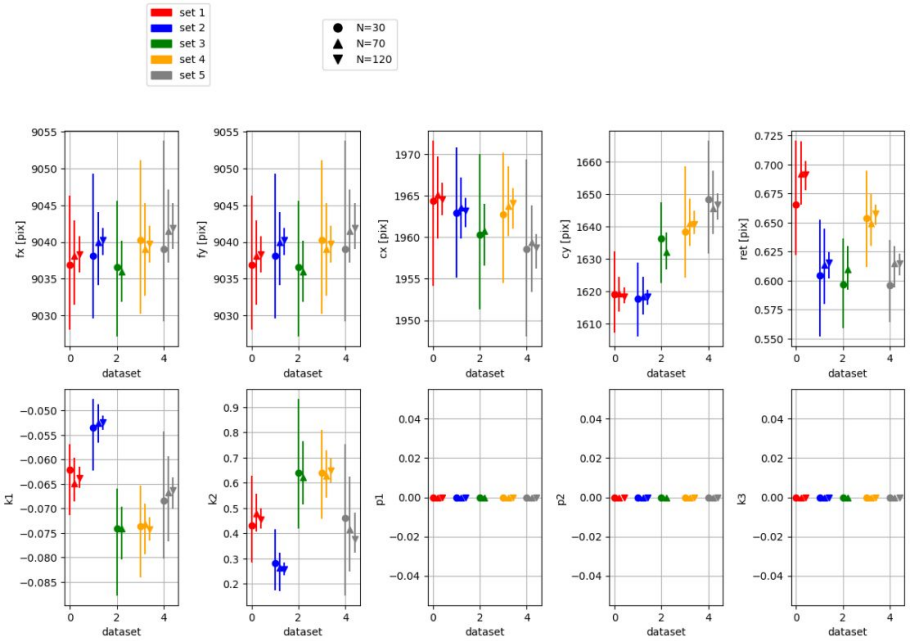
SENSORS CALIBRATION - CAMERA

$$\mathbf{K} = \begin{pmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$

$$x_d = x'(1+k_1r^2+k_2r^4+k_3r^6)+2p_1x'y'+p_2(r^2+2x'^2)$$
$$y_d = y'(1+k_1r^2+k_2r^4+k_3r^6)+p_1(r^2+2y'^2)+2p_2x'y'$$





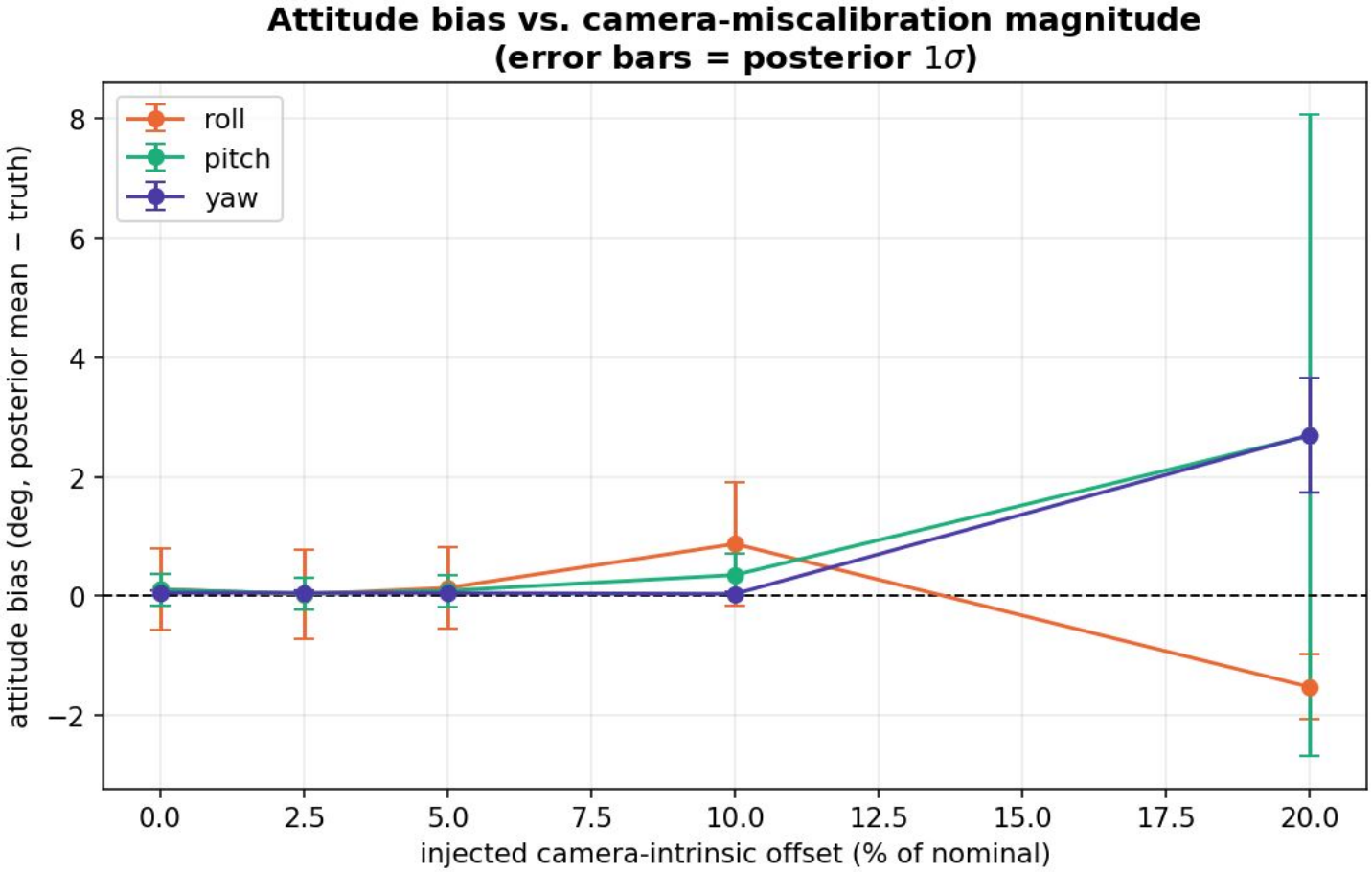


$$F_j = F^j \text{mm}, \frac{\Sigma^j \text{pix}}{\Sigma^j_{\text{mm}}}$$

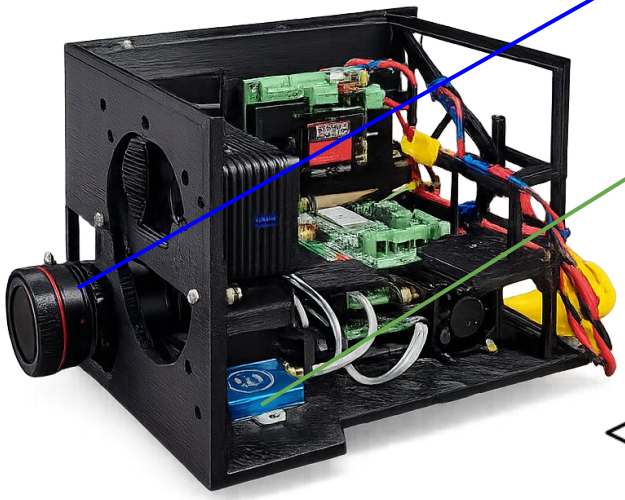
$$C_j = \frac{\Sigma^j_{\text{pix}}}{2}$$

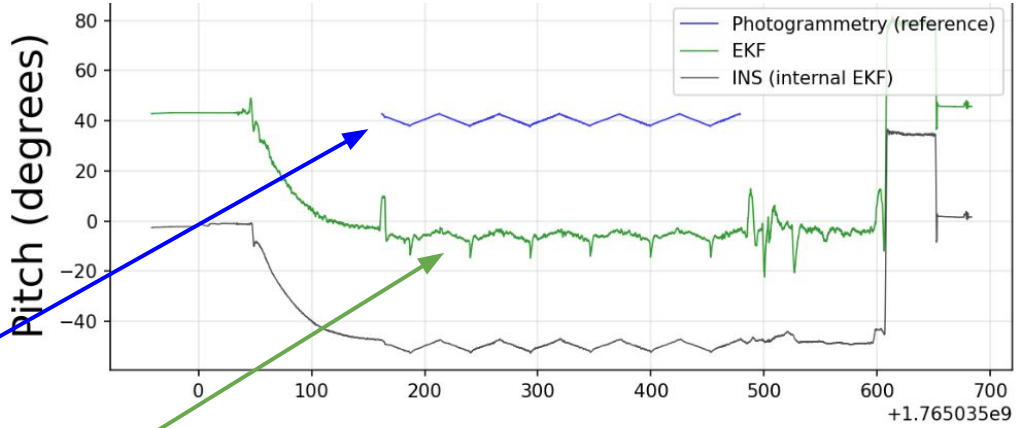
SENSORS CALIBRATION - CAMERA

Sensitivity test: how much the camera calibration affect the attitude reconstruction

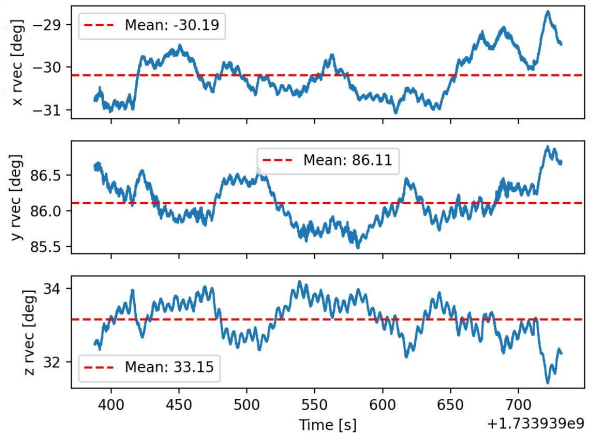


SENSORS CALIBRATION - IMU



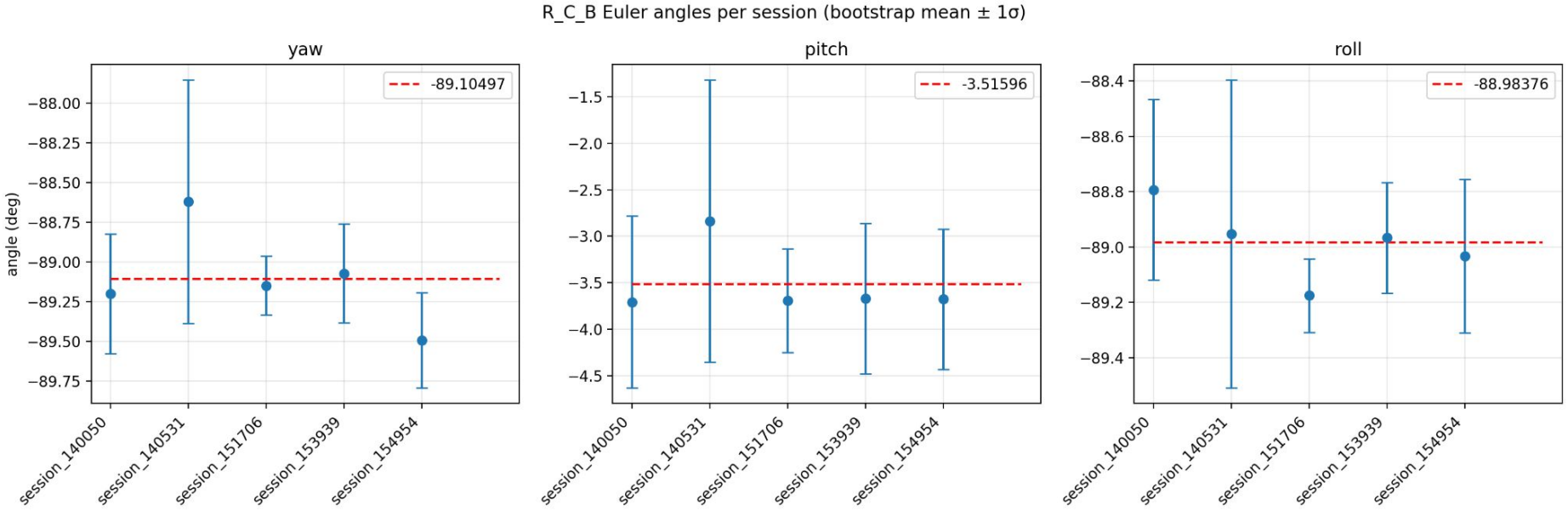


$$< R_{i2c} = R_c @ R_i >$$



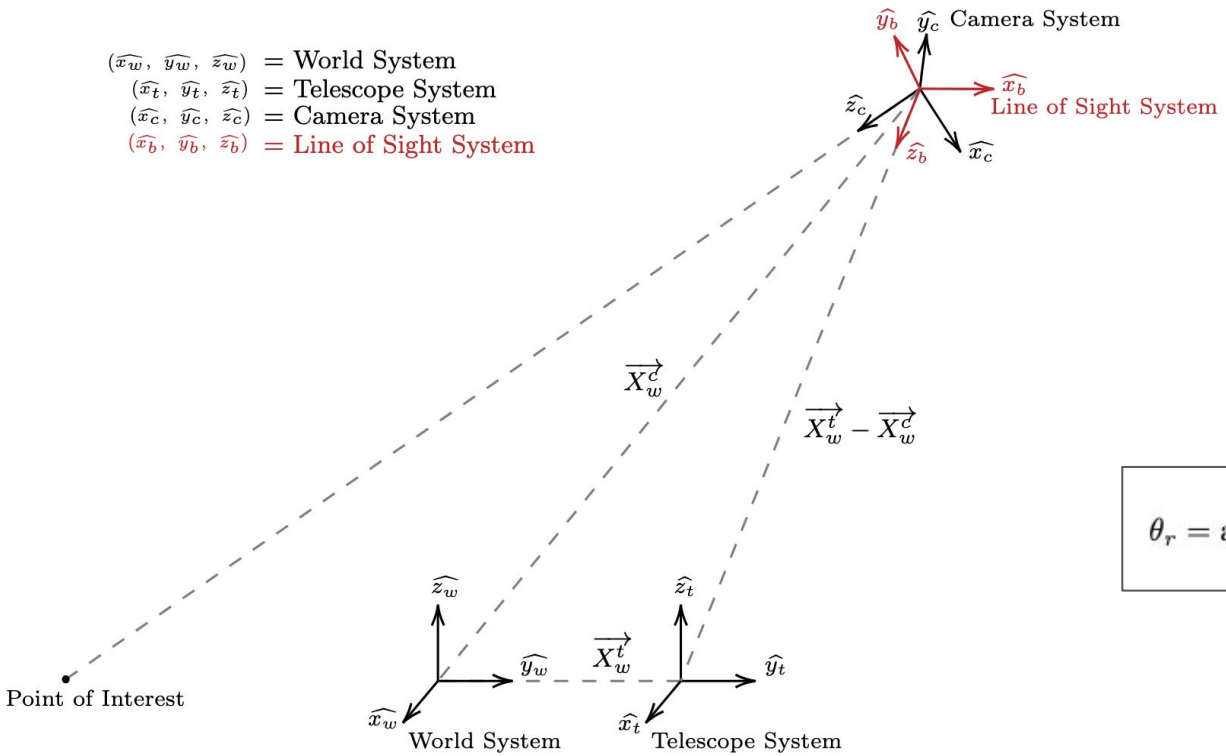
SENSORS CALIBRATION - IMU (Kalibr)

- Rigid IMU-camera link assumed constant rotation R_CB.
- Poisson kinematics + Rodrigues → integrate gyro ω into ΔR_gyro.
- PnP solver on ChArUco board → ΔR_cam over same time window.
- Hand-eye relation → conjugation collapses to **v_cam = R_CB · v_gyro**.
- Wahba (1965) → weighted least-squares over rotation vector pairs.
- IRLS → iterative re-weighting to down-weight outlier frames.
- Markley (1988) → closed-form SVD solution: **$R_{CB} = U \cdot \mathrm{diag}(1,1,\det(UV^T)) \cdot V^T$**.



ARS: Coordinate systems

$(\widehat{x_w},\ \widehat{y_w},\ \widehat{z_w})$ = World System
$(\widehat{x_t},\ \widehat{y_t},\ \widehat{z_t})$ = Telescope System
$(\widehat{x_c},\ \widehat{y_c},\ \widehat{z_c})$ = Camera System
$(\widehat{x_b},\ \widehat{y_b},\ \widehat{z_b})$ = Line of Sight System



$$\hat{z}_b = \frac{\vec{X}_w^t - \vec{X}_w^c}{\left\|\vec{X}_w^t - \vec{X}_w^c\right\|}$$

$$\hat{x}_b = \frac{[0,0,1] \times \hat{z}_b}{\left\|[0,0,1] \times \hat{z}_b\right\|}$$

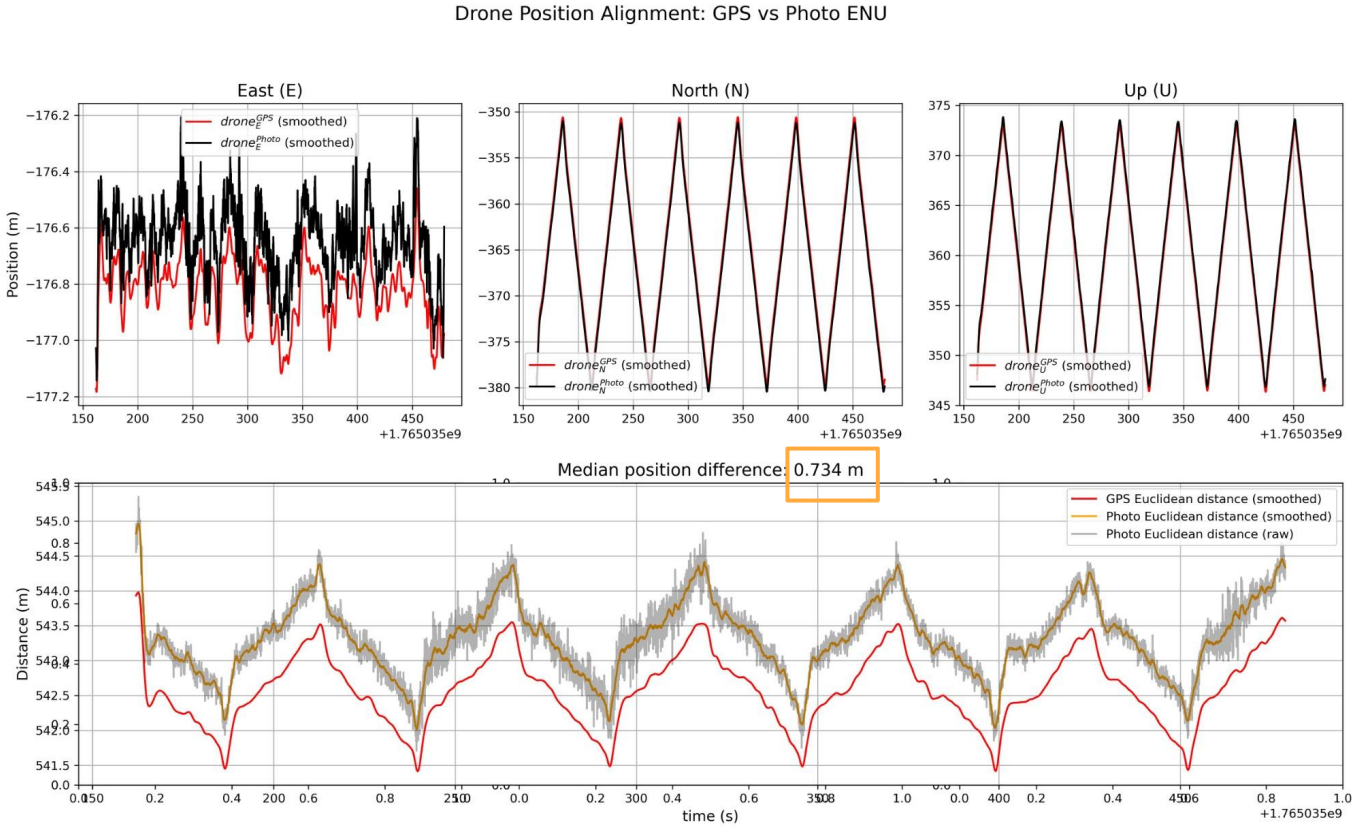
$$\hat{y}_b = \frac{\hat{z}_b \times \hat{x}_b}{\left\|\hat{z}_b \times \hat{x}_b\right\|}$$

$$\theta_r = \arctan\left(\frac{S_{bc}^{2,1}\cos(\theta_c) + S_{bc}^{2,2}\sin(\theta_c)}{S_{bc}^{1,1}\cos(\theta_c) + S_{bc}^{1,2}\sin(\theta_c)}\right)$$

ARS: Photogrammetry

Position:

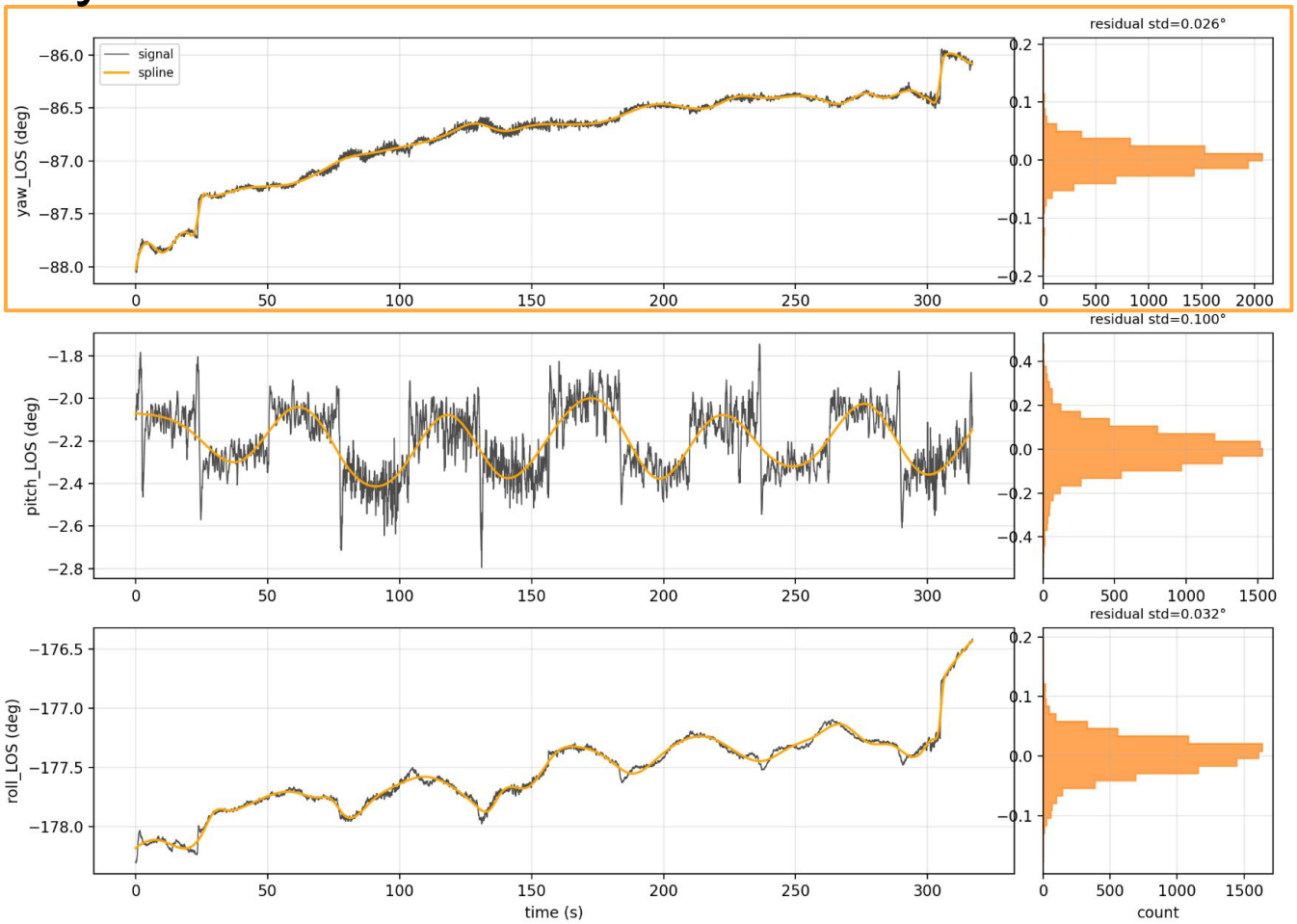
GPS vs Photogramm



ARS: Photogrammetry

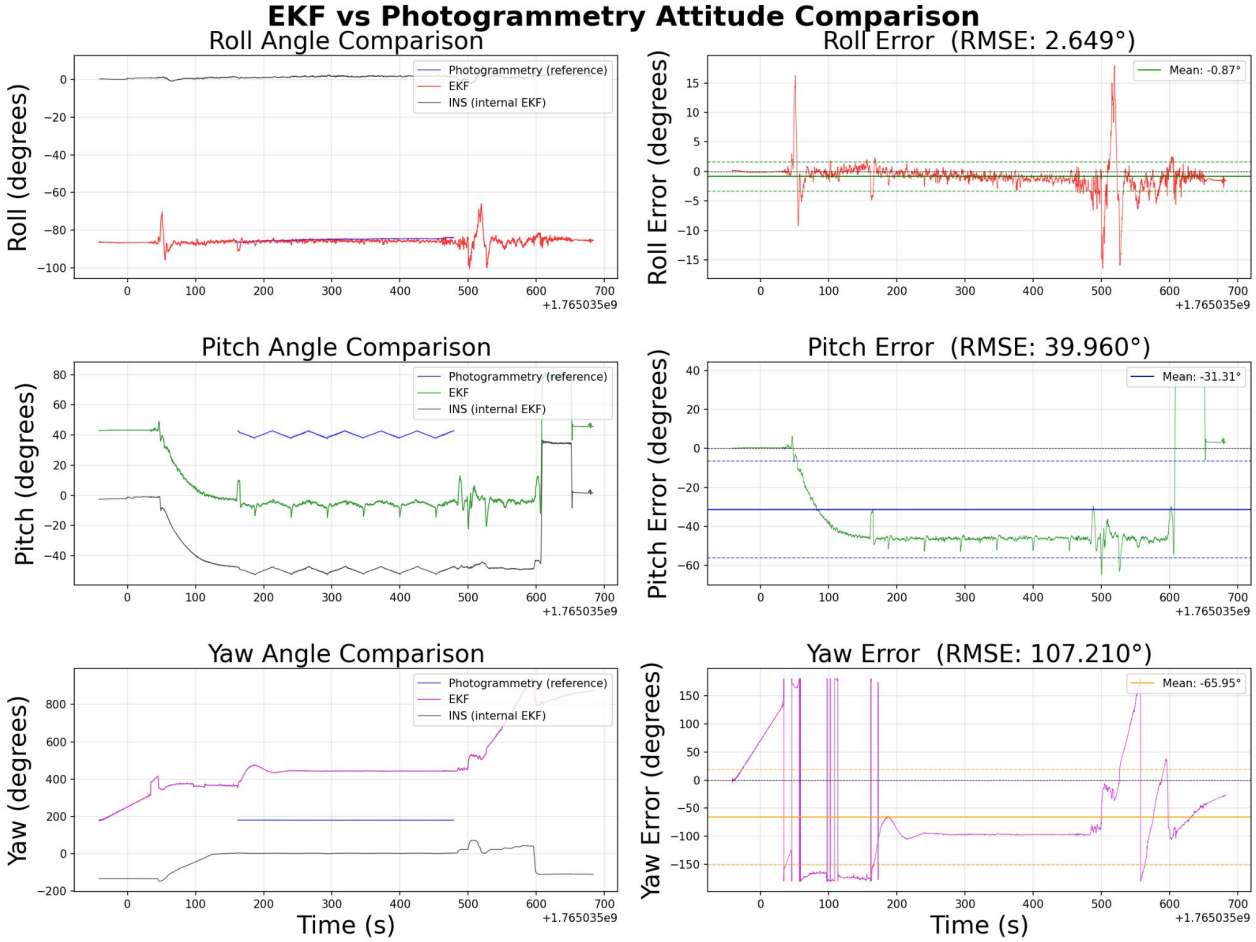
Orientation:

- ❑ Euler's angles
- ❑ Concerning Telescope - drone LOS
- ❑ Yaw → Ψ

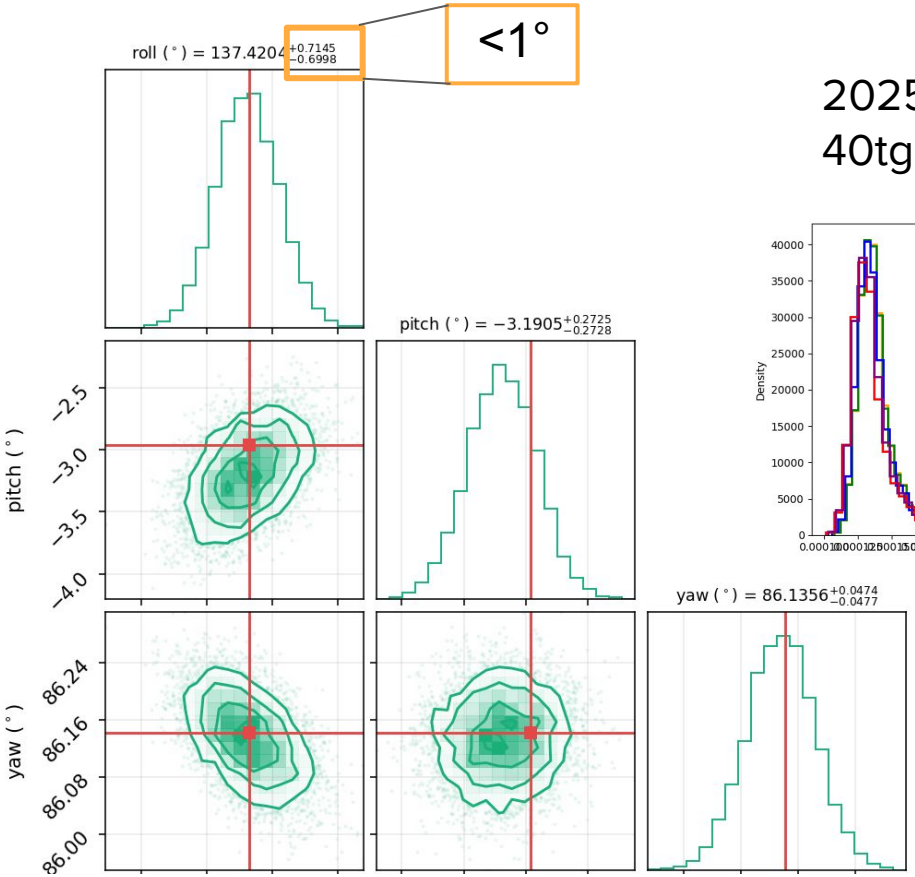


ATTITUDE VALIDATION - EKF

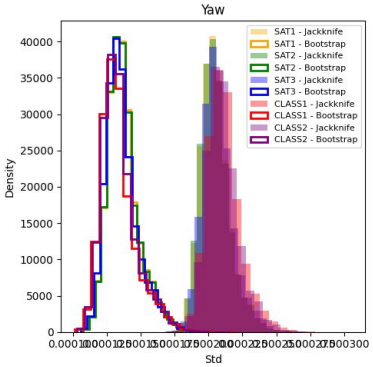
$$\hat{x}_k = \underbrace{f(\hat{x}_{k-1}, u_k)}_{\text{IMU propagation}} + K_k\Big[\underbrace{z_k^{\text{photo}}}_{\text{photogrammetry}} - \underbrace{h\big(f(\hat{x}_{k-1}, u_k)\big)}_{\text{predicted observation}}\Big]$$

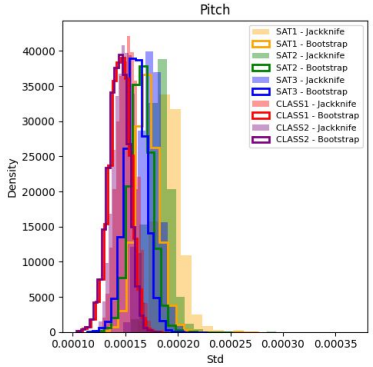


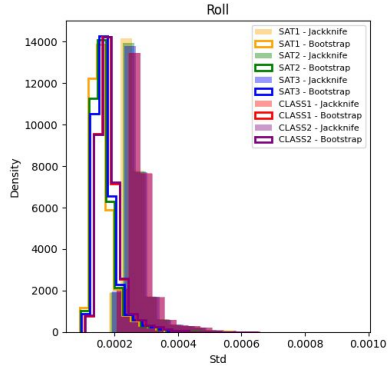
Uncertainties



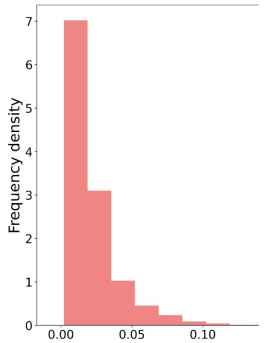
2025
40tgs







2024
20tgs

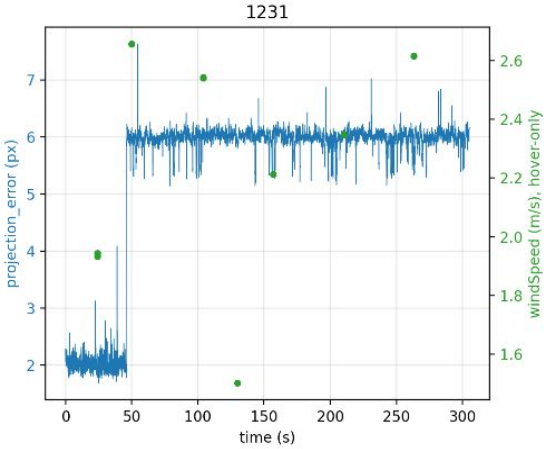


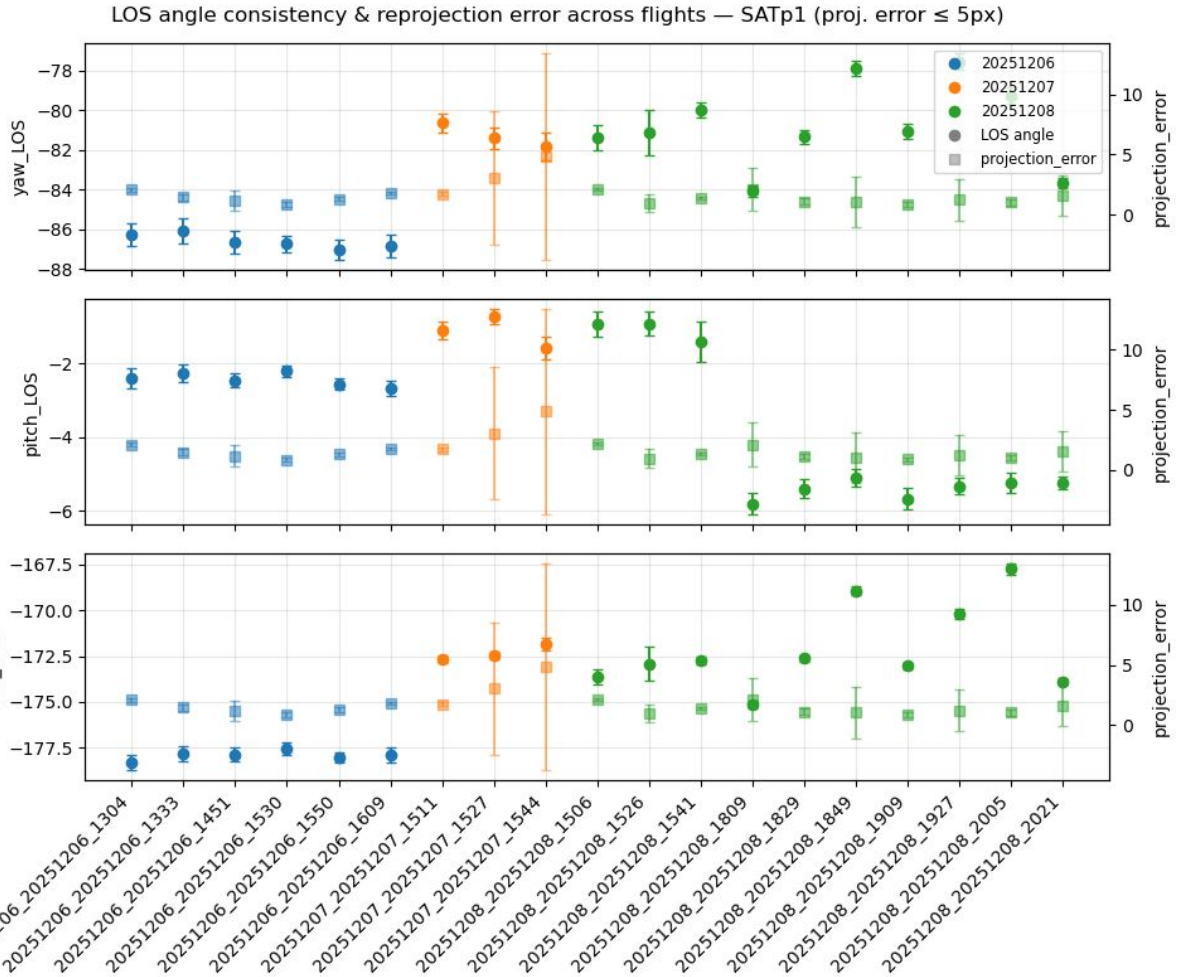
CM
Resolution

Reproducibility

pol ang avg solution in 3 days:

- different POI
- W or S flights
- reprojErr < 5px
- wind





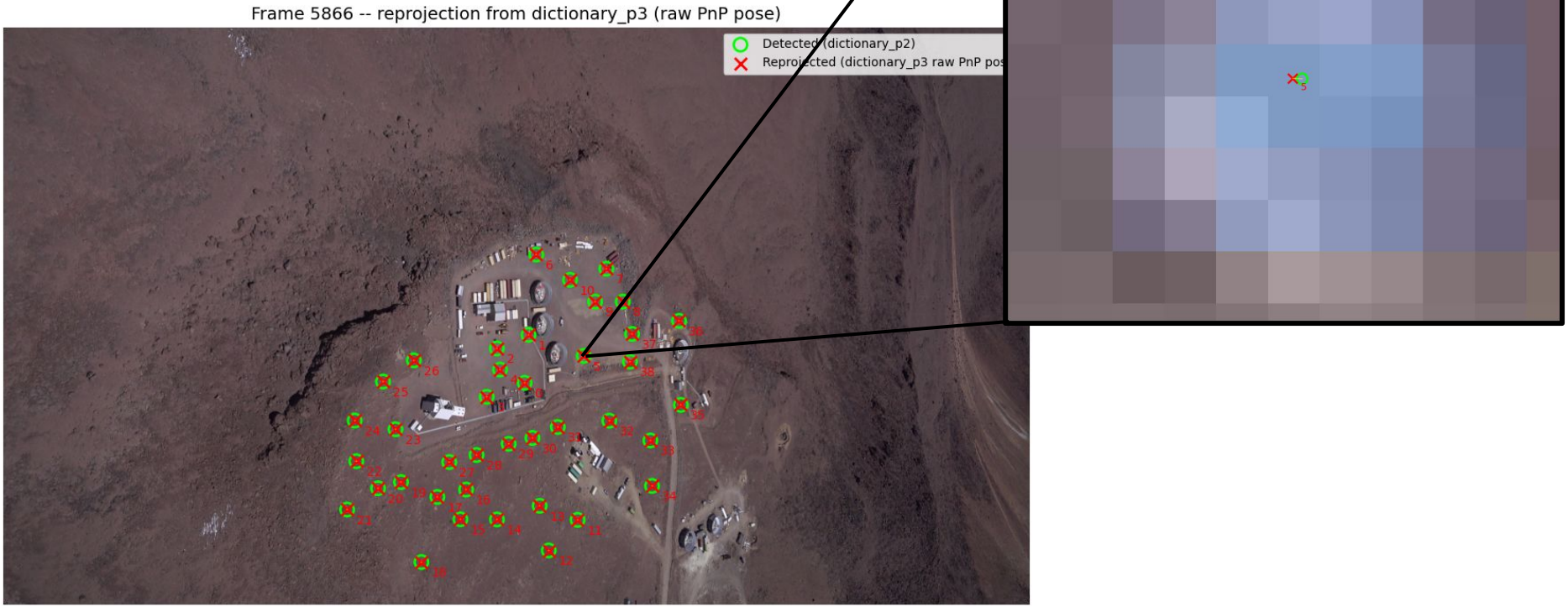
What’s next?

1) Improve the hardware
2) Improve the camera calibration strategy
3) Improve the consistency among the flights and their analysis
4) Include EKF solution in the ARS (better lever-arm calibration)
5) Include in the EKF the uncertainties

Thanks!

Backup Slides

Reprojection Error



EKF - Phases

Prediction phase:

1) Theoretical dynamic model;
2) by reading the gyroscope move the model in the body RF;
3) by using the model the state vector is computed;
4) Update sensor noise and covariance computing the jacobian of the real world model.

Update phase:

1) Measured model estimate is compared to accelerometer values;
2) Compute the residuals between the theoretical and the measured model (how much the state vec need to shift to match the world)
3) Compute the gain to weight the uncertainty (how much we trust the sensor)
4) Jacobian to estimate uncertainties
5) Update state vector by correcting the prediction
6) gain+jacobian update the covariance matrix

EKF - model

Dynamic Model Equations

The dynamic model uses the following equations for quaternion estimation:

$$\dot{\mathbf{q}} = \frac{1}{2}\,\Omega(\boldsymbol{\omega}_{\text{measured}} - \mathbf{b}) \cdot \mathbf{q}$$

Where:

- $\boldsymbol{\omega}_{\text{measured}} = [\omega_x, \omega_y, \omega_z]^T$ is the **measured angular velocity** from the gyroscope.
- $\mathbf{b} = [b_x, b_y, b_z]^T$ is the **estimated gyroscope bias** from the state vector.
- $\Omega(\cdot)$ is the skew-symmetric matrix for quaternion dynamics, defined as:

$$\Omega(\boldsymbol{\omega}) = \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix}$$

The quaternion is integrated over time using the gyroscope data, normalized to ensure it remains a valid unit quaternion.