MASSACHUSETTS INSTITUTE OF TECHNOLOGY
MIT CENTER FOR THEORETICAL PHYSICS – A LEINWEBER INSTITUTE

FWF Österreichischer Wissenschaftsfonds

Ready, Jet, Quench: Jets in the Early Nonequilibrium Stages of Heavy-Ion Collisions

Mainly based on
Phys.Rev.D 113 (2026) 11, 114009 [Barata, Boguslavski, FL, Sadofyev]

Florian Lindenbauer (MIT Center for Theoretical Physics – a Leinweber Institute)

August 27, 2026, Jyväskylä, Finland

Outline

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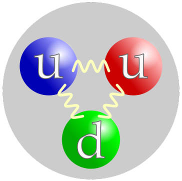
Standard model and QCD

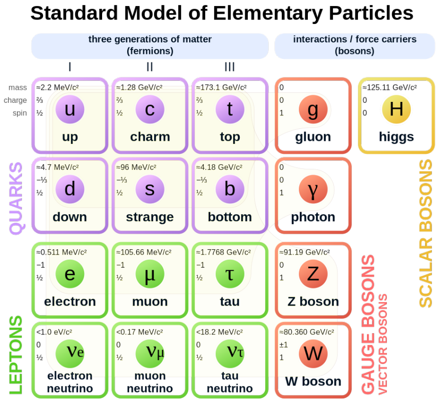
[Cush, Wikipedia]

[Arpad Horvath, Wikipedia]

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \bar{\psi}(i\not{D} - m)\psi$$





At “room” temperature quarks **confined** into hadrons

Standard model and QCD

[Arpad Horvath, Wikipedia]

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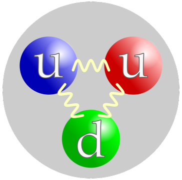

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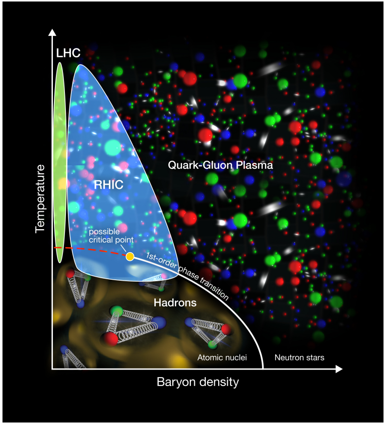
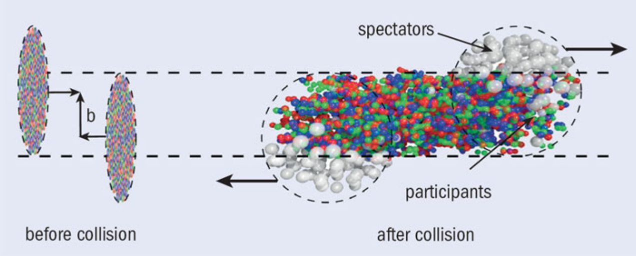


[BNL]

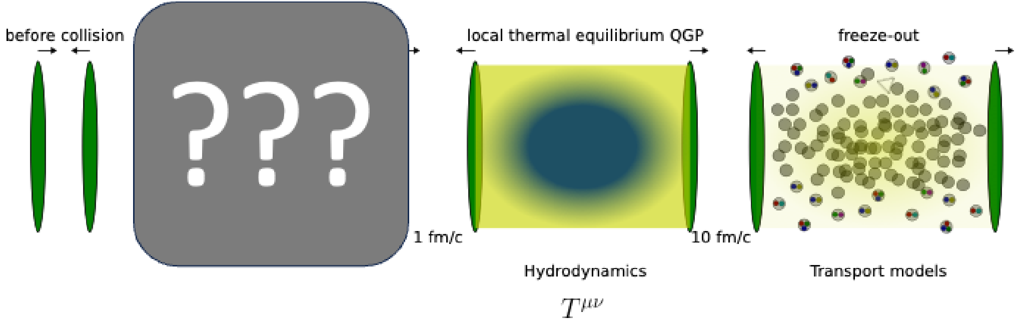
Heavy-ion collisions and the quark-gluon plasma

- Study high-temperature properties of the strong interaction (QCD)
- Collision of atomic nuclei at LHC and RHIC
- Creates high-temperature QCD matter = Quark-Gluon plasma (QGP)



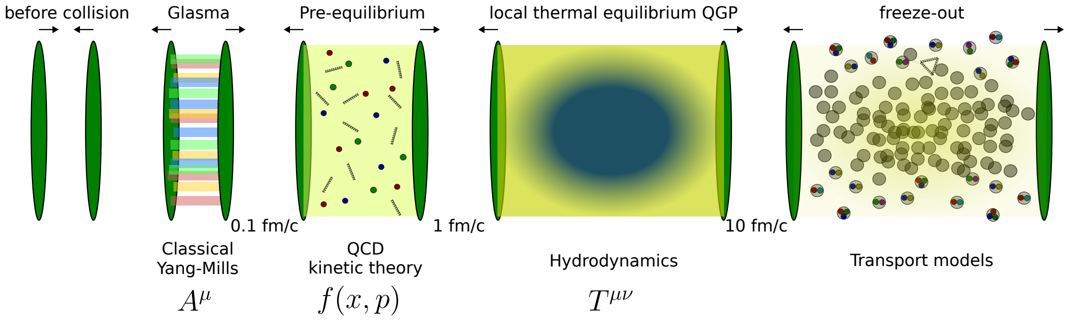
[Alberica Toia 2013, CERN COURIER]

Time-evolution of the QGP in heavy-ion collisions



Far-from-equilibrium initial state $\rightarrow$ (close-to-equilibrium?) **hydrodynamic plasma**

Weak-coupling picture of thermalization



Interested in pre-equilibrium stages ("Initial stages")

$\rightarrow$ **QCD out of equilibrium** $\rightarrow$ Use **QCD kinetic theory** to simulate plasma evolution

[Rev.Mod.Phys. 93 (2021) [Berges, Heller, Mazeliauskas, Venugopalan]]

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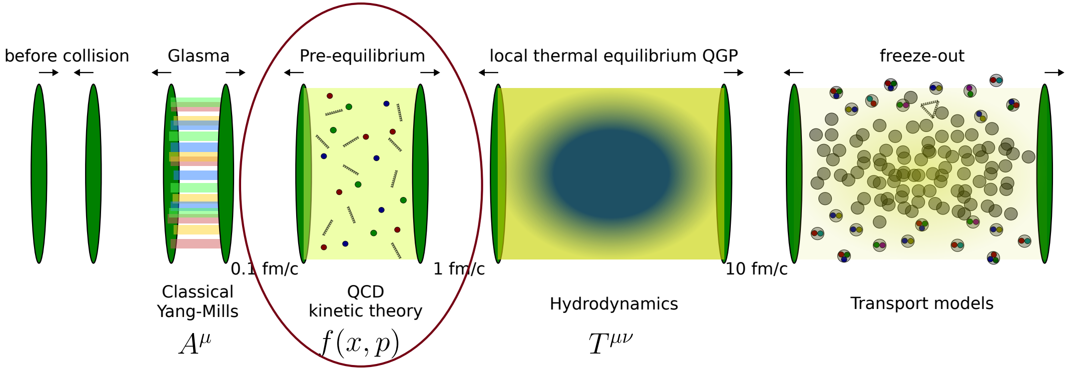

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Effective kinetic theory (EKT) description of the QGP

- Gluons with **distribution function** $f(t, \mathbf{x}, \mathbf{p}) \leftarrow$ How many particles/gluons

[1] [JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]]

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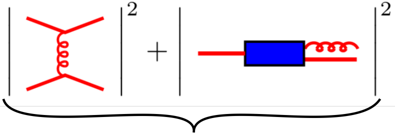
$$(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla})f =$$



Collision term

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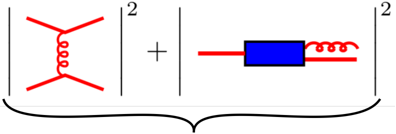

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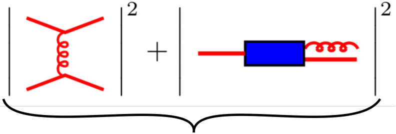
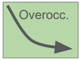
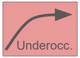


Collision term

- 'Stable' particles between interactions, mean free path $\gg$ duration of scattering processes
- Consider 3 types of plasmas:



Isotropic
Initially overocc.



Isotropic
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Bjorken
expanding

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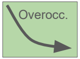
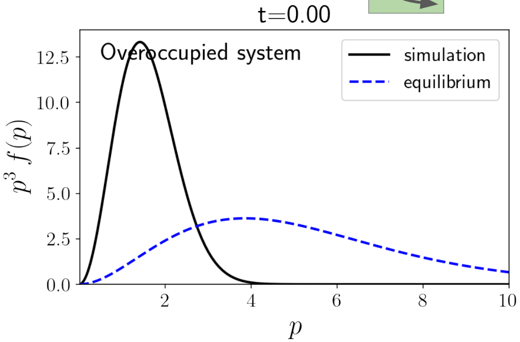
Thermalization in isotropic plasmas





- (Left): Initially over-occupied system
- (Right): Underocc. system: Soft thermal bath formed (bottom-up thermalization)

First studied in Phys.Rev.Lett. 133 (2014) [Kurkela, Lu], see also Phys.Rev.D 105 (2022) [Fu, Ghiglieri, Iqbal, Kurkela]

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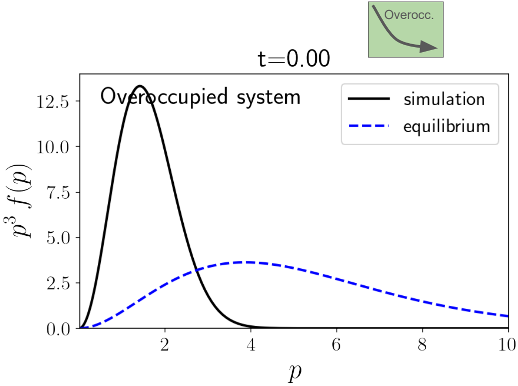

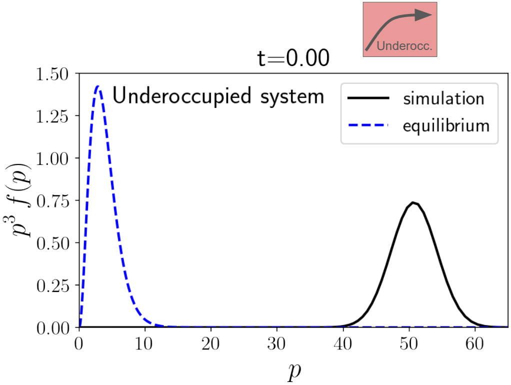

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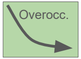

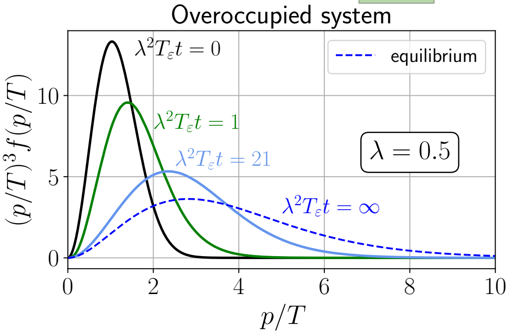

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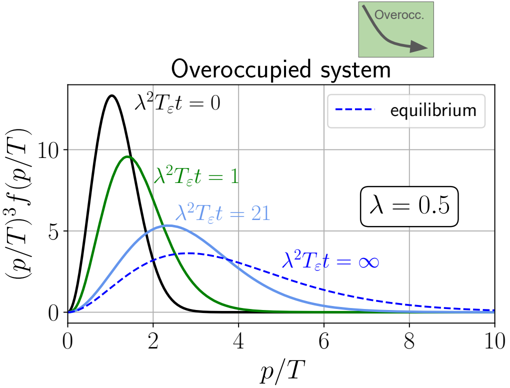

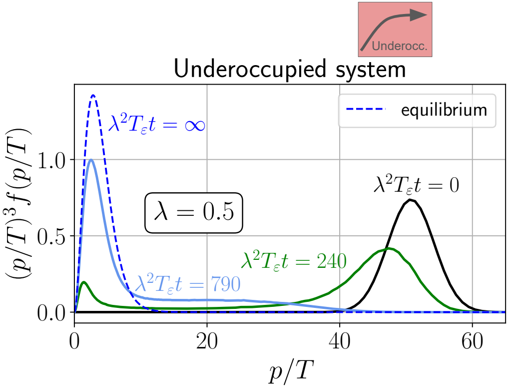

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Bottom-up thermalization in heavy-ion collisions

Initial condition[2], with $\lambda = g^2 N_\mathrm{C}$:

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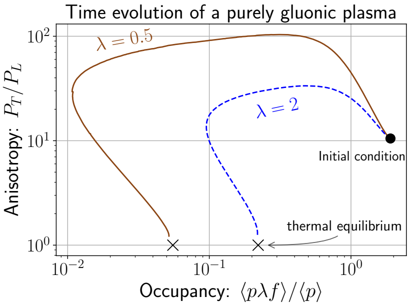
Expand.



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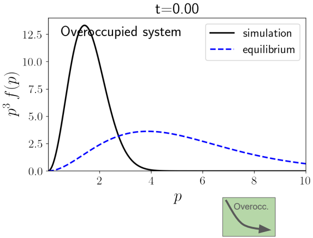

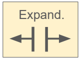

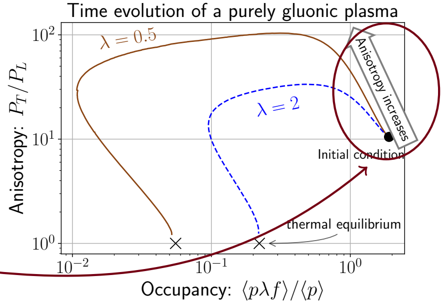

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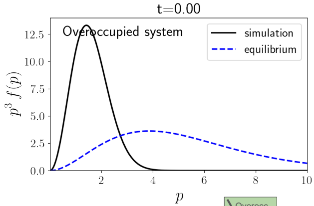

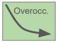

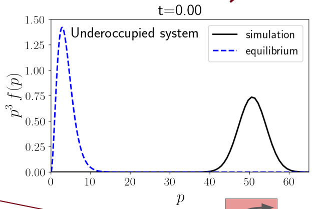

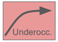

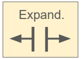

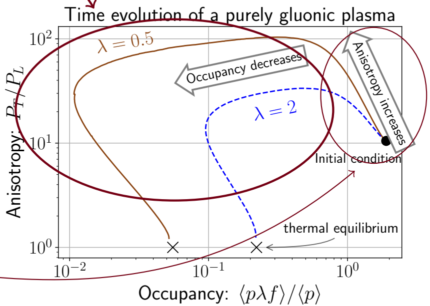

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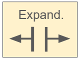

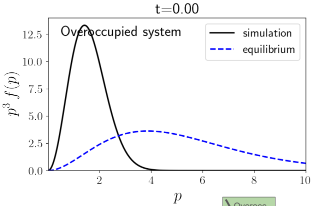

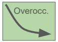

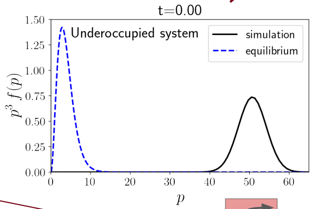

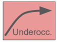

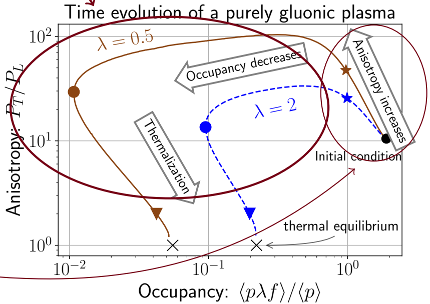

- **Markers** represent **different stages**
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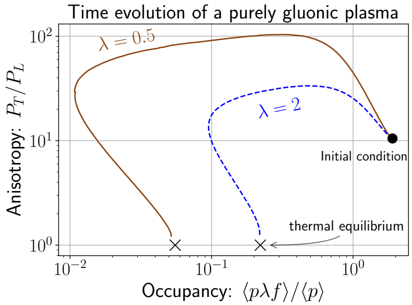

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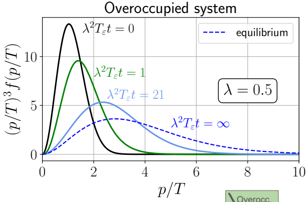

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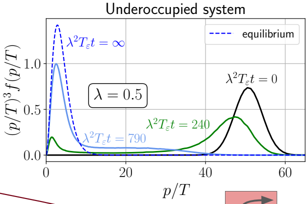

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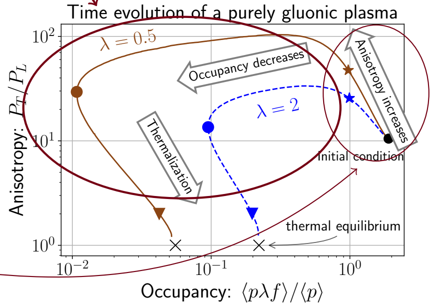

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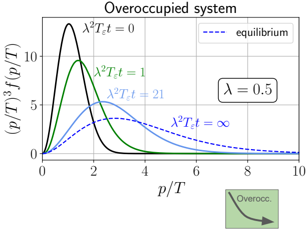

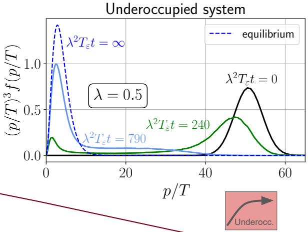

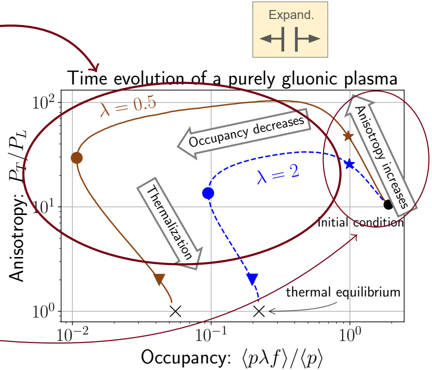

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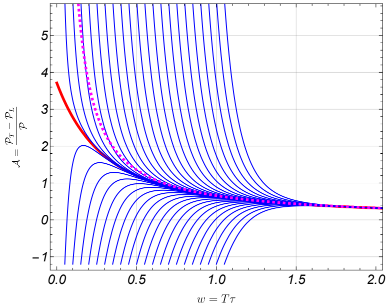
Hydrodynamization: Hydrodynamic attractors

- Collapse to **lower-dimensional surface** for wide range of initial conditions
- Complexity of initial state quickly reduced, rapid **loss of information**



[Phys.Rev.Lett. 115 (2015) [Heller, Spalinski]]

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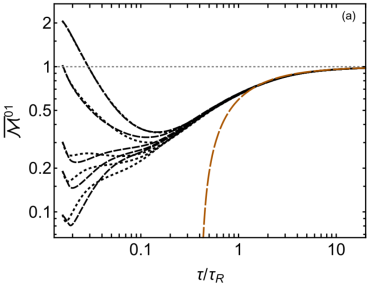

[Phys.Rev.Lett. 125 (2020) [Almaalol, Kurkela, Strickland]]

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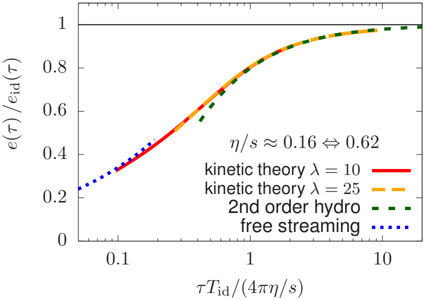

[Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, Phys.Rev.Lett. 122 (2019)]

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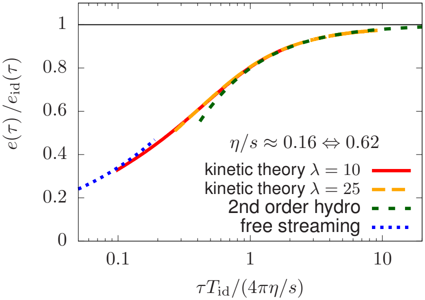
- Only time scale in conformal first-order hydro,

$$\frac{P_L}{P_T} = 1 - 8\frac{\tau_R}{\tau}$$



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Possible probes of initial stages

- Produced early (energetic or heavy):
  - Jets

    Phys.Lett.B 803 (2020) [Andres, Armesto, Niemi, Paatelainen, Salgado], JHEP 08 (2023) 027 [Hauksson, Iancu], Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron], JHEP 12 (2024) [Barata, Salgado, Silva] arXiv:2509.03868 [Altenburger, Boguslavski, FL], arXiv:2509.19430 [Pablos, Takacs], arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev], arXiv:2512.17009 [Barata, Milhano, Sadofyev, Silva], . . .
  - Heavy quarks, quarkonium

    Phys.Rept. 858 (2020) [Rothkopf], Phys.Rev.D 109 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron], Phys.Rev.C 109 (2024) [Du], Phys.Rev.Lett. 134 (2025) [Avramescu, Greco, Lappi, Mäntysaari, Müller], . . .
- Weakly interacting with background:
  - Lepton production

    Nucl.Phys.A 1030 (2023) [Coquet, Du, Ollitrault, Schlichting, Winn], Phys.Rev.D 111 (2025) [Garcia-Montero, Plaschke, Schlichting]
  - Photon production/polarization?

    JHEP 03 (2024) [Garcia-Montero, Mazeliauskas, Plaschke, Schlichting], Phys.Rev.C 109 (2024) [Hauksson, Gale]

Possible probes of initial stages

- Produced early (energetic or heavy):
  - **Jets**

    Phys.Lett.B 803 (2020) [Andres, Armesto, Niemi, Paatelainen, Salgado], JHEP 08 (2023) 027 [Hauksson, Iancu], Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron], JHEP 12 (2024) [Barata, Salgado, Silva] arXiv:2509.03868 [Altenburger, Boguslavski, FL], arXiv:2509.19430 [Pablos, Takacs], arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev], arXiv:2512.17009 [Barata, Milhano, Sadofyev, Silva], . . .
  - Heavy quarks, quarkonium

    Phys.Rept. 858 (2020) [Rothkopf], Phys.Rev.D 109 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron], Phys.Rev.C 109 (2024) [Du], Phys.Rev.Lett. 134 (2025) [Avramescu, Greco, Lappi, Mäntysaari, Müller], . . .
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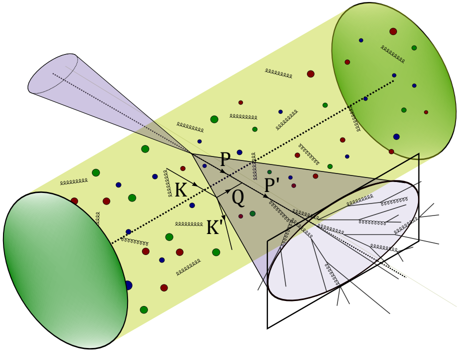
Jets in heavy-ion collisions

- **Jet** = Particle shower from **energetic parton** created in initial collision

→ Imprints of **pre-equilibrium plasma**

- Many event generators use simplifying assumptions (isotropic, thermal, . . .)



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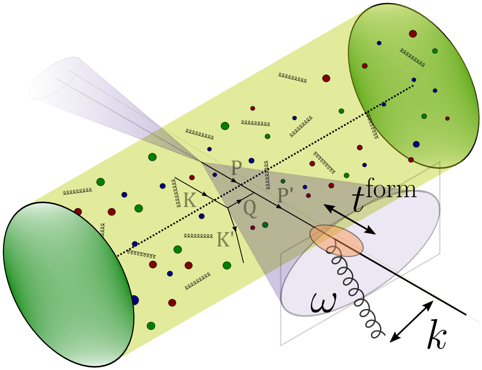

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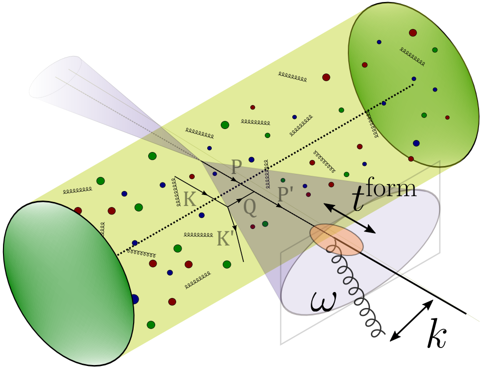
  - $\mathrm{d}I/\mathrm{d}\omega$: **Spectrum** $=$Probability of emitting gluon with energy ω (relative to vacuum)



- ω: emitted gluon energy
- k: transverse momentum
- $t^{\mathrm{form}} \sim \sqrt{\omega/\hat{q}}$: formation time

Outline

Obtaining the gluon emission spectrum

Phys.Rev.D 79 (2009) [Arnold]
JETP Lett. 65 (1997) [Zakharov]
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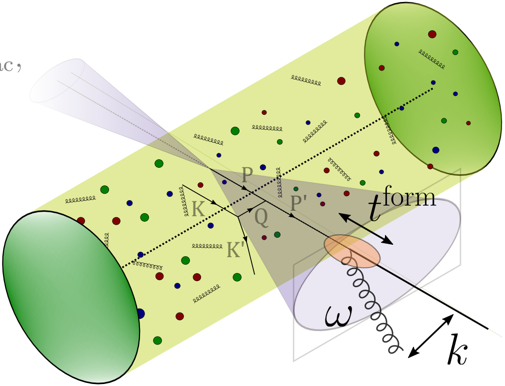

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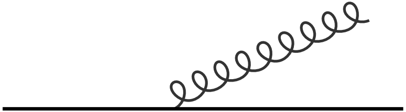

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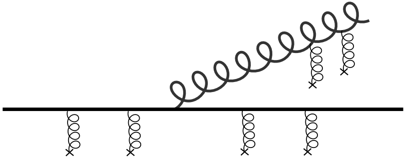

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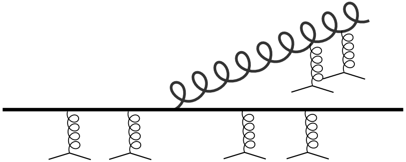

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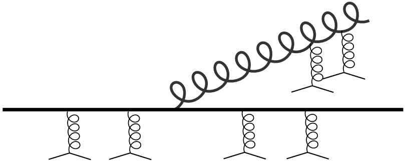

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energy shift

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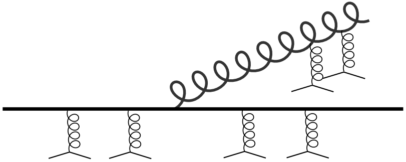

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$$C(\mathbf{x}) = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \left(1 - e^{i\mathbf{x}\cdot\mathbf{q}_\perp}\right), \qquad C(\mathbf{q}_\perp) = (2\pi)^2 \frac{\mathrm{d}\Gamma^{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

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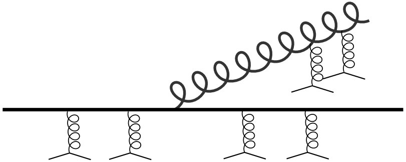

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'broadening probability'
Probability of receiving momentum kick $\mathbf{q}_\perp$

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Harmonic approximation for radiation

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- $\hat{q}$ quantifies **momentum broadening**, "jet quenching parameter"
- Schrödinger equation for harmonic oscillator $\to$ analytic solution!

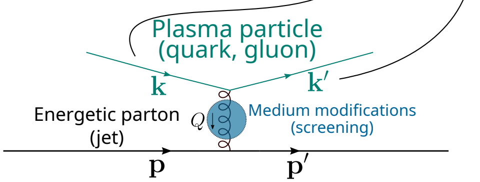
$$\text{Spectrum} = \text{function of } c(t) \quad \xrightarrow{\text{“simple formula”}} \quad \ddot{c}(t) = i\frac{\hat{q}(t)}{2\omega} c(t)$$

Results for $\hat{q}$: Broadening

Phys.Rev.D 110 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]
Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

- Obtain **Jet quenching parameter** from kinetic theory, input $f(\mathbf{k})$,

$$\hat{q}^{ij} = \int_{\substack{q_\perp < \Lambda_\perp \\ p\to\infty}} \mathrm{d}\Gamma\, q^i q^j |\mathcal{M}|^2 f(k)(1+f(k'))$$



Note: Isotropic screening to prevent plasma instabilities! Phys.Lett.B 214 (1988) [Mrowczynski], Phys.Rev.D 68 (2003) [Romatschke, Strickland]

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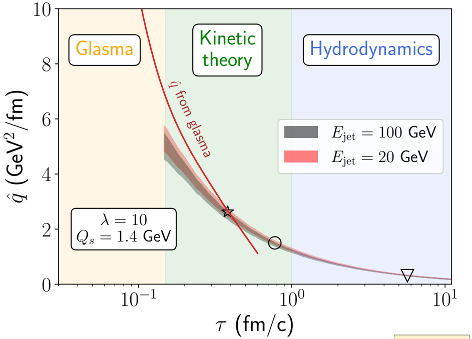
- Little jet energy dependence

$\hat{q}$ during pre-equilibrium stage



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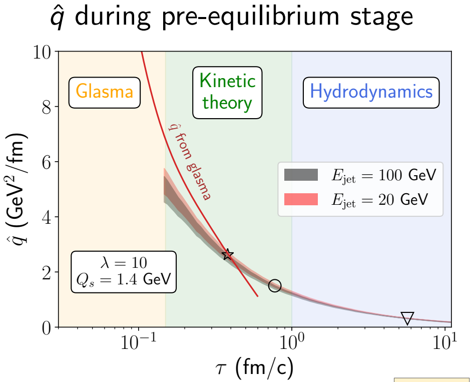
- **Bands:** Vary cutoff and initial conditions
- Little jet energy dependence
- Supports hydrodynamic values and **large values** in **Glasma**

Glasma values from Phys.Lett.B 810 (2020) [Ipp, Müller, Schuh]]



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scattering rate

Probability of receiving kick $q_\perp$

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Can we do better? $\to$ Consider full kernel $C(\mathbf{q}_\perp, t)$ for $C(\mathbf{x}, t)$

Angular dependence and contribution to $\hat{q}$

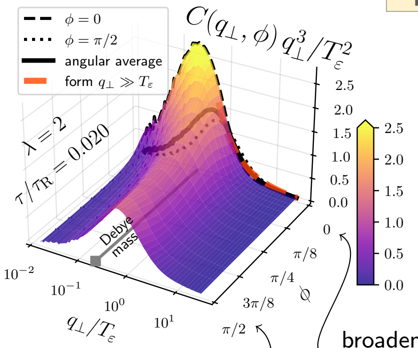
arXiv:2509.03868 [Altenburger, Boguslavski, FL]

Expand.

- Momentum broadening kernel:

$$C(\mathbf{q}_\perp) = \int \mathrm{d}\Gamma_{\mathrm{PS}}\, |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k}-\mathbf{q}))$$

- $C(q_\perp, \phi)$ is 2D function



broadening along beam

broadening transverse to beam

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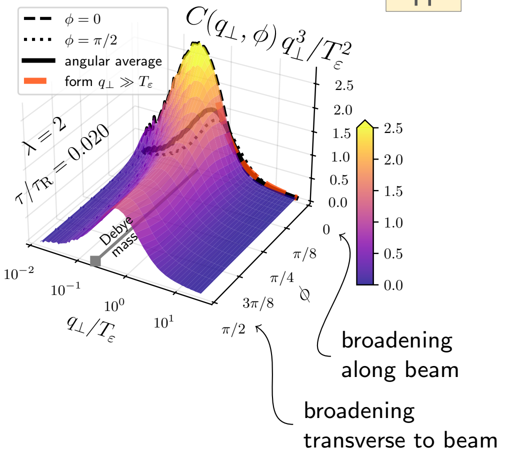
- $C(q_\perp, \phi)$ is 2D function
- More differential than $\hat{q}$,

$$\hat{q}^{ij} = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} q^i q^j C(\mathbf{q}_\perp),$$

- Along beam ($\phi_{pq} = 0$): Much larger and different form at early times

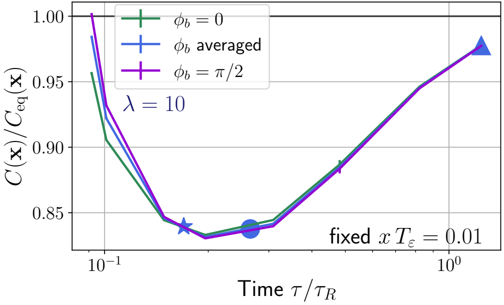


Small-distance behavior of $C(\mathbf{x})$

arXiv:2509.03868 [Altenburger, Boguslavski, FL]

Expand.

- Obtained via Fourier transform
  $C(\mathbf{x}) = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^3}\left(1 - e^{i\mathbf{q}_\perp\cdot\mathbf{x}}\right)C(\mathbf{q}_\perp)$
- Small-distance behavior important for jet quenching!
- For $\lambda = 10$, always smaller than in equilibrium



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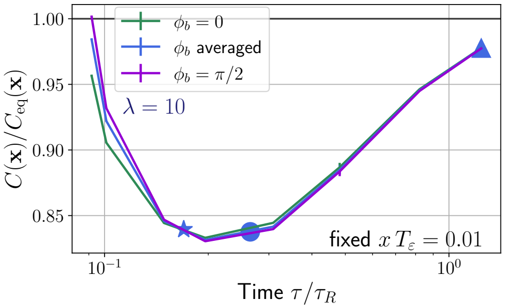
- Small-distance behavior important for jet quenching!
- For $\lambda = 10$, always smaller than in equilibrium
- Take small distance behavior and calculate jet emission spectrum

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]



Small-distance form of $C(x)$

- Proper expansion for small distance:

medium input,
obtained from $\hat{q}(\Lambda_\perp)$

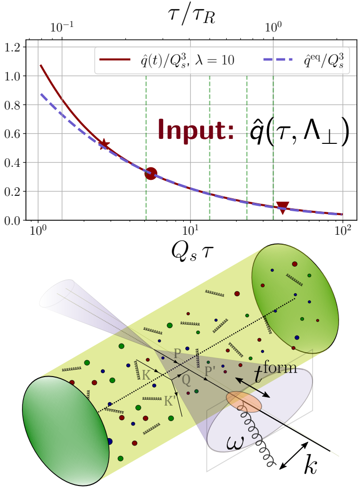
$$C(\mathbf{x},t) = \frac{1}{4}\hat{q}_0(t)\mathbf{x}^2 \log\frac{1}{\mathbf{x}^2\mu_*^2(t)}$$

- **Expand** perturbatively **around harmonic oscillator solution**
  $\rightarrow$ "improved opacity expansion" JHEP 09 (2021) [Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk]

$$C(\mathbf{x},t) = \underbrace{\frac{\hat{q}_0(t)}{4}\mathbf{x}^2\log\frac{Q^2}{\mu_*^2}}_{\text{harmonic approximation}} - \underbrace{\frac{\hat{q}_0(t)}{4}\mathbf{x}^2\log(Q^2\mathbf{x}^2)}_{\text{perturbation}}$$

Obtaining the spectrum – Schematic

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]



Input: $\hat{q}(\tau, \Lambda_\perp)$

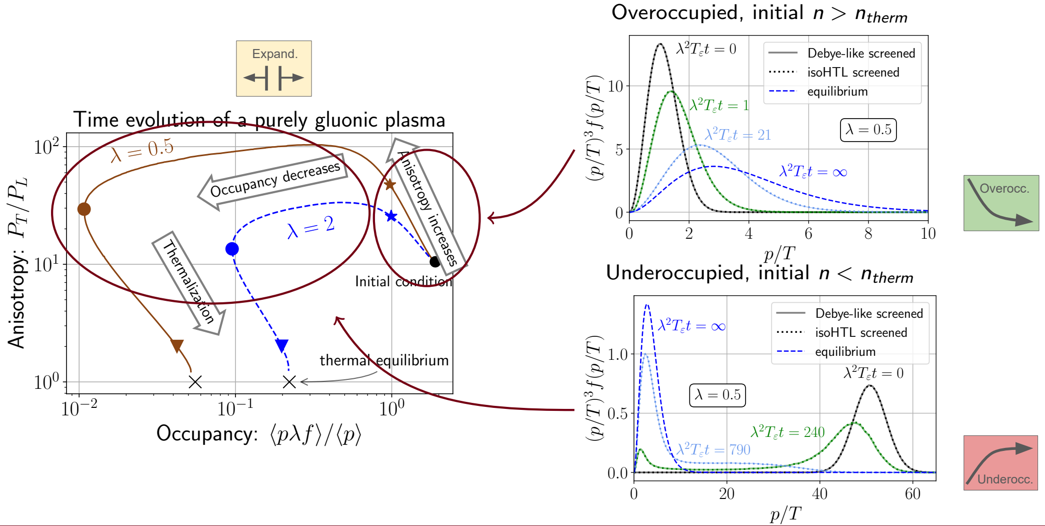
$\hat{q}_0(\tau)$, $\mu_*(\tau)$

Small-distance form of potential

"Improved opacity expansion"

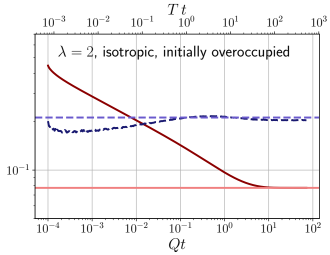
Output: Spectrum $\frac{\mathrm{d}I}{\mathrm{d}\omega\,\mathrm{d}^2\mathbf{k}}$

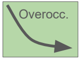
Recall: Bottom-up thermalization

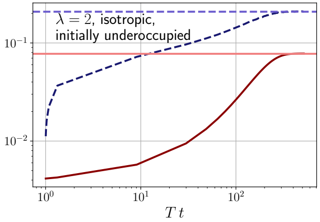


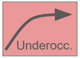
Isotropic toy models

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

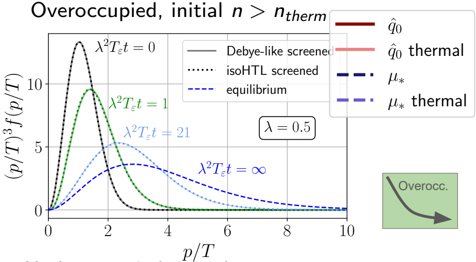




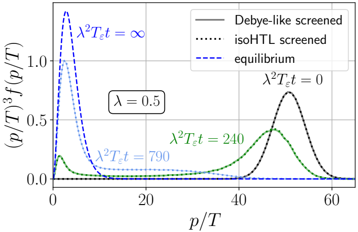


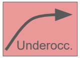


Overoccupied, initial $n > n_{therm}$



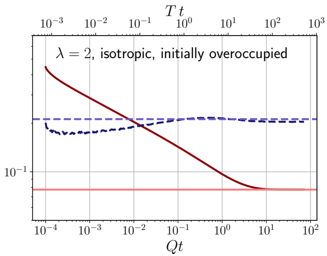
Underoccupied, initial $n < n_{therm}$

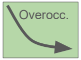


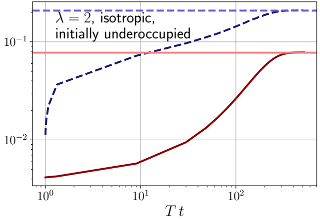


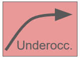
Comparison with isotropic toy models

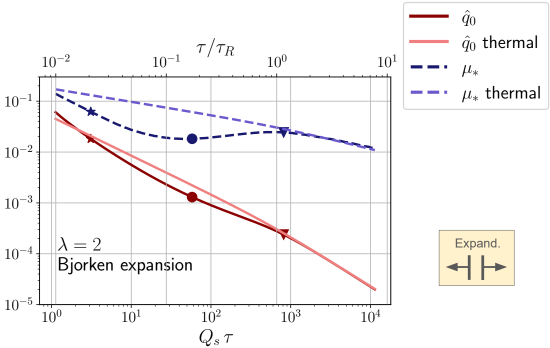
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]







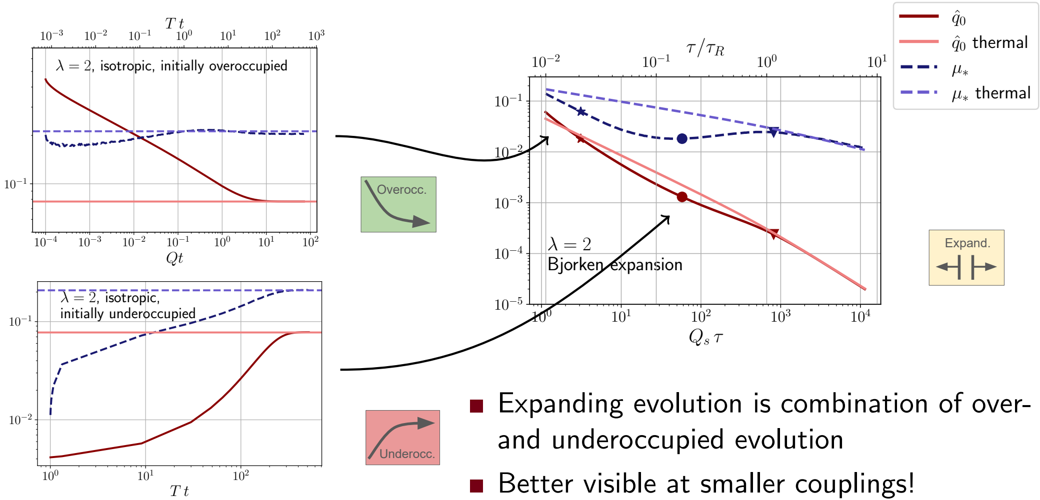




- Expanding evolution is combination of over- and underoccupied evolution
- Better visible at smaller couplings!

Comparison with isotropic toy models

Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



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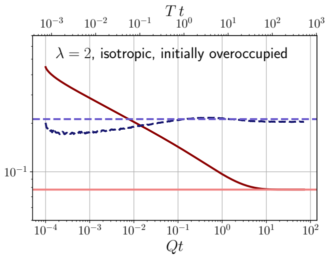

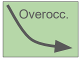

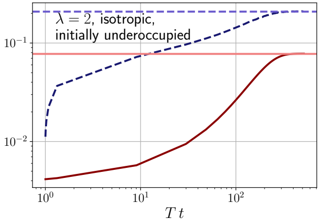

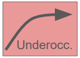

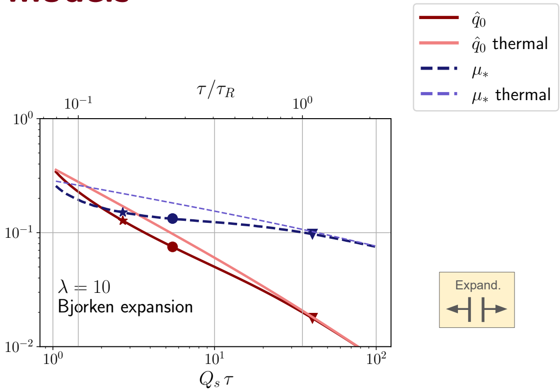

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Outline

Gluon spectrum

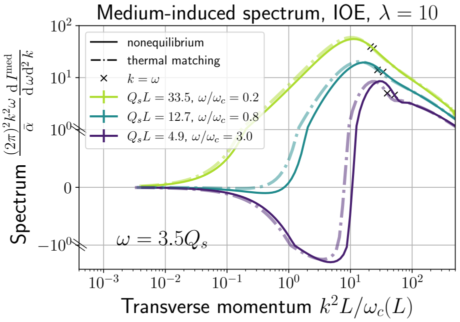
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]

Expand.

- $\hat{q}_0(t),\ \mu_*(t) \to$ spectrum $\mathrm{d}I/\mathrm{d}\omega$
- Consider different medium lengths (colors in plot: green, blue purple)

$\rightarrow$ Set $\hat{q}_0(t) = 0$ for $t > L$

- Form of spectrum dominated by ratio ω/ω_c, $\omega_c = \int_{t_{min}}^{t_{min}+L} \mathrm{d}t\,(t - t_{min})\hat{q}(t)$
- All information, difficult to analyze



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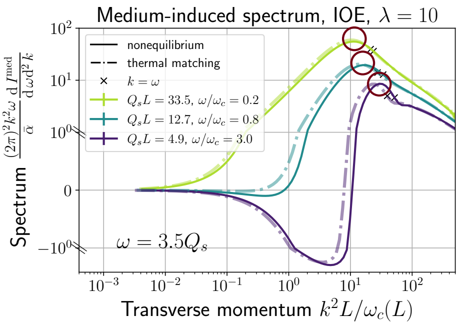


Comparison to equilibrium

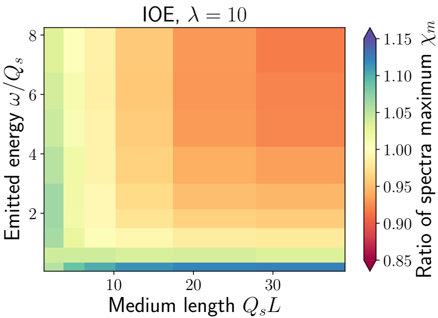
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]

Expand.

- Comparison to equilibrium: Maximum of spectrum plot

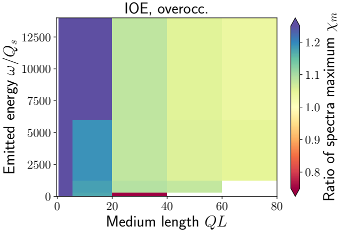
$$\chi_m = \frac{\text{Maximum spectrum nonequ.}}{\text{Maximum spectrum equ.}}$$

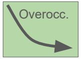
- Clear systematics visible
- Does not approach 1 even for $L \to \infty$!



Comparison to equilibrium II

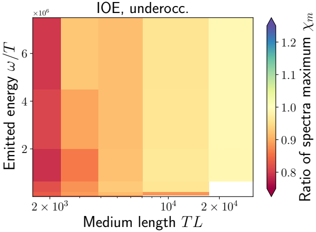
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]

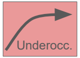




initially overoccupied

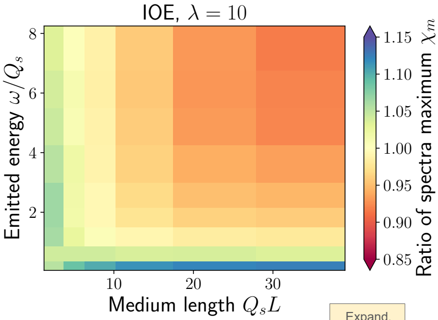
$\chi > 1$

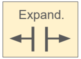




then underoccupied

$\chi < 1$





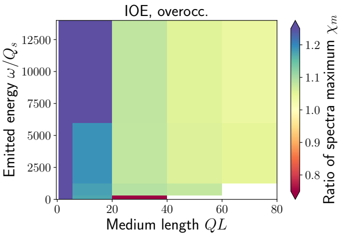
- For fixed L:

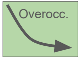
  $\chi > 1 \rightarrow \chi < 1$

$\rightarrow$ Can be explained with toy models

Comparison to equilibrium II

Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]





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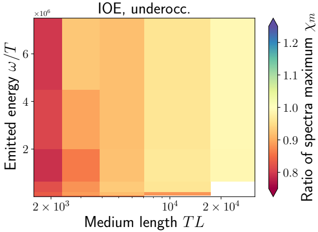

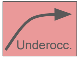

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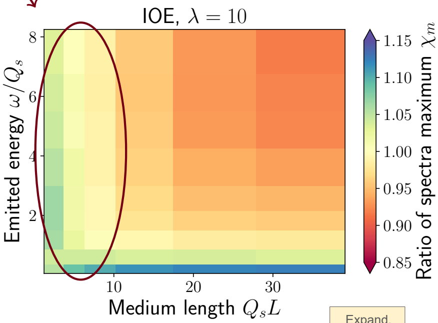

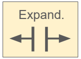

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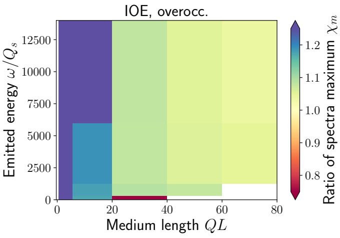

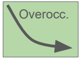

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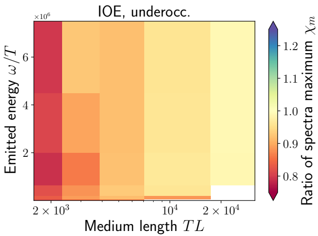

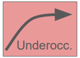

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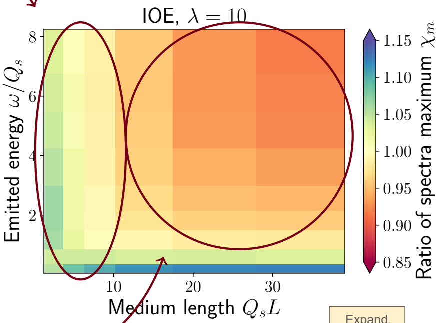

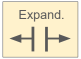

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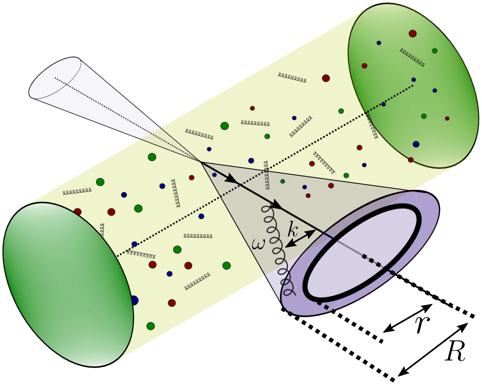
Jet shape

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

- Energy deposited in *annulus* with radii r and R

$$\frac{\mathrm{d}\rho(r,R)}{\mathrm{d}\omega} = \int_{r\omega}^{R\omega} \mathrm{d}^2\mathbf{k}\,\omega \frac{\mathrm{d}I}{\mathrm{d}\omega\,\mathrm{d}^2\mathbf{k}}$$



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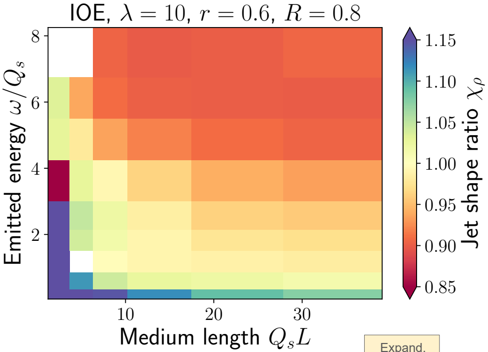
- Qualitatively same behavior as spectra maximum



Expand.

Outline

Bottom-up vs. hydrodynamic attractor

- **Bottom-up** expects thermalization around $\tau_{\mathrm{BMSS}} = \alpha_s^{-13/5}/Q_s$
- **Hydrodynamics** expects thermalization around $\tau_R = \frac{4\pi\eta/s}{T}$

How to reconcile these different time scales?

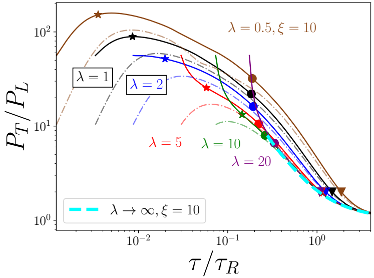
Method: Kinetic theory simulations for

- **different couplings**: $0.5 \leq \lambda \leq 20$
- **initial conditions**: vary anisotropy parameter ξ

Pressure ratio

$$\tau_R = \frac{4\pi\eta/s}{T}, \ \tau_{\mathrm{BMSS}} = \alpha_s^{-13/5}/Q_s$$

- Rescale quantities with these different time scales
- Define limiting attractors by extrapolating to zero and infinite couplings



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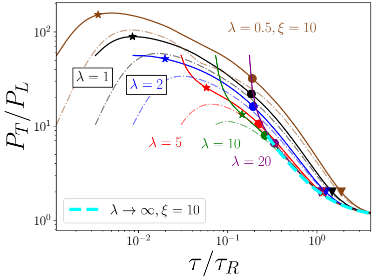
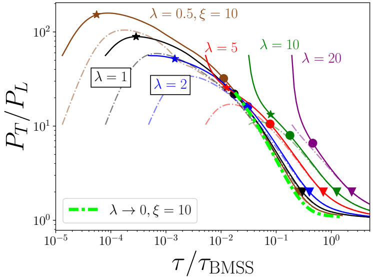




- At weak coupling: Runs approach hydrodynamic attractor at very late times
- Curves **approach limiting attractors** after •

Extrapolation to limiting attractors

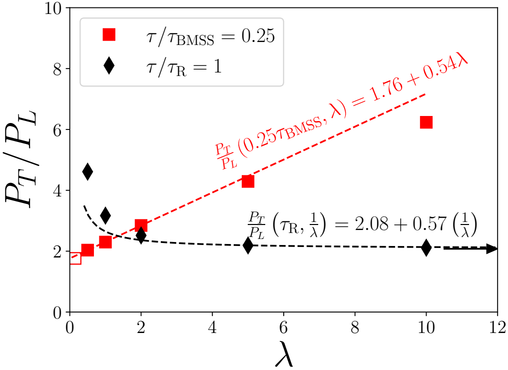
$$\tau_R = \frac{4\pi\eta/s}{T}$$
$$\tau_{\rm BMSS} = \alpha_s^{-13/5}/Q_s$$

- Obtain limiting attractors by extrapolating at fixed τ/τ_R or $\tau/\tau_{\rm BMSS}$

- **Bottom-up attractor**: Linear extrapolation to $\lambda \to 0$, i.e.,

$$\frac{P_T}{P_L}(\tau/\tau_{\rm BMSS}) = a(\tau/\tau_{\rm BMSS}) + \lambda b(\tau/\tau_{\rm BMSS})$$

- **Hydro attractor**: Linear extrapolation to $1/\lambda \to 0$



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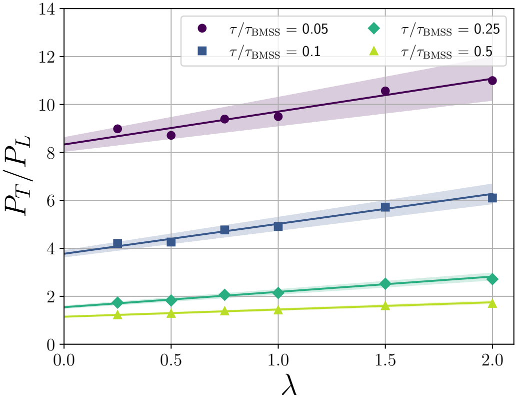
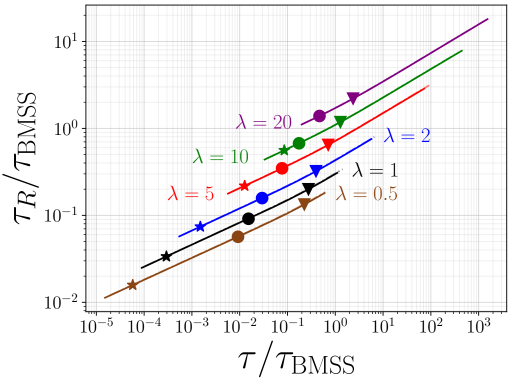


Different thermalization time scales for hydro and kinetic theory

[Phys.Lett.B 852 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

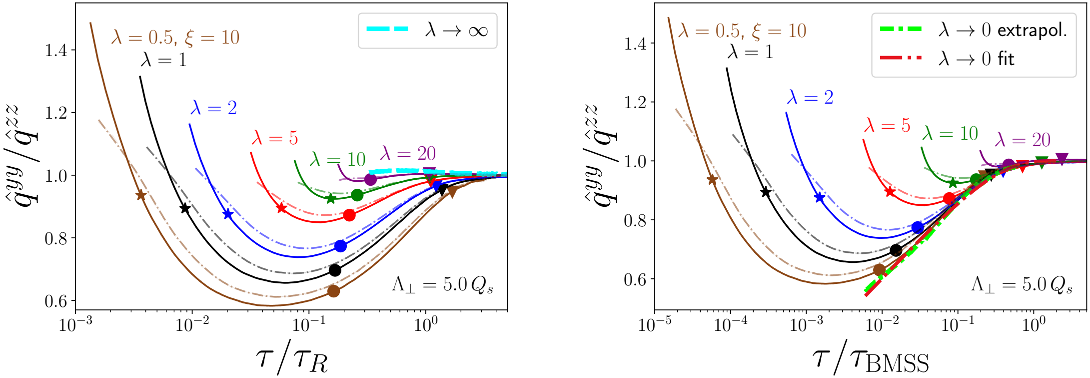
- $\tau_R = \frac{4\pi\eta/s}{T(\tau)}$
  $\tau_{\mathrm{BMSS}} = \alpha_s^{-13/5}/Q_s$
- Difficult comparison, τ_R function of τ
- Also: Bottom-up[4]: $T_{\max} \sim \alpha_s^{2/5}$, $\eta/s \sim \alpha_s^{-2} \to \tau_R \sim \alpha_s^{-12/5}$
  But: **Strong coupling attractor** only valid for large λ, where $\eta/s \neq \alpha_s^{-2}$
- **Thermalization dominated by largest time scale**



[4] [Phys.Lett.B 502 (2001) [Baier, Mueller, Schiff, Son]]

$\hat{q}$ and the limiting attractors

[Phys.Lett.B 852 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]



- $\hat{q}$ allows extrapolation to **both limiting attractors**
- **Weak-coupling attractor** approached even at **moderate couplings**

Outline

Medium

QCD kinetic theory

Thermalization of QCD plasma

Distribution function $f(p,t)$

Momentum broadening

Isotropic	Anisotropic
$\hat{q}(t), C(q_\perp)$	$\hat{q}_x(t) \neq \hat{q}_y(t)$

Potential

$$C(x,t) = \frac{\hat{q}_0(t)}{4} x^2 \log\frac{1}{x^2 \mu_*^2(t)}$$

$$C(x,t) = \frac{\hat{q}_x(t)x^2 + \hat{q}_y(t)y^2}{2}$$

Medium-induced radiation

"beyond harmonic"

Gluon spectrum $\frac{dI}{d\omega d^2k}$

"harmonic"

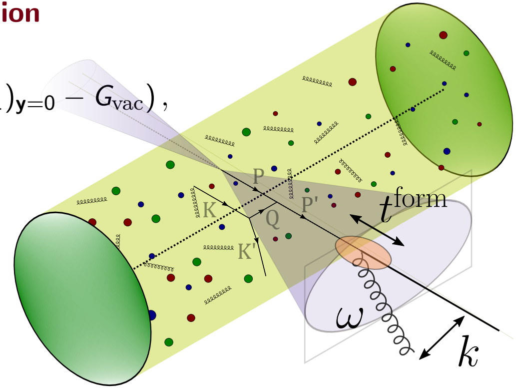
Gluon spectrum $\frac{dI}{d\omega}$

Obtaining the gluon emission spectrum

JETP Lett. 65 (1997) [Zakharov]
Nucl.Phys.B 483 (1997) [Baier, Dokshitzer, Mueller, Peigne, Schiff]

- **Energy loss** dominated by **gluon radiation**

$$\frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \mathrm{Re} \int_{t_0}^{\infty} \mathrm{d}t_2 \int_{t_0}^{t_2} \mathrm{d}t_1 \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} \left(G(\mathbf{x}, t_2; \mathbf{y}, t_1)_{\mathbf{y}=0} - G_{\mathrm{vac}} \right),$$



- ω: emitted gluon energy
- $t^{\mathrm{form}} \sim \sqrt{\omega/\hat{q}}$: formation time

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- Greens function G of 2D Schrödinger equation

$$\left(\partial_t - \frac{\partial_x^2}{2\omega} + \frac{i}{4}\hat{q}(t)x^2\right) = i\delta^2(\mathbf{x}-\mathbf{y})\delta(t_1 - t)$$

[5] [Phys.Rev.D 79 (2009) [Arnold]]

Obtaining the gluon emission spectrum

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Nucl.Phys.B 483 (1997) [Baier, Dokshitzer, Mueller, Peigne, Schiff]

- **Energy loss** dominated by **gluon radiation**

$$\frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \mathrm{Re} \int_{t_0}^{\infty} \mathrm{d}t_2 \int_{t_0}^{t_2} \mathrm{d}t_1 \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} \left(G(\mathbf{x}, t_2; \mathbf{y}, t_1)_{\mathbf{y}=0} - G_{\mathrm{vac}} \right),$$

- Greens function G of 2D Schrödinger equation

$$\left(\partial_t - \frac{\partial_x^2}{2\omega} + \frac{i}{4}\hat{q}(t)x^2\right) = i\delta^2(\mathbf{x} - \mathbf{y})\delta(t_1 - t)$$

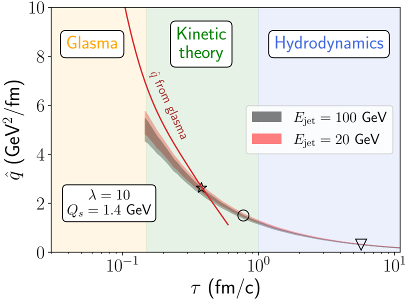
- Isotropic (in transverse plane): Trick: Find spectrum using 'simple formula'[5]

$$\ddot{c}(t) = i\frac{\hat{q}(t)}{2\omega}c(t), \qquad \rightarrow \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \ln|c(0)|$$

5 [Phys.Rev.D 79 (2009) [Arnold]]

Simple formula for jet energy loss (isotropic $\hat{q}$)

Input: $\hat{q}(\tau)$, any time-dependence



'Simple formula'

Output: Spectrum $\frac{\mathrm{d}I}{\mathrm{d}\omega}$

Naive picture:
Mean energy of
single emitted gluon

mean energy loss: $E = \int \mathrm{d}\omega\, \omega \frac{\mathrm{d}I}{\mathrm{d}\omega}$

Generalizing 'simple formula' to anisotropic $\hat{q}$

arXiv:2603.09912 [Boguslavski, Hörl, FL]

- Previous trick[6] not applicable
- Need to perform time integrals numerically

[6] [Phys.Rev.D 79 (2009) [Arnold]]

Generalizing ‘simple formula’ to anisotropic $\hat{q}$

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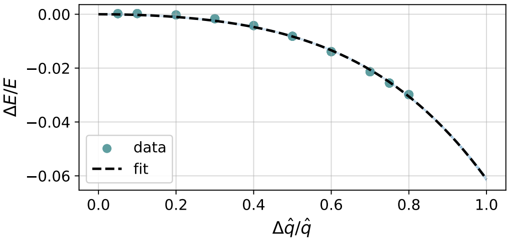
- Previous trick[6] not applicable
- Need to perform time integrals numerically
- Calculate in static brick
- Only **small effect** $< 2\%$ for realistic $\hat{q}$ $\rightarrow$ (may be **observable dependent!**)
- $\hat{q} = \frac{\hat{q}^{zz}+\hat{q}^{yy}}{2}$, $\Delta\hat{q} = \frac{\hat{q}^{zz}-\hat{q}^{yy}}{2}$
- $\Delta E = E_{\mathrm{aniso}} - E_{\mathrm{iso}}$

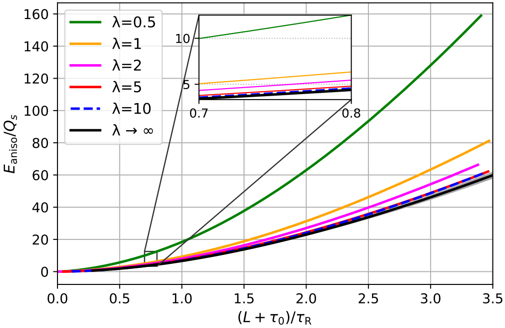


[6] [Phys.Rev.D 79 (2009) [Arnold]]

Plugging $\hat{q}(\tau)$ evolution

Other estimates using hydro attractor:
Phys.Rev.Lett. 122 (2019) [Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney],
Phys.Rev.Lett. 123 (2019) [Giacalone, Mazeliauskas, Schlichting],
JHEP 06 (2024) [Zhou, Brewer, Mazeliauskas], ...

- Now use $\hat{q}(t)$ from previous simulation
- Use 'corresponding brick'[7]
  $\hat{q}_x^{\mathrm{eff}}(L) = \frac{2}{L^2}\int_{\tau_0}^{L+\tau_0}(\tau-\tau_0)\,\hat{q}_x(\tau)\,\mathrm{d}\tau$



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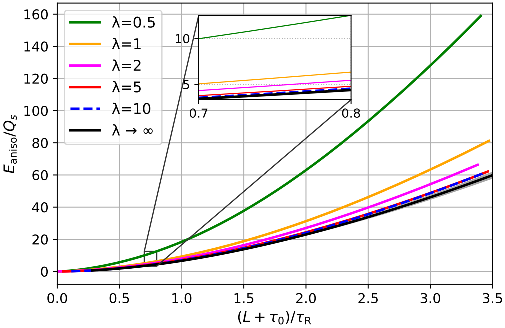
- Extrapolation to **limiting attractor**

$$E_{\text{aniso}}/Q_s \approx 6.7 \times \tilde{w}^{1.8},$$



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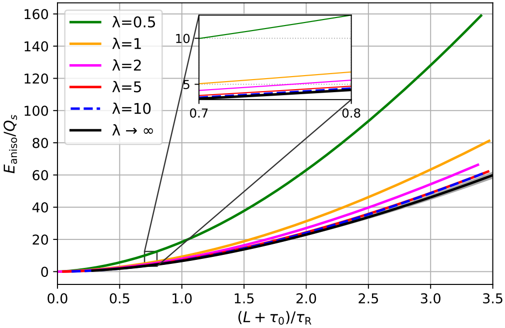
- Extrapolation to **limiting attractor**

$$E_{\text{aniso}}/Q_s \approx 6.7 \times \tilde{w}^{1.8},$$

- Obtain estimate

$$E_{\text{aniso}} \approx 54\,\text{GeV} \times \left(\frac{Q_s}{1.4\,\text{GeV}}\right)\left(\frac{\langle s\tau\rangle}{4.1\,\text{GeV}^2}\right)^{3/5} \times \left(\frac{\nu}{40}\right)^{-3/5}\left(\frac{\tau}{5\,\text{fm}}\right)^{6/5}\left(\frac{4\pi\eta/s}{2}\right)^{-9/5}$$



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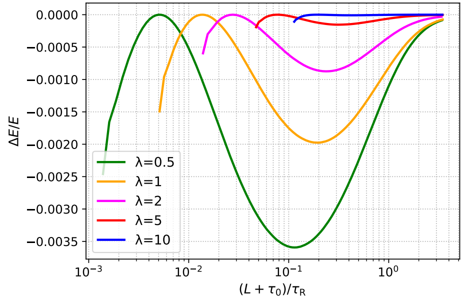
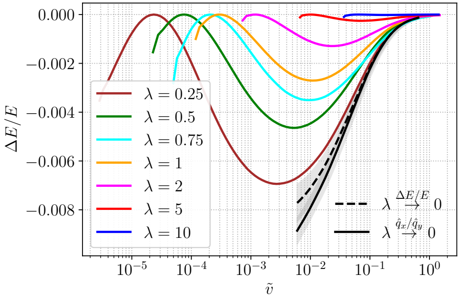
Relative change $\Delta E/E$

arXiv:2603.09912 [Boguslavski, Hörl, FL]





- **Bottom-up attractor** visible in relative change (despite averaging)
- **Both** attractors visible in **mean energy loss**

→ Attractors ‘survive’ procedure to obtain spectrum!

Outline

Conclusions

- Use **QCD kinetic theory** to describe initial **nonequilibrium QCD plasma** in heavy-ion collisions
- Calculate **momentum broadening of jets** $\rightarrow \hat{q}(t)$ and $C(\mathbf{q}_\perp, t)$
- Taken as input to calculate **gluon emission spectrum**
- Clear differences to thermal medium evolution, even at **large medium lengths**
- Mean jet energy loss exhibits both weak and strong-coupling limiting attractor!

Thank you very much for your attention!

Observables in kinetic theories

- Fundamental quantity: Distribution function $f(\mathbf{p})$
- **Energy-Momentum tensor**:

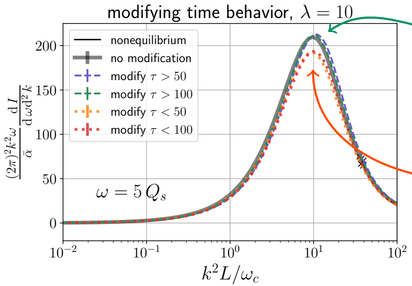
$$T^{\mu\nu} = \nu_g \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{p} f(\mathbf{p}) \qquad \leftarrow \quad \text{“Moments of distribution function”}$$

- Longitudinal pressure $P_L = T_{zz}$
- Transverse pressure $P_T = T_{xx} = T_{yy}$
- How many ‘hard’ particles

$$\frac{\langle pf \rangle}{\langle p \rangle} = \frac{\int \mathrm{d}^3\mathbf{p}\, p\, f(\mathbf{p})^2}{\int \mathrm{d}^3\mathbf{p}\, p\, f(\mathbf{p})} \qquad \leftarrow \quad \text{“particle number weighted with energy”}$$

Effect of modifications

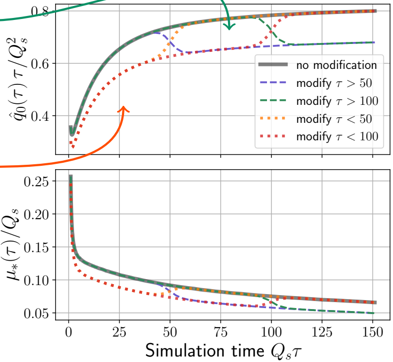
Gluon spectrum



late-time modification

early-time modification

Input $\hat{q}_0$ and μ_*



- Only modifications at early time influence spectrum!

Spectrum more explicit

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

IOE Spectrum is obtained via

$$\frac{\mathrm{d}I^{(0)}}{\mathrm{d}\omega\,\mathrm{d}^2k} = \frac{2\bar{\alpha}}{\pi\omega}\mathrm{Re}\left\{k^{-2}\left(e^{-i\frac{k^2}{2\omega\mathrm{Cot}(L,t_0)}}-1\right)+\int_{t_0}^{L}\mathrm{d}t\,\frac{\mathrm{Cot}(t,t_0)}{\hat{P}^2(t,t_0)}e^{-\frac{k^2}{\hat{P}^2(t,t_0)}}\right\},$$

$$\begin{aligned}\frac{\mathrm{d}I^{(1)}}{\mathrm{d}\omega\,\mathrm{d}^2k} = \frac{\bar{\alpha}\pi}{k^4}\mathrm{Re}\int_{t_0}^{L}\mathrm{d}t_2\Bigg\{&\frac{i}{2\omega}\int_{t_0}^{t_2}\mathrm{d}t_1\,\frac{\hat{q}_0(t_1)}{\hat{R}_{21}^2}e^{-\frac{k^2}{\hat{P}_{21}^2}}\left[Q_s^2(L,t_2)I_a\left(\frac{k^2}{\hat{K}_{21}},\frac{k^2\hat{R}_{21}^2}{Q_r^2}\right)+2C_{12}\hat{R}_{21}k^2I_b\left(\frac{k^2}{\hat{K}_{21}},\frac{k^2\hat{R}_{21}^2}{Q_r^2}\right)\right]-\mathrm{Cot}_{20}\,Q_{s0}^2(L,t_2)\,I_a\left(\frac{k^2}{\hat{P}^2(t_2,t_0)},\frac{k^2}{Q_b^2}\right)\\ &+2\,\hat{q}_0(t_2)C^2(t_2,L)I_b\left(\frac{ik^2}{2\omega C^2(t_2,L)\left(\mathrm{Cot}_{20}-\frac{1}{C(t_2,L)S(L,t_2)}-\mathrm{Cot}(t_2,L)\right)};\frac{k^2}{Q_r^2C^2(t_2,L)}\right)e^{-\frac{ik^2}{2\omega\mathrm{Cot}(L,t_2)}}\Bigg\}\end{aligned}$$

medium input,
obtained from $\hat{q}(\Lambda_\perp)$

scale,
obtained self-consistently

And requires solving a differential equation

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}t^2}+\Omega^2(t)\right]F(t,t_0)=0\,,\qquad \Omega^2(t)=-i\frac{\hat{q}_r(t)}{\omega},\qquad \hat{q}_r(t)=\hat{q}_0(t)\log\frac{Q_r^2}{\mu_*^2(t)}$$

with specific boundary conditions: $S(t_0,t_0)=\partial_t C(t,t_0)_{t=t_0}=0$,
$\partial_t S(t,t_0)_{t=t_0}=C(t_0,t_0)=1$

Small-distance form of $C(x)$

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

- Introduce cutoff

$$C(\mathbf{x}) = \underbrace{\int_{|\mathbf{q}_\perp|<\Lambda} \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \underbrace{\left(1 - e^{i\mathbf{x}\cdot\mathbf{q}_\perp}\right)}_{\text{expand: } \sim \frac{1}{4}x^2 q_\perp^2 + \ldots}} + \underbrace{\int_{|\mathbf{q}_\perp|>\Lambda_\perp} \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^3} C(\mathbf{q}_\perp)(1 - e^{i\mathbf{x}\cdot\mathbf{q}_\perp})}$$

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- $\hat{q}_0$ is constant related to large $q_\perp$ behavior of $C(q_\perp \to \infty) \to \frac{\hat{q}_0}{2\pi}\frac{1}{q_\perp^4}$

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$$C(\mathbf{x}, t) = \frac{1}{2}x^2 b(t) + \frac{x^2\hat{q}_0(t)}{2}\left(1 - \gamma_E - \log\frac{x\,Q}{2}\right)$$

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- **Expand** perturbatively **around harmonic oscillator solution**
  $\to$ "improved opacity expansion" JHEP 09 (2021) [Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk]

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$\hat{q}_0$ and μ_* from EKT

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

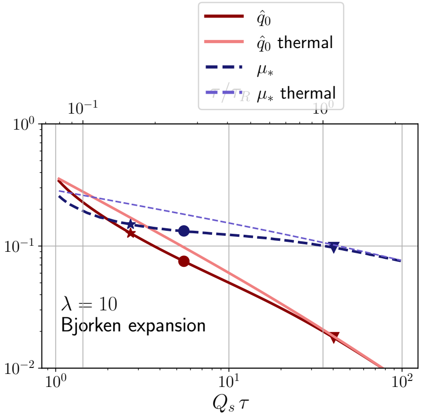
- Recall: $\hat{q}_0$, μ_* characterize small-x of $C(\mathbf{x})$,
- Get $\hat{q}_0$ from large-cutoff behavior,
  $\hat{q}(t) \approx 2\hat{q}_0(t) \log \frac{\Lambda_\perp}{Q_s} + b(t)$
- Get μ_* via

$$\mu_*^2(t) = \frac{1}{4} \exp(-b(t)/\hat{q}_0(t) + 2\gamma_E - 2)$$

- Thermal:

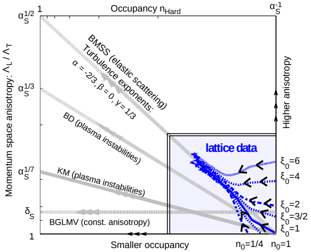
$$\hat{q}_0 = T^3 \frac{N_C C_R g^4 \zeta(3)}{2\pi^3}$$

$$\mu_*^2 = \frac{m_D^2}{4} \left(\frac{m_D}{2T}\right)^{2\zeta(2)/\zeta(3)-2} e^{2\gamma_E - 2 - \left(\frac{\zeta(2)}{\zeta(3)} - 1\right)(1-2\gamma_E) + \frac{\sigma_+}{\pi\zeta(3)}}$$



Plasma instabilities

- Anisotropic QCD plasmas generally lead to plasma instabilities Phys.Lett.B 214 (1988) [Mrowczynski], Phys.Rev.D 68 (2003) [Romatschke, Strickland]
- Soft modes grow exponentially, until perturbation theory (HTL) breaks down
- Although recent progress[8], no clear avenue for implementation in kinetic theory
- Classical statistical simulations seem to indicate that instabilities not dominant Phys.Rev.D 89 (2014) [Berges, Boguslavski, Schlichting, Venugopalan]



Phys.Rev.D 89 (2014) [Berges, Boguslavski, Schlichting, Venugopalan]

8 [Phys.Rev.C 105 (2023) [Hauksson, Jeon, Gale]]

Momentum broadening kernel vs. jet quenching parameter

Momentum broadening kernel:

$$C(\mathbf{q}_\perp) = \int \mathrm{d}\Gamma_{\mathrm{PS}}\, |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k}-\mathbf{q}))$$

Compare to $\hat{q}$,

weighted with
momentum transfer

$$\hat{q}^{ij} = \int_{\substack{q_\perp < \Lambda_\perp \\ p \to \infty}} \mathrm{d}\Gamma\, q^i q^j |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k}-\mathbf{q}))$$

different
integration measure

momentum cutoff
needed

- Similar quantity, less differential,

$$\hat{q}^{ij} = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} q^i q^j C(\mathbf{q}_\perp),$$

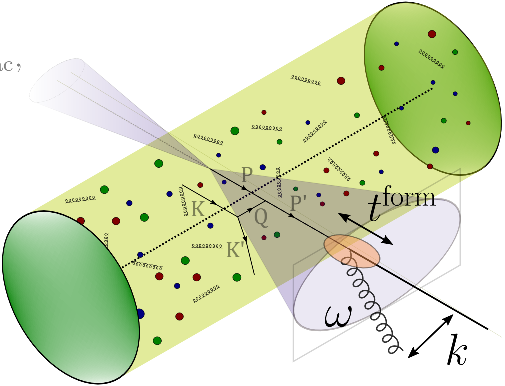
- $C(q_\perp)$ is **more general** than $\hat{q}$

Obtaining the gluon emission spectrum

Phys.Rev.D 79 (2009) [Arnold]
JETP Lett. 65 (1997) [Zakharov]
Nucl.Phys.B 483 (1997) [Baier, Dokshitzer, Mueller, Peigne, Schiff]

- **Energy loss** dominated by **gluon radiation**

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \mathrm{Re} \int_{t1,t2} \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} G(\mathbf{x}, t_2; \mathbf{y}, t_1)_{\mathbf{y}=0} - G_{\mathrm{vac}},$$



- $\mathrm{d}I/\mathrm{d}\omega$: **Spectrum**
  =Probability of emitting gluon
  with energy ω (relative to vacuum)
- ω: emitted gluon energy
- k: transverse momentum
- $t^{\mathrm{form}} \sim \sqrt{\omega/\hat{q}}$: formation time

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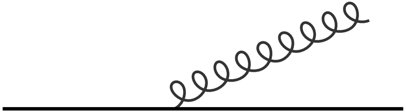

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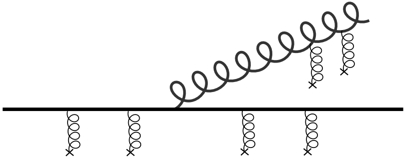

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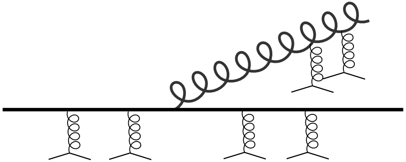

Obtaining the gluon emission spectrum

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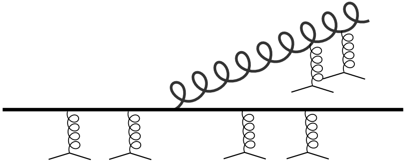

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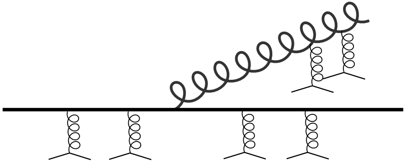

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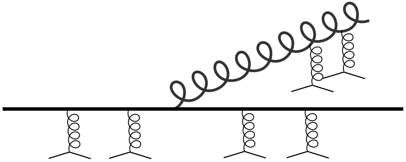

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'broadening probability'
Probability of receiving momentum kick $\mathbf{q}_\perp$

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scattering rate probability of receiving $q_\perp$ kick

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- Schrödinger equation for harmonic oscillator $\rightarrow$ analytic solution!

Small-distance form of $C(x)$

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

- Proper expansion for small distance:

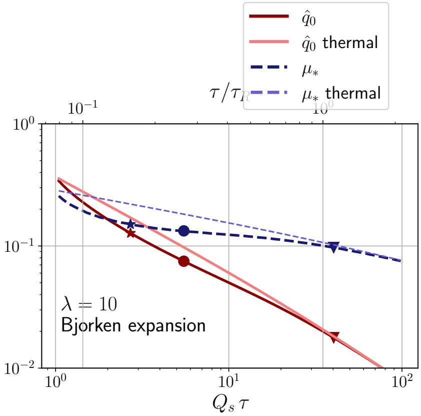
$$C(\mathbf{x}, t) = \frac{1}{4}\hat{q}_0(t)\mathbf{x}^2 \log\frac{1}{\mathbf{x}^2\mu_*^2(t)} + \mathcal{O}(\hat{q}_0\mathbf{x}^4\mu_*^2)$$

- μ_* and $\hat{q}_0$ are medium-parameters uniquely determined from large-cutoff form of $\hat{q}(\Lambda_\perp)$,

$$\hat{q}(t) \approx 2\hat{q}_0(t)\log\frac{\Lambda_\perp}{Q_s} + b(t)$$

- Obtain μ_* via

$$\mu_*^2(t) = \frac{1}{4}\exp(-b(t)/\hat{q}_0(t) + 2\gamma_E - 2)$$



Problems with $\hat{q}$ for radiation

- Recall: $\hat{q}$ appeared in small-distance expansion:

expansion parameter

physical quantity, enters equations as potential

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Need cutoff $\Lambda_\perp$ to restrict momentum transfer $q_\perp < \Lambda_\perp$
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Can we do better? $\rightarrow$ Consider full kernel $C(\mathbf{q}_\perp)$

Comparing to thermal system

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

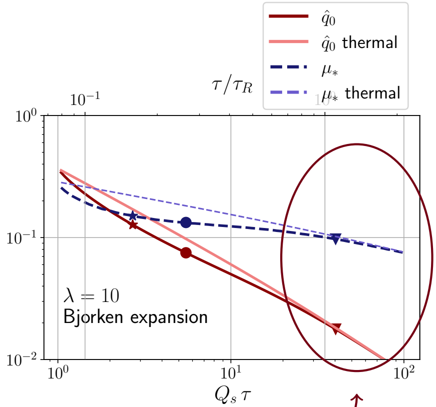
- Recall: $\hat{q}_0$, μ_* obtained from large cutoff behavior of $\hat{q}$
- Compare with an evolving thermal system

$$T_\varepsilon(t) = \left(\frac{30\epsilon(t)}{\pi^2}\right)^{1/4}$$

with energy density from kinetic theory,

$$\epsilon = \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}|\mathbf{p}|f(\mathbf{p})$$

- Thermal forms of $\hat{q}_0$, μ_* known[9]



Late time
approache to thermal

[9] [Phys.Rev.D 78 (2008) [Arnold, Xiao]]