

Localization and universality of three-dimensional pseudospin-s fermions

[Phys. Rev. B 114, 034209 (2026)]

Gargee Sharma

Indian Institute of Technology Delhi

Symposium on Women in Physics: Networking, Innovation, Opportunity and Equity

(SWPNIOE 2026)



Acknowledgements



Arpan Gupta

PhD (2025 --)



Talk based on our recent work:

Localization and universality of
three-dimensional pseudospin-s fermions,
Arpan Gupta and GS, Phys. Rev. B 114, 034209 (2026)

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON

Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

Prog. Theor. Phys. Vol. 63, No. 2, February 1980, Progress Letters

Spin-Orbit Interaction and Magnetoresistance in the Two Dimensional Random System

Shinobu HIKAMI, Anatoly I. LARKIN*¹ and Yosuke NAGAOKA

Research Institute for Fundamental Physics

Kyoto University, Kyoto 606

(Received November 5, 1979)

Effect of the spin-orbit interaction is studied for the random potential scattering in two dimensions by the renormalization group method. It is shown that the localization behaviors are classified in the three different types depending on the symmetry. The recent observation of the negative magnetoresistance of MOSFET is discussed.

Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

E. Abrahams

Serlin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08854

and

P. W. Anderson,^(a) D. C. Licciardello, and T. V. Ramakrishnan^(b)

Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08540

(Received 7 December 1978)

Arguments are presented that the $T=0$ conductance G of a disordered electronic system depends on its length scale L in a universal manner. Asymptotic forms are obtained for the scaling function $\beta(G) = d \ln G / d \ln L$, valid for both $G \ll G_c \approx e^2/h$ and $G \gg G_c$. In three dimensions, G_c is an unstable fixed point. In two dimensions, there is no true metallic behavior; the conductance crosses over smoothly from logarithmic or slower to exponential decrease with L .

Diagrammatic, self-consistent treatment of the Anderson localization problem in $d \leq 2$ dimensions

D. Vollhardt and P. Wölfle

*Max-Planck-Institut für Physik und Astrophysik, Munich, Federal Republic of Germany
and Physik Department, Technische Universität München, Garching, Federal Republic of Germany*

(Received 21 July 1980)

A standard diagrammatic theory is formulated for the density response function $\chi(\vec{q}, \omega)$ of a system of independent particles moving in a random potential. In the limit of small $\vec{q}, \omega$ the Bethe-Salpeter equation for the particle-hole vertex function may be solved for $\chi(\vec{q}, \omega)$ in terms of a current relaxation kernel $M(\vec{q}, \omega)$ [essentially the inverse of the diffusion coefficient $D(\vec{q}, \omega)$]. $M(\vec{q}, \omega)$ is obtained as the sum of the imaginary part of the single-particle self-energy and the current matrix element of the irreducible kernel and is determined diagrammatically. A theorem is formulated, stating that *any* diagram for $M(0, \omega)$ or $D(0, \omega)$ containing a (bare) diffusion propagator belongs to a well-defined class of diagrams whose divergencies cancel each other, and an exact proof is presented. In particular, this implies that there are no divergent contributions to $M(0, \omega)$ or $D(0, \omega)$ from a diffusion propagator. However, in the presence of time-reversal invariance, $M(\vec{q}, \omega)$ is shown to have infrared divergencies in $d \leq 2$, signalling a breakdown of the perturbation expansion in terms of the scattering potential which has first been discussed by Abrahams *et al.* A self-consistent treatment in the weak-coupling limit yields a finite static polarizability α , a dynamical conductivity $\text{Re} \sigma(\omega) \propto \omega^2$ for $\omega \rightarrow 0$, and a finite localization length in $d \leq 2$ for arbitrarily weak disorder. In $d = 1$ our results agree remarkably well with the exact solutions by Berezinsky and also Abrikosov and Ryskin.

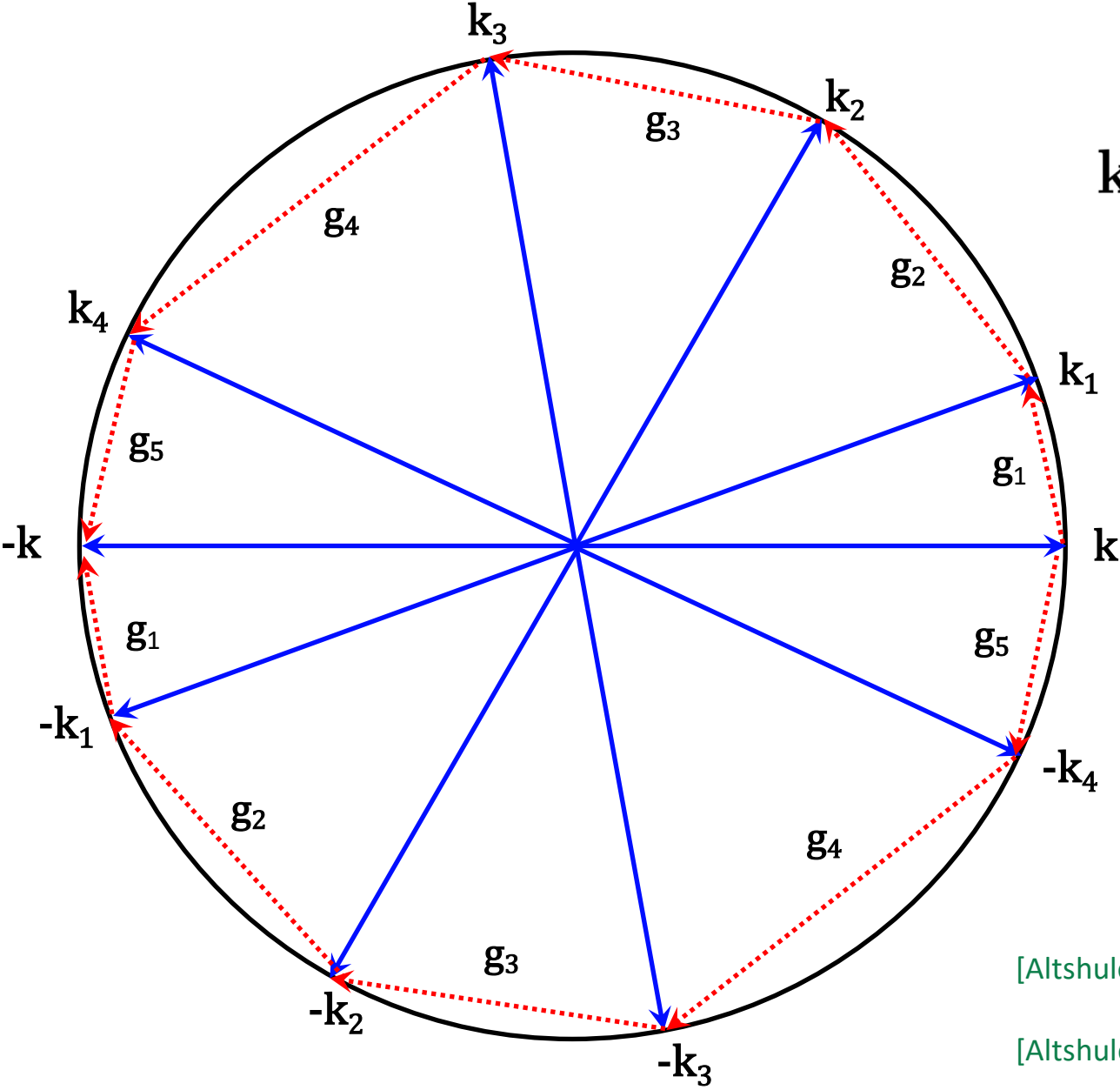
[Efetov (1983)]

[Suzura & Ando (2002)]

[Evers & Mirlin, RMP(2010)]

[Altland & Bagrets (2016)]

Let us move our quantum state $|\mathbf{k}\rangle$ on the Fermi surface in the presence of disorder



$$\mathbf{k} \rightarrow \mathbf{k}_1 \rightarrow \mathbf{k}_2 \rightarrow \mathbf{k}_3 \rightarrow \mathbf{k}_4 \rightarrow -\mathbf{k}$$

backscattering path

a sequence of momentum transfers:
 $\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3, \mathbf{g}_4, \mathbf{g}_5$

$$\mathbf{k} \rightarrow -\mathbf{k}_4 \rightarrow -\mathbf{k}_3 \rightarrow -\mathbf{k}_2 \rightarrow -\mathbf{k}_1 \rightarrow -\mathbf{k}$$

a time-reversed backscattering path

opposite sequence of momentum transfers:
 $\mathbf{g}_5, \mathbf{g}_4, \mathbf{g}_3, \mathbf{g}_2, \mathbf{g}_1$

If scattering is elastic (*phase coherent*), the two paths can constructively interfere. Enhanced probability of backscattering.

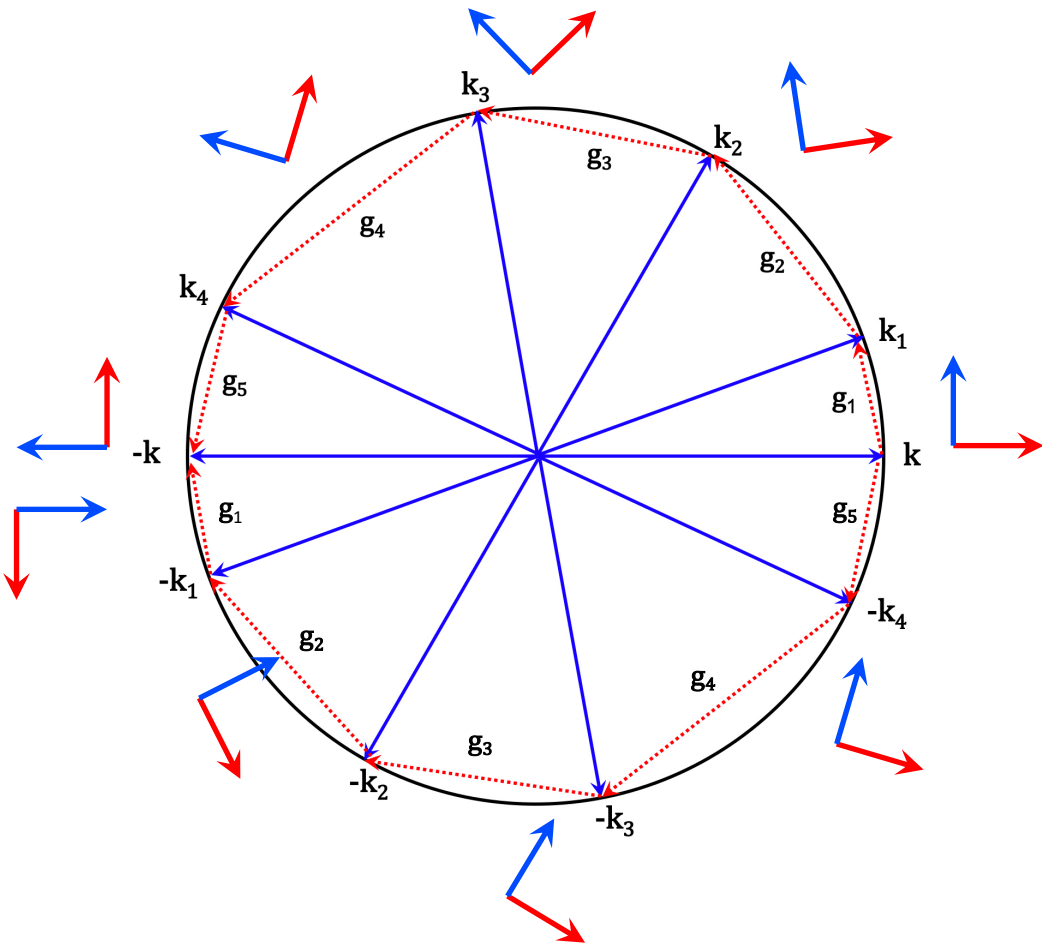
Quantum corrections to the conductivity (weak localization).

[Altshuler, Anorov, and Lee, PRL(1980)]

[Altshuler, Khmelnitzkii, Larkin, & Lee, PRB (1980)] [Langer & Neal, PRL (1966)]

The quantum state may acquire an *extra* phase along the path.

This phase may additionally contribute with the interference of two paths.



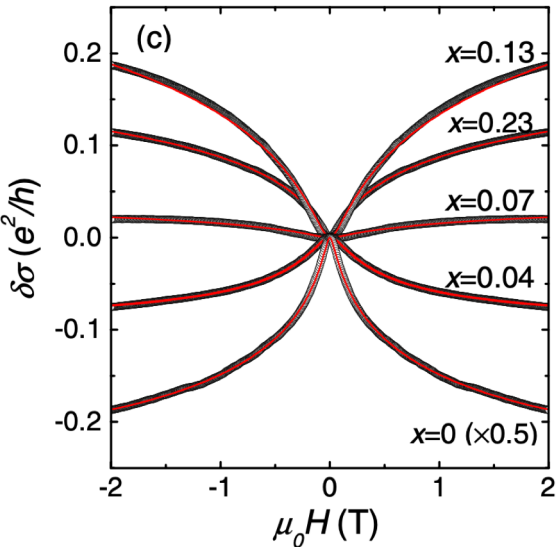
Weak antilocalization: reduced backscattering probability

Spin-orbit coupling

Berry phase

[E. McCann,..., PRL (2006)]

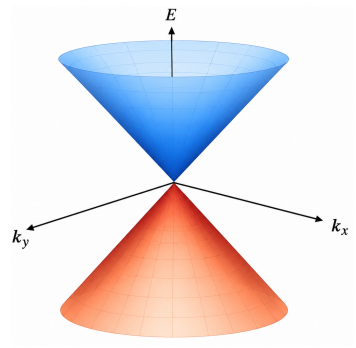
[Hikarni, Larkin, & Nagaoka, Prog. Theor. Phys. (1980)]



Localization effects can be measured by magnetoconductance experiments (from Liu,...,PRL 2012).

Novel excitations in solids

Going beyond the conventional *spin-1/2 Dirac and Weyl varieties* familiar from high-energy physics.



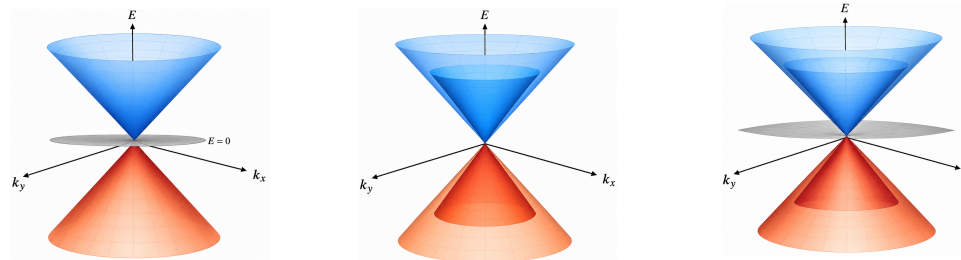
In crystalline solids, certain symmetries can stabilize multifold band degeneracies (three-, four-, six-fold and so on), whose *low-energy descriptions are generalized chiral Hamiltonians characterized by an effective pseudospins*.

These *higher-fold chiral fermions*, which have no direct counterpart in relativistic vacuum field theory, exhibit qualitatively new features.

higher topological charge.

unconventional optical selection rules.

exhibit anomalous transport responses that go beyond those of simple Weyl/Dirac fermions.



Example systems: CoSi, RhSi, AlPt, ..., engineered structures proposed in cold-atom optical lattice settings.

Our question -- *How does quantum localization evolve across the full pseudospin-s hierarchy?*

weak-localization

strong-localization

Localization in pseudospin-1/2 Weyl fermions and pseudospin-1 fermions have been studied in isolation, *a unified analytical framework capable of treating an arbitrary pseudospin-s has been missing.*

We wish to:

extract semiclassical transport coefficients,
identify the surviving diffusive Cooperon modes,
compute the WL/WAL correction,
examine the role of interband and intervalley mixing,

all in a unified framework for arbitrary -s

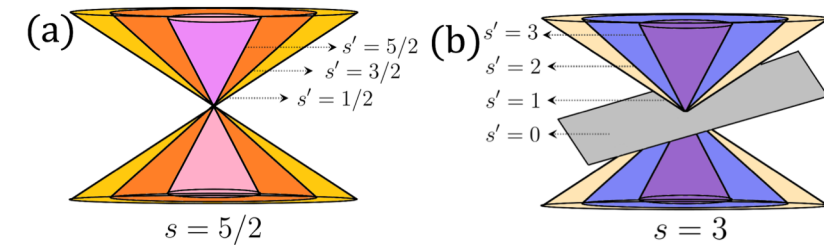
We consider a system hosting low-energy fermionic excitations of arbitrary pseudospin- s , in which the spin degrees of freedom are linearly coupled to the particle momentum.

The Hamiltonian $H_s(\mathbf{k}) = \hbar v \mathbf{S} \cdot \mathbf{k}$

$\mathbf{S} = (S_x, S_y, S_z)$ are $(2s+1) \times (2s+1)$ spin- s matrices satisfying the SU(2) algebra

$$[S_i, S_j] = i\epsilon_{ijk} S_k, \quad \mathbf{S}^2 = s(s+1)\mathbf{1}$$

Energy Spectrum $E_{s'}(\mathbf{k}) = \hbar v s' k, \quad s' = -s, -s+1, \dots, s$



Energy eigenmodes are helicity eigenstates

$$|\mathbf{k}; s, s'\rangle = \sum_{m=-s}^s e^{-im\phi} d_{ms'}^s(\theta) |s, m\rangle$$

$d_{ms'}^s(\theta)$ are reduced Wigner functions

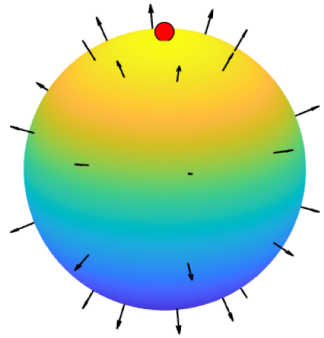
$$\sum_{m=-s}^s d_{ms'}^s(\theta) d_{ms''}^s(\theta) = \delta_{s's''},$$

$$\sum_{s'=-s}^s d_{ms'}^s(\theta) d_{m's'}^s(\theta) = \delta_{mm'}.$$

Each eigenmode is a coherent superposition of $2s+1$ spin-projections.

With increasing pseudospin (s) – the spinor becomes more semiclassical;
near-backscattering is exponentially suppressed;
phase winding around the Fermi surface becomes denser.

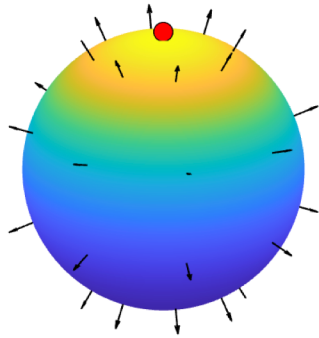
$$s = \frac{1}{2}$$



$$P_s(\theta) = \left(\frac{1 + \cos \theta}{2}\right)^1$$

$$e^{i1\phi}$$

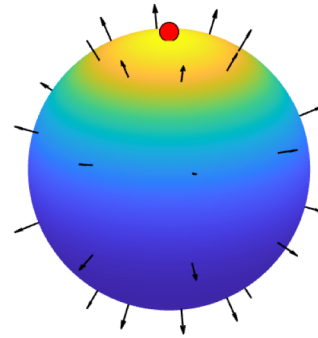
$$s = 1$$



$$P_s(\theta) = \left(\frac{1 + \cos \theta}{2}\right)^2$$

$$e^{i2\phi}$$

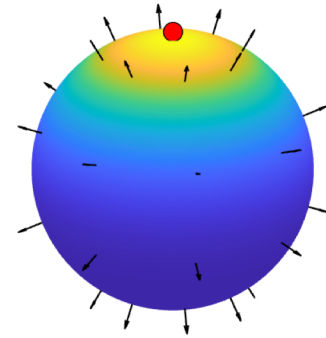
$$s = \frac{3}{2}$$



$$P_s(\theta) = \left(\frac{1 + \cos \theta}{2}\right)^3$$

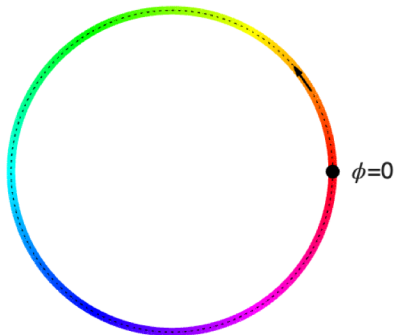
$$e^{i3\phi}$$

$$s = 2$$

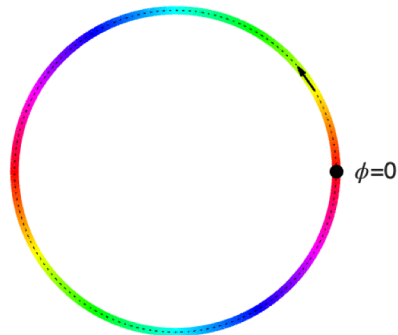


$$P_s(\theta) = \left(\frac{1 + \cos \theta}{2}\right)^4$$

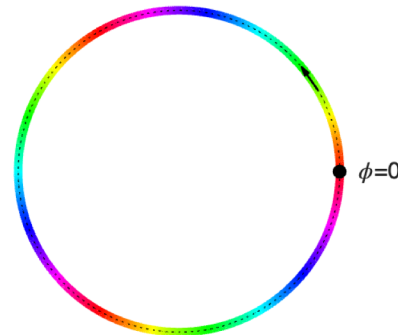
$$e^{i4\phi}$$



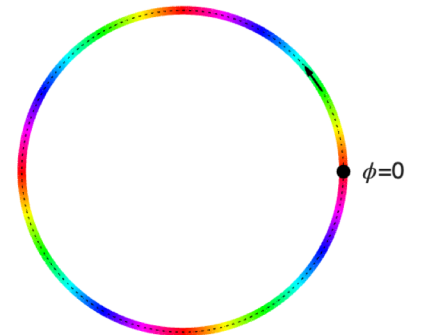
winding = 1



winding = 2



winding = 3



winding = 4

Introduction of (Gaussian correlated) disorder

$$U(\mathbf{r}) = \sum_i u_0 \delta(\mathbf{r} - \mathbf{R}_i) \mathcal{M}$$

$$\overline{U(\mathbf{r})U(\mathbf{r}')} = n_i u_0^2 \delta(\mathbf{r} - \mathbf{r}') \mathcal{M}^2$$

$\mathcal{M}$ is a general square matrix of dimension $(2s + 1)$

It couples the different bands by off-diagonal elements

$$\mathcal{M} = \sum_{L=0}^{2s} \sum_{Q=-L}^L M_{LQ} T_Q^{(L)}$$

We expand M in terms of irreducible tensor operators

$$\langle s, s'' | T_Q^{(L)} | s, s' \rangle = \sqrt{2L + 1} (-1)^{s+s''-L} \begin{pmatrix} s & L & s \\ s' & Q & -s'' \end{pmatrix}$$

(.....) is the Wigner's 3-j symbol, which is related to the Clebsch-Gordon coefficients.

Wigner decomposition of the disorder allows us to compute the standard quantities relevant for transport *(for an arbitrary disorder matrix M)*

$$\Sigma_{s'}^R(\mathbf{k}) = n_i u_0^2 \sum_{s''} \sum_{\mathbf{k}'} |\langle \mathbf{k}', s'' | \mathcal{M} | \mathbf{k}, s' \rangle|^2 G_{s''}^0(\mathbf{k}') \quad \text{self-energy (Born approx.)}$$

$$\frac{1}{\tau_{s'}} = \frac{2\pi}{\hbar} n_i u_0^2 \frac{1}{(2s+1)^2} \sum_{s''} N_F^{(s'')} \sum_Q |\mathcal{A}_{s'}(Q)|^2 \quad \text{scattering time}$$

$$\mathcal{A}_{s'}(Q) = \sum_L \sqrt{2L+1} M_{LQ} C_{s, s', L, Q}^{s, s'+Q}$$

In the scalar disorder limit,
we recover the standard known expressions

$$\frac{1}{\tau_{s'}} = \frac{2\pi}{\hbar} n_i u_0^2 \frac{1}{(2s+1)} \sum_{s''} N_F^{(s'')}$$

Similarly, the transport lifetime
can also be evaluated in a closed form

$$\frac{1}{\tau_{s'}^{\text{tr}}} = \frac{1}{\tau_{s'}} \left(1 - \frac{s'^2}{s(s+1)} \right)$$

[Singh & **GS**, PRB (2023)]

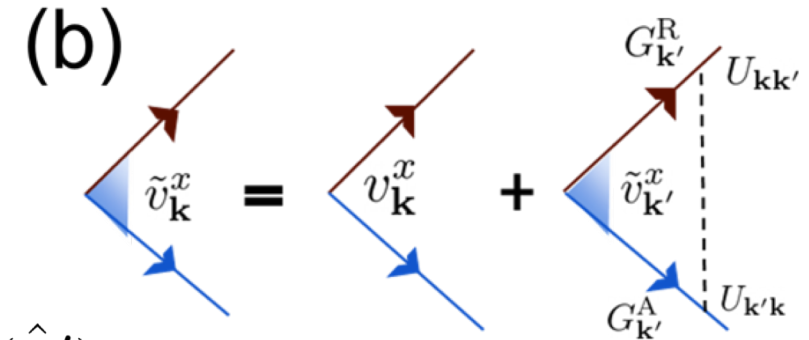
[Lu & Shen, PRB (2015)]

[Gupta & **GS**, PRB (2026)]

Disorder –

generates finite scattering time, broadens the quasiparticle spectrum,
renormalizes the current operator through multiple scattering.

Diagrammatically the renormalization corresponds to –
 summing the ladder series of impurity lines in the particle–hole channel,
 this leads to a Bethe–Salpeter equation for the *dressed current vertex*.



$$\Gamma_{s',\beta}(\hat{\mathbf{k}}) = v_{s',\beta}(\hat{\mathbf{k}}) + n_i u_0^2 \sum_{s''} \mathcal{J}_{s''} \int \frac{d\Omega'}{4\pi} W_{s's''}(\hat{\mathbf{k}}, \hat{\mathbf{k}}') \Gamma_{s'',\beta}(\hat{\mathbf{k}}')$$

What ansatz to choose? For a generic matrix disorder M , the kernel W is not purely a function of the relative angle, & thus the dressed vertex may not be characterized by a single scalar renormalization factor.

$$\Gamma_{s',\beta}(\hat{\mathbf{k}}) = \sum_{\ell m} \eta_{s';\ell m}^{(\beta)} Y_{\ell m}(\hat{\mathbf{k}})$$

We therefore to expand the vertex
in spherical harmonics.

$$\eta_{s'} = \frac{1}{1 - \mu_{s'}} \quad \mu_{s'} = \frac{s'}{s(s+1)} \frac{\sum_Q (s' + Q) |\mathcal{A}_{s'}(Q)|^2}{\sum_Q |\mathcal{A}_{s'}(Q)|^2}$$

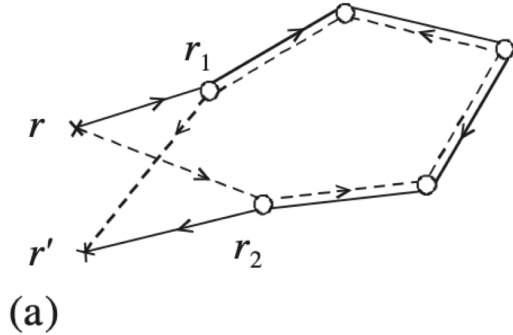
$$\eta^{(\beta)} = \left(I - \hat{\mathcal{K}} \right)^{-1} \eta^{(\beta,0)}$$

[Gupta & GS, PRB (2026)]

[Akkermans & Montambaux (2007)]

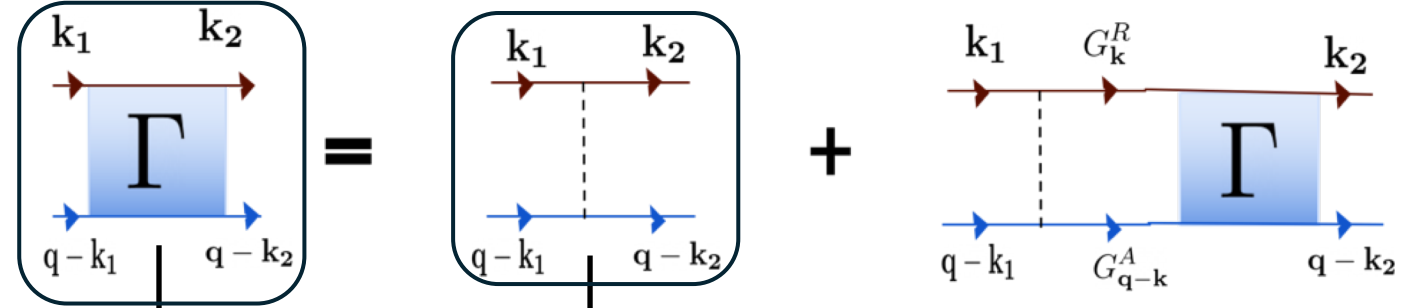
We recover standard expressions for scalar disorder.

Quantum corrections to the Drude conductivity arise from coherent backscattering of time-reversed trajectories.



Diagrammatically -- described by the Cooperon propagator; this captures the diffusive dynamics of two particles moving along time-reversed paths.

(c)



$$\Gamma_{k_1 k_2} = \Gamma_{k_1 k_2}^0 + \sum_k \Gamma_{k_1 k}^0 G_k^R G_{q-k}^A \Gamma_{k k_2}$$

$$\Gamma_{k_1 k_2}^0 = n_0 u_0^2 \sum_p \sum_{m,n} C_{mnp}^0 e^{i(m\theta_1 + n\theta_2)} e^{ip(\phi_1 + \phi_2)};$$

$$\Gamma_{k_1 k_2} = n_0 u_0^2 \sum_{m,n,p} C_{mnp} e^{i(m\theta_1 + n\theta_2)} e^{ip(\phi_1 + \phi_2)}$$

[Gupta & GS, PRB (2026)]

[Akkermans & Montambaux (2007)]

[assumed scalar disorder here]

[matrix disorder later on]

The coefficients C_{mnp} are functions of q , and diverge for weak localization channels

$$C_{mnp} \rightarrow \infty \text{ as } q \rightarrow 0$$

After solving the Bethe-Salpeter eqn., we find that C_{mnp} for each channel takes the form:

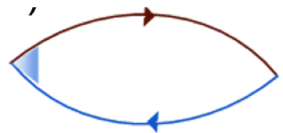
$$C_{mnp} = \frac{1}{q^2 + Q_{mnp}^2}$$

$$Q_{mnp}^2 \neq 0 \text{ except when } p = 2s, m = n = 0$$

Therefore, only one channel is divergent in the diffusive limit.

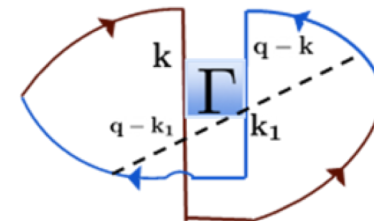
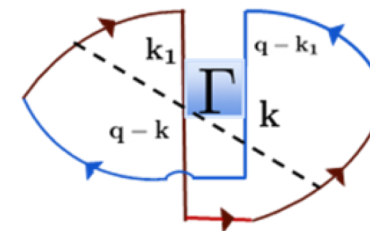
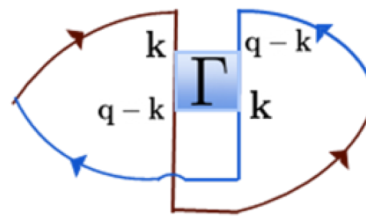
$$\Gamma(\mathbf{q}) = \frac{e^{p(i\phi_1 - i\phi_2)}}{D_{s'} q^2} \quad \text{Cooperon}$$

We then calculate the localization correction by summing the bare and dressed Hikami boxes (put the Cooperon in the Drude diagram).



Drude conductivity

$$\sigma_{\alpha\beta}^{(s')} = \frac{e^2}{2\pi} \int \frac{d^d k}{(2\pi)^d} v_\alpha(\hat{\mathbf{k}}) G_{s'}^R \Gamma_{s',\beta}(\hat{\mathbf{k}}) G_{s'}^A$$



[Gupta & GS, PRB (2026)]

[Akkermans & Montambaux (2007)]

The quantum correction to conductivity: $\sigma^{\text{qi}} = -\frac{e^2}{h} \frac{1}{\pi^2} \left(\frac{1}{\ell} - \frac{1}{\ell_\phi} \right) e^{i\pi\alpha}$ $\alpha = 2s$

The **magnitude** of the quantum-interference correction to the conductivity for *higher-pseudospin fermions* is identical to that of *conventional Weyl fermions* and to that of a *standard 3D electron gas*. Independent of s and s' !

The **nature** of the quantum-interference correction depends sensitively on the pseudospin: systems with half-integer pseudospin exhibit weak antilocalization, whereas those with integer pseudospin display weak localization; independent of helicity.

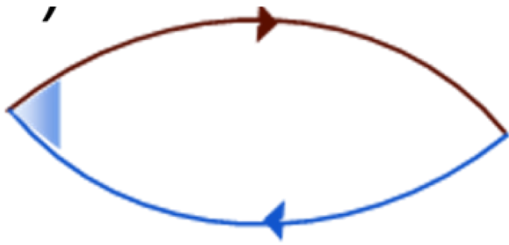
$$\mathcal{T} = e^{-i\pi S_y} \kappa \quad \mathcal{T}^2 = (-1)^{2s} \quad \zeta_s = \text{sgn}(\mathcal{T}^2)$$

time-reversal operator

$$\zeta_s = \begin{cases} -1, & s \text{ is integer (weak localization),} \\ +1, & s \text{ is half-integer (weak antilocalization).} \end{cases}$$

For integer pseudospin- s , $T^2=+1$, and the system belongs to the *orthogonal symmetry class*.

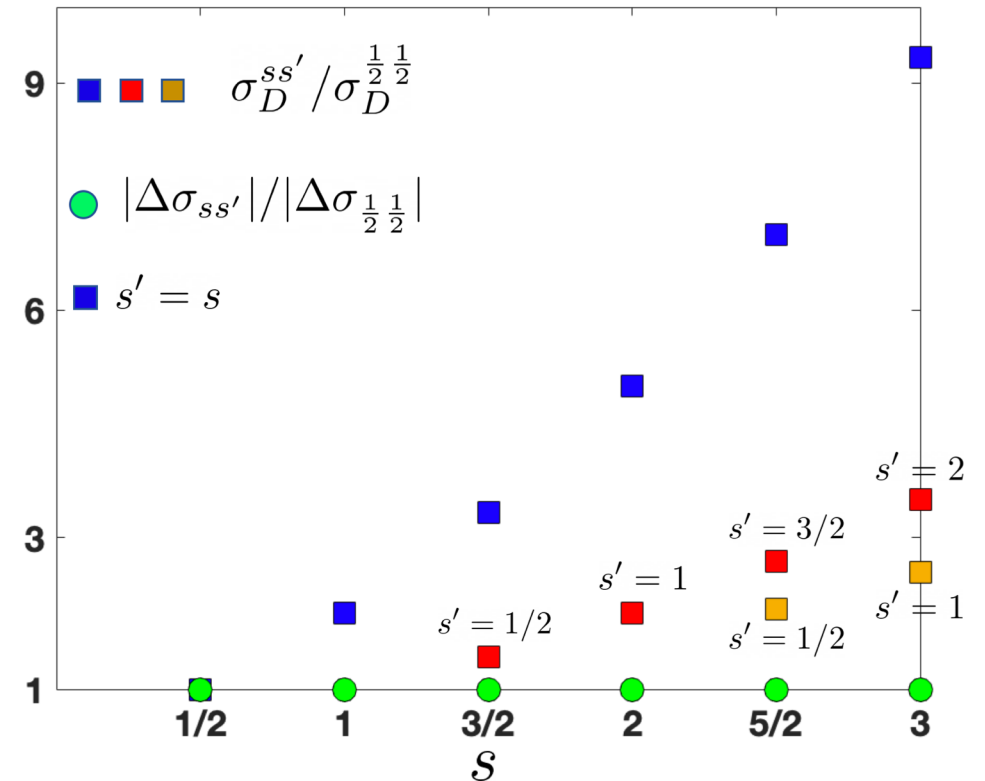
For half-integer pseudospin- s , $T^2=-1$, placing the system in the *symplectic class*.



- Drude conductivity

$$\sigma_{\alpha\beta}^{(s')} = \frac{e^2}{2\pi} \int \frac{d^d k}{(2\pi)^d} v_\alpha(\hat{\mathbf{k}}) G_{s'}^R \Gamma_{s',\beta}(\hat{\mathbf{k}}) G_{s'}^A$$

Universality in the quantum correction to conductivity.



The universal correction assumes – scalar diagonal disorder and no intervalley or interband scattering.

Disorder can, in principle, couple various bands and valley indices, which changes the helicity and/or chirality states of the pseudospin.

Examples –

pseudospin-s, with multiple chirality flavors, may exist at different valleys as expected by the Nielsen-Ninomiya theorem.

pseudospin-3/2 has multiple bands crossing the Fermi level at an arbitrary value of chemical potential.

The Cooperon is no longer a single scalar object associated with a fixed channel but instead becomes a tensor in the combined band/valley space.

Different scattering processes mix these channels, and the interference correction is governed by the coupled dynamics of all such modes.

This leads to a Bethe–Salpeter formulation in which the Cooperon is built up iteratively from the bare disorder correlators.

$$\Gamma_{n,m}^{m,n}(\mathbf{k}_1, \mathbf{k}_2) = \gamma_{n,m}^{m,n}(\mathbf{k}_1, \mathbf{k}_2) + \sum_{\mathbf{k}} \sum_{\nu=m,n} \gamma_{n,\bar{\nu}}^{m,\nu}(\mathbf{k}_1, \mathbf{k}) \times G_{\mathbf{k}}^{i\epsilon_n} G_{\mathbf{q}-\mathbf{k}}^{i\epsilon_n - i\omega_m} \Gamma_{\bar{\nu},m}^{\nu,n}(\mathbf{k}, \mathbf{k}_2),$$

$$\Gamma_{m,m}^{n,n}(\mathbf{k}_1, \mathbf{k}_2) = \gamma_{m,m}^{n,n}(\mathbf{k}_1, \mathbf{k}_2) + \sum_{\mathbf{k}} \sum_{\nu=m,n} \gamma_{m,\bar{\nu}}^{n,\nu}(\mathbf{k}_1, \mathbf{k}) \times G_{\mathbf{k}}^{i\epsilon_n} G_{\mathbf{q}-\mathbf{k}}^{i\epsilon_n - i\omega_m} \Gamma_{\bar{\nu},m}^{\nu,n}(\mathbf{k}, \mathbf{k}_2)$$

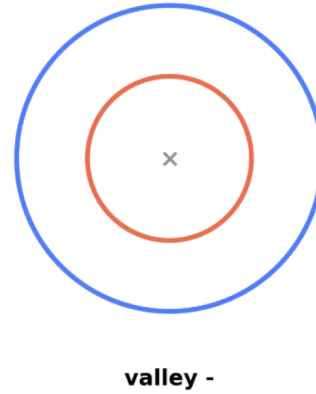
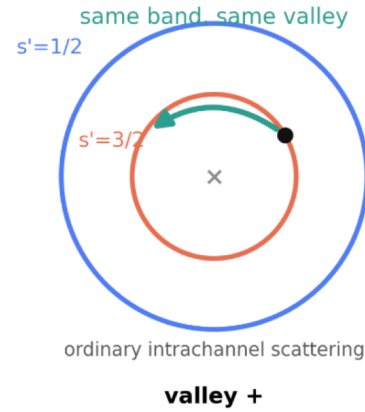
$\gamma_{\bar{\nu}\nu}^{\nu\bar{\nu}}(\mathbf{k}_1, \mathbf{k}_2)$ and $\gamma_{\bar{\nu}\bar{\nu}}^{\nu\nu}(\mathbf{k}_1, \mathbf{k}_2)$ are the bare disorder correlators defining the zeroth-order Cooperons

- (i) Substitute the bare cooperons into the Bethe–Salpeter integral. This gives the zeroth order solution.
- (ii) Insert the zeroth-order solution into the Bethe–Salpeter integral. This yields a first-order correction that contains additional terms coming from one extra impurity scattering.
- (iii) Insert the first order solution to obtain the second order solution that contains further extra terms.
- (iv) Repeat till all the angular structures converge.

Scattering channels for pseudospin-3/2 fermions

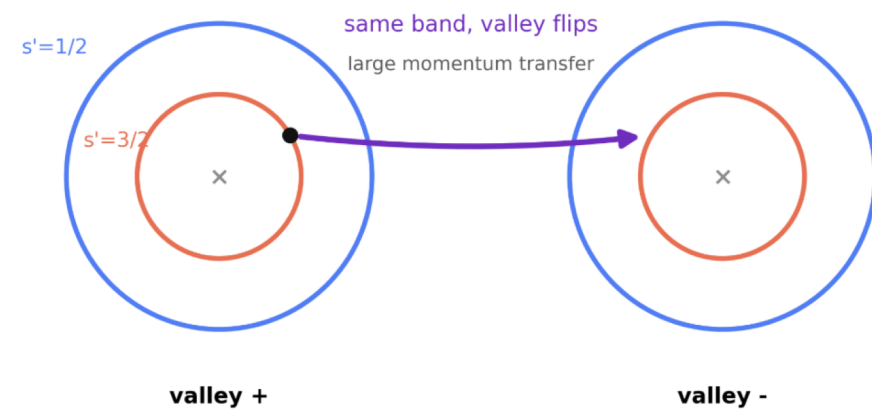
(a) Intraband - intravalley

(band, valley): $(3/2, +) \rightarrow (3/2, +)$



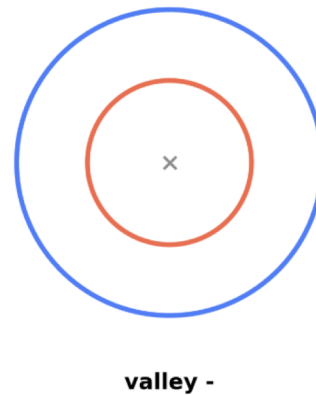
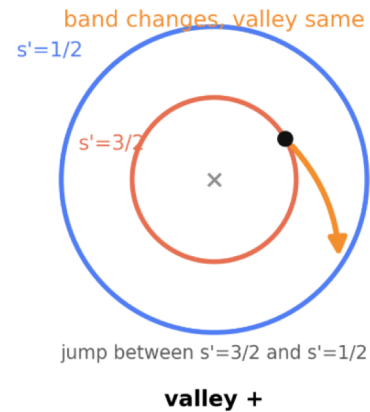
(b) Intraband - intervalley

(band, valley): $(3/2, +) \rightarrow (3/2, -)$



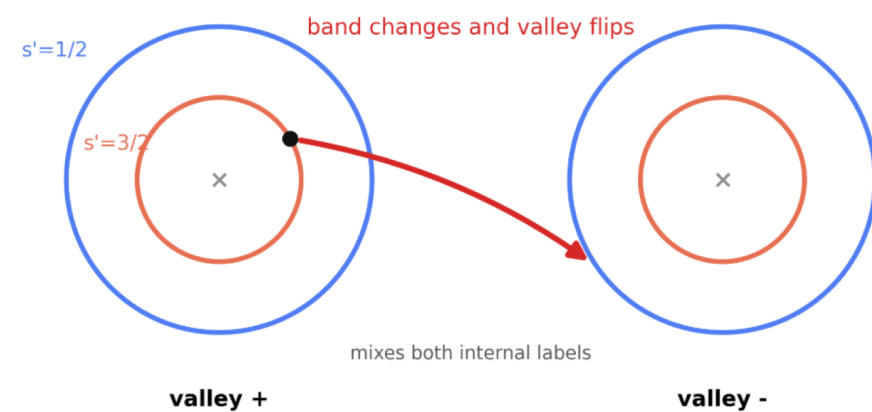
(c) Interband - intravalley

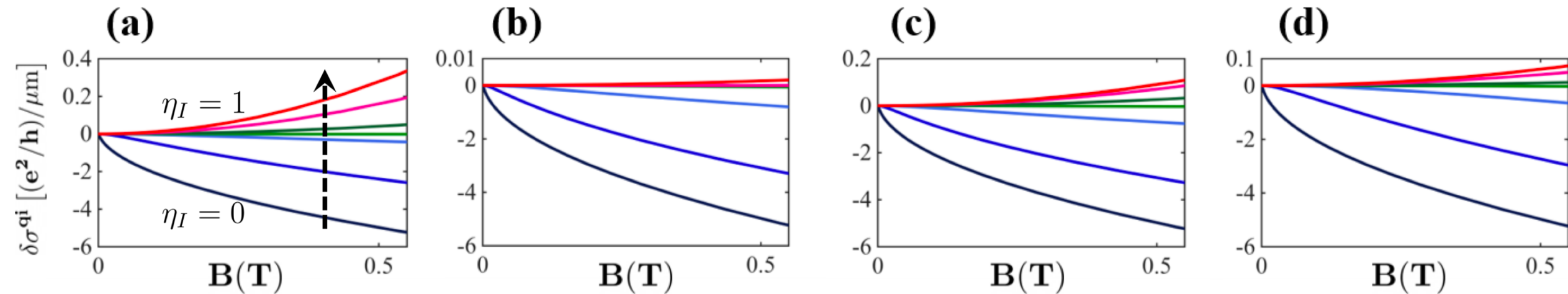
(band, valley): $(3/2, +) \rightarrow (1/2, +)$



(d) Interband - intervalley

(band, valley): $(3/2, +) \rightarrow (1/2, -)$





Intervalley-intraband
scattering for $s' = 3/2$

Intravalley-Interband
scattering

Intervalley-Interband
scattering

Intervalley-intraband
scattering for $s' = 1/2$

Magnetoconductivity for pseudospin $s = 3/2$ fermions for different scattering channels.

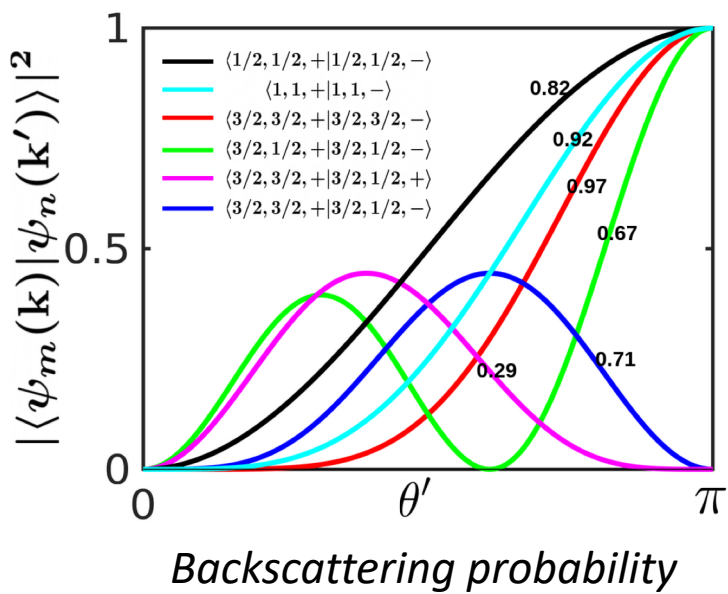
As $\eta_I \rightarrow 1$, the magnetoconductivity changes sign at a critical value η_C , signaling a crossover from WAL to WL.

Channel	η_I^C	$\mathcal{P}_{r's'}^{\lambda\lambda'}$	WAL→WL behavior
Intervalley-intraband ($s' = r' = 3/2, \lambda \neq \lambda'$)	0.58	0.97	Strong interference, early crossover
Intervalley-interband ($s' = 3/2, r' = 1/2, \lambda \neq \lambda'$)	0.63	0.71	Moderate interference and crossover
Intervalley-intraband ($s' = r' = 1/2, \lambda \neq \lambda'$)	0.65	0.67	Moderate interference and crossover
Intravalley-interband ($s' = 3/2, r' = 1/2, \lambda = \lambda'$)	0.91	0.29	Weak interference, delayed crossover

TABLE II. Critical intervalley/interband strength η_I^C where the crossover from negative (WAL) to positive magnetoconductance (WL) occurs, and the corresponding backscattering probability in each channel for pseudospin-3/2 fermions.

An inverse correlation emerges between the critical scattering strength η_I^C and the corresponding backscattering probability.

$$\eta_I^C \sim \mathcal{P}^{-1}$$



We developed a unified theory of semiclassical transport and quantum interference for three-dimensional chiral fermions with arbitrary pseudospin- s .

The *Drude response is strongly pseudospin dependent*, reflecting a geometric suppression of backscattering and a systematic enhancement of transport with increasing s .

The *leading quantum-interference correction retains a universal magnitude* identical to that of conventional diffusive metals and conventional Weyl-Dirac fermions.

Its *sign* determined solely by the *parity of $2s$* , placing half-integer (integer) pseudospins in the symplectic (orthogonal) class.

The *universality is fragile* -- interband and intervalley scattering render the Cooperon intrinsically multichannel.

We show explicitly for $s=3/2$ that channel mixing suppresses weak antilocalization and drives a crossover toward localization, with a scale set by the underlying backscattering probability.

Future works – the fate of strong localization in these systems?

Thank You