

# Phase-Boundary Pinning Across a Topological Phase Transition in the Presence of Interactions

**Sudeshna Sen**  
IIT(ISM) Dhanbad

SWPNIOE, SNU, Delhi NCR  
September 2026

Ref: “*Probing Topological Phases in a Strongly Correlated Ladder Model via Entanglement*”,  
Aminul Hussain, Nisa Ara, Rudranil Basu, Sudeshna Sen, **arXiv:2510.26713**



# Topology in Condensed Matter Physics



## Topology:

- Branch of Mathematics concerned with the properties of a geometric object that are preserved under continuous deformation, such as stretching, twisting, crumpling, and bending.

## Topological Invariant: The preserved quantity.

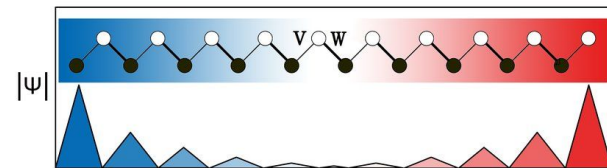
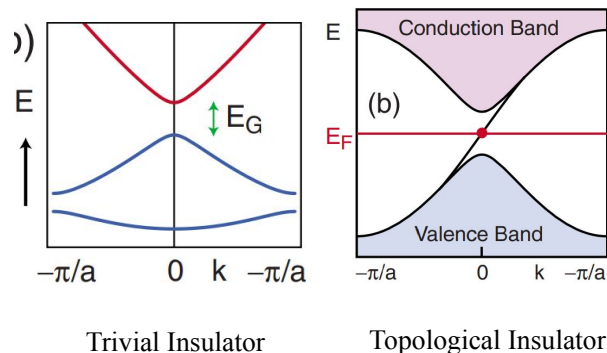
In the cup-doughnut example, the number of hole is preserved.

## Topology in Condensed Matter Physics:

- Hidden in its **electronic band structure**; can have its own “shape” and is classified by a topological invariant, “hole” is in the electronic band “shape”.

## Significance of Topological Phases:

- Robust & Protected:** Properties are protected by this invariant and aren't disturbed by small defects or impurities.
- Exotic Properties:** This leads to unique behaviors, like "topological insulators" that conduct electricity perfectly on their surface but are insulators in the middle(bulk).



# The Simplest model....

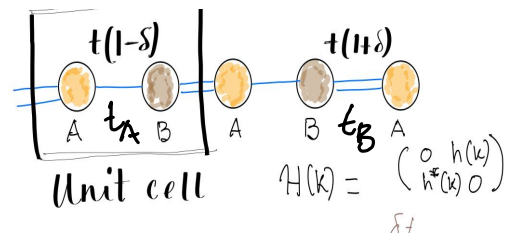
Su-Schrieffer-Heeger (SSH) model:

$$H_{\text{SSH}} = \varepsilon \sum_j c_j^\dagger c_j + \left( t_A \sum_{j \text{ odd}} c_{j+1}^\dagger c_j + t_B \sum_{j \text{ even}} c_{j+1}^\dagger c_j + \text{H.c.} \right) \quad \underline{\underline{E=0}}$$

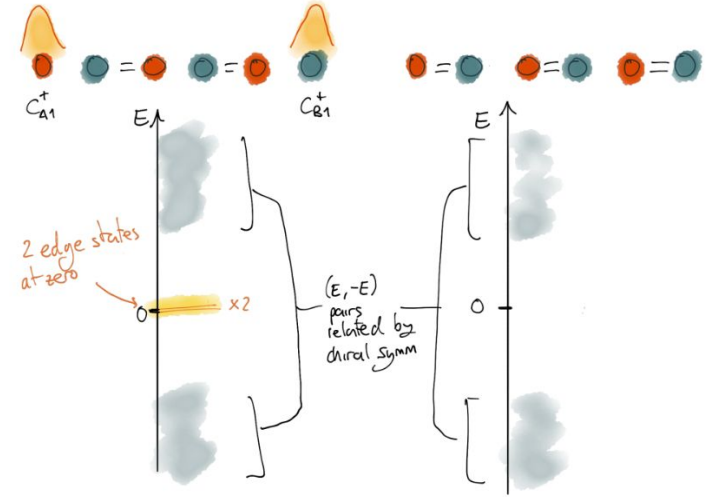
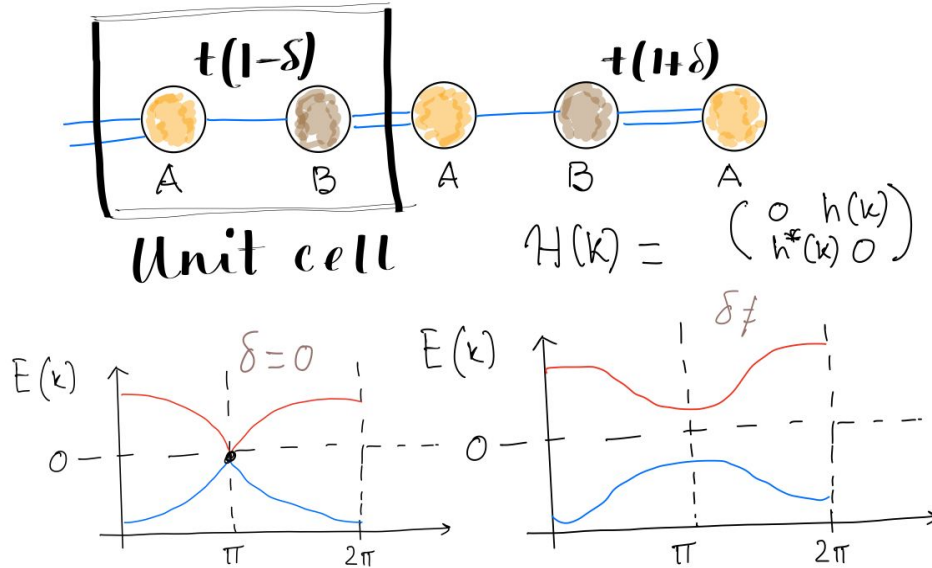
Trivial band insulator for all  $t_A > t_B$

Topological band insulator for all  $t_A < t_B$ ; end site exponentially localised

Non trivial aspect: end state localised for all  $t_A < t_B$



# Band gap closing to Topology

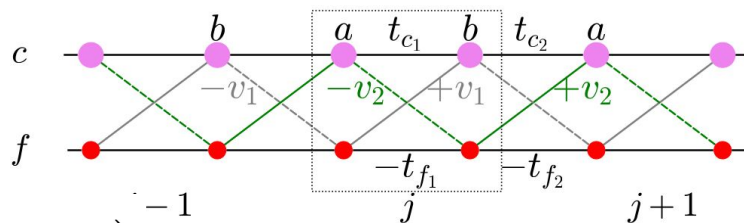


Topological Invariant : 
$$\nu = \frac{1}{2\pi} \int_{BZ} dk \, h(k) \partial_k h(k)$$

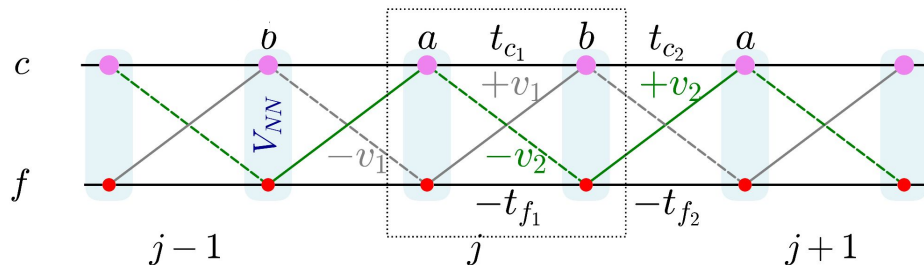
# Beyond the SSH model.....

- Possibility of Multiple phase boundaries

- Role of interactions

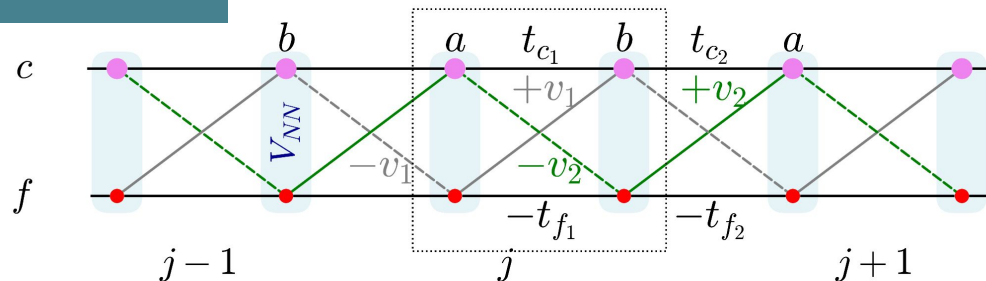


- $$H_{int} = \pm V_{NN} \sum_j \left( c_{aj}^\dagger c_{aj} f_{aj}^\dagger f_{aj} + c_{bj}^\dagger c_{bj} f_{bj}^\dagger f_{bj} \right)$$



- Stability of the phases

# The Interacting model



## Challenges with Interacting problem:

- (i) Solution like non-interacting is not possible.
- (ii) Failure of conventional Winding number as a topological marker.

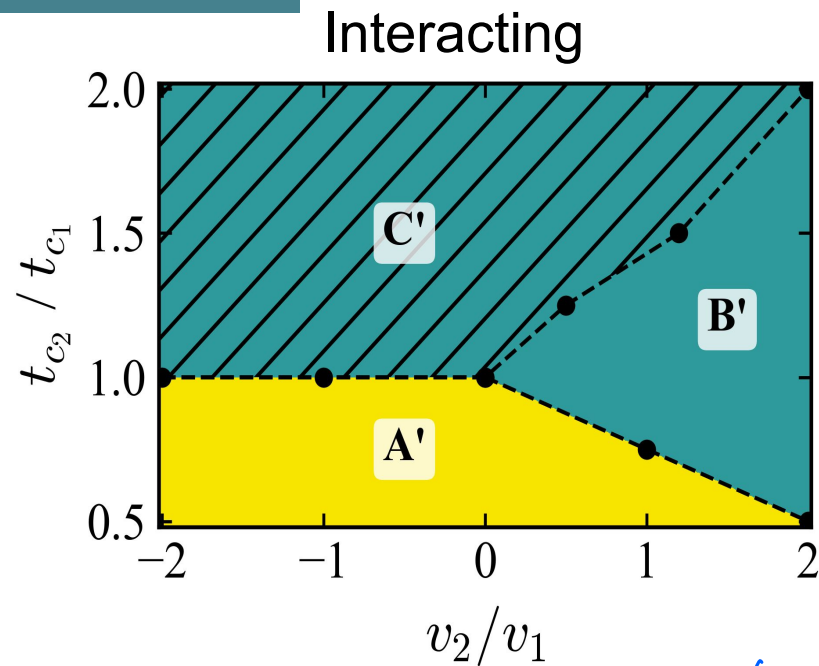
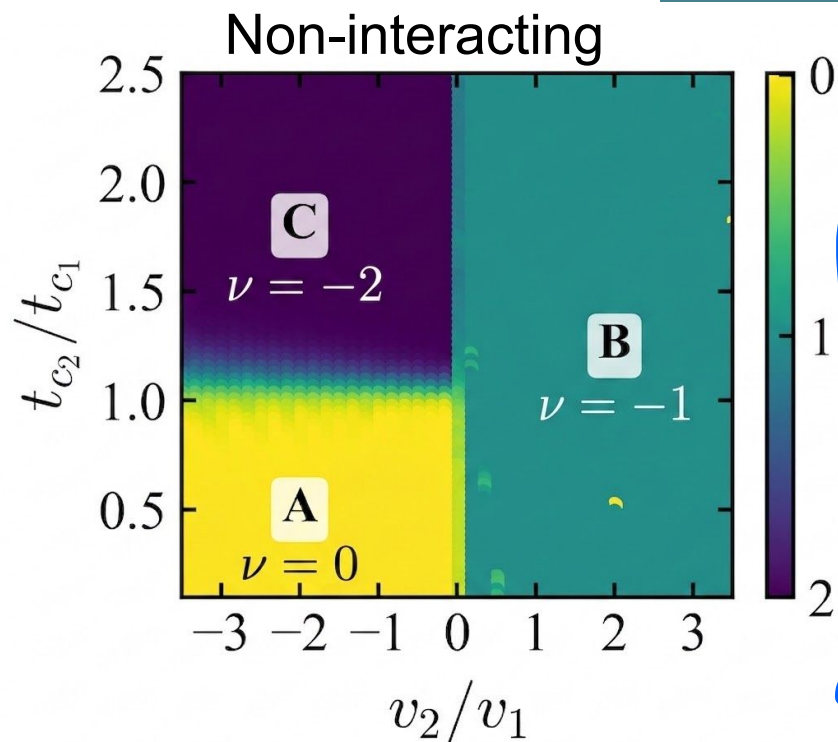
## Tools to study Interacting model:

- (i) A tool which can solve interacting model: Density matrix renormalization group (DMRG).
- (ii) A topological marker which work for interacting case: entanglement entropy diagnostics

Entanglement entropy  $S_A = -\text{Tr}(\rho_A \ln \rho_A)$ ;  $\mathcal{H}_E = -\ln \rho_A \neq$  Entanglement Spectrum

Refs: Physical review letters 69, 2863 (1992), Annals of Physics 326, 96 (2011), special Issue, Phys. Rev. B 83, 075102 (2011)

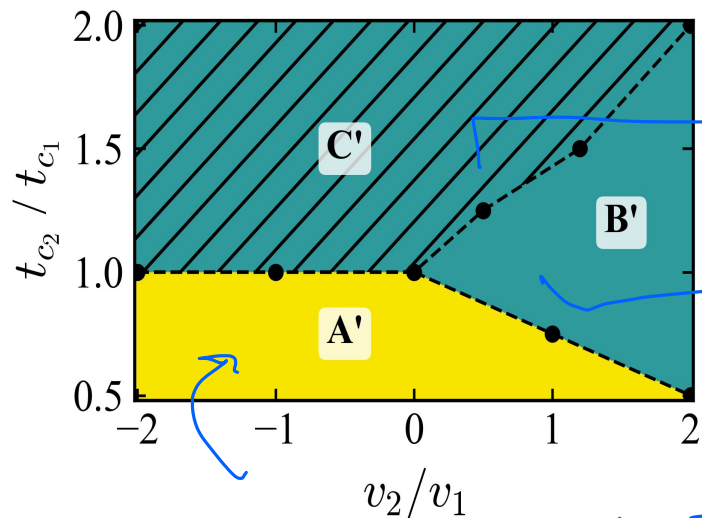
# Phase Diagram



*Correlated with the # of  $E=0$  states/edge*

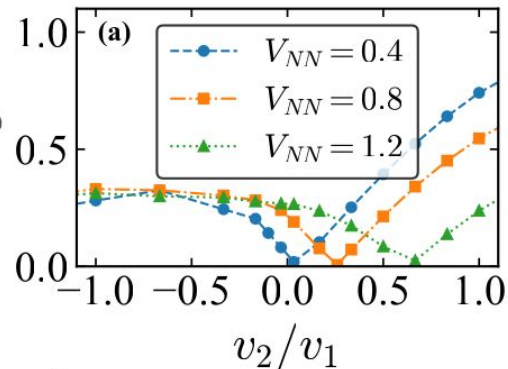
A-C Phase boundary pinning, B-C, A-B shifts  
in the presence of interaction

# Interacting Phase Diagram

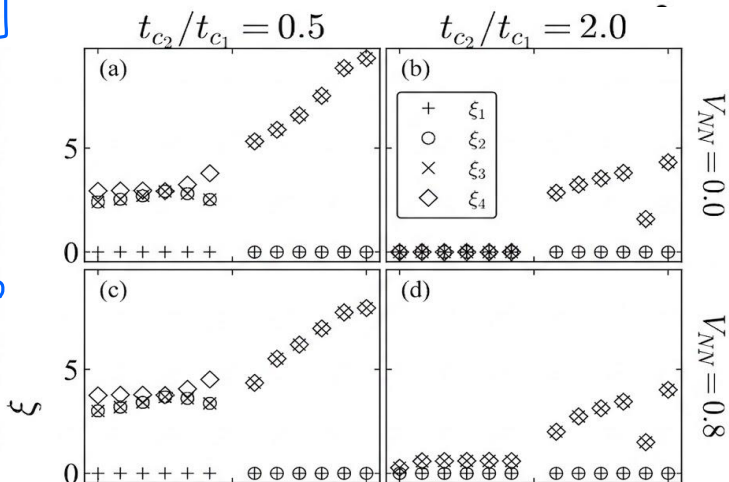


Vanishing Spectral gap  $\Delta g$

2-fold degenerate ES

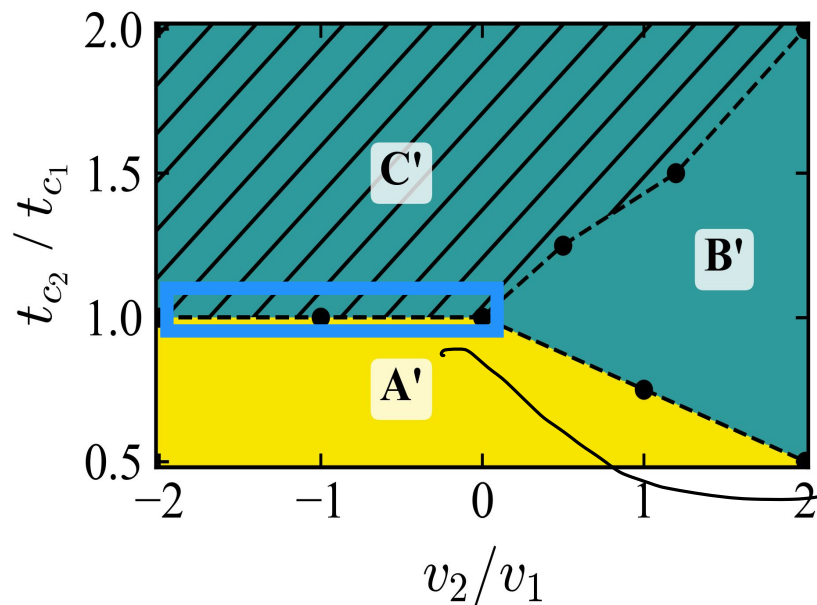


Entanglement Spectrum (ES)

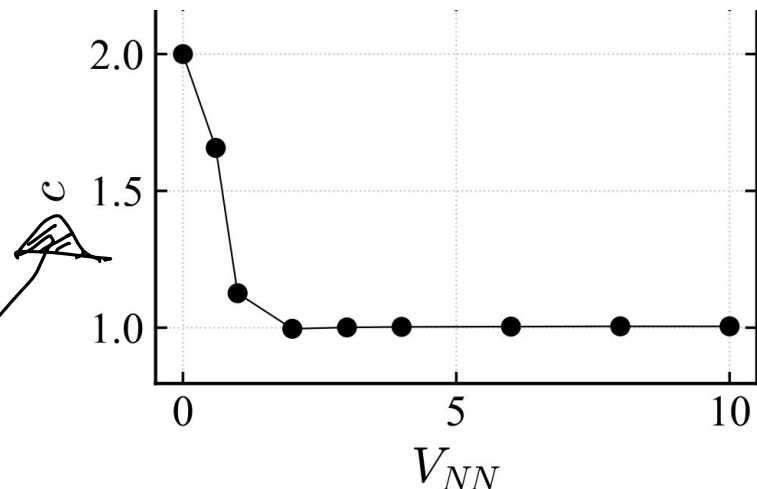


$V_{NN}$   
 $\Rightarrow$  Interaction  
 Degeneracy  
 $\rightarrow$  Non trivial state

# Interacting Phase Diagram



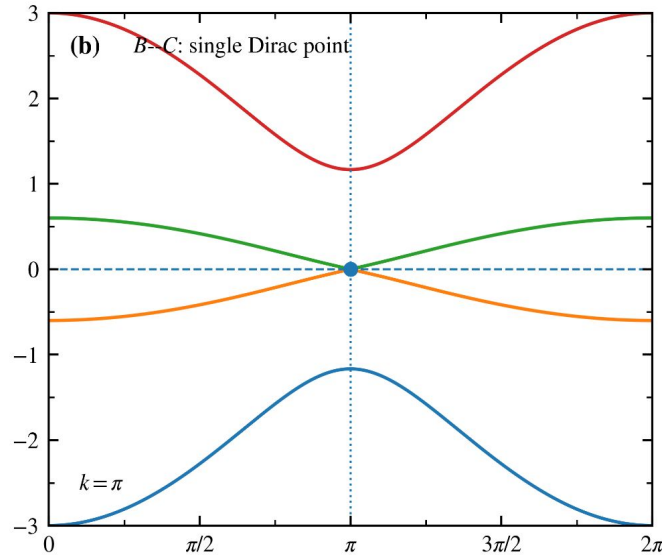
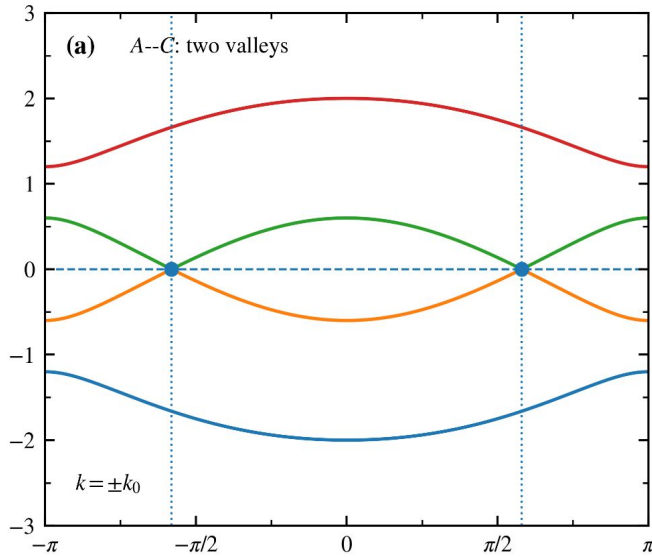
$$S(l) = \frac{c}{3} \log \left[ \frac{L}{\pi} \sin \left( \frac{\pi l}{L} \right) \right] + s_0$$



Central Charge ( $c$ ): How many gapless modes on the critical line?

The critical theory tells us **how many independent degrees of freedom become gapless at that point.**

# Low energy theory



A-C  $\Rightarrow$  2 Dirac modes

$$h \sim k \sigma_x + m_{A-C} \sigma_z$$

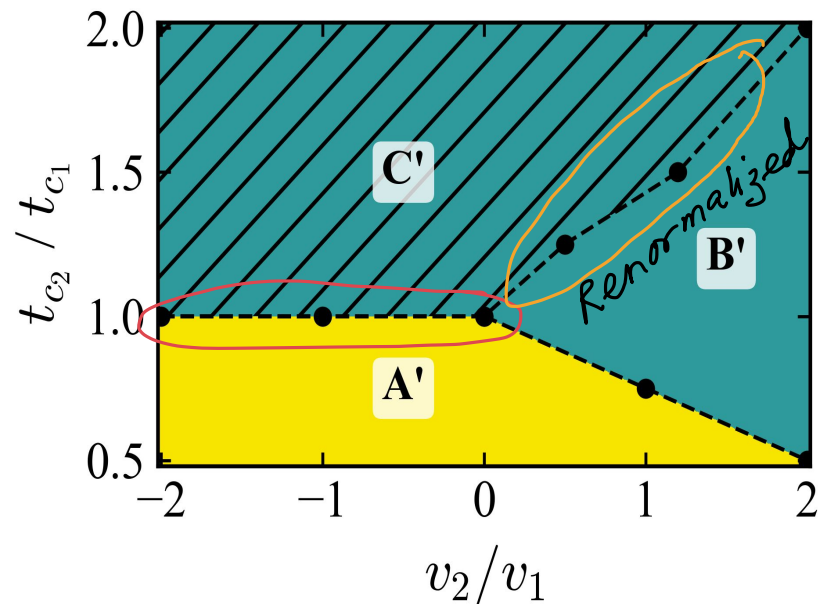
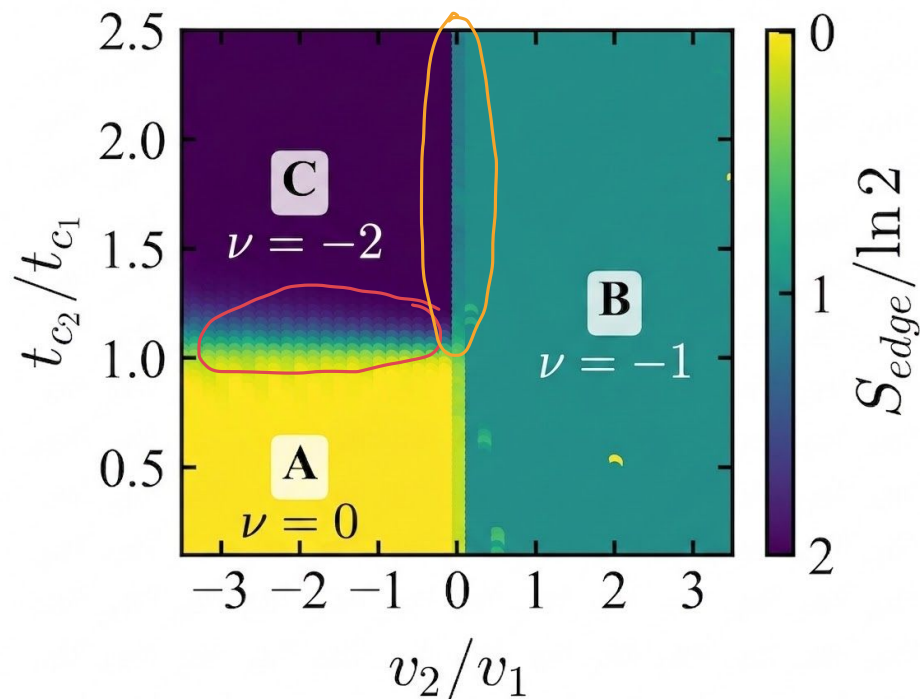
B-C  $\Rightarrow$  1 Dirac mode

$$h \sim k \sigma_y + m_{B-C} \sigma_x$$

For A-C boundary:

- Interaction couples the two valleys, creating two **collective** low energy Dirac modes.
- Interactions reduce the number of critical gapless modes from **two to one**.

# Boundary pinning

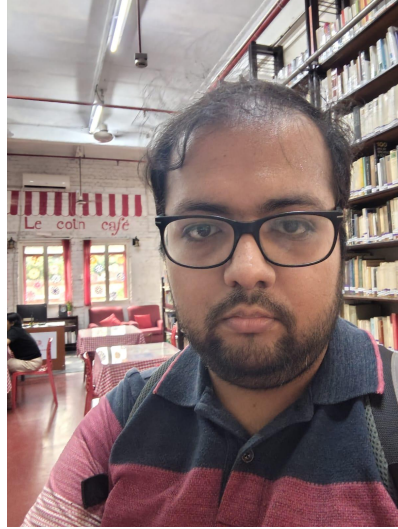


A'-C' Symmetry pinned, but different criticality  $m_{A-C} \propto |t_{c2} - t_{c1}|$   
 B'-C' : Interaction renormalized

# Conclusions

- Different topological phase boundaries of the same interacting system can exhibit qualitatively different responses to the same interaction.
- One of them remains at its non-interacting location while the other boundary shifts and is interaction-renormalized.
- Valley coupling mechanism to control the stability of the phase boundaries in an interacting topological system.
- The response of an interacting topological phase boundary depends on the symmetry and operator content of the corresponding low-energy critical theory.

# Our Group at IIT(ISM) Dhanbad



**Collaborators:** Prof. Rudranil Basu, BITS Goa, Dr. Nisa Ara, TIFR

*Thank you!*