

Cosmic Connectivity

Pravabati Chingangbam

Indian Institute of Astrophysics, Bengaluru

@ SWPNIOE, Shiv Nadar University, 11-12 Sep 2026

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Credit: Mike MacKinven



Credit: Dark Energy Spectroscopic Instrument

Shapley 1933

On the distribution of Galaxies

100,000 galaxies

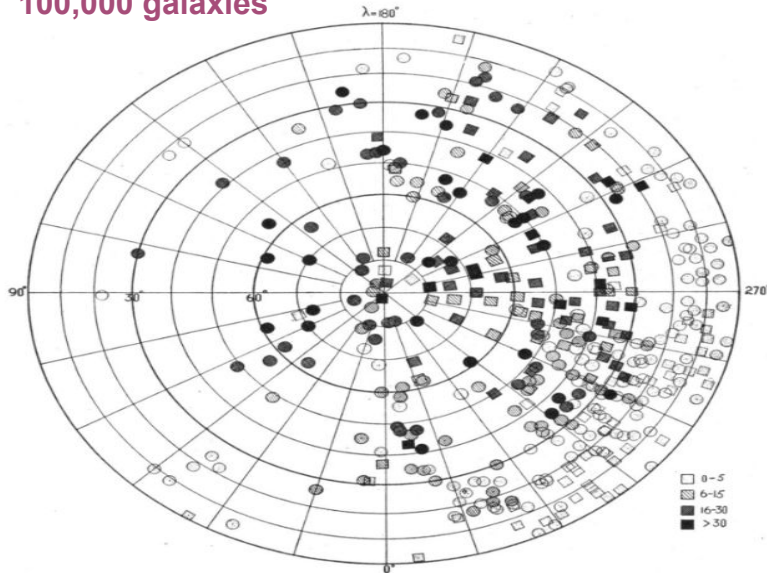


FIGURE 2

Distribution of fields in galactic coördinates. Squares indicate northern latitudes; circles, southern latitudes. Shading represents density per square degree.

Hubble 1934

The distribution of extra-Galactic nebulae

44,000 galaxies

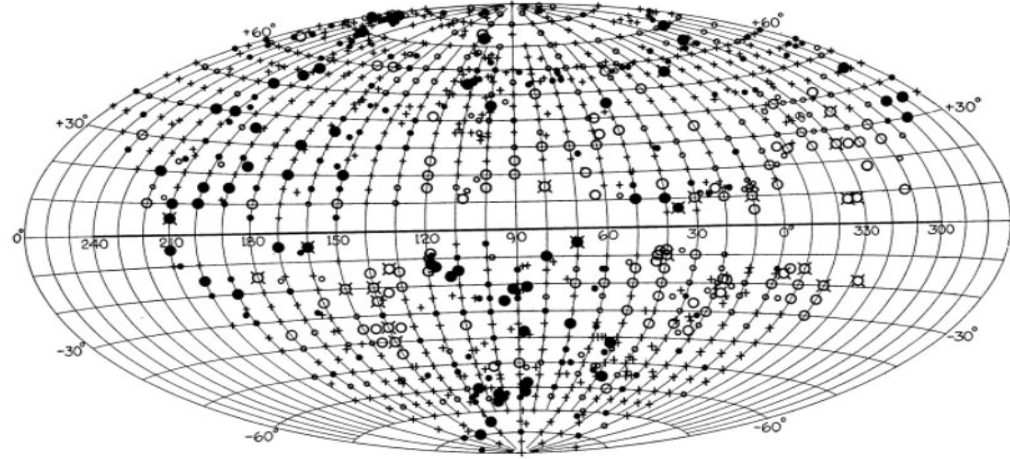


FIG. 6.—Distribution of extra-galactic nebulae when observed data are corrected for the latitude effect. Small crosses represent normal distribution ($\log N = 1.82-2.11$); small disks and circles, moderate excesses ($\log N = 2.12-2.26$) and deficiencies ($\log N = 1.67-1.81$); large disks and circles, considerable excesses ($\log N = 2.27-2.56$) and deficiencies ($\log N = 1.37-1.66$), with crosses added for $\log N > 2.56$ and $\log N < 1.37$. Fields with no nebulae are omitted.

Hinks, *The galactic distribution of the spiral nebulae*, 1911

Hardcastle and Fath, 1914

Curtis 1918

Seares 1925

Wirtz 1925

Reynolds 1923

Star maps have been around for more than three millennia.

Cosmic Microwave Background Radiation

Cosmic Dawn

~100 GHz $3.8 \times 10^5 \text{ years}$

~100 MHz $3 \times 10^8 \text{ years}$

Epoch of Reionization

~ 400 MHz $6 \times 10^8 \text{ years}$

Cosmic web: Structure formation ~ 500 THz $6 \times 10^9 \text{ years}$

Galactic emissions

Central assumption:

Each cosmological observable is a random field.

Deciphering the patterns - 1

Counts

Counting - *peak statistics*

Counting in pairs - *correlations*

Counting in triplets - *3-pt correlations*

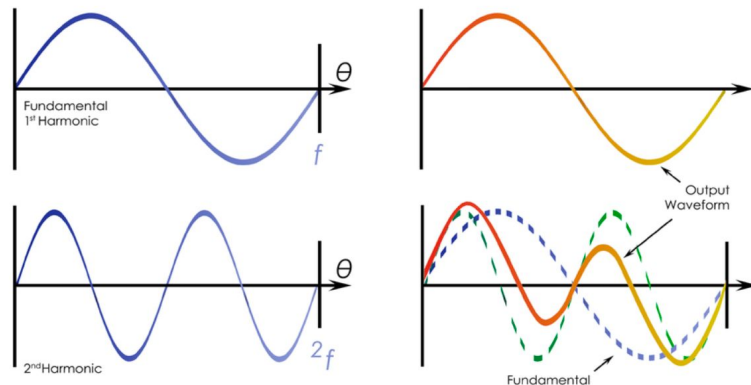
.....

Harmonics of the patterns

Peebles 1970, 1973 - 1980,

Harrison 1970, Zeldovich 1970

Amounts of different harmonics present carries characteristic signatures of physical processes.

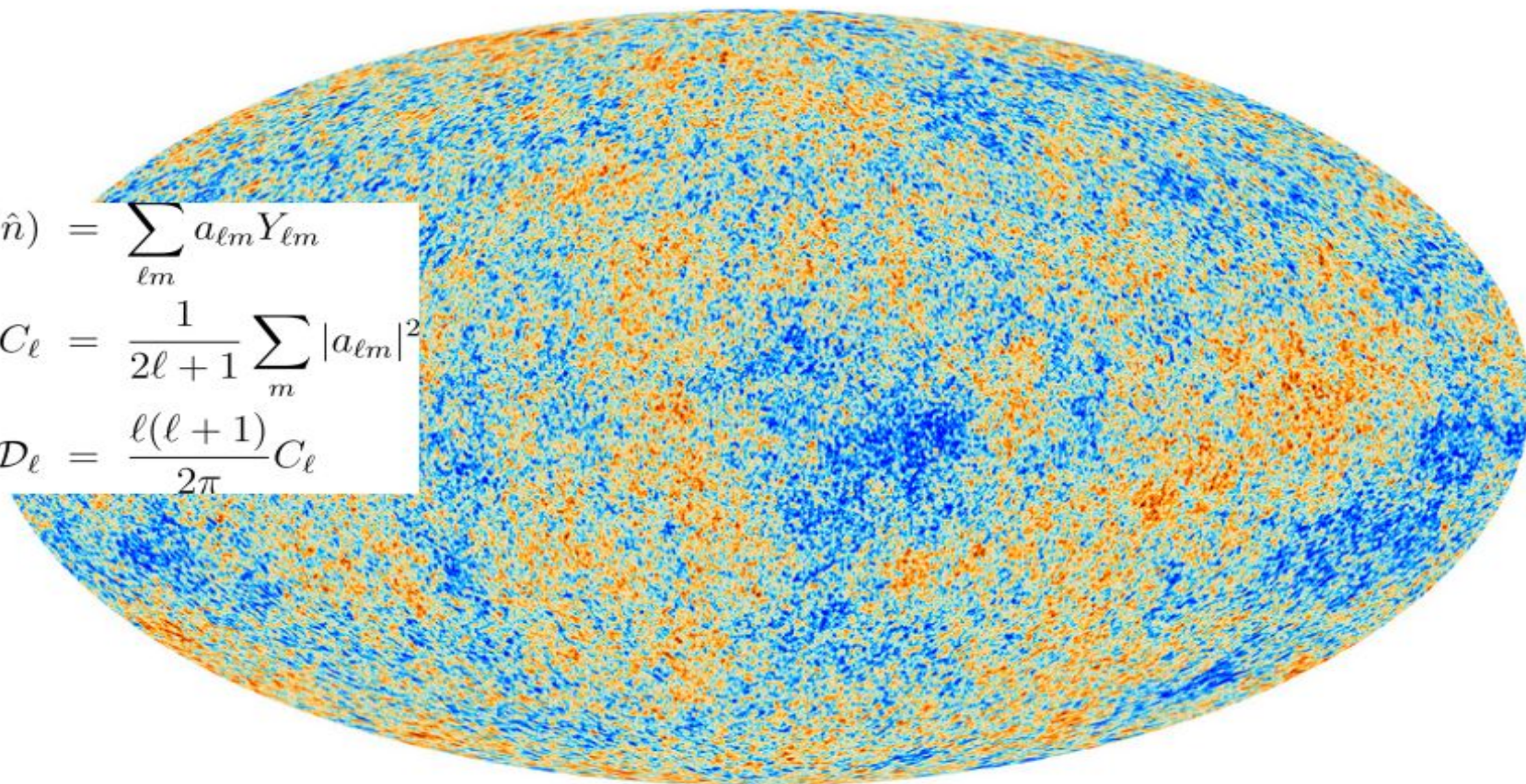


CMB Power spectrum

$$\Delta T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}$$

$$C_{\ell} = \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2$$

$$\mathcal{D}_{\ell} = \frac{\ell(\ell + 1)}{2\pi} C_{\ell}$$

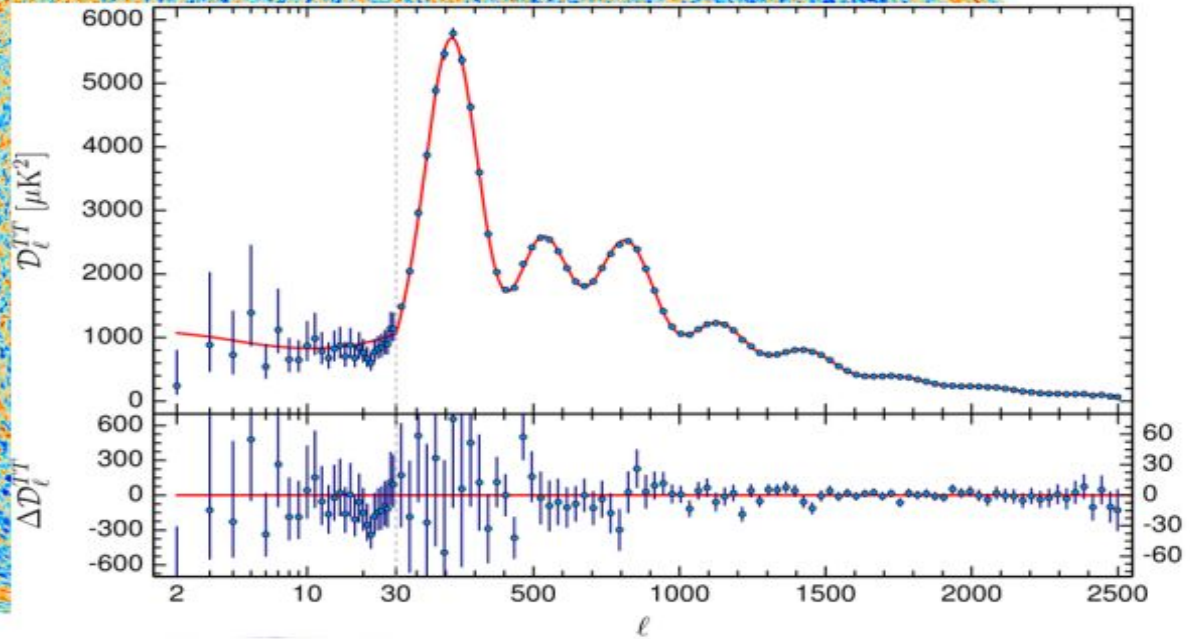


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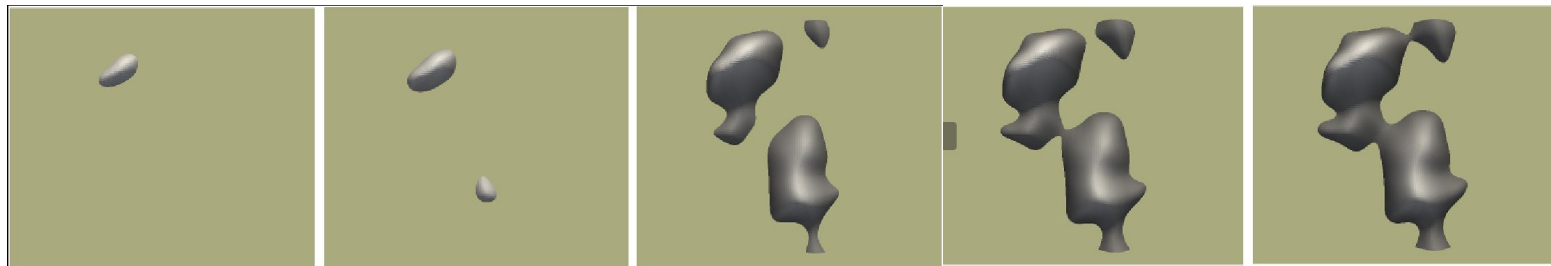
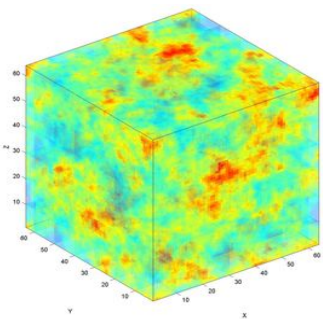
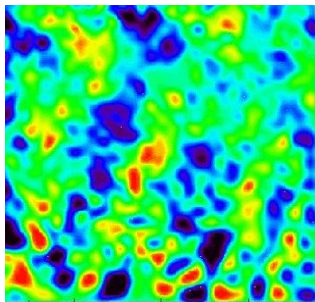
Deciphering the patterns using *excursion sets* - 2

Adler 1981

Bardeen, Bond, Kaiser & Szalay 1986

Gott+ 1986

Vary the field level ν



Geometry & topology of excursion sets $\leftrightarrow$ *nature of the field*

Ensemble expectations

- **Gaussian field** —
Tomita 1986
Area fraction = $\frac{1}{2} \operatorname{erfc} \left(\frac{\nu}{\sqrt{2}} \right)$
Perimeter $\sim \frac{1}{r_c} e^{-\nu^2/2}$
Euler characteristic $\sim \left(\frac{1}{r_c} \right)^2 \nu e^{-\nu^2/2}$

- **Mildly non-Gaussian** $\rightarrow$ perturbative corrections to the

Matsubara+ 1993 - Gaussian form in powers of the

Pogosyan + 2000 - standard deviation

- Rayleigh, lognormal fields, ...

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**Beyond
ensemble expectations?**



Focus on topology

Geometry & topology of excursion sets $\leftrightarrow$ *nature of the field*

Ensemble expectations

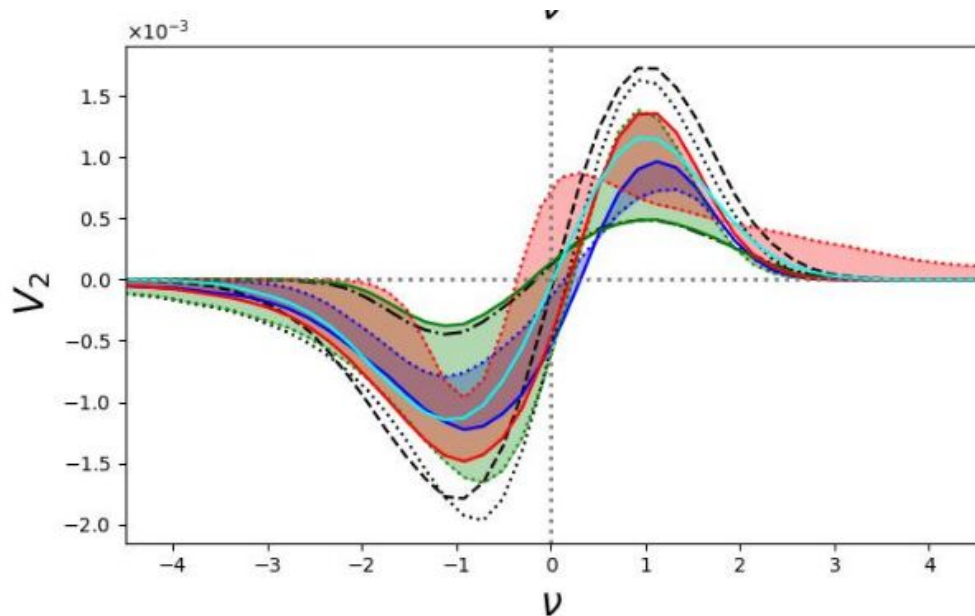
- Ga

Torr

- Mil

Mat

Pog



Beyond
ensemble expectations?

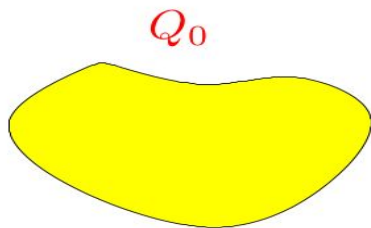


Focus on topology

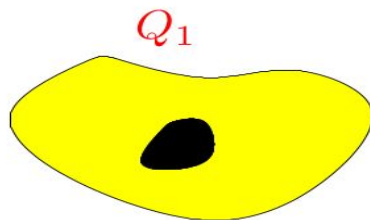
- Rayleigh, lognormal fields, ...

Topology of excursion sets - *Betti numbers and Euler characteristic*

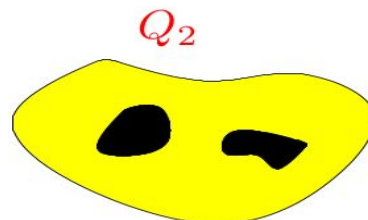
Betti 0	$b_0 =$	number of connected components
Betti 1	$b_1 =$	number of holes
Euler characteristic	$\chi =$	$b_0 - b_1$



$$\begin{aligned} b_0 &= 1 \\ b_1 &= 0 \\ \chi &= 1 \end{aligned}$$



$$\begin{aligned} b_0 &= 1 \\ b_1 &= 1 \\ \chi &= 0 \end{aligned}$$



$$\begin{aligned} b_0 &= 1 \\ b_1 &= 2 \\ \chi &= -1 \end{aligned}$$

$\dots Q_j$

$$\begin{aligned} b_0 &= 1 \\ b_1 &= j \\ \chi &= -(j - 1) \end{aligned}$$

Topological basis for excursion sets

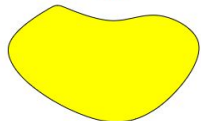
$Q^{(\nu=0)}$



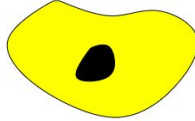
Building blocks



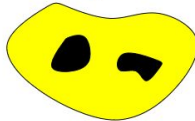
Q_0



Q_1



Q_2



$$\begin{aligned} Q^{(\nu)} &= \bigcup_{j=0}^{j_{\max}} m_j Q_j \\ &= m_0 Q_0 \cup m_1 Q_1 \cup m_2 Q_2 \cup \dots \cup m_{j_{\max}} Q_{j_{\max}} \end{aligned}$$



$$f(\mathbf{x}) \longleftrightarrow \underset{-\infty < \nu < \infty}{Q^{(\nu)}} \longleftrightarrow \left\{ m_0(\nu), m_1(\nu), m_2(\nu), \dots \right\}$$

Betti numbers and Euler characteristic in terms of the basis elements

$$b_0(\nu) = \sum_{j=0}^{j_{\max}(\nu)} m_j(\nu),$$

$$b_1(\nu) = \sum_{j=1}^{j_{\max}(\nu)} j m_j(\nu),$$

$$\chi(\nu) = \sum_{j=0}^{j_{\max}(\nu)} (1 - j) m_j(\nu)$$

$$= m_0(\nu) - \sum_{j=2}^{j_{\max}(\nu)} (j - 1) m_j(\nu)$$

- Clean separation of randomness from topology
- For actual use need to know statistical properties of m_j 's

Statistical nature of m_j 's

$Q^{(\nu)}$ can be considered as a realization of m_j number of each distinct ball in N trials of drawing balls from the box.

p_j : probability that the ball is type Q_j .

In N trials, the probability that a Q_j type ball will be picked k times is given by the Binomial distribution.

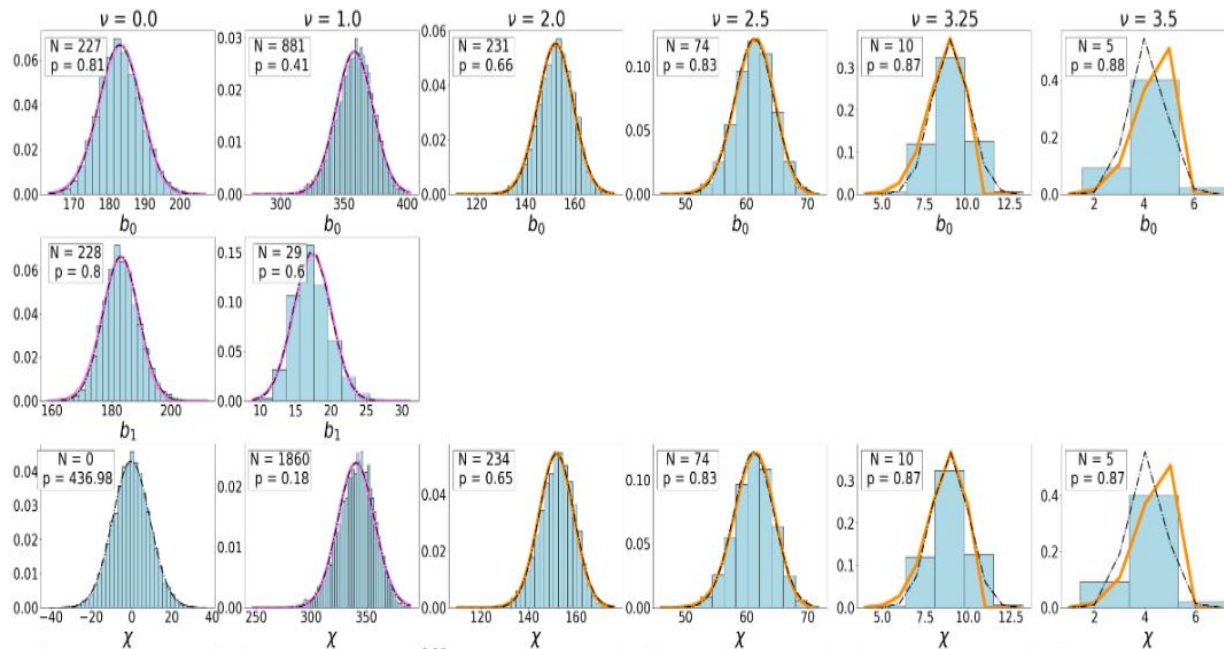
$$P(m_j = k; p_j, N) = {}^N C_k p_j^k (1 - p_j)^{N-k}.$$

Shortcomings:

- A priori N , p_j are not known. Depends on the nature of the field.
- Correlations amongst m_j 's are not known.

Testing the Binomial modeling for Gaussian field - CMB

Positive ν



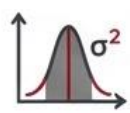
- Agrees well with numerical results.
- Betti numbers and Euler characteristic are Gaussian to good approx at intermediate thresholds, not at high/extreme thresholds.

Summary

 Random fields are central to interpreting cosmological data and inferring physical information.

 Need precision tools for statistical inference.

 Morphology of random fields provides a powerful methodology for statistical inference.

 How to sharpen morphological tools - develop analytic predictions for their statistical behaviour.



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