Modeling linear coupling and nonlinear effects from experimental solenoids in the FCC-ee

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Motivation

- The FCC-ee interaction region (IR) will be equipped with a detector solenoid
  - The solenoid strongly distorts the optics and must be compensated
  - Studies so far have used a $2\mathrm{T}$ solenoid
  - A $3\mathrm{T}$ solenoid is now under consideration for a prospective new experiment
  - The established compensation scheme is expected to run into limits at $3\mathrm{T}$ [2]

- We compare the established scheme (local) with an alternative (non-local)

- Can a $3\mathrm{T}$ solenoid be compensated non-locally?

Outline

- Effects of solenoidal field
- Compensation methods
  - Local
  - Non-local
- Results so far
  - Linear optics and coupling
  - Vertical equilibrium emittance
  - Dynamic Aperture and Momentum Acceptance
- Future work

Effects of a Solenoid

- In the linear limit, the solenoid field is approximated by

$$\mathbf{B} \approx -\frac{r}{2} B_z' \, \hat{\mathbf{r}} + B_z \hat{\mathbf{z}}$$

- The Lorentz force in cylindrical coordinates is:

$$\mathbf{F} = q \left(v_\varphi B_z \, \hat{\mathbf{r}} - \left(v_z \frac{r}{2} B_z' + v_r B_z \right) \hat{\boldsymbol{\varphi}} + v_\varphi \frac{r}{2} B_z' \, \hat{\mathbf{z}} \right)$$

- If $v_\varphi = v_r = 0$ initially, it will become nonzero and negative in the fringe
- With a $v_\varphi < 0$, the radial force is negative and hence, inward

- The inward force causes a focusing effect
  - Like a quadrupole of $k \propto B_z^2$, but in both planes
- The longitudinal field rotates the beam
  - Solenoid rotates beam by an angle θ
- Coupling terms
  - In a realistic solenoid, higher-order field components may enter and cause additional coupling
- **The coupling and rotation must be compensated**

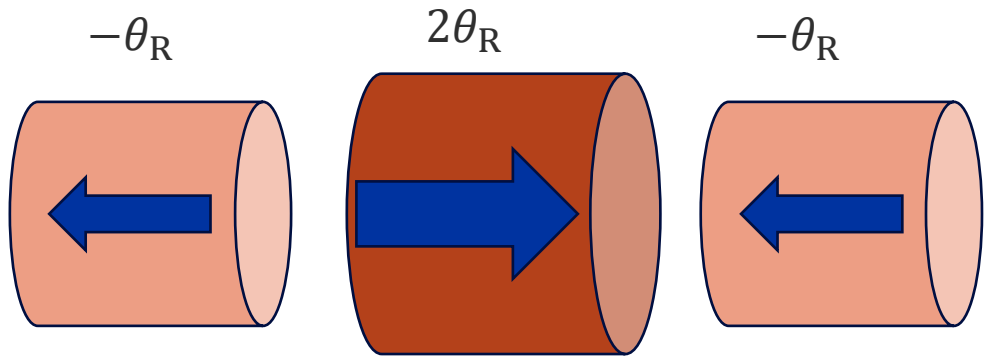
$$\theta = \frac{1}{2B\rho} \int_0^L B_z(s) \, \mathrm{d}s$$

$\theta = 8.09$ mrad for 2T

$\theta = 12.8$ mrad for 3T

Correction

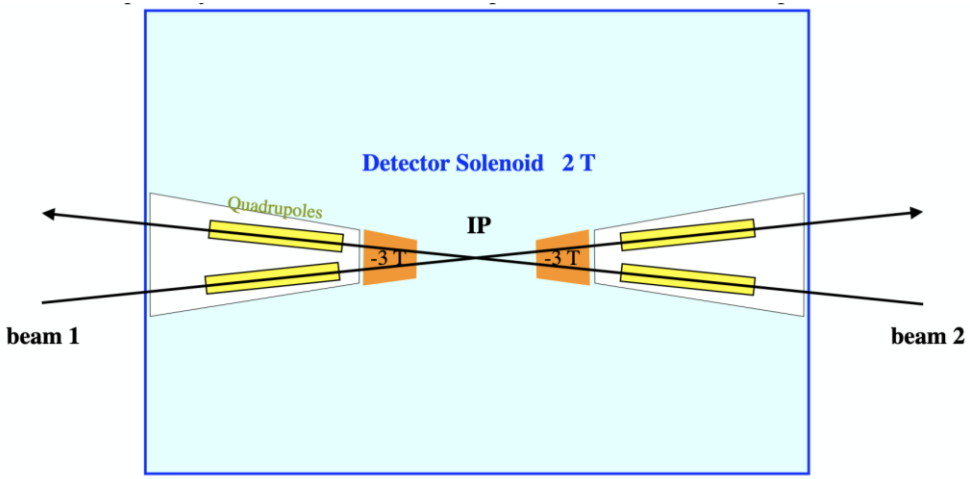
- We can correct θ by placing *compensation solenoids* at each side of the detector solenoid
- These solenoids carry a field in the opposite direction to the detector solenoid, each having half the field integral of the detector solenoid
- Residual coupling can be corrected by using skew quadrupoles

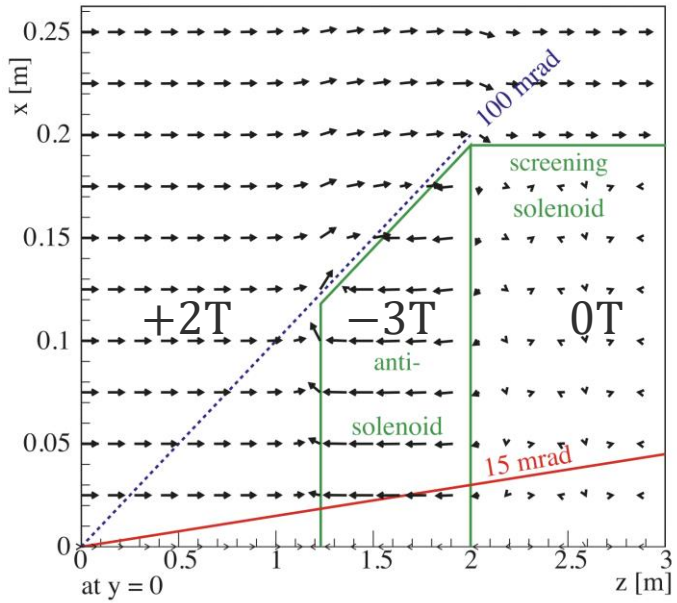


Local Correction [1]

- We can place the compensation solenoids right next to the IP
- ✓ Simple method
  - θ is compensated right outside the IP
  - Remaining coupling is corrected elsewhere
- ❖ Fringe fields have strong transverse component:
  - $B_x = 0.263\ \mathrm{T}$ (main bends: $B_y = 0.016\ \mathrm{T}$)
  - Causes strong SR

- ❖ A $3\mathrm{T}$ main solenoid has been under consideration, which would make these drawbacks even worse





Source: [1,3]

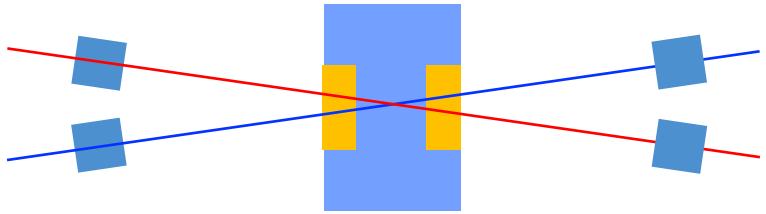
Work so far

- M. Boscolo, A. Ciarma and H. Burkhardt [1] have already studied aspects of the local and non-local schemes and the scheme has already been applied in DAΦNE in Frascati, Italy
  - Fringe field strength, SR power, orbit
  - Mainly for the 2T solenoid
  - MAD-X

- Our goal is to expand on this study as follows:
  - Implement the local and non-local schemes in Xsuite
  - Study linear optics and coupling
  - Nonlinear effects
  - Field errors
  - Misalignments

- Main question: Can a 3T detector solenoid be compensated with non-local compensation solenoids?

Non-Local Correction [1,8]

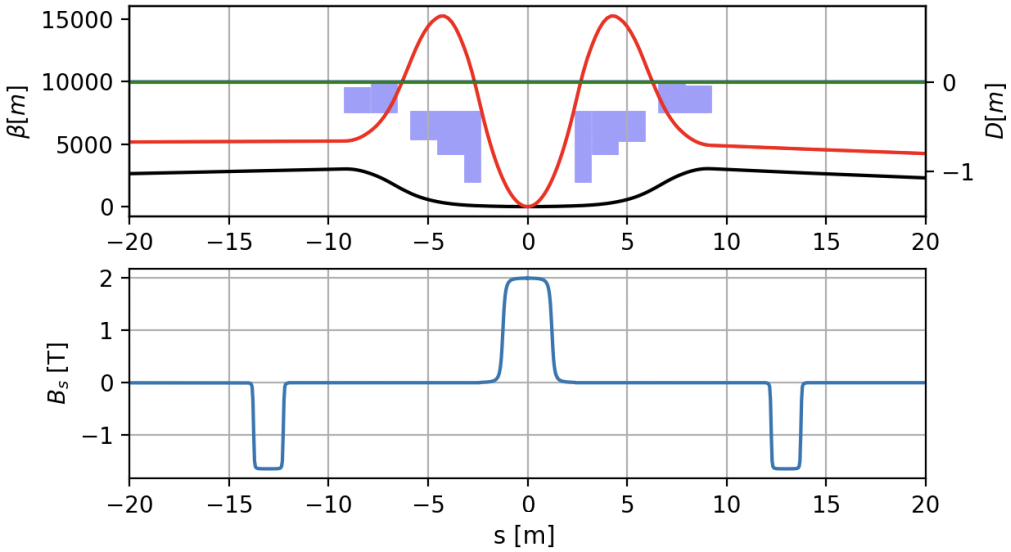


- We can place the compensation solenoids outside the final focusing quadrupoles

❖ Slightly more complicated

✓ But:

- Normal quadrupole as seen by a rotated beam has a skew component related to θ
- We can add a skew quadrupole coil inside the Final Focusing Quadrupoles so that the beam effectively sees a normal quadrupole



Source: [8]

$$k_{1n} \rightarrow k_{1n}\cos(-2\theta_{\mathrm{R}})$$

$$k_{1s} \rightarrow k_{1n}\sin(-2\theta_{\mathrm{R}})$$

✓ Transverse fringe field can be reduced to $B_x = 0.0576\ \mathrm{T}$

- This can reduce SR power and vertical emittance

Modeling the Solenoid Field

- Exact expressions:

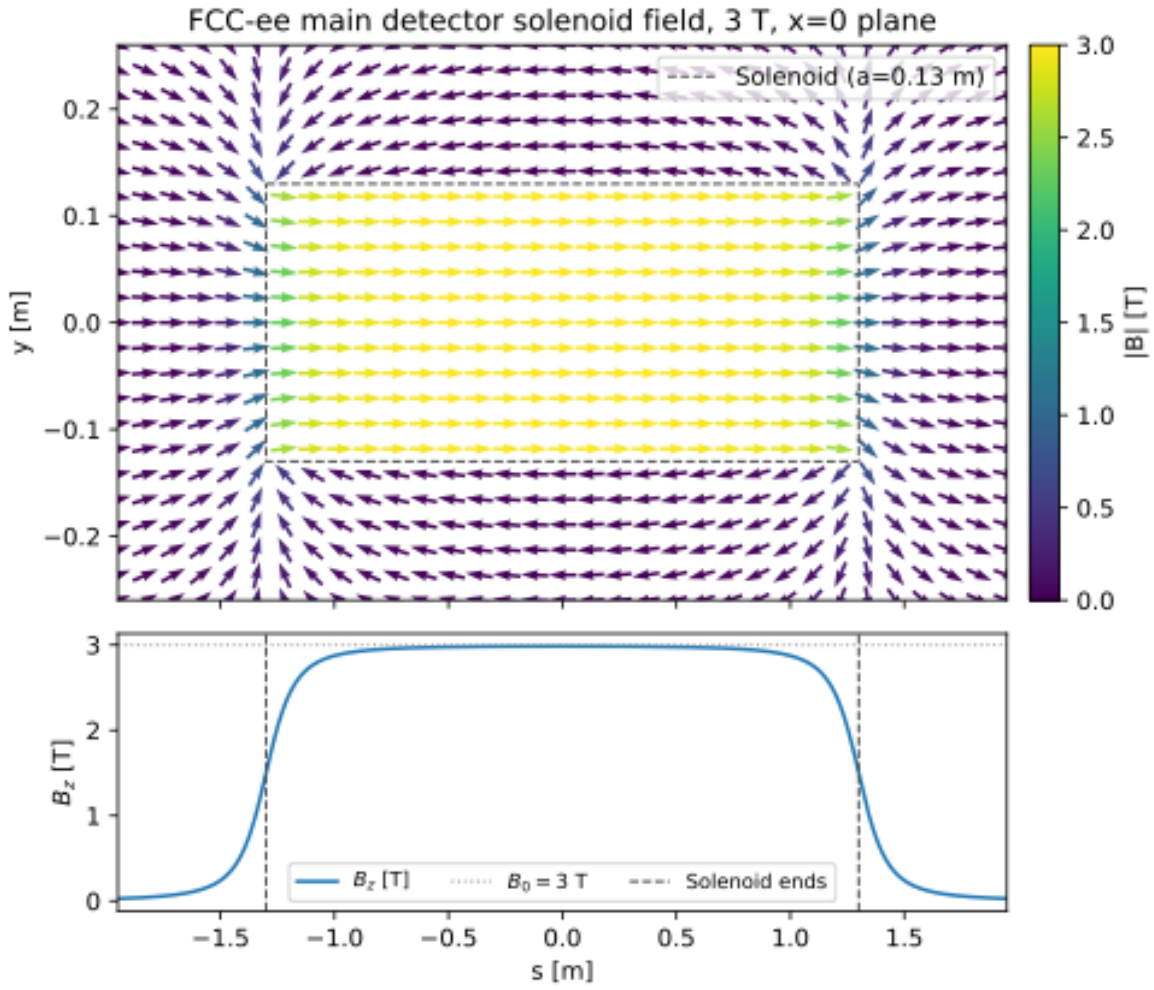
$$B_r(r,\theta,z) = \frac{B_0}{\pi}\sqrt{\frac{a}{rm_+}}\left(E(m_+) - \left(1-\frac{m_+}{2}\right)K(m_+)\right)$$

$$-\frac{B_0}{\pi}\sqrt{\frac{a}{rm_-}}\left(E(m_-) - \left(1-\frac{m_+}{2}\right)K(m_-)\right)$$

$$B_z(r,\theta,z) = \frac{B_0\zeta_+}{4\pi}\sqrt{\frac{m_+}{ar}}\left(K(m_+) + \left(\frac{a-r}{a+r}\right)\Pi(u,m_+)\right)$$

$$-\frac{B_0\zeta_-}{4\pi}\sqrt{\frac{m_-}{ar}}\left(K(m_-) + \left(\frac{a-r}{a+r}\right)\Pi(u,m_-)\right)$$

- a, L: Radius and length of solenoid
- $m_\pm$, $\zeta_\pm$, u: Geometric factors dependent on a, L, r, z
- K, E, Π: Incomplete elliptic integrals of the first, second and third kind respectively



Bpmeth and SplineBoris [6, 4]

- Following method from Ref. [6] and references therein. Define functions

$$a_m(z) = \left.\frac{\partial^{m-1} B_x}{\partial x^{m-1}}\right|_{x=y=0}$$

$$b_m(z) = \left.\frac{\partial^{m-1} B_y}{\partial x^{m-1}}\right|_{x=y=0}$$

$$b_z(z) = B_z(0,0,z)$$

- It can be shown that the scalar potential $\Phi(x,y,z)$ can be expressed as

$$\Phi(x,y,z) = \sum_{n=0}^{\infty} \phi_n(x,z)\frac{y^n}{n!}$$

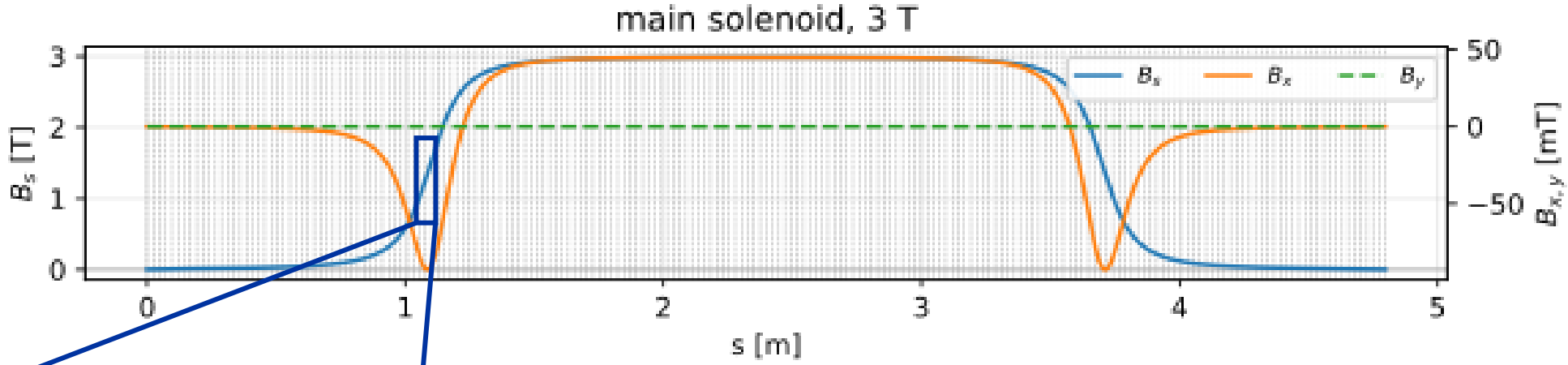
- Where $\phi_n(x,z)$ depends on $a_m(z)$, $b_m(z)$, $b_z(z)$ and their derivatives. The magnetic field is then found from

$$\vec{B}(x,y,z) = -\nabla\Phi$$

Provided a_m, b_m and b_z, we are able to compute $\vec{B}$ anywhere and we are sure it satisfies Maxwell's Equations.

These functions can be found from data, or defined arbitrarily (e.g., in design stage when a field map is not available).

Split Field into Regions



$$B_{s,\text{segment } j} = P_{4,s,0,j}(s)$$

$$B_{x,\text{segment } j} = P_{4,x,0,j}(s)$$

$$B_{y,\text{segment } j} = P_{4,y,0,j}(s)$$

$$\partial_x B_{x,\text{segment } j} = P_{4,x,1,j}(s)$$

$$\partial_x B_{y,\text{segment } j} = P_{4,y,1,j}(s)$$

$$\vdots$$

SplineBoris

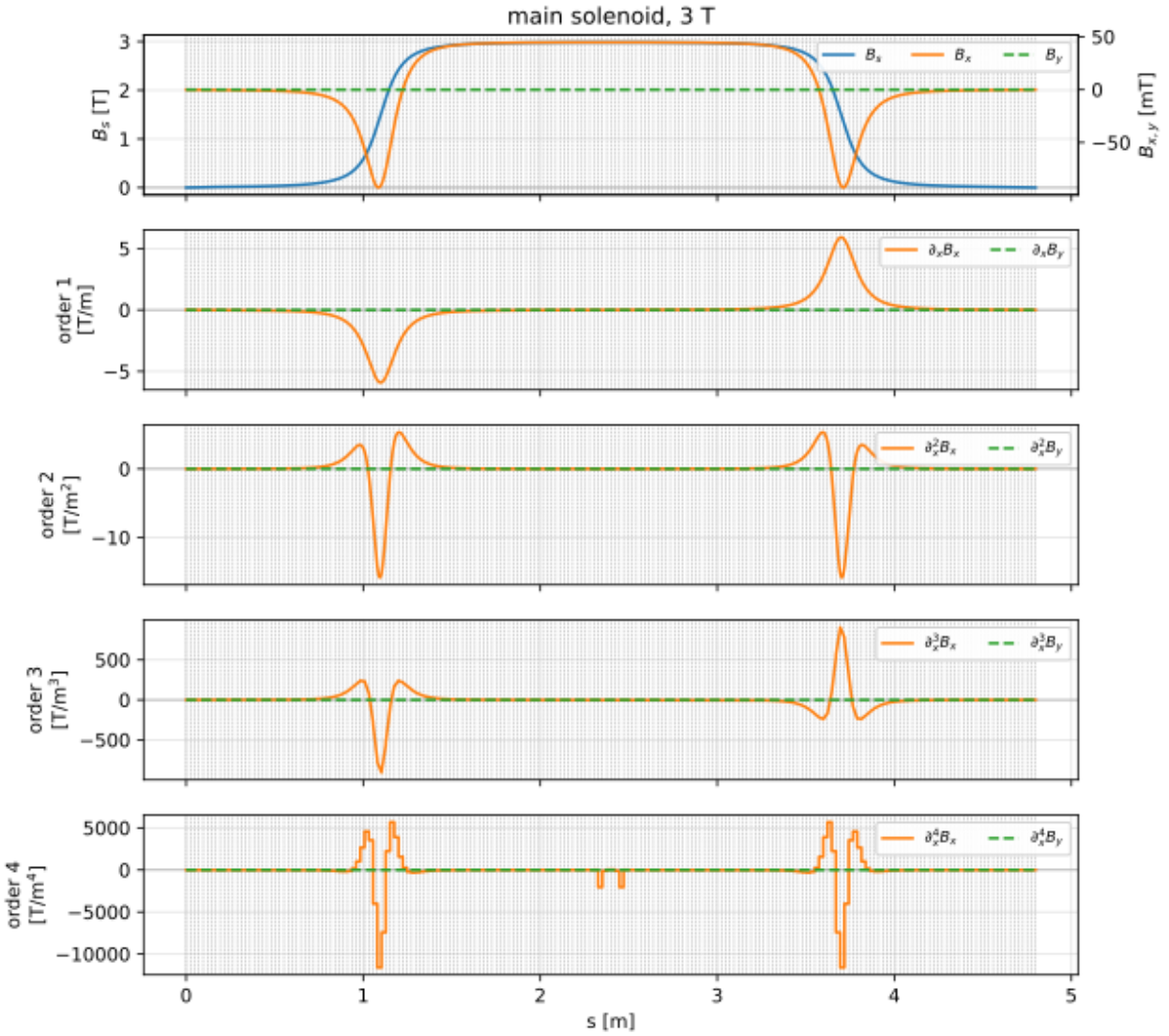
$$\vec{B}(x, y, s; \text{coefficients})$$

Power of the SplineBoris

- The analytic model can:
  - Describe high-order magnetic field components not described by the linearized model from before
  - Can be used for optics and tracking in the same way as any magnetic element

- On top of that, the SplineBoris can:
  - Model the magnetic field from a field map for tracking through a realistic field
  - We can manipulate specific field components to study their effects (as we will see later)

- SplineBoris has already been applied to study undulators, see [4] for a more detailed description.



Our Setup

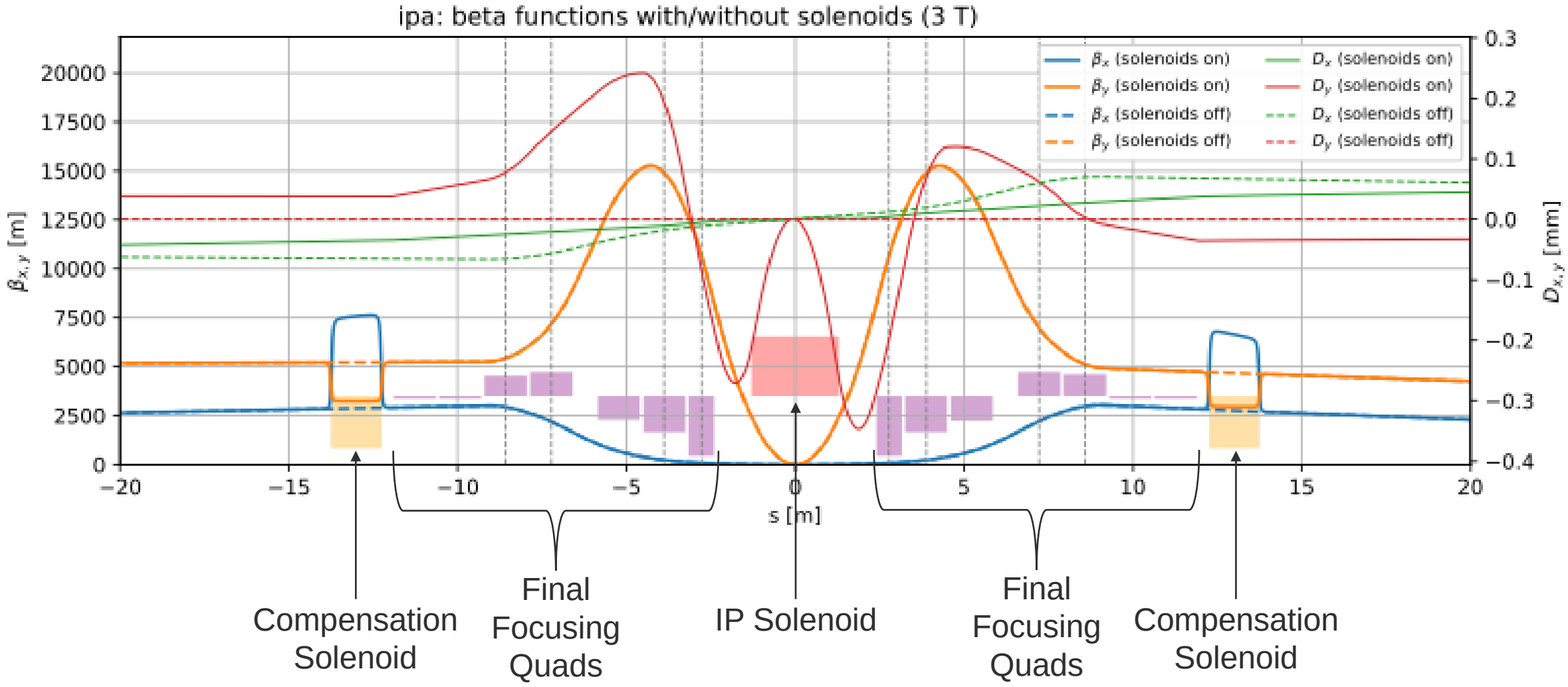
- We study the non-local correction scheme, where we will mostly examine the $3\mathrm{T}$ solenoid, but are sometimes interested in its comparison to the $2\mathrm{T}$ solenoid

- The solenoids are modeled as SplineBoris-based solenoids, which themselves are built from the analytical field expressions (no field maps yet)

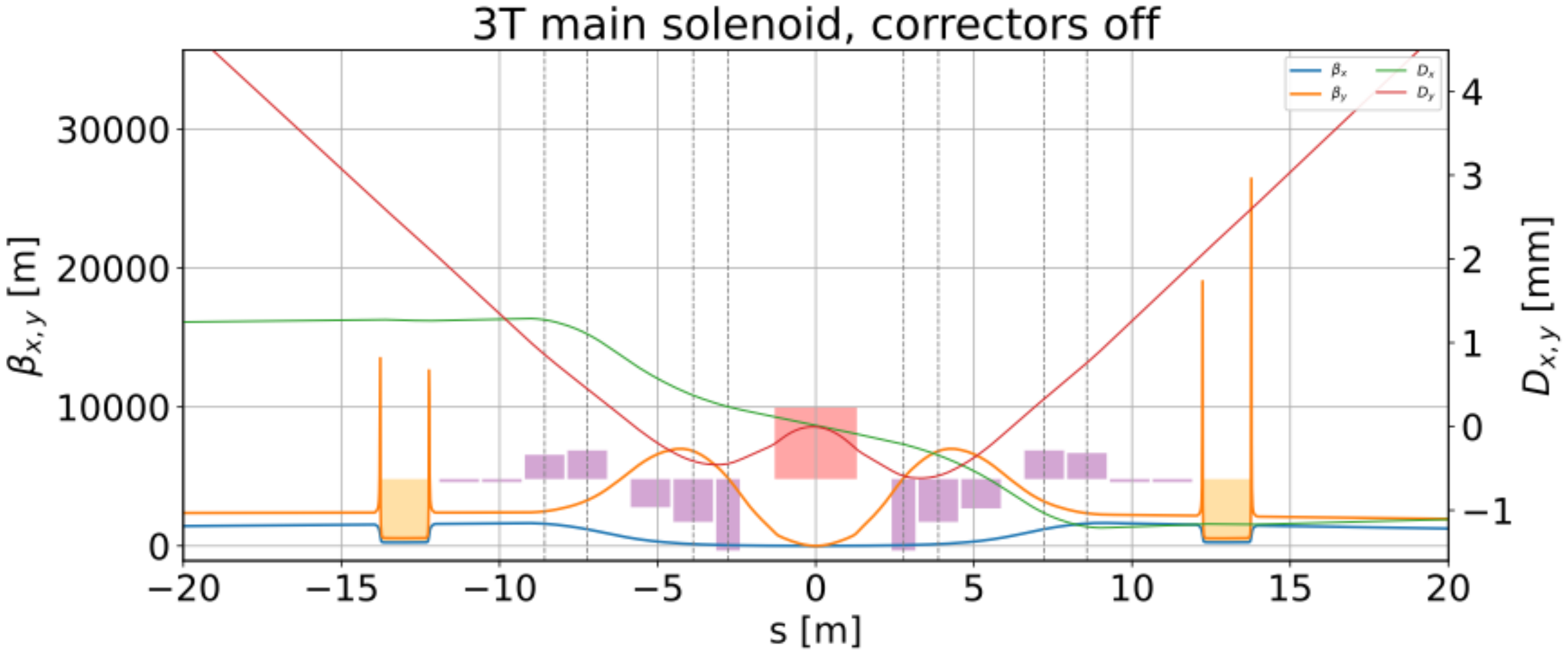
- All modeling is done in Xsuite

- Final focusing: 7 quadrupoles on each side
  - All 7 have a skew component related to θ, 4 of them have a dipole component for orbit correction
  - Additional 5 (+ 3) quads for optics correction
  - All quads in the straight are equipped with a skew corrector for coupling correction

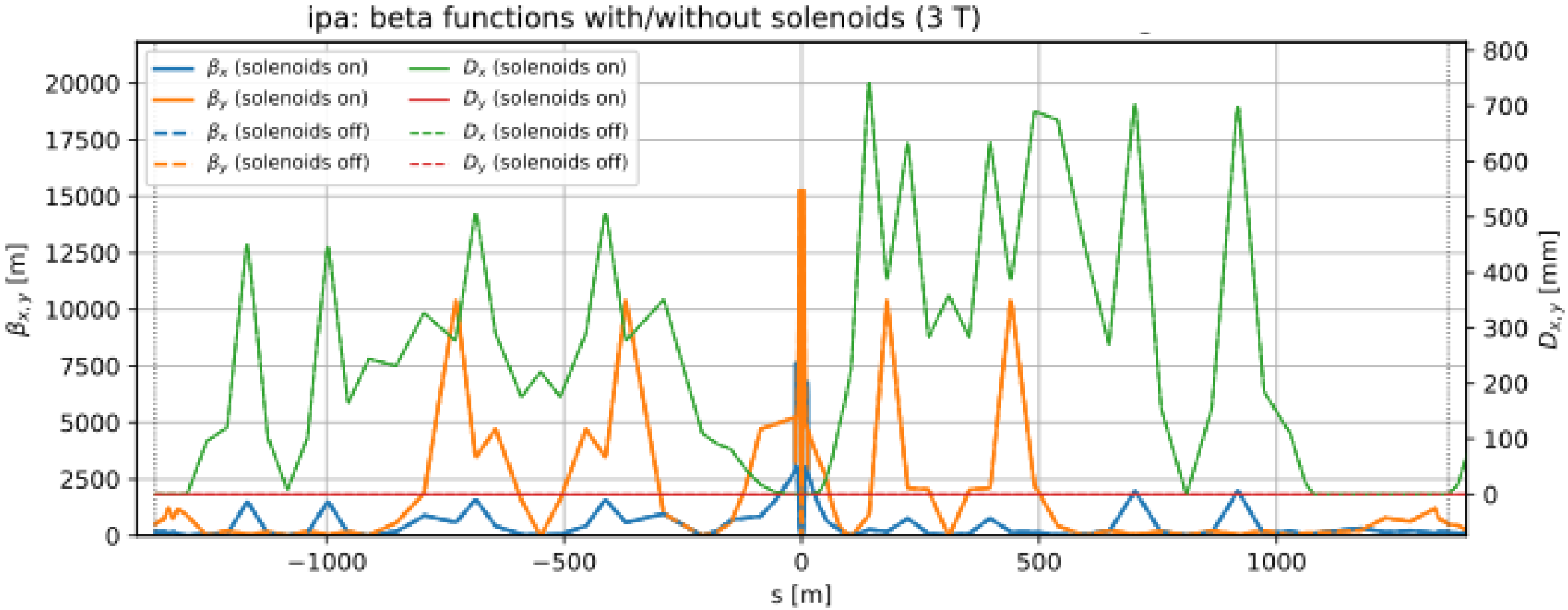
Our Setup (Close to IR, Corrected)



Our Setup (Close to IR, Uncorrected)

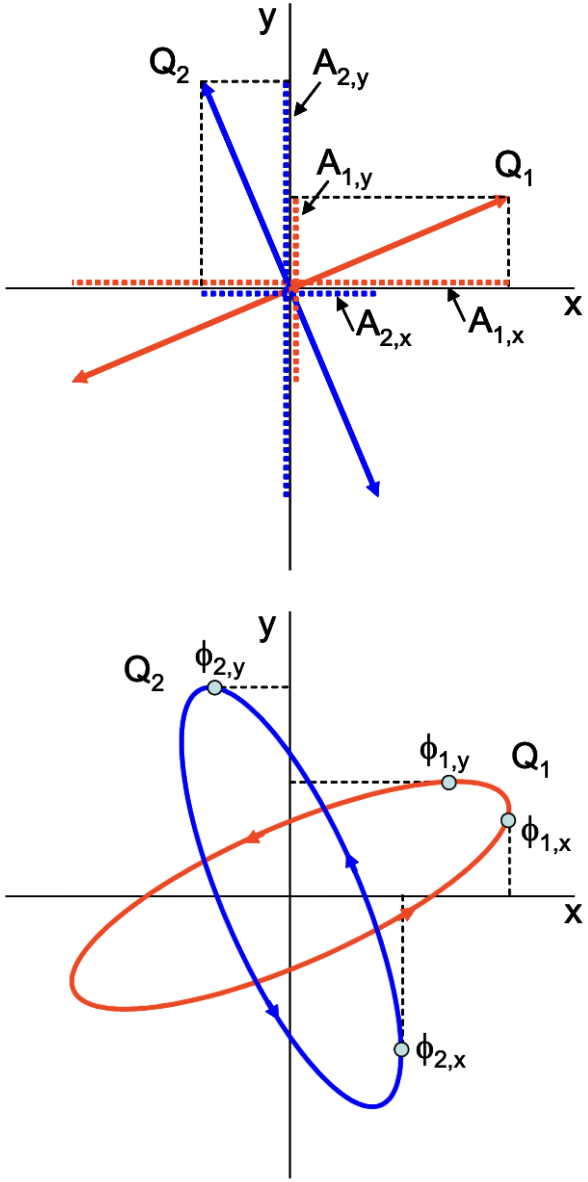


Our Setup (Full Straight, Corrected)



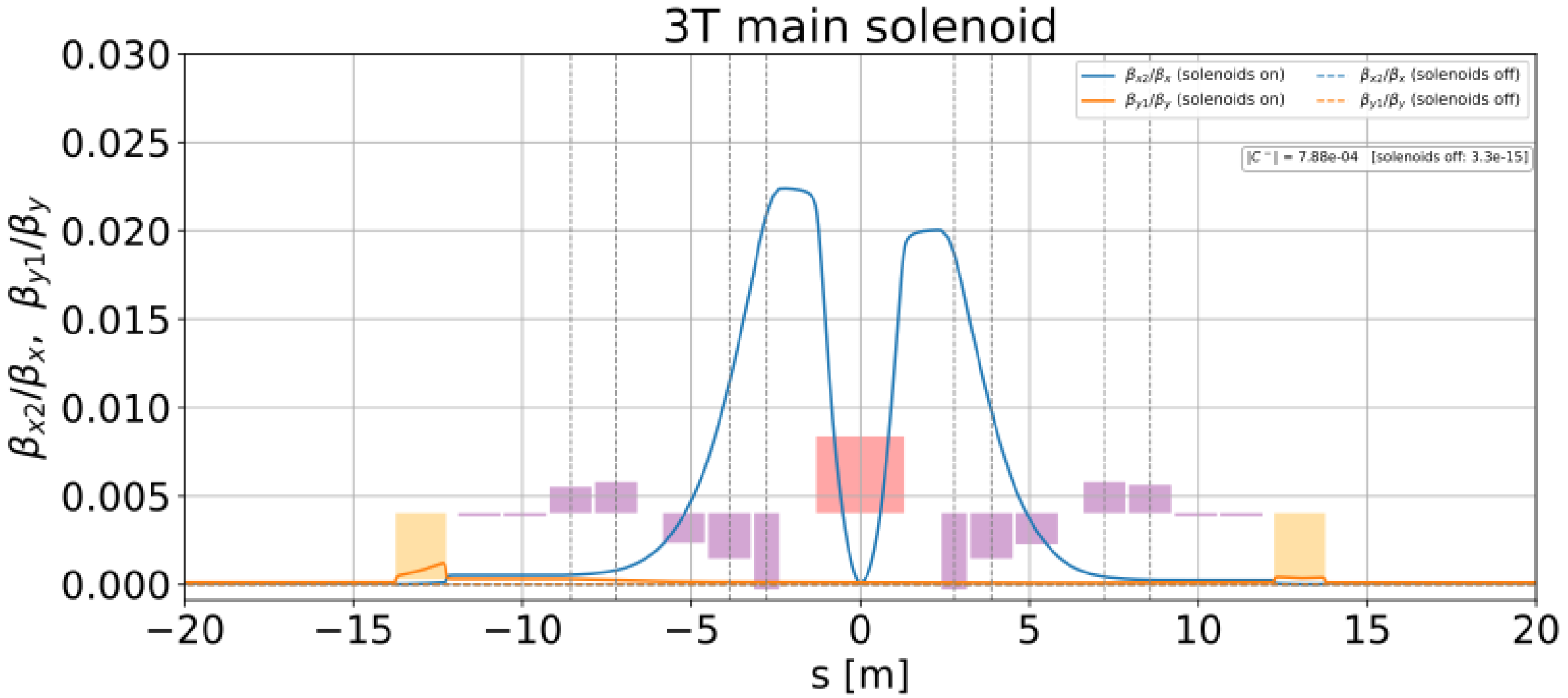
Coupling Correction [5,7]

- **Coupled optics**
  - Skew fields and solenoids rotate the planes of oscillation, so horizontal motion shows up in the vertical plane and vice versa
  - The eigenmodes of the one-turn map are rotated with respect to x and y: mode 1 is usually mostly horizontal, mode 2 mostly vertical
  - Each mode has a beta function in both planes: β_{x1}, β_{y1} and β_{x2} and β_{y2}. Without coupling the cross terms β_{y1} and β_{x2} vanish
  - We plot β_{x2}/β_x and β_{y1}/β_y : each is a measure for local coupling
- **Coupling coefficient**
  - The tunes of the two modes (Q_1 and Q_2) can be brought closer together by varying the arc quadrupole strength (not demonstrated here). The tunes will never cross, but reach a minimum separation at maximum coupling, which is defined as $|C^-|$
  - So $|C^-|$ is a measure for global coupling

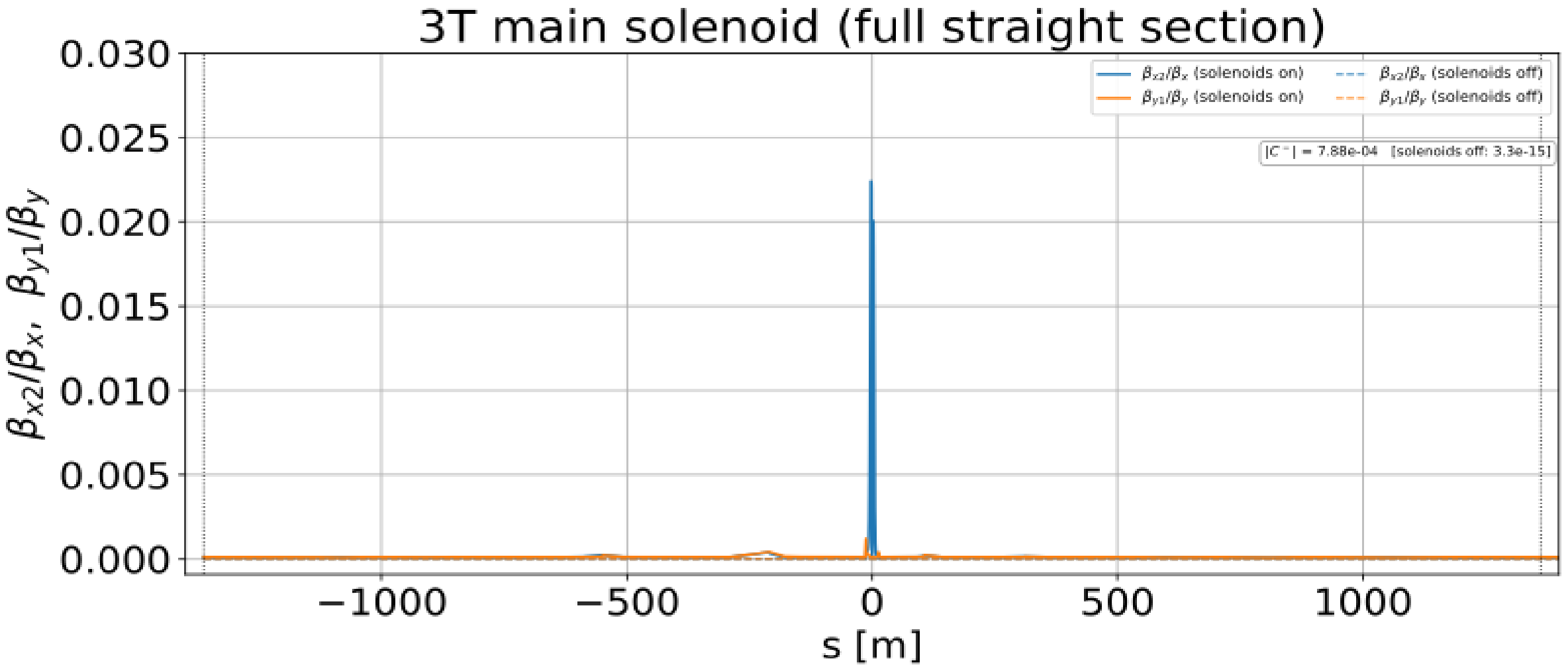


Source: [5]

Coupling Correction



Coupling Correction



Coupling Correction

- The coupled beta functions, β_{x2} and β_{y1} were successfully matched at the edges of the straight for both the $2\mathrm{T}$ and the $3\mathrm{T}$ cases
  - At edges and IP, $3\mathrm{T}$: $\beta_{x2}/\beta_x \sim \beta_{y2}/\beta_y \sim 10^{-4}$

- If the coupling is not corrected, β_{x2} and β_{y1} “leak” into the arcs
  - This is reflected in $|C^-|$, which grows by two orders of magnitude if the coupling is not corrected

Vertical Equilibrium Emittance

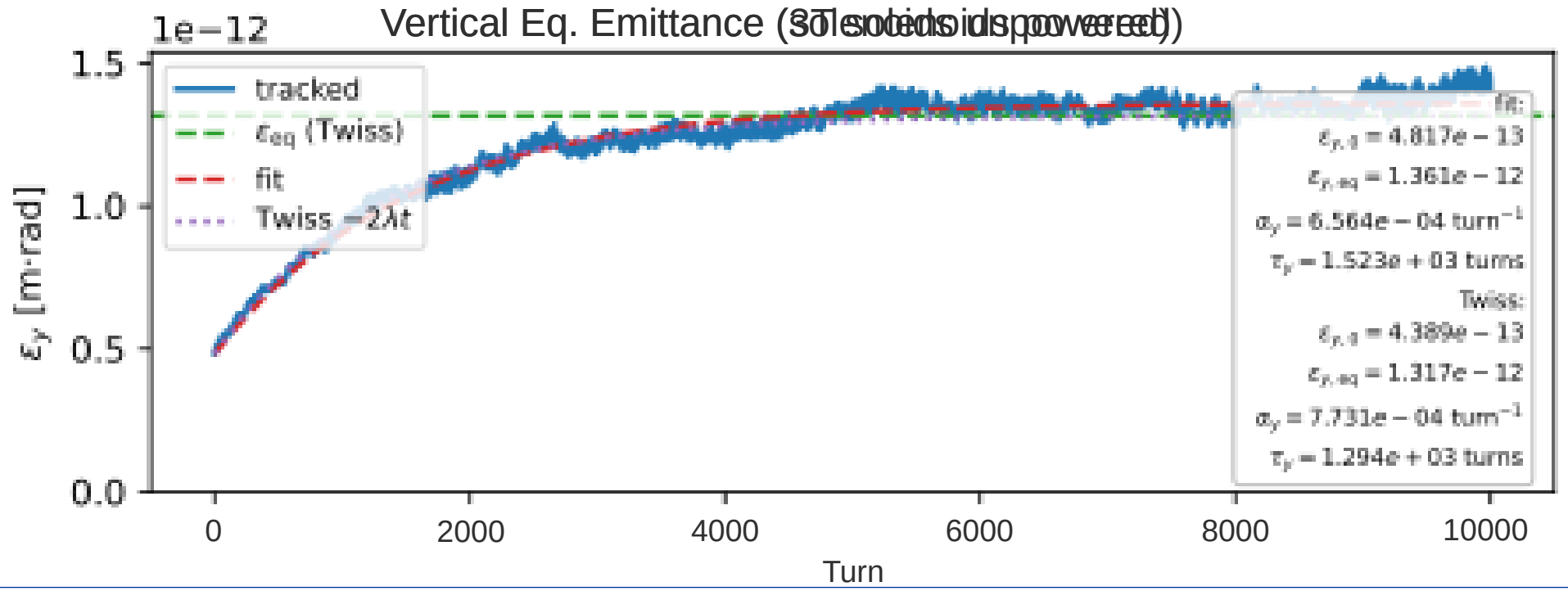
- Without the solenoids and the correction scheme, the FCC-ee is planar, which means the vertical equilibrium emittance (ε_y) is effectively zero
- In the presence of coupling, or field errors/misalignments, the vertical ε_y becomes nonzero
- With the linear optics corrected, we studied the effect on ε_y
- We can calculate ε_y as well as the damping rate per turn, α_y, from the Twiss. The ε_y then evolves with the number of turns, n, as

$$\varepsilon_y(n) = \left(\varepsilon_{y,0} - \varepsilon_{y,\mathrm{eq}}\right)\exp\left(-2\alpha_y n\right) + \varepsilon_{y,\mathrm{eq}}$$

- We initialize a bunch of particles with $\varepsilon_{y,0} = \frac{1}{3}\varepsilon_{y,\mathrm{eq}}$ and track the particles turn-by-turn for 10000 turns, with quantum radiation enabled
- We can fit the function with $\varepsilon_{y,\mathrm{eq}}$ and α_y as free parameters and compare these to the Twiss output

Vertical Equilibrium Emittance

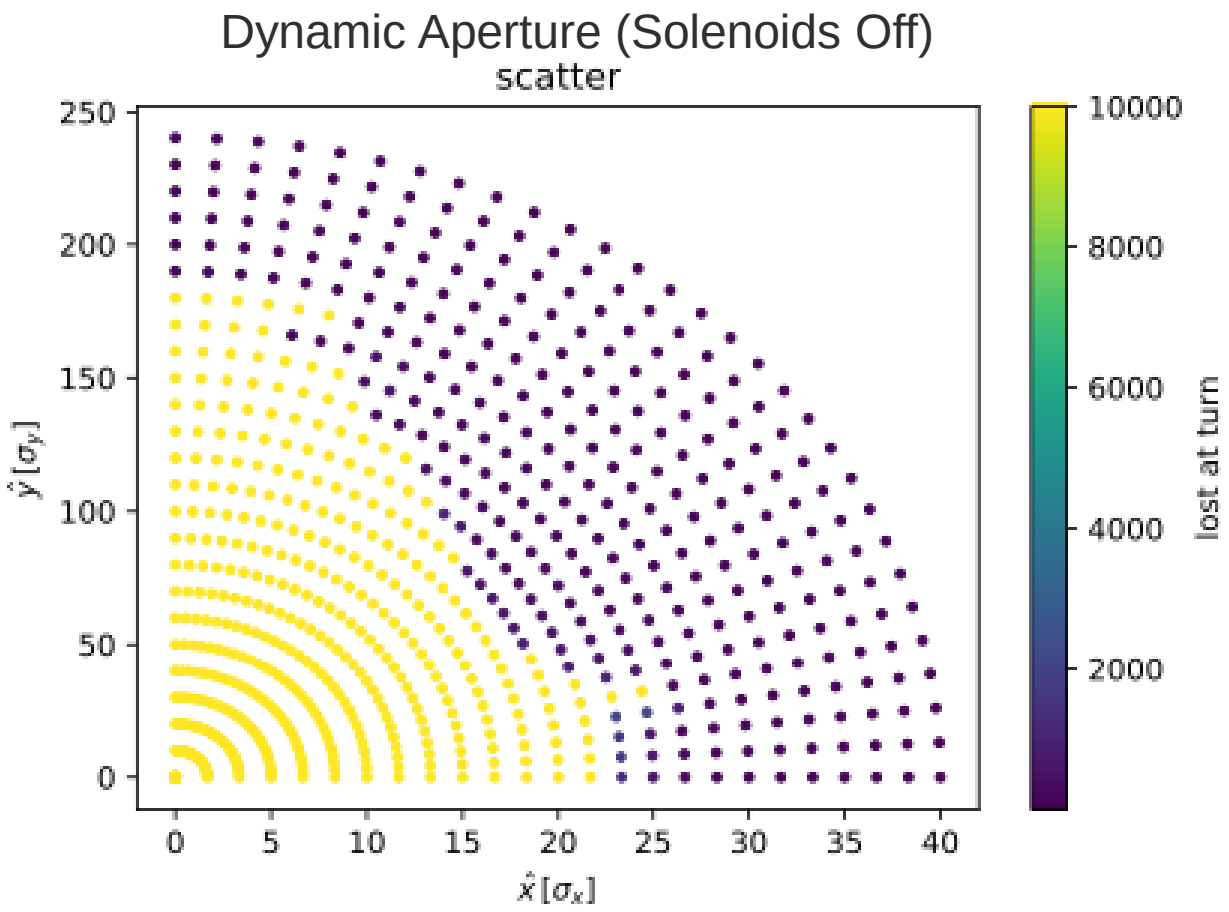
- Design vertical equilibrium emittance $\varepsilon_y \sim 1\ \mathrm{pm \cdot rad} = 10^{-12}\ \mathrm{m \cdot rad}$ [2]
- Twiss and tracking agree well

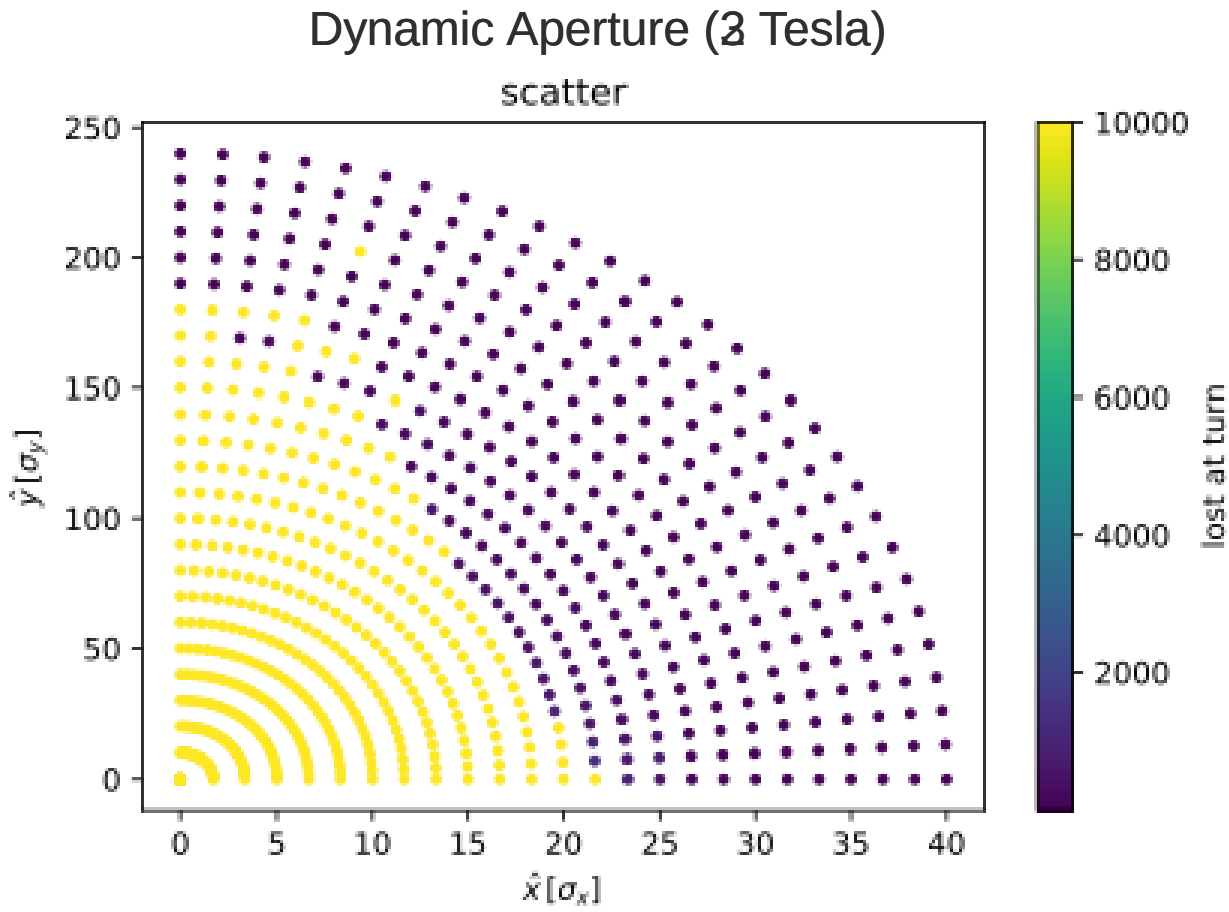


Nonlinear Effects

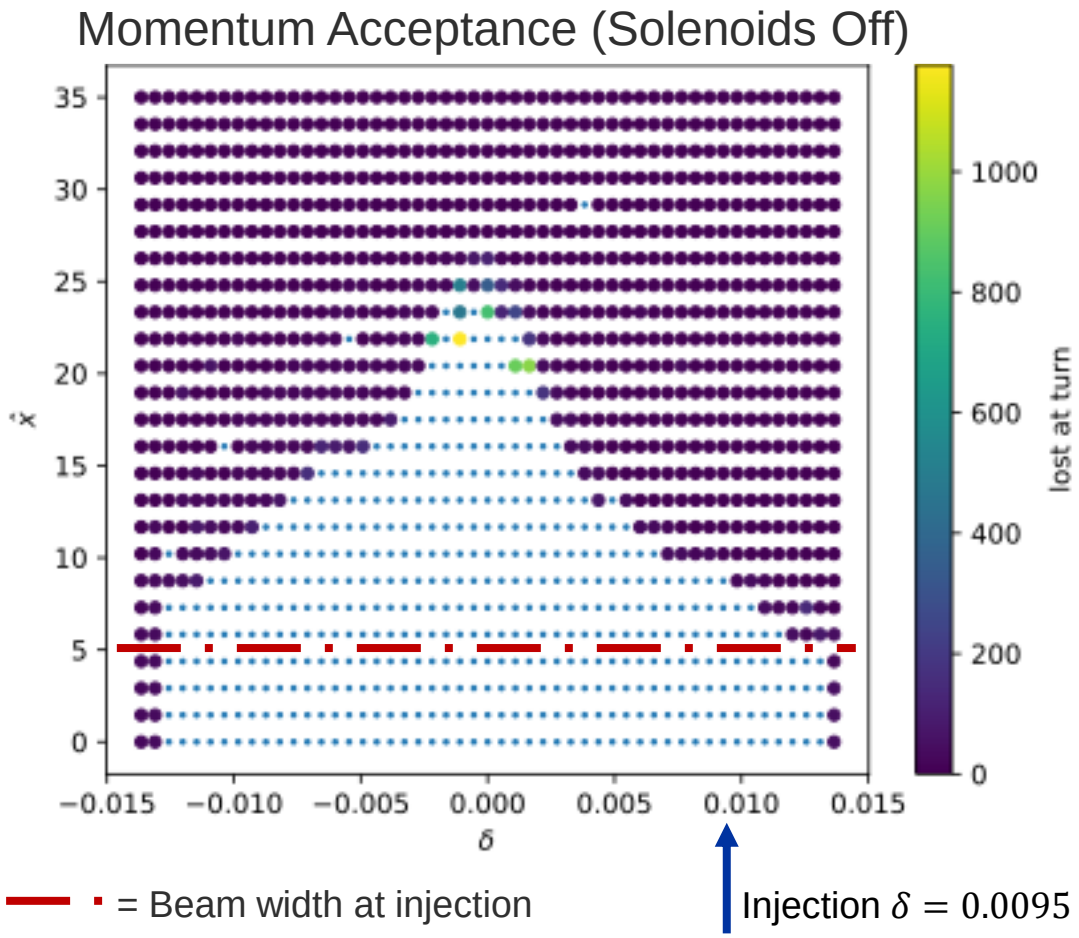
- Linear effects are sufficiently understood to start investigating nonlinear effects
- We calculated the Dynamic Aperture (DA) and Momentum Acceptance (MA) for 1000 particles and 10000 turns to reveal effects that the linear optics does not show
- We observed that the $3\mathrm{T}$ solenoid + correction had a detrimental effect on the MA

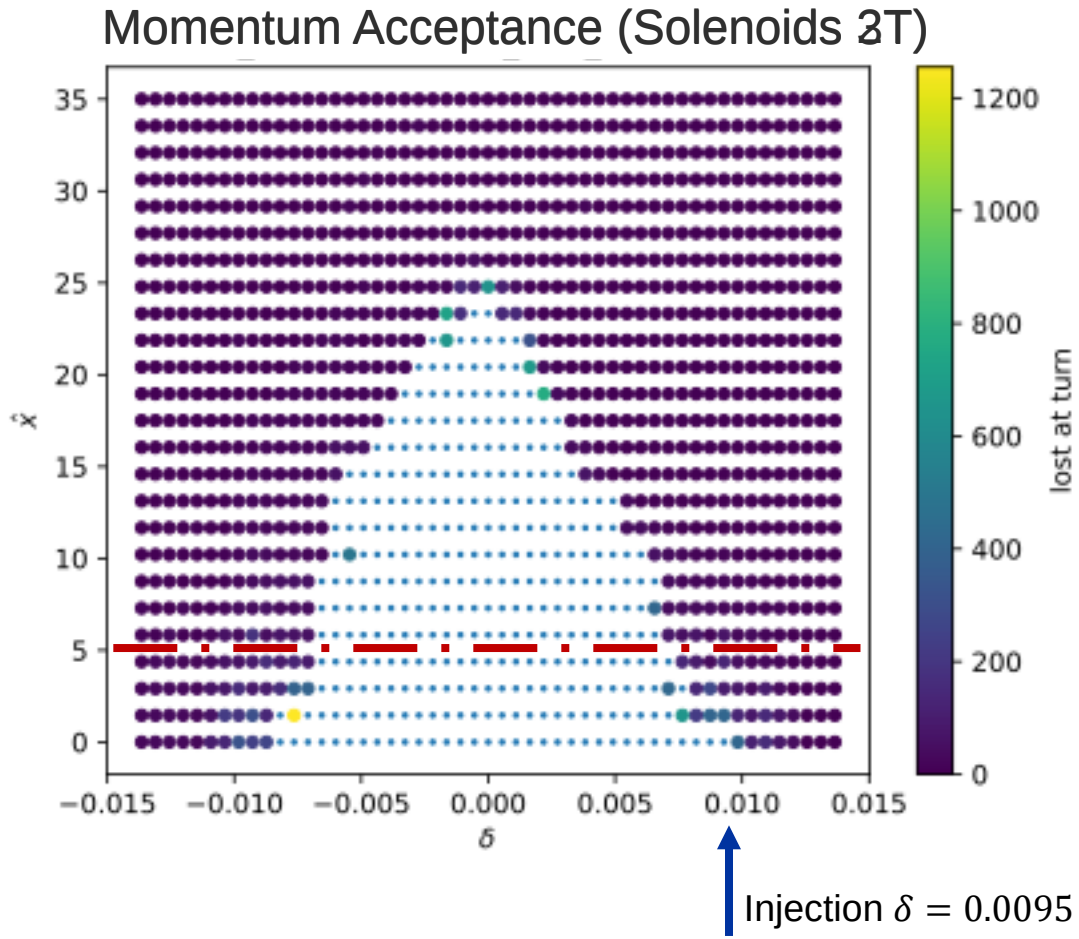
Dynamic Aperture (2 Tesla)





Momentum Acceptance (2 Tesla)





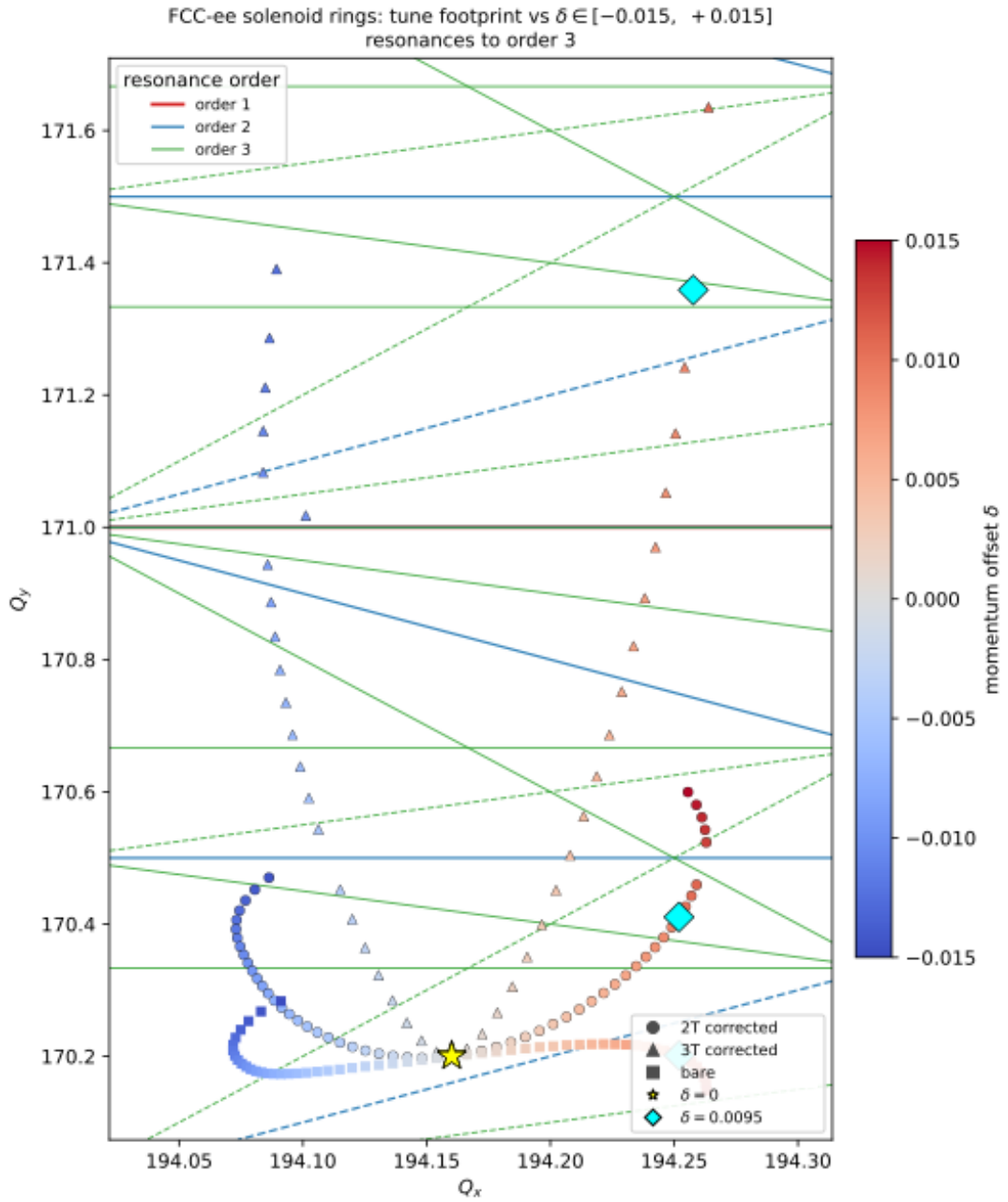
Nonlinear Effects

- We found that the second-order chromaticity blows up:

$$\xi^{(2)} = \frac{\mathrm{d}^2 Q}{\mathrm{d}\delta^2}$$

- Tune diagram shows that ramping δ with the 3T solenoid crosses a half-integer and integer resonance respectively

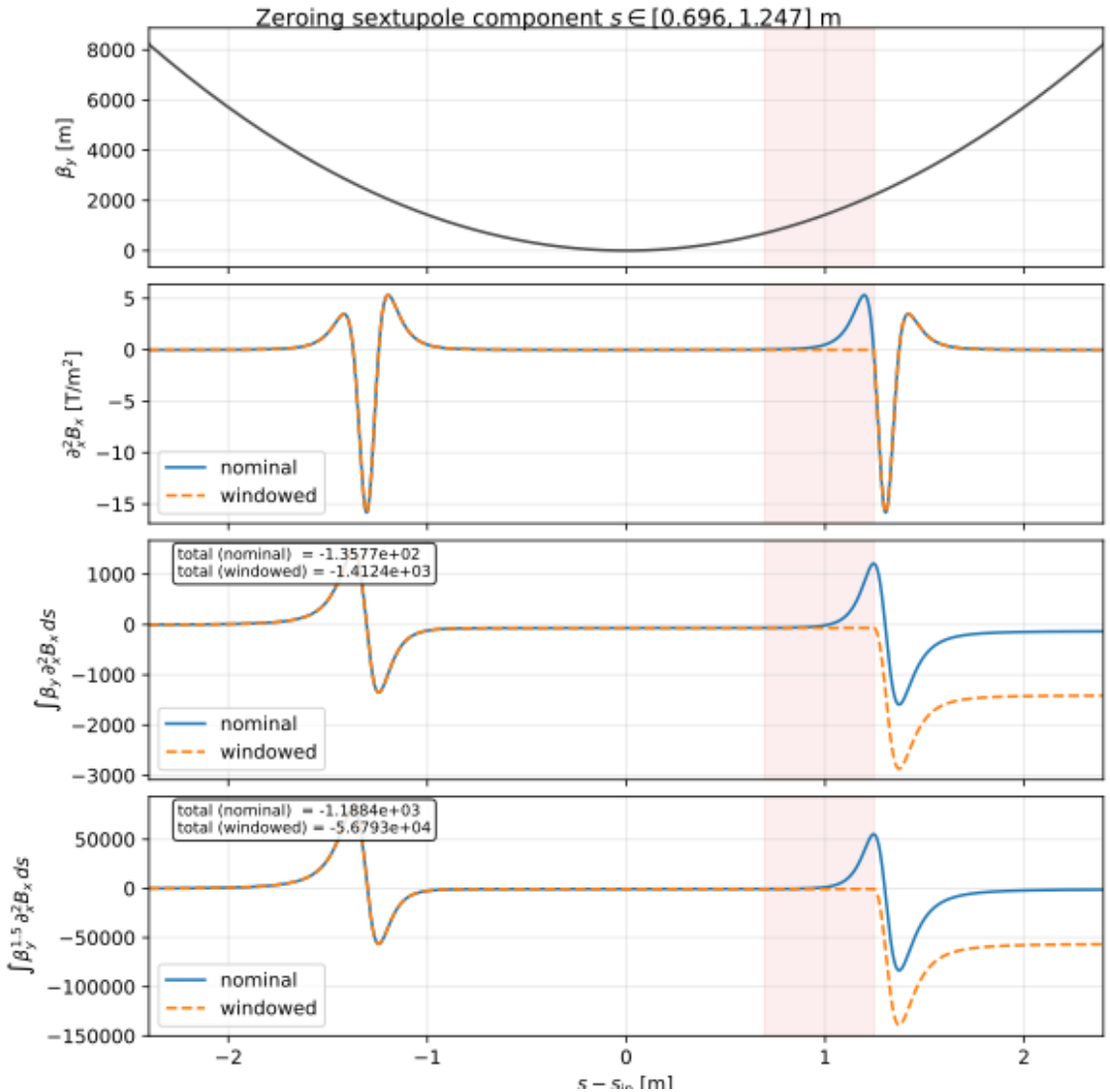
- Some culprits:
  - Fringes have weak sextupole component, but β-functions are very large there
  - Beta-beat at sextupole outside of IR



Sextupole-like Component in Fringe

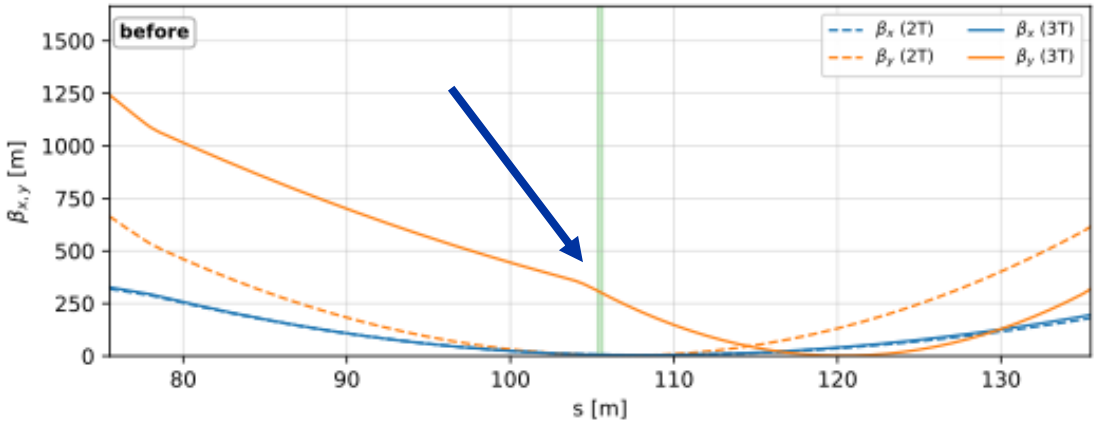
- One hypothesis:
  - Skew sextupole in fringe ~ 15
  - While $\beta_y \sim 2000\ \mathrm{m}$
  - RDT involving skew sextupole $\propto \int \beta_y^{3/2}\, \partial_x^2 B_x\, \mathrm{d}s$
  - This integral exactly cancels in the region
  - Is this the cause for the blowup?
  - If not, is it a cancellation, or a genuine small effect?

- We removed part of the sextupole component
  - Removes the canceling of the RDT
  - However, it had no observable effect on the MA
  - Conclusion: The effect is genuinely small

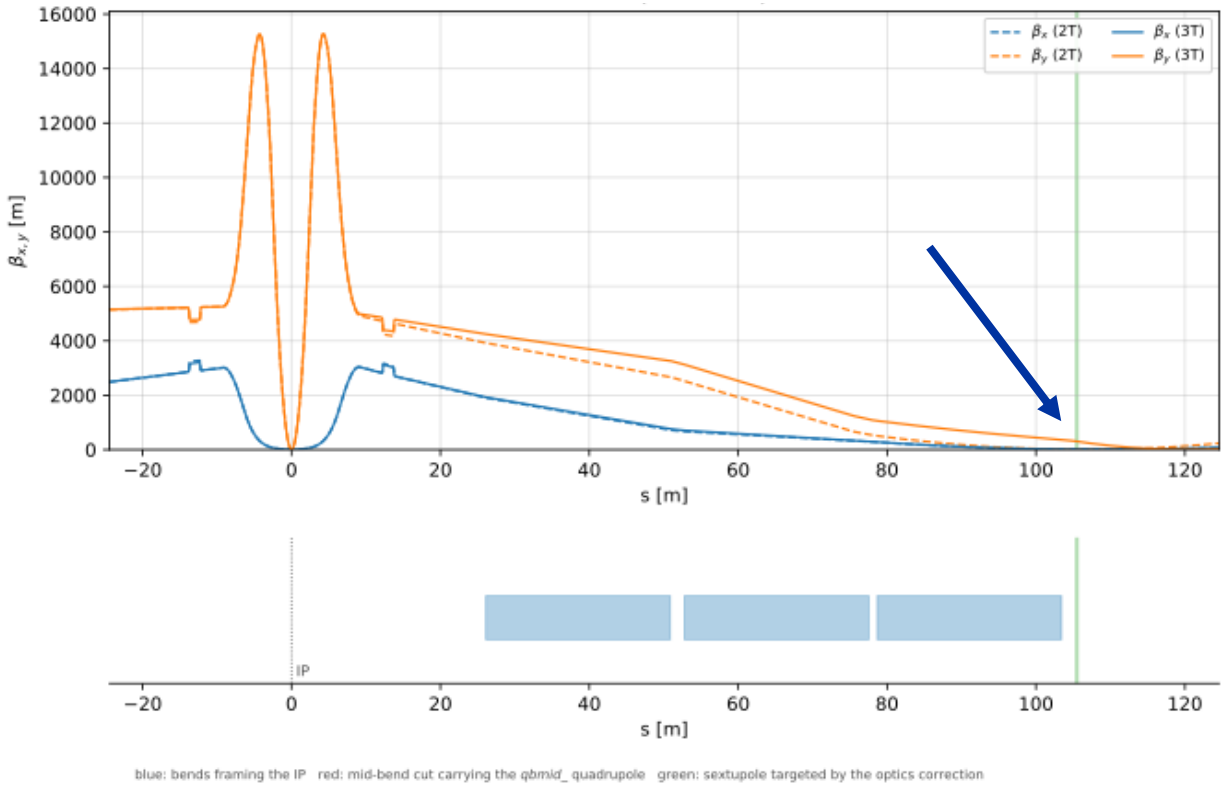


Beta Beat

- After correction, a beta-beat emerged around one of the sextupoles (solid orange curve)
- This beta-beat was only present for the 3T solenoid



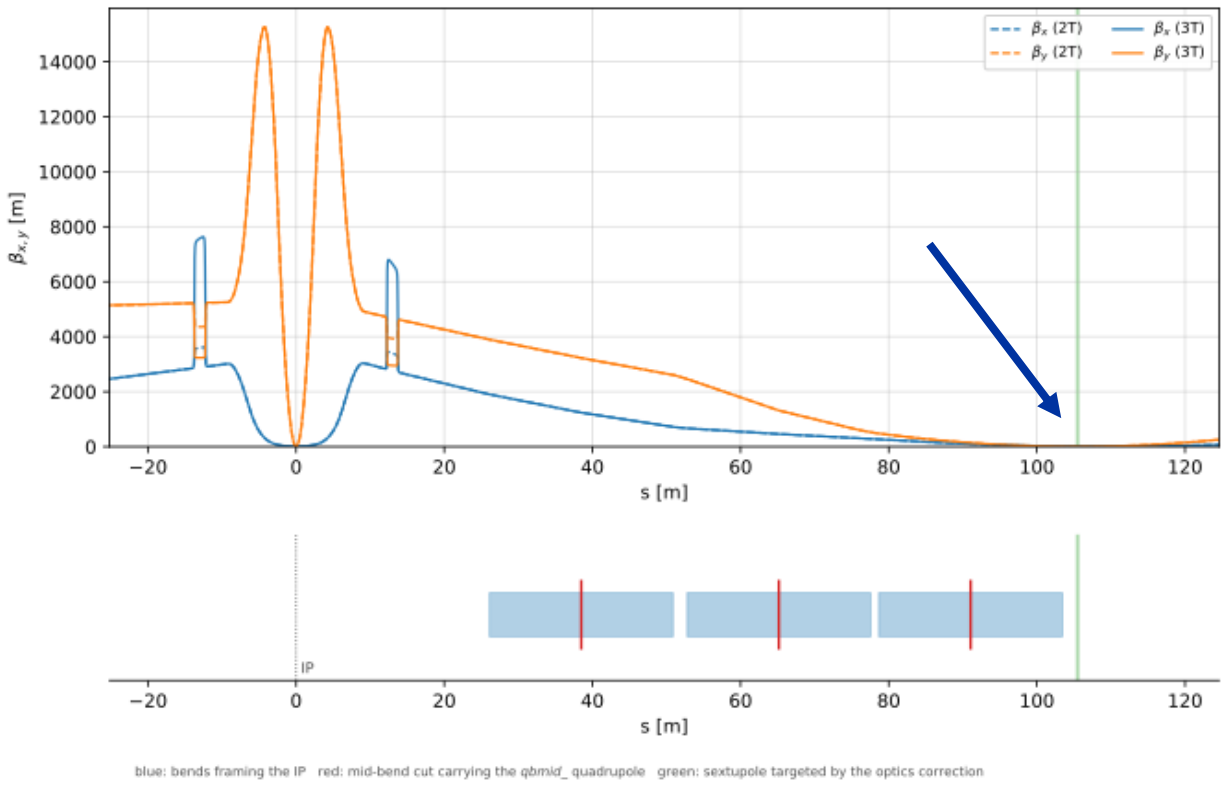
Beta-beat at sextupole



Beta Beat

- We added some additional corrector quadrupoles (red lines) inside the bends between the IP and the sextupole (blue rectangles)
- This way, we were able to correct the beta-beat
- We investigated if that solved our chromaticity problem

Beta-beat at sextupole

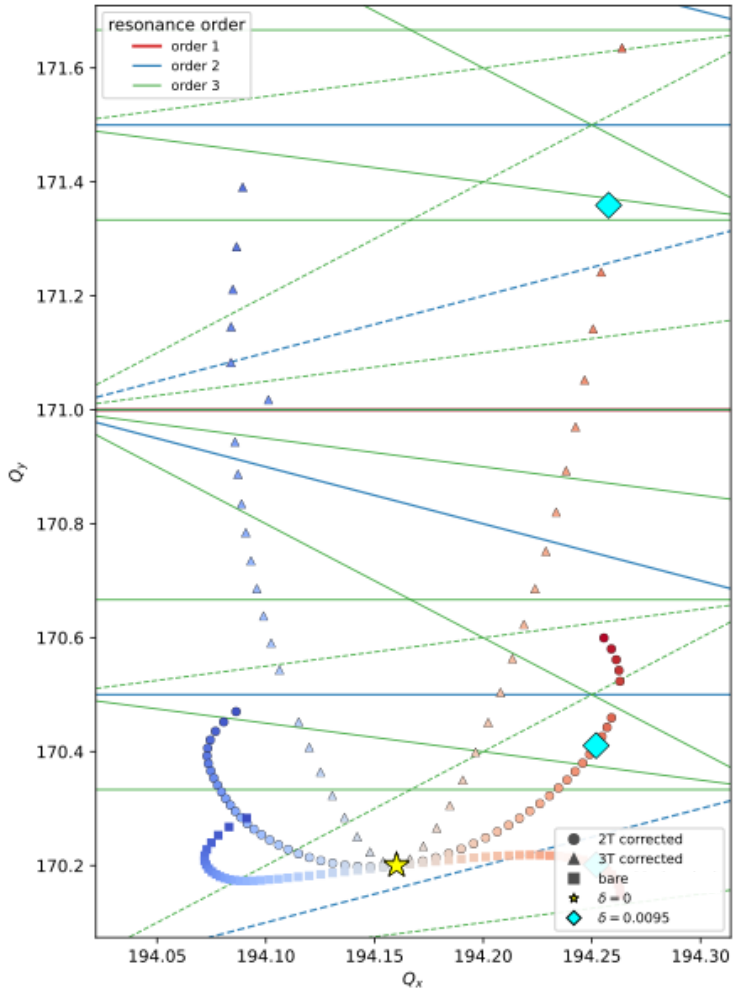


Second-Order Chromaticity

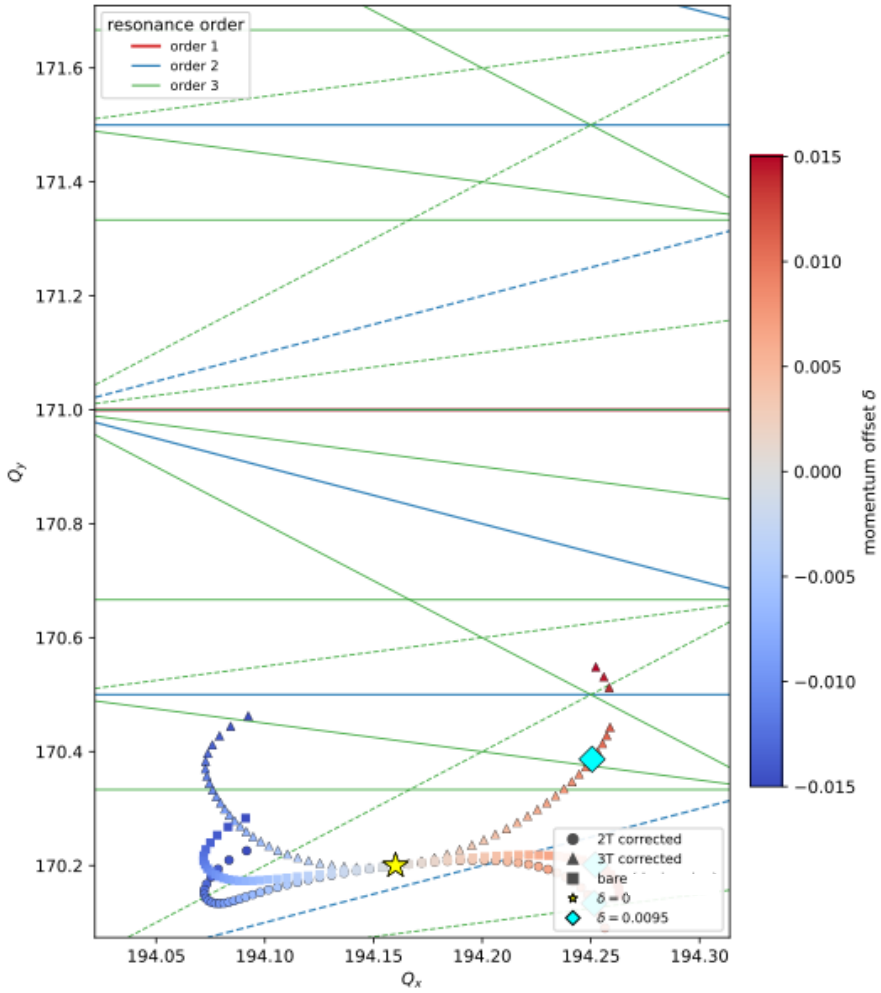
- Before:
$$\frac{d^2Q_y}{d\delta^2} = 57 \cdot 10^3$$

- After:
$$\frac{d^2Q_y}{d\delta^2} = 4.7 \cdot 10^3$$

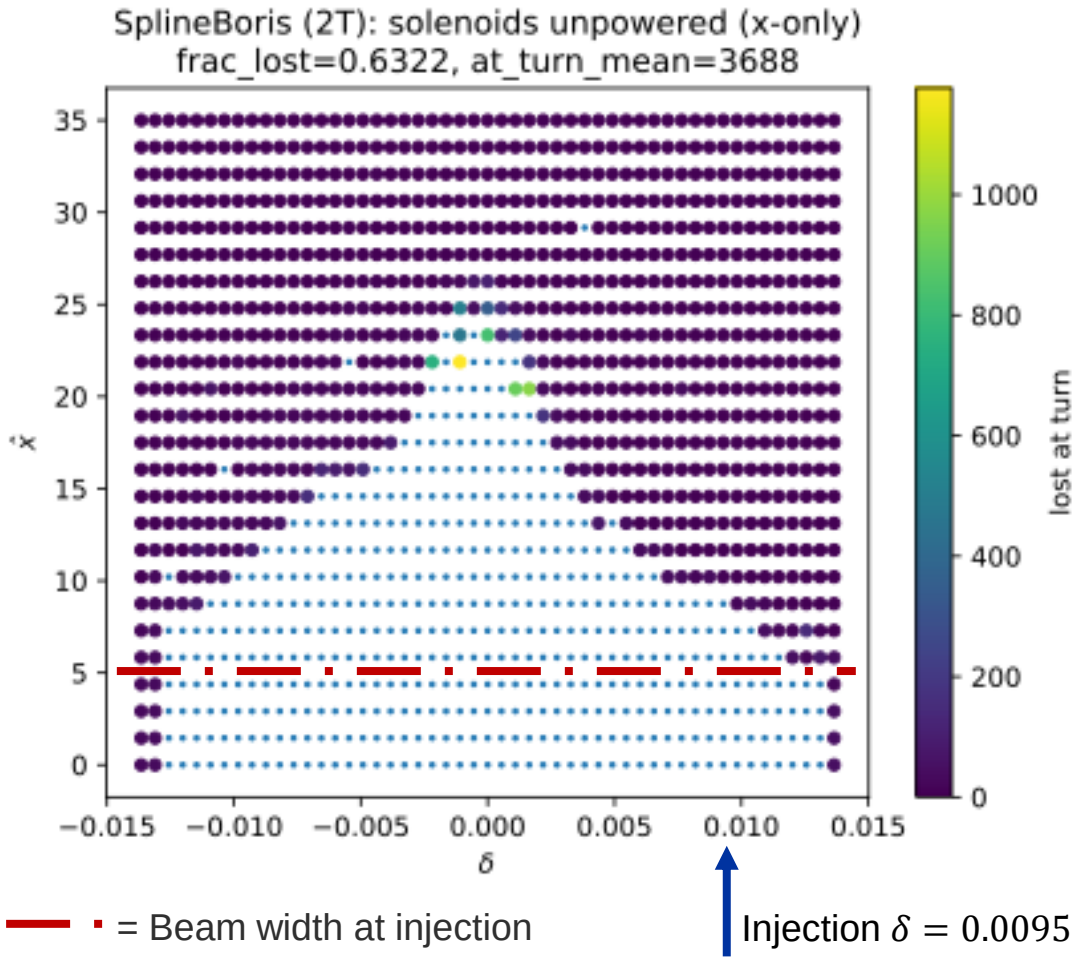
Before Correction

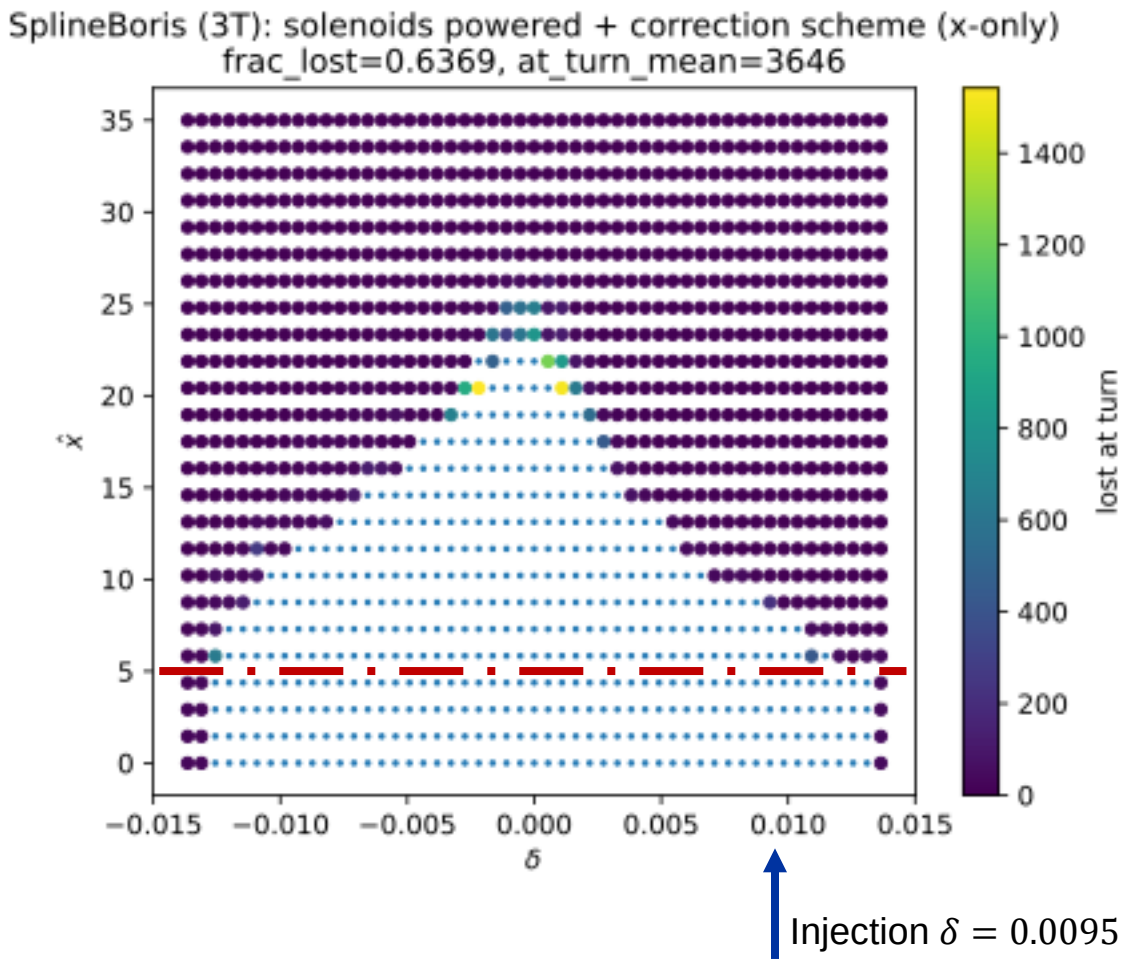


After Correction



Momentum Acceptance (3 Tesla)





Achievements so far

- **Linear Optics**
  - We demonstrated some features of the non-local IP solenoid correction scheme
  - The analytic model for the solenoid field causes coupling that the linearized model does not exhibit
  - Linear coupling can still be corrected very well
  - Equilibrium emittance close to tolerance limit

- **DA, MA and Nonlinear Optics**
  - After correction, the second-order chromaticity blew up, which was caused by a beta-beat at a sextupole outside the IR.
  - We could resolve it by placing additional quadrupole correctors inside the three bends directly out of the IP

- **Non-local solenoid correction method is promising, even for the $3\mathrm{T}$ solenoid**

Future steps

- Use a generated field map
  - The SplineBoris method in Xsuite has already been used to model these ideal solenoids, but can also be used to build a solenoid from field map data
  - We have already kindly received a field map for the 2 Tesla setup from Vikas Teotia (BNL)

- Study polarization due to the presence of the solenoid and its correction

- Research the effects of field errors and misalignments on the MA and DA

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