Jacobson's thermodynamic approach to classical gravity applied to non-Riemannian geometries: *remarks on the simplicity of Nature*

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THREE DEEP CLUES ABOUT GRAVITY

1 Bekenstein–Hawking Entropy

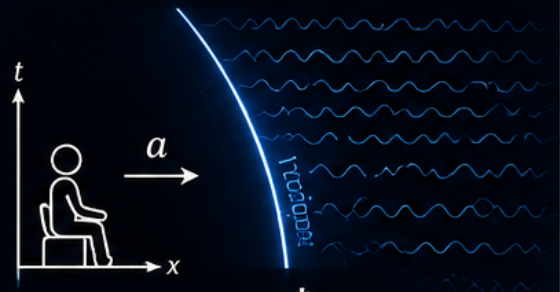


$$S_{BH} = \frac{k_B c^3}{4G\hbar} A$$

Entropy is proportional to the horizon area.

Information is hidden behind horizons.

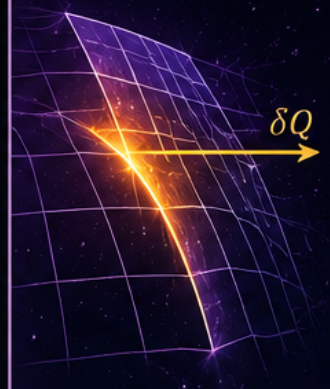
2 Unruh Temperature



$$T_U = \frac{\hbar a}{2\pi k_B c}$$

An accelerated observer perceives the vacuum as a thermal bath.

3 Jacobson's Thermodynamic Derivation



$$\delta Q = T\,\delta S$$

Applying the first law of thermodynamics to local Rindler horizons yields the gravitational field equations.

Which gravitational theory would Nature select if spacetime were non-Riemannian?

Torsion (T) twisting of spacetime

Non-metricity (Q) change of length under parallel transport

Thermodynamics strongly constrains the geometry of spacetime.
Simplicity may be the deepest principle Nature follows.

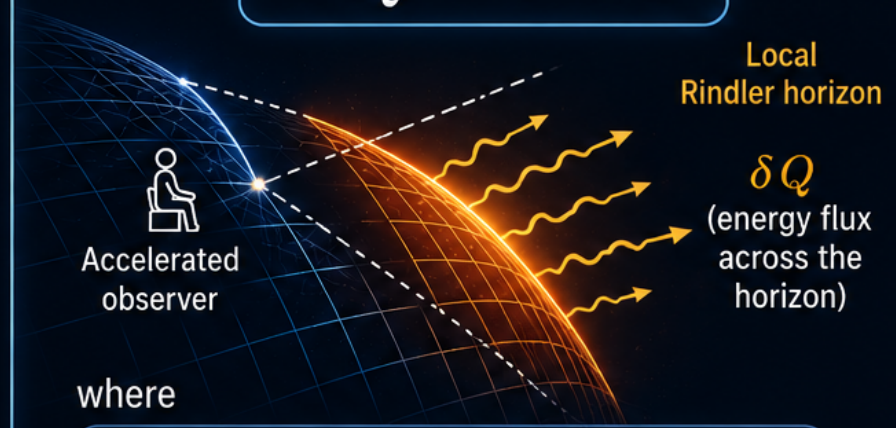
Jacobson (1995)

Gravity as an Emergent Thermodynamic Relation

FUNDAMENTAL IDEA

For any local Rindler horizon:

$$\delta Q = T \delta S.$$



where

- δQ: energy flux across the horizon
- T: Unruh temperature
- δS: entropy associated with the horizon

RESULT

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}.$$



The Einstein equations emerge as a **fundamental thermodynamic relation**.

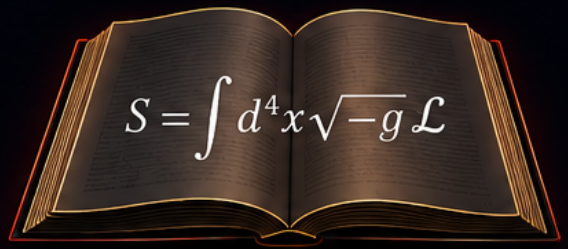
Gravity is not a fundamental force,
but an **emergent thermodynamic relation** encoded in spacetime.

The Message of Jacobson

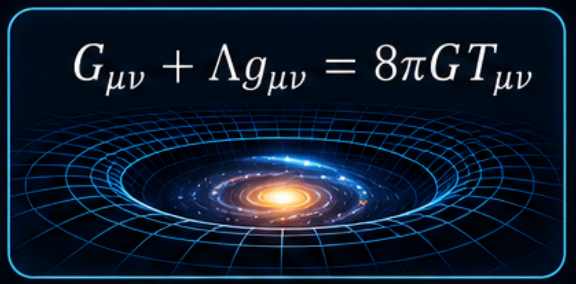
A PARADIGM SHIFT

No:

Action → Equations



Action



Field Equations

But:

Thermodynamics → Equations

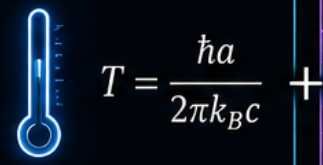
Heat flux



Energy crossing a local Rindler horizon

+

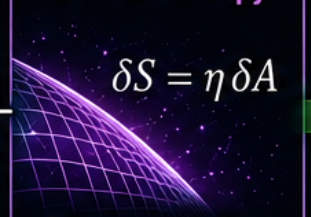
Unruh temperature



Temperature perceived by an accelerated observer

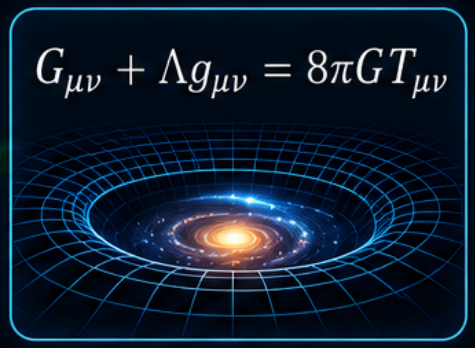
+

Horizon entropy



Entropy proportional to the horizon area

$$\delta Q = T\,\delta S$$



Field Equations

Gravity appears to be an *emergent theory.*

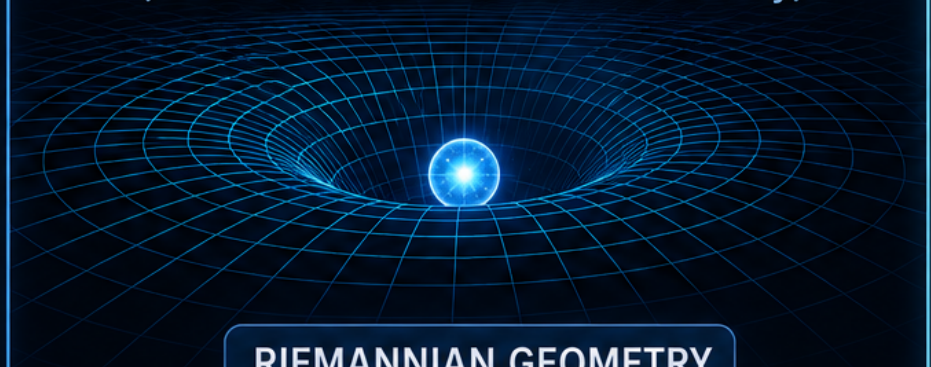
BEYOND RIEMANN

Exploring gravity in the most general spacetime geometry

JACOBSON WORKS WITH:

$$T^{\lambda}_{\mu\nu} = 0, \; Q^{\lambda}_{\mu\nu} = 0.$$

He assumes a Riemannian spacetime (zero torsion and zero non-metricity).



RIEMANNIAN GEOMETRY

Curvature only

?

WE ASK:

What happens when there are



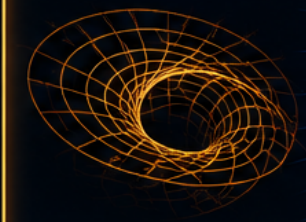
torsion,



non-metricity,

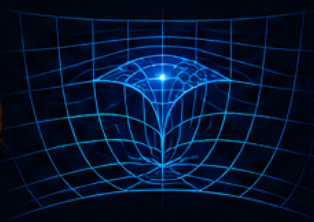


or both?



Torsion

$T^{\lambda}_{\mu\nu} \neq 0$



Non-metricity

$Q^{\lambda}_{\mu\nu} \neq 0$



Torsion + Non-metricity

$T^{\lambda}_{\mu\nu} \neq 0, Q^{\lambda}_{\mu\nu} \neq 0$



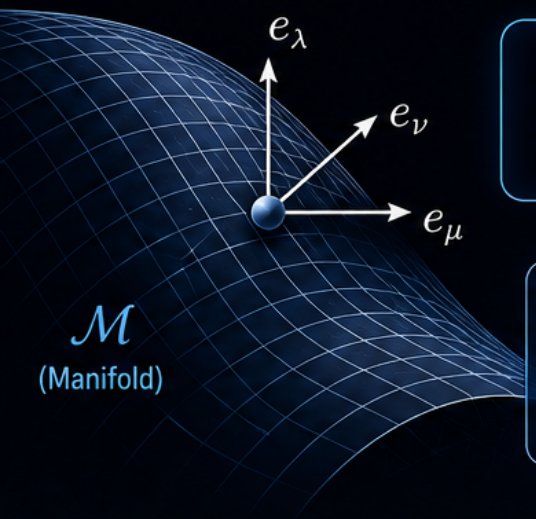
To understand gravity in its most general form,

we must go beyond Riemann.



NON-RIEMANNIAN GEOMETRIES

GEOMETRIC STRUCTURE



$$\Gamma^{\lambda}_{\ \mu\nu} = \left\{ {\lambda \atop \mu\nu} \right\} + K^{\lambda}_{\ \mu\nu} + L^{\lambda}_{\ \mu\nu}$$

LEVI-CIVITA CONNECTION (RIEMANNIAN PART)
Metric-compatible and torsion-free

CONTORSION
$K^{\lambda}_{\ \mu\nu}$
Encodes torsion

DISFORMATION
$L^{\lambda}_{\ \mu\nu}$
Encodes non-metricity

WHERE

$K^{\lambda}_{\ \mu\nu}$: contorsion

$L^{\lambda}_{\ \mu\nu}$: disformation

APPEAR



Vectors do not close (path dependent)

TORSION

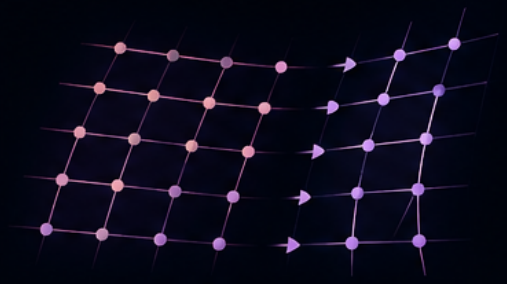
$$T^{\lambda}_{\ \mu\nu} = \Gamma^{\lambda}_{\ [\mu\nu]}$$

Measures the antisymmetric part of the connection

NON-METRICITY

$$Q^{\lambda}_{\ \mu\nu} = -\nabla_\lambda\, g_{\mu\nu}$$

Measures the failure of metric compatibility



Lengths and angles vary under parallel transport

Non-Riemannian geometries generalize Riemannian geometry by allowing *torsion* and/or *non-metricity*.

OBJECTIVE

CENTRAL PROBLEM

1 Given a non-Riemannian gravitational action,

can it be derived from Jacobson's thermodynamic proposal?



Non–Riemannian Gravitational Action

$$S[g_{\mu\nu}, \Gamma^{\lambda}_{\mu\nu}]$$

Metric $g_{\mu\nu}$ | General connection $\Gamma^{\lambda}_{\mu\nu}$ | Torsion $T^{\lambda}_{\mu\nu}$ | Non-metricity $Q_{\lambda\mu\nu}$

2 And if several theories are possible:

is there any criterion that selects a **privileged theory?**

Possible Non-Riemannian Theories

Theory A | Theory B | Theory C | ...

Jacobson's Thermodynamic Proposal

δQ

Local Rindler horizon

Unruh temperature T | Accelerated observer | Entropy δS

$$\delta Q = T\,\delta S$$

Our goal is to understand how thermodynamics can determine gravity in the most general geometric setting.

STRATEGY

TWO INDEPENDENT CRITERIA:

1 CRITERION 1

Generalization of
Jacobson's program.



Local Rindler horizon

$$\delta Q = T\,\delta S$$

Local thermodynamic equilibrium

Field equations (with torsion, non-metricity, or both)

2 CRITERION 2

Hypotheses that lead to
Lanczos–Lovelock theories.



Geometric hypotheses (diffeomorphism invariance, locality, second-order equations, etc.)

Lanczos–Lovelock Lagrangian

$$\sum_k \alpha_k \mathcal{L}_k$$

Field equations (Lanczos–Lovelock type)









The agreement between both criteria is interpreted as a **selection principle.**

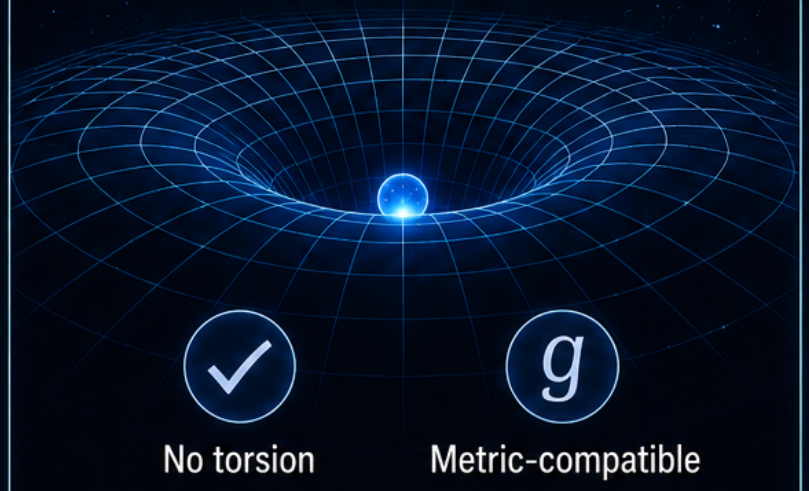
RIEMANNIAN CASE

A REMINDER

RIEMANNIAN GEOMETRY

Torsion-free and metric-compatible

$$T^{\lambda}{}_{\mu\nu} = 0,\ Q^{\lambda}{}_{\mu\nu} = 0$$



No torsion

Metric-compatible



Jacobson + entropy
proportional to area

$$\delta Q = T\,\delta S$$
$$\delta S = \frac{\eta}{4}\,\delta A$$



Einstein–Hilbert action

$$S_{EH} = \frac{1}{16\pi G}\int R\sqrt{-g}\,d^4x$$



Einstein equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$



Geometric simplicity matches **thermodynamic simplicity.**



TORSION CASE

We study Jacobson's proposal when torsion is present and non-metricity is absent.

ASSUMPTIONS



$$Q^{\lambda}{}_{\mu\nu} = 0$$

No non-metricity



$$T^{\lambda}{}_{\mu\nu} \neq 0$$

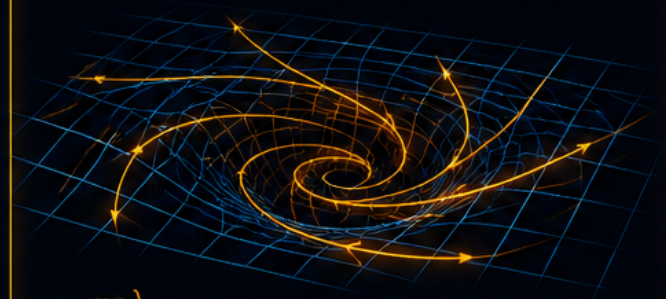
Torsion is allowed and non-vanishing

We consider spacetimes of Riemann–Cartan type.

RIEMANN–CARTAN GEOMETRY

Connection with torsion (and no non-metricity):

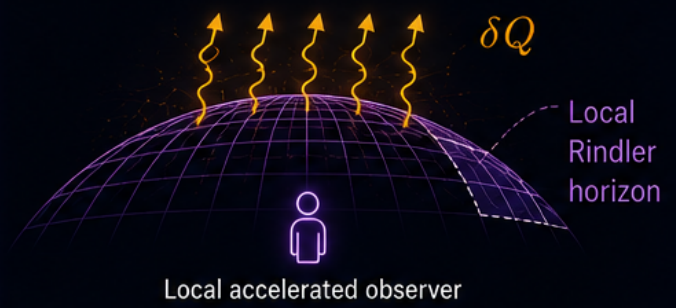
$$\Gamma^{\lambda}{}_{\mu\nu} = \left\{{}^{\lambda}{}_{\mu\nu}\right\} + K^{\lambda}{}_{\mu\nu}$$



$K^{\lambda}{}_{\mu\nu}$ is the contorsion tensor, encoding the antisymmetric part of the connection.

JACOBSON'S FORMULATION

Apply the thermodynamic reasoning of Jacobson in Riemann–Cartan spacetime.



Heat flux δQ, entropy $\delta S = \frac{\eta}{4}\,\delta A$, and Clausius relation $\delta Q = T\delta S$ are imposed on local horizons.



We analyze the thermodynamic derivation of gravitational field equations allowing torsion $\left(T^{\lambda}{}_{\mu\nu} \neq 0\right)$ while keeping $Q^{\lambda}{}_{\mu\nu} = 0$.



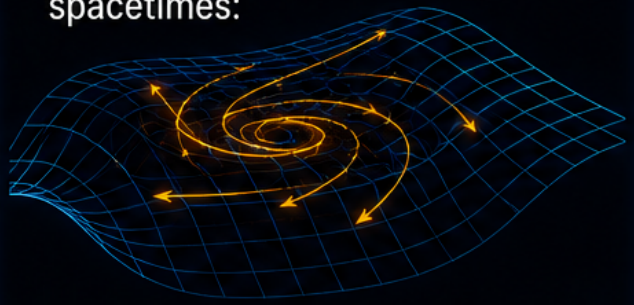
Goal: Understand how torsion modifies Jacobson's derivation and what gravitational dynamics it implies.

SURPRISING RESULT

EINSTEIN–HILBERT NO LONGER WORKS

SETTING

We work in **Riemann–Cartan** spacetimes:



- No non-metricity: $Q^{\lambda}_{\mu\nu} = 0$
- Torsion present: $T^{\lambda}_{\mu\nu} \neq 0$

THE ACTION

Einstein–Hilbert action

$$S_{EH} = \int \sqrt{-g}\, R\, d^4x$$



Built from the Ricci scalar R of the full connection Γ.

IT DOES NOT WORK

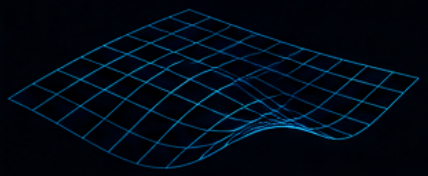
The Einstein–Hilbert action **fails** to satisfy the requirements imposed by the thermodynamic construction.

THERMODYNAMIC REQUIREMENTS



Heat flux	Entropy	Clausius relation
δQ	δS	$\delta Q = T\delta S$

EXCEPT IN THE RIEMANNIAN LIMIT



- $T^{\lambda}_{\mu\nu} \to 0$
- $\Gamma^{\lambda}_{\mu\nu} \to \{^{\lambda}_{\mu\nu}\}$ (Levi–Civita connection)

In this limit, the thermodynamic requirements are recovered and the Einstein–Hilbert action **works**.

Torsion changes the thermodynamic constraints: the Einstein–Hilbert action is **no longer compatible**.

CONCEPTUAL CONSEQUENCE

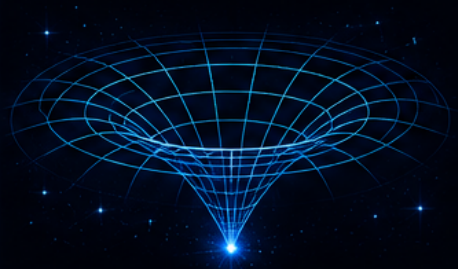
THE LESSON FROM TORSION

GEOMETRIC SIMPLICITY

The simplest action from the geometric viewpoint

$$S_{EH} = \int \sqrt{-g}\, R\, d^4x$$

Einstein–Hilbert action



Riemannian geometry

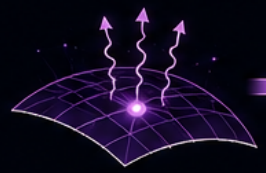
- No torsion $T^{\lambda}_{\ \mu\nu} = 0$
- No non-metricity $Q^{\lambda}_{\ \mu\nu} = 0$

GEOMETRICALLY THE MOST ECONOMICAL

DOES NOT BELONG TO

THERMODYNAMIC SELECTION

Jacobson's proposal in the presence of torsion



Local Rindler horizon → δQ Heat flux → δS Entropy (proportional to area) → Claussius relation $\delta Q = T\,\delta S$

Only actions whose field equations are compatible with this thermodynamic construction are selectable.

With torsion: $T^{\lambda}_{\ \mu\nu} \neq 0$



CONCLUSION: The geometrically simplest theory, S_{EH} does not belong to the set of theories selectable by Jacobson's proposal when torsion is present.

SEARCH FOR A UNIQUE ALTERNATIVE

We look for an **alternative action** compatible with Jacobson's thermodynamic proposal in the presence of torsion.

WE CONSIDER THE HYPOTHESES USED IN LANCZOS–LOVELOCK THEORIES:



1. COVARIANCE

The action is generally covariant.



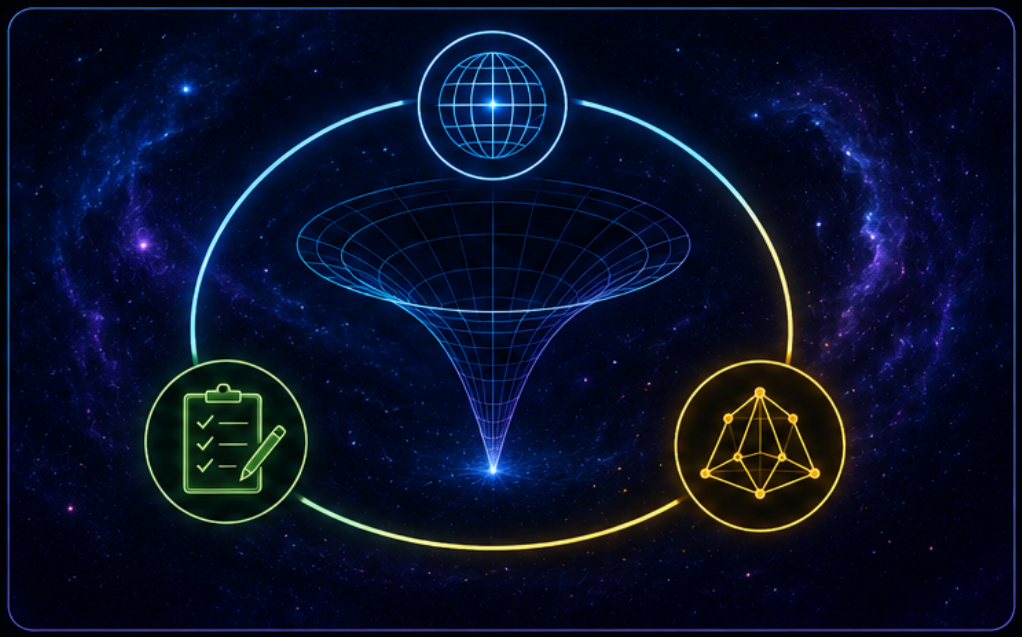
2. SECOND-ORDER EQUATIONS

The field equations derived from the action are of second order in derivatives.



3. SIMPLE GEOMETRIC STRUCTURE

The action is constructed from curvature tensors using simple contractions, without introducing additional fields.





QUESTION:

WHAT ACTION SURVIVES?

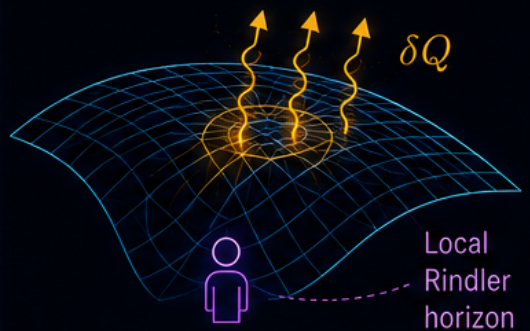
$$S = \int L \, d^D x$$

SELECTED THEORY

BOTH CRITERIA POINT TOWARD

CRITERION 1

Generalization of Jacobson's program



Thermodynamic reasoning in Riemann–Cartan spacetime with torsion leads to field equations containing a torsion quadratic term.

CRITERION 2

Hypotheses leading to Lanczos–Lovelock theories

- Covariance
- ∂^2 Second-order equations
- Simple geometric structure



$$S = \int \mathrm{d}^4x \sqrt{-g} \left(R + \beta T_\mu T^\mu \right)$$

The action is Einstein–Hilbert plus a quadratic torsion term.

where $T_\mu = T^\nu_{\mu\nu}$.

β is a coupling constant determined by the theory.

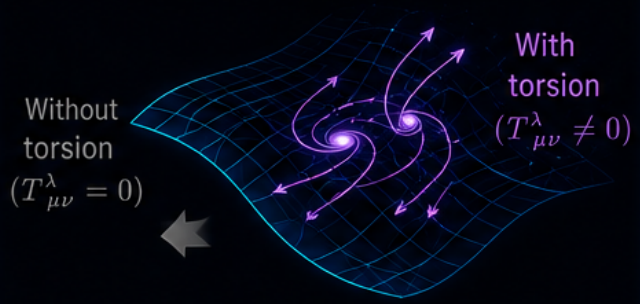
This is the unique action selected by the convergence of the two independent criteria.

PHYSICAL INTERPRETATION

WHAT DOES THE SELECTION TELL US?

1 Torsion does not appear arbitrarily.

The thermodynamic construction imposes stringent constraints that rule out many possible actions.



Torsion emerges as a **necessary** degree of freedom, not an ad hoc addition.

2 The thermodynamic structure specifically favors $T_\mu T^\mu$.

Among all torsion contributions, thermodynamics singles out the simplest scalar built from the torsion trace vector:

$$T_\mu T^\mu \quad \text{where } T_\mu = T^{\nu}_{\mu\nu}.$$

Covariance

Second-order equations

Simple geometric structure

Minimal, consistent and selected.

3 The simplicity of Nature seems preserved,

but in a form different from Einstein–Hilbert.



Einstein–Hilbert (no torsion)

Einstein–Hilbert + $\beta T_\mu T^\mu$ (with torsion)

Nature remains **simple**, but its simplicity allows for **torsion** through the specific combination $T_\mu T^\mu$.

PHYSICAL MESSAGE:

Thermodynamics does not merely allow torsion. It reveals the **unique, simplest** way in which torsion can be consistently part of gravitational dynamics.

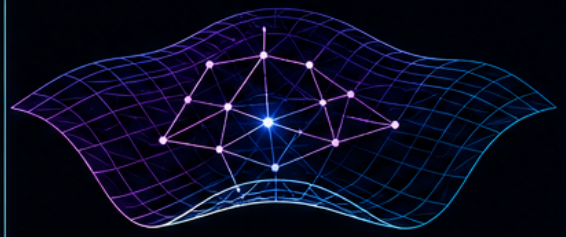


GENERAL METRIC–AFFINE CASE

WE NOW CONSIDER THE FULL METRIC–AFFINE SETTING

GEOMETRIC FRAMEWORK

Spacetime with independent metric $g_{\mu\nu}$ and connection $\Gamma^{\lambda}_{\mu\nu}$.



No restrictions imposed:

- **Torsion:** $T^{\lambda}_{\mu\nu} \neq 0$
- **Non-metricity:** $Q^{\lambda}_{\mu\nu} \neq 0$

We allow both torsion and non-metricity:

$$T^{\lambda}_{\mu\nu} \neq 0, \quad Q^{\lambda}_{\mu\nu} \neq 0.$$

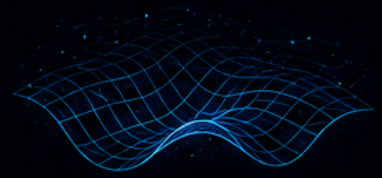
TORSION



Measures the antisymmetric part of the connection.

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{[\mu\nu]}$$

NON-METRICITY



Measures the failure of the connection to preserve the metric.

$$Q^{\lambda}_{\mu\nu} = -\nabla_{\lambda} g_{\mu\nu}$$

THERMODYNAMIC PROGRAM

We apply Jacobson's thermodynamic construction in this general setting.



Local Rindler horizon

Heat flux δQ

Entropy δS

Clausius relation $\delta Q = T\delta S$

Local Lorentz invariance + thermodynamic equilibrium $\Rightarrow$ field equations

OUR GOAL

Determine which gravitational action(s) are compatible with the thermodynamic requirements in the presence of $T^{\lambda}_{\mu\nu} \neq 0$ and $Q^{\lambda}_{\mu\nu} \neq 0$.

- Covariance
- Second-order field equations

REPEAT THE ANALYSIS.

We do not assume a *priori* any relation between $g_{\mu\nu}$ and $\Gamma^{\lambda}_{\mu\nu}$: metric and connection are independent.

MAIN RESULT

IN THE GENERAL METRIC–AFFINE CASE $(T^{\lambda}{}_{\mu\nu} \neq 0,\ Q^{\lambda}{}_{\mu\nu} \neq 0)$

THE TWO CRITERIA BECOME INCOMPATIBLE.

1. JACOBSON SELECTION

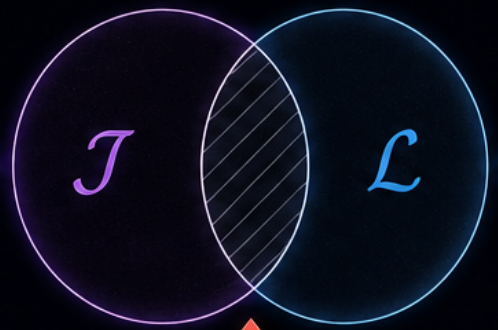
Thermodynamic construction in the presence of torsion and non-metricity.



- Heat flux δQ
- Entropy δS
- Clausius relation $\delta Q = T\,\delta S$

SELECTS A SET $\mathcal{J}$
of admissible theories.

THE SELECTED SETS ARE DIFFERENT.



NO COMMON PREFERRED THEORY.

2. LANCZOS–LOVELOCK SELECTION

Geometric principles:

- Covariance
- ∂^2 Second-order field equations
- Simple geometric structure



SELECTS A SET $\mathcal{L}$
of admissible theories.

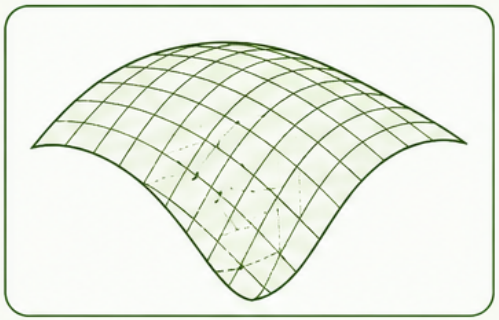


THERE IS NO COMMON PRIVILEGED THEORY.

Jacobson and Lanczos–Lovelock select different sets of theories in the general metric–affine setting.

GRAPHICAL SUMMARY

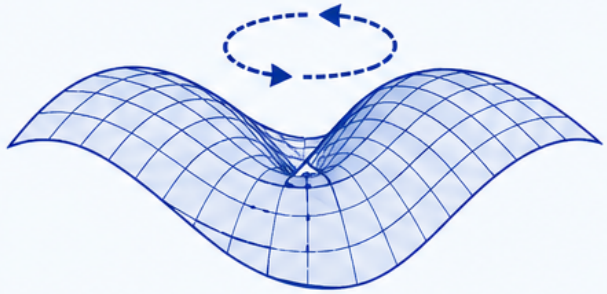
1. RIEMANN



EINSTEIN–HILBERT ✓

$$S_{EH} = \int d^4x \sqrt{-g}\, R$$

2. RIEMANN–CARTAN



EINSTEIN–HILBERT ✗

$$S_{EH} = \int d^4x \sqrt{-g}\, R$$

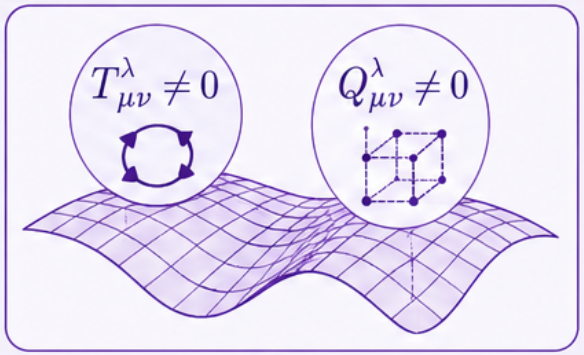
Does not satisfy the thermodynamic construction

$$R + \beta\, T^2$$ ✓

$$S = \int d^4x \sqrt{-g}\, (R + \beta\, T_\mu T^\mu)$$

Selected / Consistent

3. MÉTRIC–AFFINE GENERAL



NO UNIQUE SELECTION ?

Jacobson selects a set of theories.
Lanczos–Lovelock selects a different set.

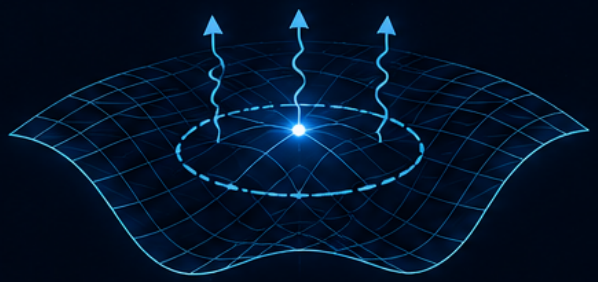
NO COMMON PRIVILEGED THEORY

✓ Selected / Consistent ✗ Not selected / Inconsistent ? No unique selection

IMPLICATIONS

1. FOR CLASSICAL GRAVITY

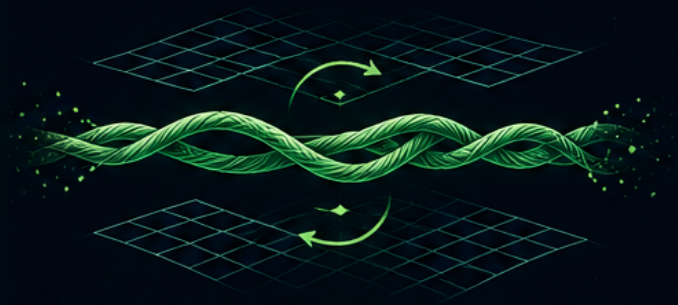
Thermodynamics restricts geometry.



Not all geometric possibilities are thermodynamically allowed.

2. FOR GAUGE THEORIES OF GRAVITY

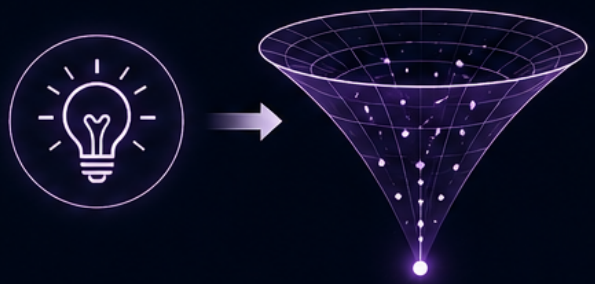
Torsion acquires a privileged role.



The thermodynamic structure singles out the torsion trace $T_\mu T^\mu$.

3. FOR QUANTUM GRAVITY

Suggests that thermodynamics can act as a **selection principle**.



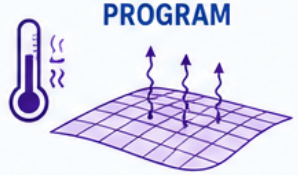
Thermodynamics may guide us toward the physically relevant theories.

Thermodynamics is not merely a tool:
it **constrains, selects** and **reveals** the underlying geometric structure of gravitational theories.

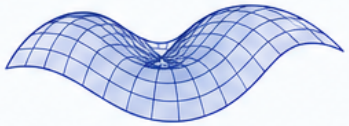
CONCLUSIONS

1

JACOBSON PROGRAM



NON-RIEMANNIAN GEOMETRIES



We extended Jacobson's thermodynamic program to **non-Riemannian geometries.**

2

EINSTEIN–HILBERT ACTION



$$S_{EH} = \int d^4x \sqrt{-g}\, R$$

TORSION



Einstein–Hilbert action is **no longer** a selectable theory when torsion is present.

3

TWO INDEPENDENT CRITERIA

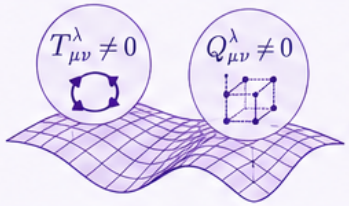
- Thermodynamic criterion (Jacobson)
- Geometric criterion (Lanczos–Lovelock)

$$R + \beta\, T_\mu T^\mu$$

where $T_\mu = T^\nu{}_{\mu\nu}$

Both criteria independently favor $R + \beta\, T_\mu T^\mu$.

4



TORSION AND NON-METRICITY SIMULTANEOUSLY

NO UNIQUE THEORY EMERGES

In geometries with torsion and non-metricity simultaneously, **no unique theory** emerges.

5



The **simplicity** of Nature might be deeply connected to fundamental **thermodynamic principles**.

» = NATURE SIMPLICITY

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