# Semi-Analytical Jeans Filtering Functions for Baryonic Density and Velocity Fields in Second-Order Cosmological Perturbation Theory

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Universidad Nacional de Colombia
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Cosmology in Colombia
Universidad ECCI
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# Overview

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- Cosmological Motivations
- Standard Cosmological Perturbation Theory
- Analytical Approach to First-Order Solutions
- Analytical Approach to Second-Order Solutions
- Semi-Analytical Modeling of Baryonic Fluctuations
- Conclusions and Outlook

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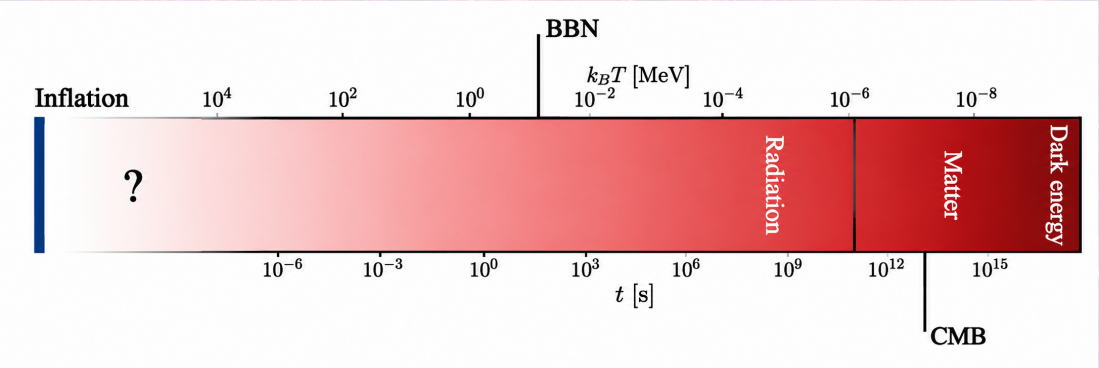
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# Cosmological Motivations

Cosmological perturbation theory provides the fundamental framework for describing the evolution of the matter-energy density field in an expanding Universe and serves as the basis for understanding the formation of large-scale structures within the $\Lambda$CDM paradigm.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1}$$

$$\frac{\partial f}{\partial t} + \frac{p}{E} \frac{\hat{p}^i}{a(t)} \frac{\partial f}{\partial x^i} - Hp \frac{\partial f}{\partial p} = C[f] \tag{2}$$

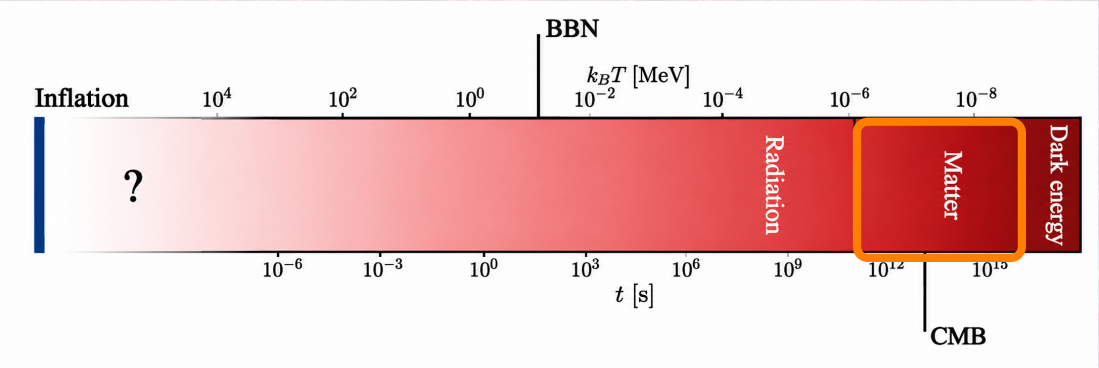


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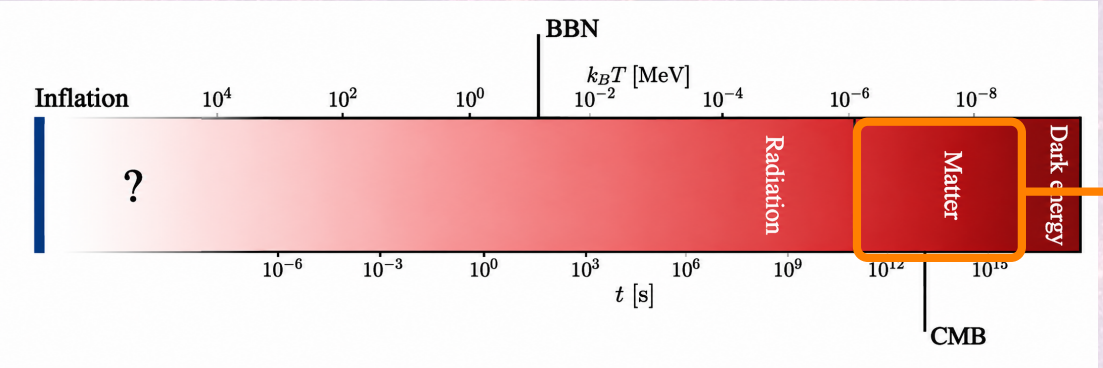


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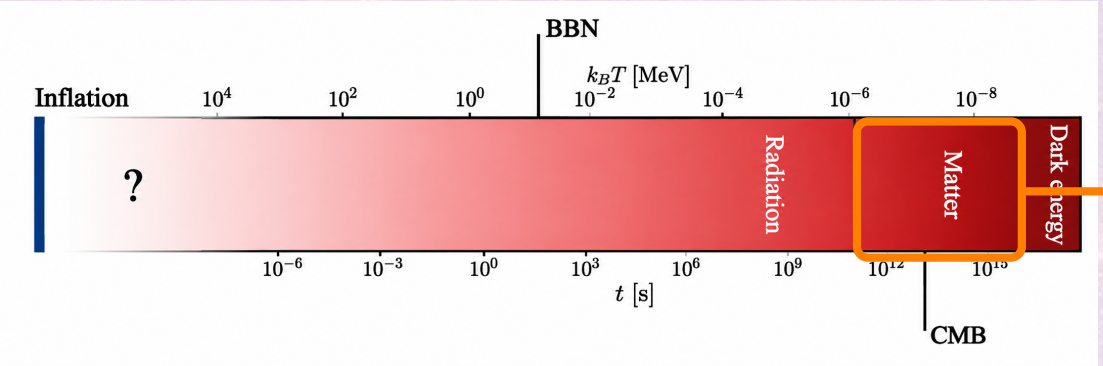


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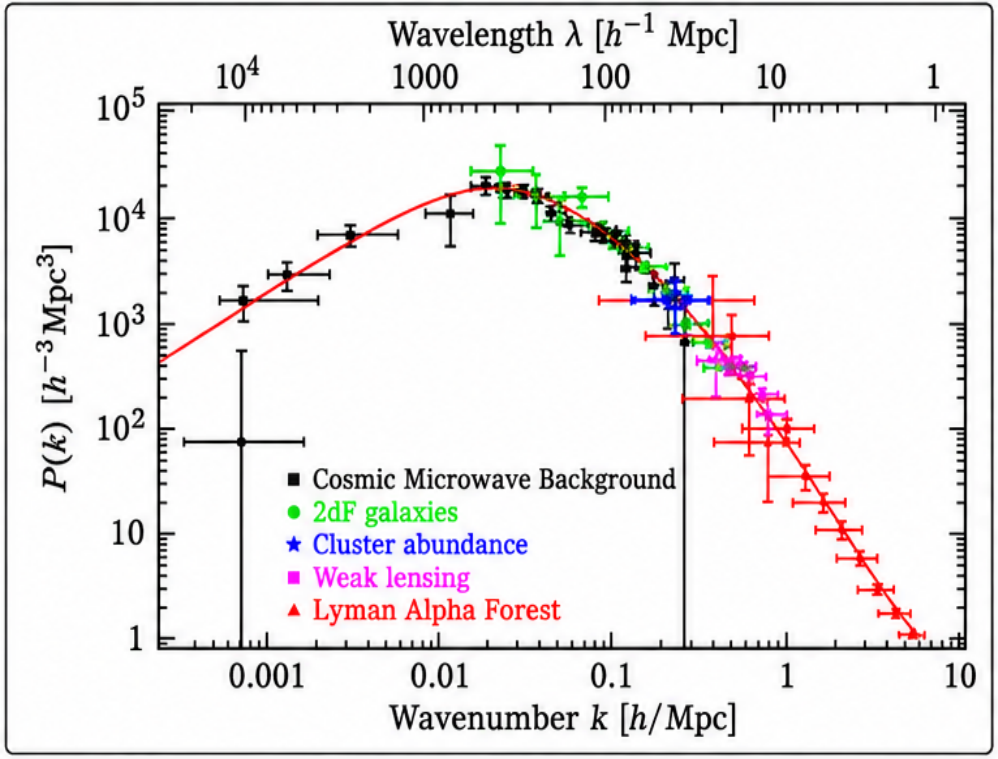
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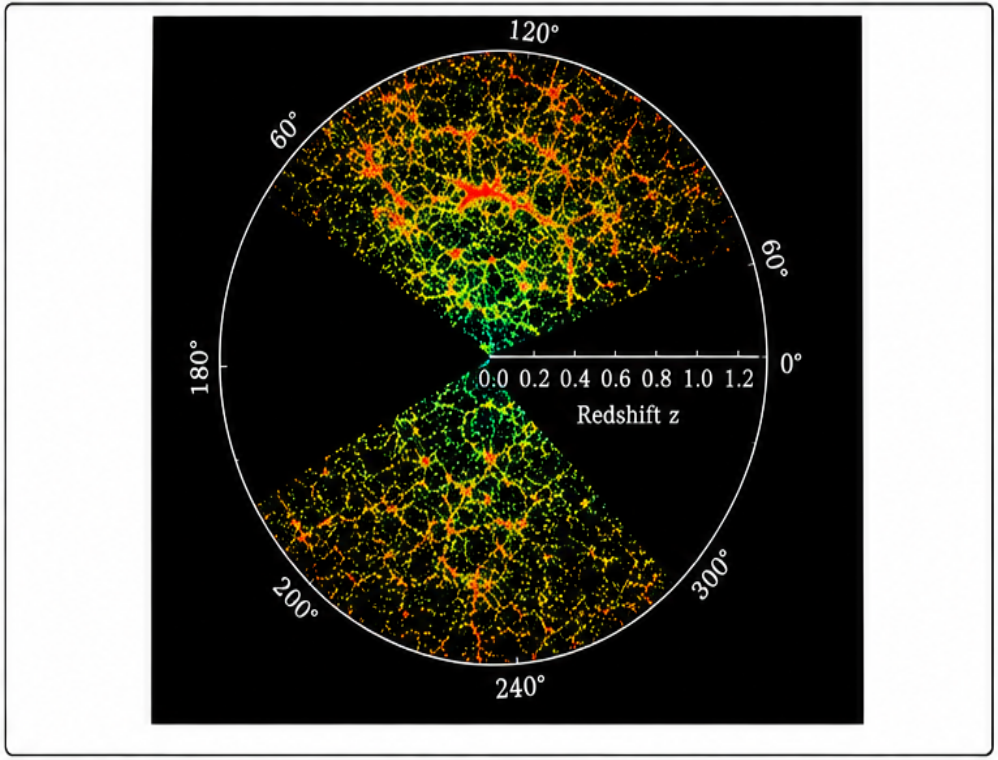
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1}$$

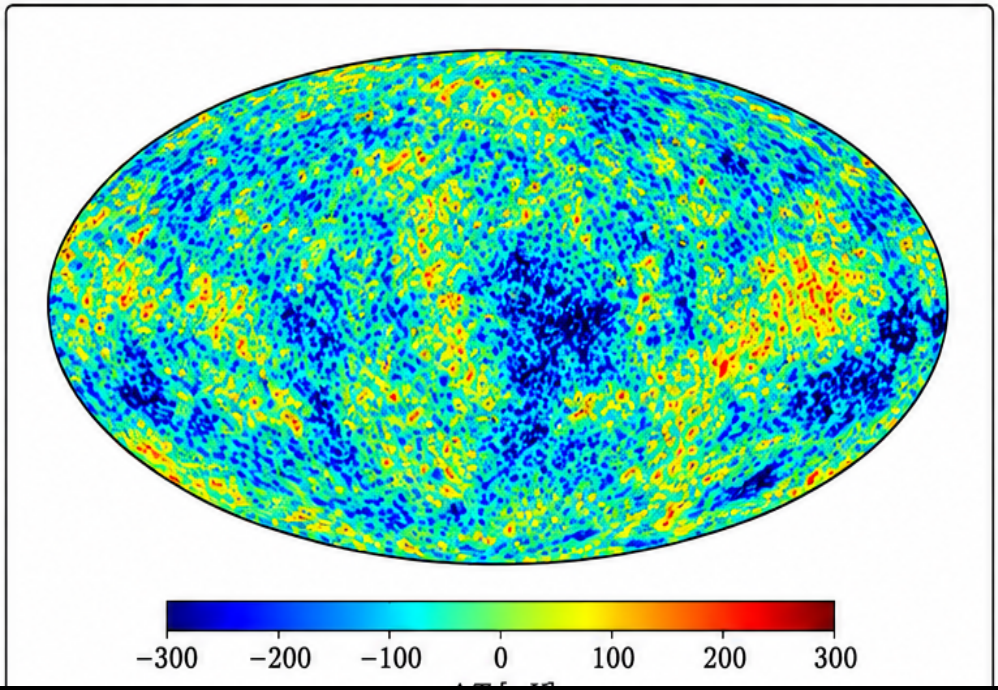
$$\frac{\partial f}{\partial t} + \frac{p}{E} \frac{\hat{p}^i}{a(t)} \frac{\partial f}{\partial x^i} - Hp \frac{\partial f}{\partial p} = C[f] \tag{2}$$

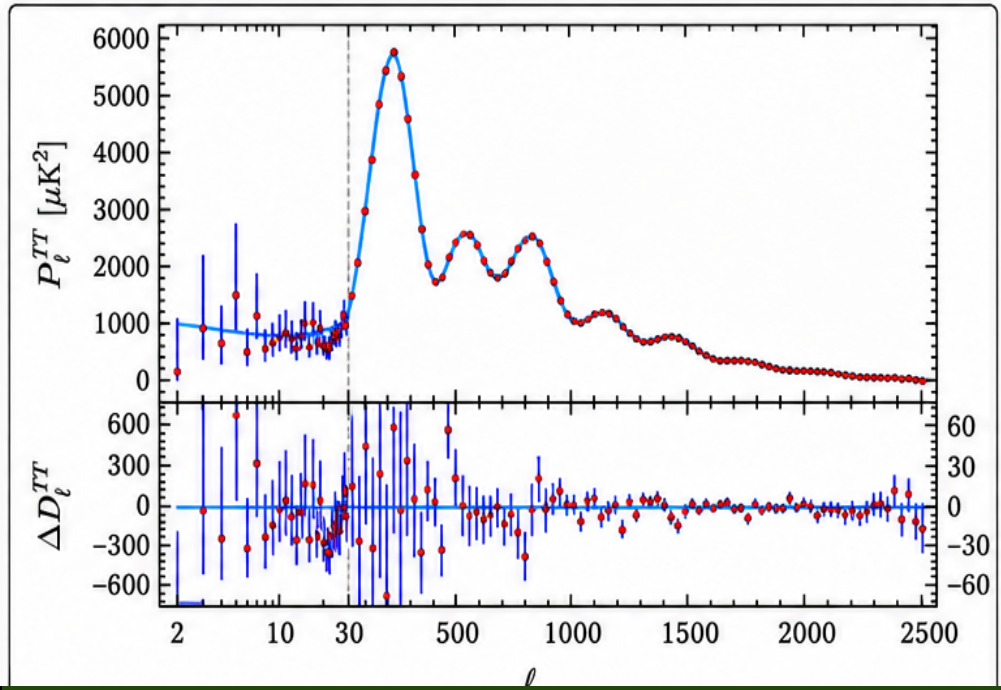


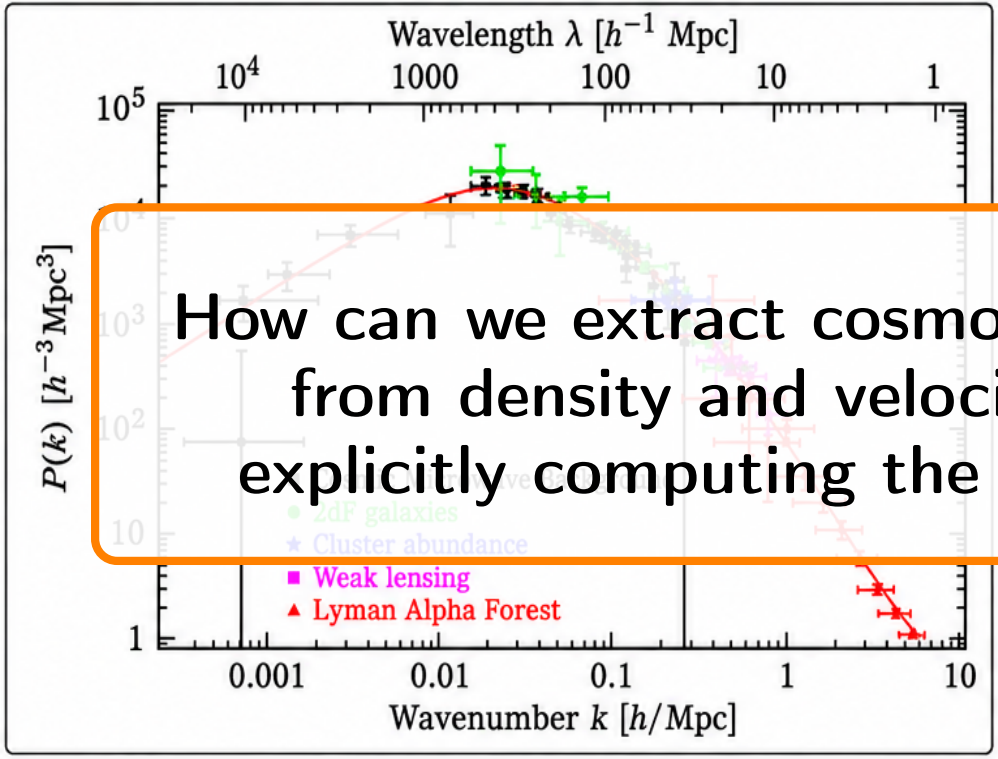

To describe the evolution of fluctuations in baryonic matter and cold dark matter within cosmological perturbation theory, using analytical tools.

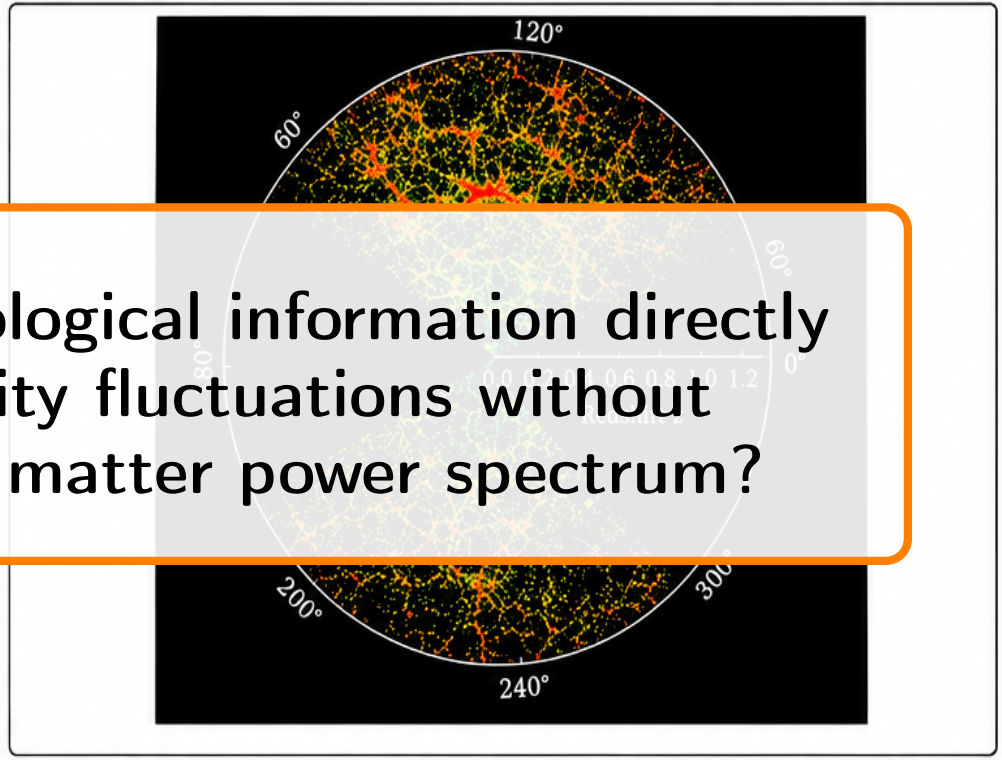

Wavelength $\lambda$ [$h^{-1}$ Mpc]
$10^4$
1000
100
10
1
$10^5$
$10^4$
$10^3$
$10^2$
10
1
$P(k)$ [$h^{-3}$Mpc$^3$]
Cosmic Microwave Background
2dF galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest
0.001
0.01
0.1
1
10
Wavenumber $k$ [$h$/Mpc]


120°
60°
60°
180°
0°
0.0 0.2 0.4 0.6 0.8 1.0 1.2
Redshift z
200°
300°
240°

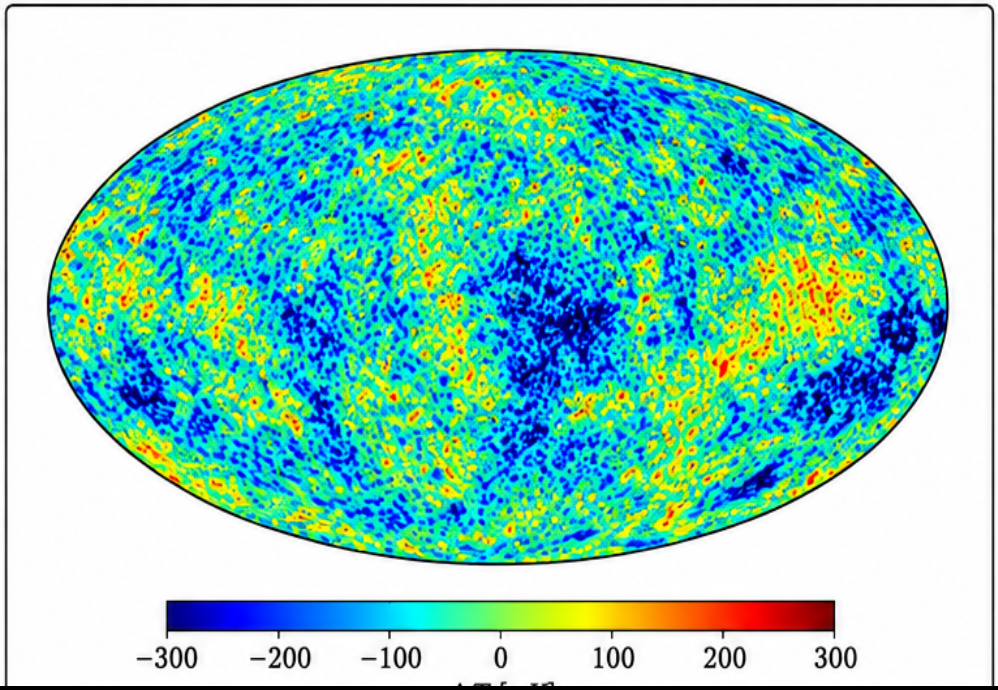

−300
−200
−100
0
100
200
300

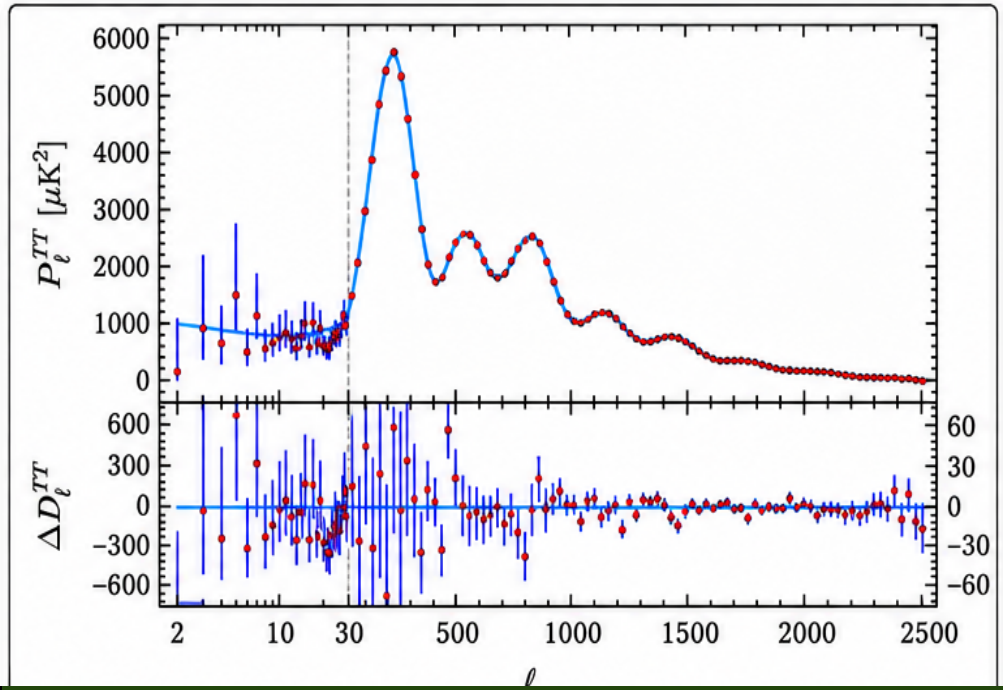

6000
5000
4000
3000
2000
1000
0
$P_\ell^{TT}$ [$\mu$K$^2$]
600
300
0
−300
−600
$\Delta D_\ell^{TT}$
60
30
0
−30
−60
2
10
30
500
1000
1500
2000
2500
$\ell$




How can we extract cosmological information directly from density and velocity fluctuations without explicitly computing the matter power spectrum?



Following Peebles (1980) and inspired by Komatsu (2009), we have the Vlasov equation

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{ma^2(t)} \cdot \nabla_{\mathbf{x}} f - m\nabla_{\mathbf{x}}\phi \cdot \nabla_{\mathbf{p}} f = 0 \tag{3}$$

where $m$ is the mass of a CDM particle, and $\phi(t, x)$ is a generalized potential. Our goal is to describe the distribution of matter (both dark and baryonic) and its dynamics in terms of deviations from the homogeneous and isotropic universe model. Therefore,

$$\frac{\partial}{\partial \tau}\delta_{\mathsf{C}}(\tau, \boldsymbol{x}) + \nabla_{\boldsymbol{x}} \cdot \left[\Big(1 + \delta_{\mathsf{C}}(\tau, \boldsymbol{x})\Big)\boldsymbol{u}_{\mathsf{C}}(\tau, \boldsymbol{x})\right] = 0, \tag{4}$$

$$\frac{\partial}{\partial \tau}\delta_{\mathsf{B}}(\tau, \boldsymbol{x}) + \nabla_{\boldsymbol{x}} \cdot \left[\Big(1 + \delta_{\mathsf{B}}(\tau, \boldsymbol{x})\Big)\boldsymbol{u}_{\mathsf{B}}(\tau, \boldsymbol{x})\right] = 0. \tag{5}$$

And Euler equations, for CDM

$$\frac{\partial}{\partial\tau}\boldsymbol{u}_{\mathsf{C}}(\tau,\boldsymbol{x})+\mathcal{H}(\tau)\boldsymbol{u}_{\mathsf{C}}(\tau,\boldsymbol{x})+\Big(\boldsymbol{u}_{\mathsf{C}}(\tau,\boldsymbol{x})\cdot\nabla_{\boldsymbol{x}}\Big)\boldsymbol{u}_{\mathsf{C}}(\tau,\boldsymbol{x}) = -\nabla_{\boldsymbol{x}}\phi(\tau,\boldsymbol{x}), \quad (6)$$

and baryonic matter,

$$\frac{\partial}{\partial\tau}\boldsymbol{u}_{\mathsf{B}}(\tau,\boldsymbol{x})+\mathcal{H}(\tau)\boldsymbol{u}_{\mathsf{B}}(\tau,\boldsymbol{x})+\Big(\boldsymbol{u}_{\mathsf{B}}(\tau,\boldsymbol{x})\cdot\nabla_{\boldsymbol{x}}\Big)\boldsymbol{u}_{\mathsf{B}}(\tau,\boldsymbol{x}) + \underbrace{\frac{\nabla_{\boldsymbol{x}}[P_{\mathsf{B}}(\rho_{\mathsf{B}})]}{\rho_{\mathsf{B}}(\tau,\boldsymbol{x})}}_{\text{Pressure Effect}} = -\underbrace{\nabla_{\boldsymbol{x}}\phi(\tau,\boldsymbol{x})}_{\text{Poisson's Equation}}. \quad (7)$$

With $\nabla^2_{\boldsymbol{x}}\phi = \frac{3}{2}\mathcal{H}(\tau)\Omega(\tau)\delta(\tau,\boldsymbol{x})$. Here, the fluctuation field is described by

$$\delta(\tau,\boldsymbol{x}) = \frac{\overline{\rho}_{\mathsf{C}}}{\rho_{\mathsf{m}}}\delta_{\mathsf{C}}(\tau,\boldsymbol{x}) + \frac{\overline{\rho}_{\mathsf{B}}}{\rho_{\mathsf{m}}}\delta_{\mathsf{B}}(\tau,\boldsymbol{x}), \quad (8)$$

# Standard Cosmological Perturbation Theory

Komatsu (2009)

$$\frac{\partial}{\partial\tau}\tilde{\delta}_{\mathsf{C}}(\boldsymbol{k},\tau)+\tilde{\theta}_{\mathsf{C}}(\boldsymbol{k},\tau)$$

$$=-\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2\,\frac{\boldsymbol{k}\cdot\boldsymbol{k}_2}{k_2^2}\delta^{\mathsf{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\tilde{\delta}_{\mathsf{C}}(\boldsymbol{k}_1,\tau)\tilde{\theta}_{\mathsf{C}}(\boldsymbol{k}_2,\tau), \tag{9}$$

$$\frac{\partial}{\partial\tau}\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k},\tau)+\tilde{\theta}_{\mathsf{B}}(\boldsymbol{k},\tau)$$

$$=-\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2\,\frac{\boldsymbol{k}\cdot\boldsymbol{k}_2}{k_2^2}\delta^{\mathsf{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k}_1,\tau)\tilde{\theta}_{\mathsf{B}}(\boldsymbol{k}_2,\tau), \tag{10}$$

$$\frac{\partial}{\partial\tau}\tilde{\theta}_{\mathsf{C}}(\boldsymbol{k},\tau)+\mathcal{H}(\tau)\tilde{\theta}_{\mathsf{C}}(\boldsymbol{k},\tau)+\frac{6}{\tau^2}\tilde{\delta}(\boldsymbol{k},\tau)$$

$$=-\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2\,k^2\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{2k_1^2k_2^2}\delta^{\mathsf{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\tilde{\theta}_{\mathsf{C}}(\boldsymbol{k}_1,\tau)\tilde{\theta}_{\mathsf{C}}(\boldsymbol{k}_2,\tau), \tag{11}$$

$$\frac{\partial}{\partial\tau}\tilde{\theta}_{\mathsf{B}}(\boldsymbol{k},\tau)+\mathcal{H}(\tau)\tilde{\theta}_{\mathsf{B}}(\boldsymbol{k},\tau)+\frac{6}{\tau^2}\tilde{\delta}(\boldsymbol{k},\tau)$$

$$=-\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2 k^2\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{2k_1^2k_2^2}\delta^{\mathsf{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\tilde{\theta}_{\mathsf{B}}(\boldsymbol{k}_1,\tau)\tilde{\theta}_{\mathsf{B}}(\boldsymbol{k}_2,\tau)$$

$$+c_s^2(\tau)k^2\Bigg[\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k},\tau)-\frac{1}{2(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2\delta^{\mathsf{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k}_1,\tau)$$

$$\times\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k}_2,\tau)+\frac{1}{3(2\pi)^6}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2 d^3\boldsymbol{k}_3\delta^{\mathsf{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3-\boldsymbol{k})$$

$$\times\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k}_1,\tau)\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k}_2,\tau)\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k}_3,\tau)\Bigg] \tag{12}$$

# Analytical Approach to First-Order Solutions

To first order, the equations of motion are written as

$$\frac{\partial}{\partial\tau}\delta_{\mathsf{C}}(\tau,\boldsymbol{x})+\nabla_{\boldsymbol{x}}\cdot\boldsymbol{u}_{\mathsf{C}}(\tau,\boldsymbol{x})=0, \tag{13}$$

$$\frac{\partial}{\partial\tau}\delta_{\mathsf{B}}(\tau,\boldsymbol{x})+\nabla_{\boldsymbol{x}}\cdot\boldsymbol{u}_{\mathsf{B}}(\tau,\boldsymbol{x})=0, \tag{14}$$

$$\frac{\partial}{\partial\tau}\boldsymbol{u}_{\mathsf{C}}(\tau,\boldsymbol{x})+\mathcal{H}(\tau)\boldsymbol{u}_{\mathsf{C}}(\tau,\boldsymbol{x})+\nabla_{\boldsymbol{x}}\phi(\tau,\boldsymbol{x})=0, \tag{15}$$

$$\frac{\partial}{\partial\tau}\boldsymbol{u}_{\mathsf{B}}(\tau,\boldsymbol{x})+\mathcal{H}(\tau)\boldsymbol{u}_{\mathsf{B}}(\tau,\boldsymbol{x})+\frac{\nabla_{\boldsymbol{x}}[P_{\mathsf{B}}(\rho_{\mathsf{B}})]}{\rho_{\mathsf{B}}(\tau,\boldsymbol{x})}=-\nabla_{\boldsymbol{x}}\phi(\tau,\boldsymbol{x}). \tag{16}$$

In Fourier space, these equations can be written as:

$$\frac{\partial}{\partial\tau}\tilde{\theta}_{\mathsf{C}}(\boldsymbol{k},\tau)+\frac{2}{\tau}\tilde{\theta}_{\mathsf{C}}(\boldsymbol{k},\tau)+\frac{6}{\tau^2}\tilde{\delta}(\boldsymbol{k},\tau)=0, \tag{17}$$

$$\frac{\partial}{\partial\tau}\tilde{\theta}_{\mathsf{B}}(\boldsymbol{k},\tau)+\frac{2}{\tau}\tilde{\theta}_{\mathsf{B}}(\boldsymbol{k},\tau)+\frac{6}{\tau^2}\left[\tilde{\delta}(\boldsymbol{k},\tau)-\frac{k^2}{k_J^2}\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k},\tau)\right]=0. \tag{18}$$

We have used the Jeans wavenumber, defined as $k_J(\tau)=\dfrac{\sqrt{6}}{c_s(\tau)\tau}$.

# Analytical Approach to First-Order Solutions

Then, to solve for the baryonic fluctuations, we introduce the Jeans filtering function.

$$g(\boldsymbol{k},\tau) \equiv \frac{\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k},\tau)}{\tilde{\delta}_{\mathsf{C}}(\boldsymbol{k},\tau)}. \tag{19}$$

$$g_1^{(0)}(k,\tau) = \underbrace{c_1\,\tau^{-\frac{5}{2}\left(1+\sqrt{1-\frac{24}{25}\left(1+\frac{k^2}{k_J^2}\right)}\right)} + c_2\,\tau^{-\frac{5}{2}\left(1-\sqrt{1-\frac{24}{25}\left(1+\frac{k^2}{k_J^2}\right)}\right)}}_{\text{Decaying mode}}$$

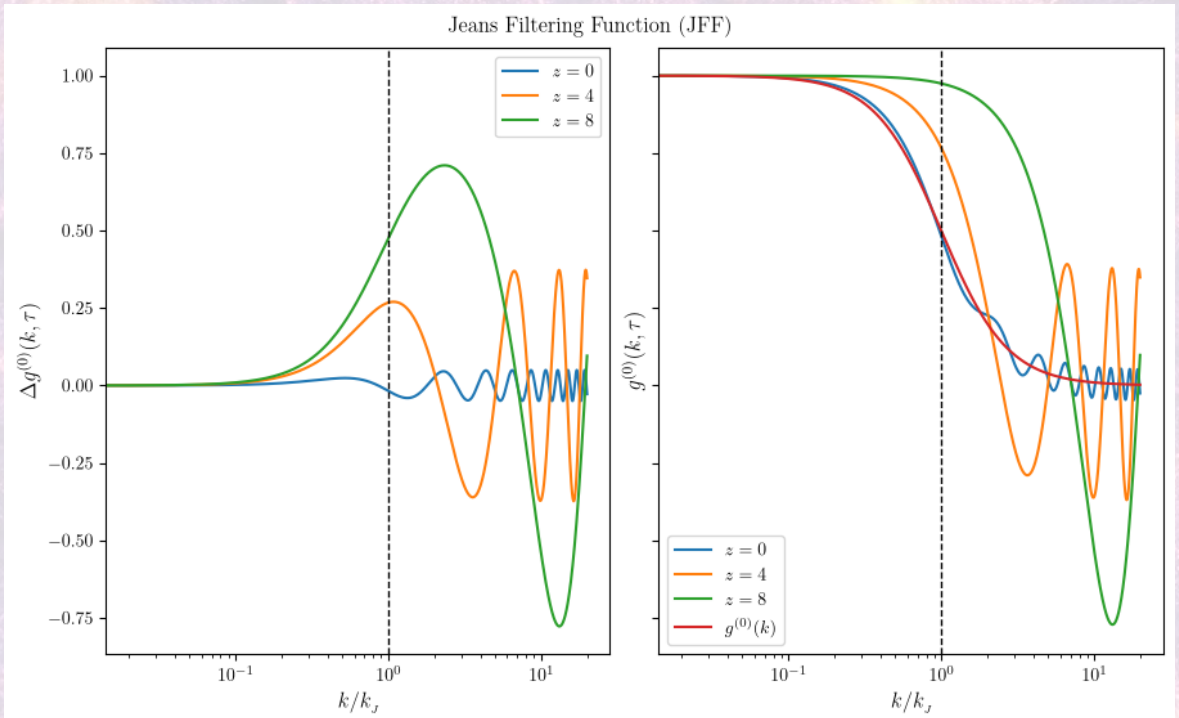
$$+ \underbrace{\frac{1}{1+\dfrac{k^2}{k_J^2}}}_{\text{Growing mode}} . \tag{20}$$

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$$+ \underbrace{\frac{1}{1+\frac{k^2}{k_J^2}}}_{\text{Growing mode}}. \tag{20}$$

Fonseca & Castañeda (2024)

# Analytical Approach to Second-Order Solutions

To obtain an analytical approach to second order, $n = 2$, in the density field and velocity divergence, we introduce the Jeans filtering functions.

$$g_n(\boldsymbol{k},\tau) \equiv \frac{\tilde{\delta}_{n,\mathsf{B}}(\boldsymbol{k},\tau)}{\tilde{\delta}_{n,\mathsf{C}}(\boldsymbol{k},\tau)}, \qquad h_n(\boldsymbol{k},\tau) \equiv \frac{\tilde{\theta}_{n,\mathsf{B}}(\boldsymbol{k},\tau)}{\tilde{\theta}_{n,\mathsf{C}}(\boldsymbol{k},\tau)}. \tag{21}$$

The equations of motion can be solved through the following perturbative expansion,

Bernardeau (2002)

$$\tilde{\delta}_{\mathsf{C}}(\boldsymbol{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau)\tilde{\delta}_{n,\mathsf{C}}(\boldsymbol{k}),$$

$$\tilde{\delta}_{\mathsf{B}}(\boldsymbol{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau)\tilde{\delta}_{n,\mathsf{B}}(\boldsymbol{k}) = \sum_{n=1}^{\infty} a^n(\tau) g_n(\boldsymbol{k},\tau)\tilde{\delta}_{n,\mathsf{C}}(\boldsymbol{k}),$$

$$\tilde{\theta}_{\mathsf{C}}(\boldsymbol{k},\tau) = \sum_{n=1}^{\infty} \dot{a}(\tau) a^{n-1}(\tau)\tilde{\theta}_{n,\mathsf{C}}(\boldsymbol{k}),$$

$$\tilde{\theta}_{\mathsf{B}}(\boldsymbol{k},\tau) = \sum_{n=1}^{\infty} \dot{a}(\tau) a^{n-1}(\tau)\tilde{\theta}_{n,\mathsf{B}}(\boldsymbol{k}) = \sum_{n=1}^{\infty} \dot{a}(\tau) a^{n-1}(\tau) h_n(\boldsymbol{k},\tau)\tilde{\theta}_{n,\mathsf{C}}(\boldsymbol{k}).$$

# Analytical Approach to Second-Order Solutions

After some algebra, we obtain

$$\ddot{g}_2(\boldsymbol{k},\tau)+\frac{10}{\tau}\dot{g}_2(\boldsymbol{k},\tau)+\frac{1}{\tau^2}\left[20+6\frac{k^2}{k_J^2}\right]g_2(\boldsymbol{k},\tau) - \frac{1}{\tau^2}\left[6+10\frac{A_2(\boldsymbol{k})}{\tilde{\delta}_{2,\mathsf{C}}(\boldsymbol{k})}-\frac{4B_2(\boldsymbol{k})}{\tilde{\delta}_{2,\mathsf{C}}(\boldsymbol{k})}\right]=0. \quad (22)$$

With $\delta^{\mathsf{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\equiv\delta^{\mathsf{D}}$, and

$$A_2(\boldsymbol{k})\equiv\frac{2}{\tau}\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2\,\frac{\boldsymbol{k}\cdot\boldsymbol{k}_2}{k_2^2}\delta^{\mathsf{D}}\,g_1(\boldsymbol{k}_1)g_1(\boldsymbol{k}_2)\tilde{\delta}_{1,\mathsf{C}}(\boldsymbol{k}_1)\tilde{\delta}_{1,\mathsf{C}}(\boldsymbol{k}_2)$$

$$B_2(\boldsymbol{k})\equiv\frac{4}{\tau^2}\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2\left[-\frac{3}{4}\frac{k^2}{k_J^2}-k^2\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{2k_1^2k_2^2}\right]\delta^{\mathsf{D}}g_1(\boldsymbol{k}_1)g_1(\boldsymbol{k}_2) \times\tilde{\delta}_{1,\mathsf{C}}(\boldsymbol{k}_1)\tilde{\delta}_{1,\mathsf{C}}(\boldsymbol{k}_2)$$

# Analytical Approach to Second-Order Solutions

Therefore, the general solution is

$$g_2^{(0)}(\boldsymbol{k},\tau) = \underbrace{c_1\,\tau^{-\frac{9}{2}\left(1+\sqrt{1-\frac{4}{81}\left(20+6\frac{k^2}{k_J^2}\right)}\right)} + c_2\,\tau^{-\frac{9}{2}\left(1-\sqrt{1-\frac{4}{81}\left(20+6\frac{k^2}{k_J^2}\right)}\right)}}_{\text{Decaying Mode}} + \underbrace{\frac{6+10\dfrac{A_2(\boldsymbol{k})}{\tilde{\delta}_{2,\mathrm{C}}(\boldsymbol{k})} - \dfrac{4B_2(\boldsymbol{k})}{\tilde{\delta}_{2,\mathrm{C}}(\boldsymbol{k})}}{20+6\frac{k^2}{k_J^2}}}_{\text{Growing mode}},$$

with $\tilde{\delta}_{2,\mathrm{C}}(\boldsymbol{k}) = \dfrac{1}{(2\pi)^3}\displaystyle\int d\boldsymbol{q} F_2^{(s)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\tilde{\delta}_{1,\mathrm{C}}(\boldsymbol{q})\tilde{\delta}_{1,\mathrm{C}}(\boldsymbol{k}-\boldsymbol{q})$, where $F_2$ is a symmetric, homogeneous function of the wave vectors $\{\boldsymbol{q}_1,\boldsymbol{q}_2\}$ of degree zero.

$$\alpha(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{\boldsymbol{k}\cdot\boldsymbol{k}_2}{k_2^2}, \qquad \beta(\boldsymbol{k}_1,\boldsymbol{k}_2) = k^2\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{2k_1^2k_2^2}. \tag{3.30}$$

# Analytical Approach to Second-Order Solutions

For the Jeans filtering function of the velocity divergence field, we can demonstrate that

$$h_2^{(0)}(\boldsymbol{k}) = \frac{1}{\tilde{\theta}_{2,\mathsf{C}}^{(0)}(\boldsymbol{k})}\left[\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2 \delta^{\mathsf{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k})\right.$$
$$\left.\left[2F_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2) - G_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2)\right] g_1^{(0)}(\boldsymbol{k}_1) g_1^{(0)}(\boldsymbol{k}_2) \tilde{\delta}_{1,\mathsf{C}}^{(0)}(\boldsymbol{k}_1)\tilde{\delta}_{1,\mathsf{C}}^{(0)}(\boldsymbol{k}_2)\right]$$
$$- 2\frac{\tilde{\delta}_{2,\mathsf{C}}^{(0)}(\boldsymbol{k})}{\tilde{\theta}_{2,\mathsf{C}}^{(0)}(\boldsymbol{k})} g_2^{(0)}(\boldsymbol{k}), \quad (23)$$

with $2F_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2) - G_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2) \equiv 1 + \dfrac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2(\boldsymbol{k}_1+\boldsymbol{k}_2)}{2\boldsymbol{k}_1\boldsymbol{k}_2}$.

# Semi-Analytical Modeling of Baryonic Fluctuations

We describe the behavior of the field $\tilde{\delta}_{2,\mathsf{B}}(\boldsymbol{k})$ using $\tilde{\delta}_{2,\mathsf{C}}(\boldsymbol{k})$ as a tracer at the zeroth-order iteration, through the JFF $g_2(\boldsymbol{k},\tau)$. We then compare its evolution with the corresponding first-order solution, considering only the growing modes.

$$\frac{\tilde{\delta}^{(0)}_{2,\mathsf{B}}(\boldsymbol{k})}{\tilde{\delta}^{(0)}_{2,\mathsf{C}}(\boldsymbol{k})} = g_2^{(0)}(\boldsymbol{k}) = \frac{\frac{10}{3} - \frac{7}{3}\left(1 - \frac{\tilde{\delta}'^{(0)}_{2,\mathsf{C}}(\boldsymbol{k})}{\tilde{\delta}^{(0)}_{2,\mathsf{C}}(\boldsymbol{k})}\right)}{\frac{10}{3} + \frac{k^2}{k_J^2}}. \tag{24}$$

Where

$$\tilde{\delta}^{(0)}_{2,\mathsf{C}}(\boldsymbol{k}) \propto \frac{1}{(2\pi)^3}\int d\boldsymbol{q} F_2^{(s)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\sqrt{P_{\mathsf{lin}}(q)}\sqrt{P_{\mathsf{lin}}(|\boldsymbol{k}-\boldsymbol{q}|)},$$

$$\tilde{\delta}'^{(0)}_{2,\mathsf{C}}(\boldsymbol{k}) \propto \frac{1}{(2\pi)^3}\int d\boldsymbol{q} \mathcal{F}_2^{(s)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\sqrt{P_{\mathsf{lin}}(q)}\sqrt{P_{\mathsf{lin}}(|\boldsymbol{k}-\boldsymbol{q}|)}.$$

# Semi-Analytical Modeling of Baryonic Fluctuations

Therefore, the second-order JFF can be written as

Fonseca, Castañeda & García (2026)

$$g_2^{(0)}(k) \propto \frac{1}{\frac{10}{3}+\frac{k^2}{k_J^2}} + \frac{7}{3} \times$$

$$\int_0^\infty \int_{-1}^{1} dx\, dr \, \frac{\left( r^{3/2} \frac{3r+7x-10rx^2}{(1+r^2-2rx)^{3/4}} + 3\frac{k^2}{k_J^2}(1+r^2-2rx)^{1/4} r^{5/2} \right) \overline{T}(k,r,x)}{\left(1+r^2\frac{k^2}{k_J^2}\right)\left(1+(1+r^2-2rx)\frac{k^2}{k_J^2}\right)}$$

$$\left(\frac{10}{3}+\frac{k^2}{k_J^2}\right) \int_0^\infty \int_{-1}^{1} dx\, dr\, r^{3/2} \frac{3r+7x-10rx^2}{(1+r^2-2rx)^{3/4}} \overline{T}(k,r,x)$$

Defining the product $\overline{T}(k,r,x) \equiv T(kr)\; T\left(k\sqrt{1+r^2-2rx}\right)$

# Semi-Analytical Modeling of Baryonic Fluctuations

We must compute the nonlinear density field,

$$\tilde{\delta}(\boldsymbol{k},\tau) = \tilde{\delta}_1(\boldsymbol{k},\tau) + \tilde{\delta}_2(\boldsymbol{k},\tau) \sim \frac{\tilde{\delta}_{1,\mathrm{B}}(\boldsymbol{k}) + \tilde{\delta}_{2,\mathrm{B}}(\boldsymbol{k})}{\tilde{\delta}_{1,\mathrm{C}}(\boldsymbol{k}) + \tilde{\delta}_{2,\mathrm{C}}(\boldsymbol{k})} \propto g_{1+2}(\boldsymbol{k}) \tag{25}$$

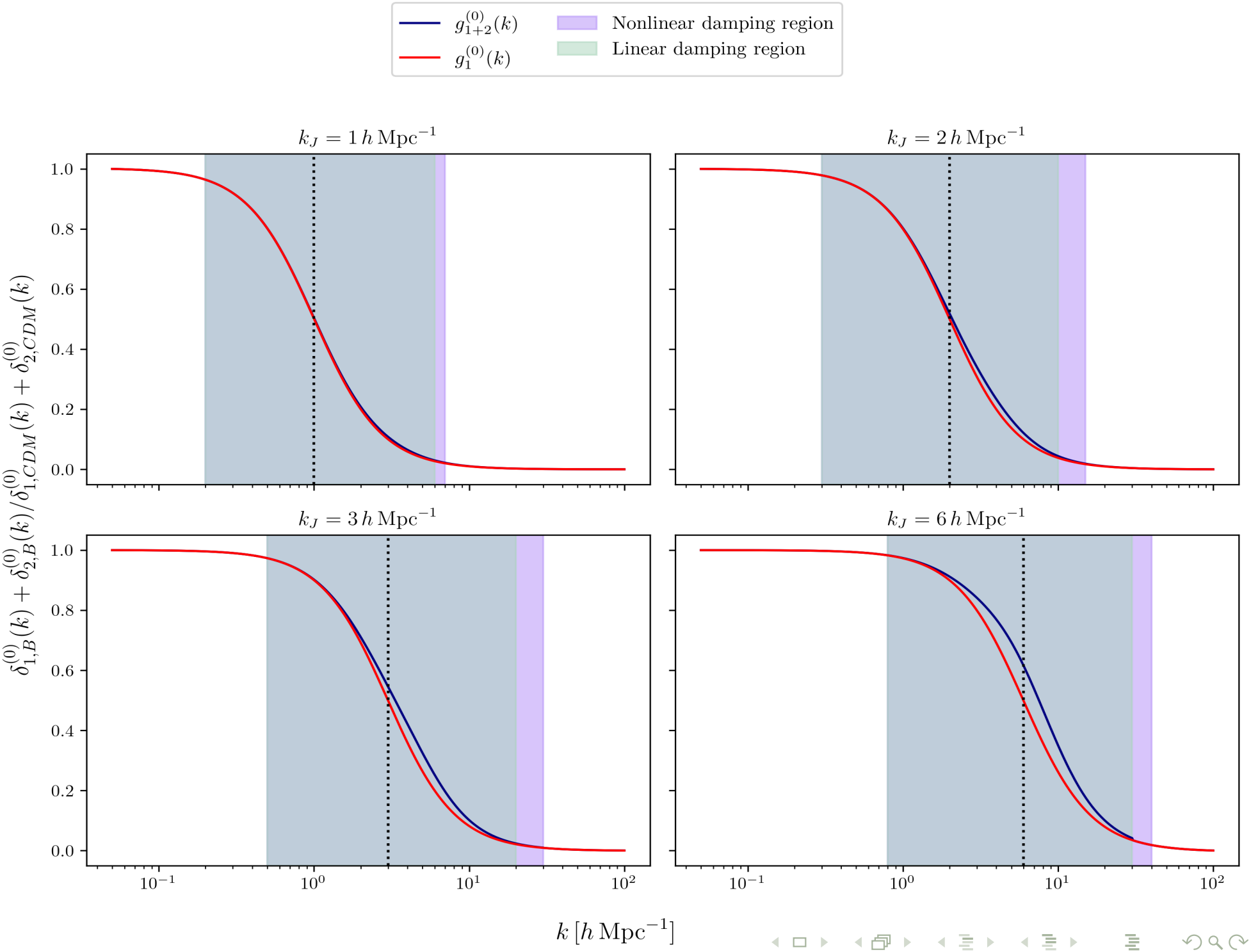
Therefore,

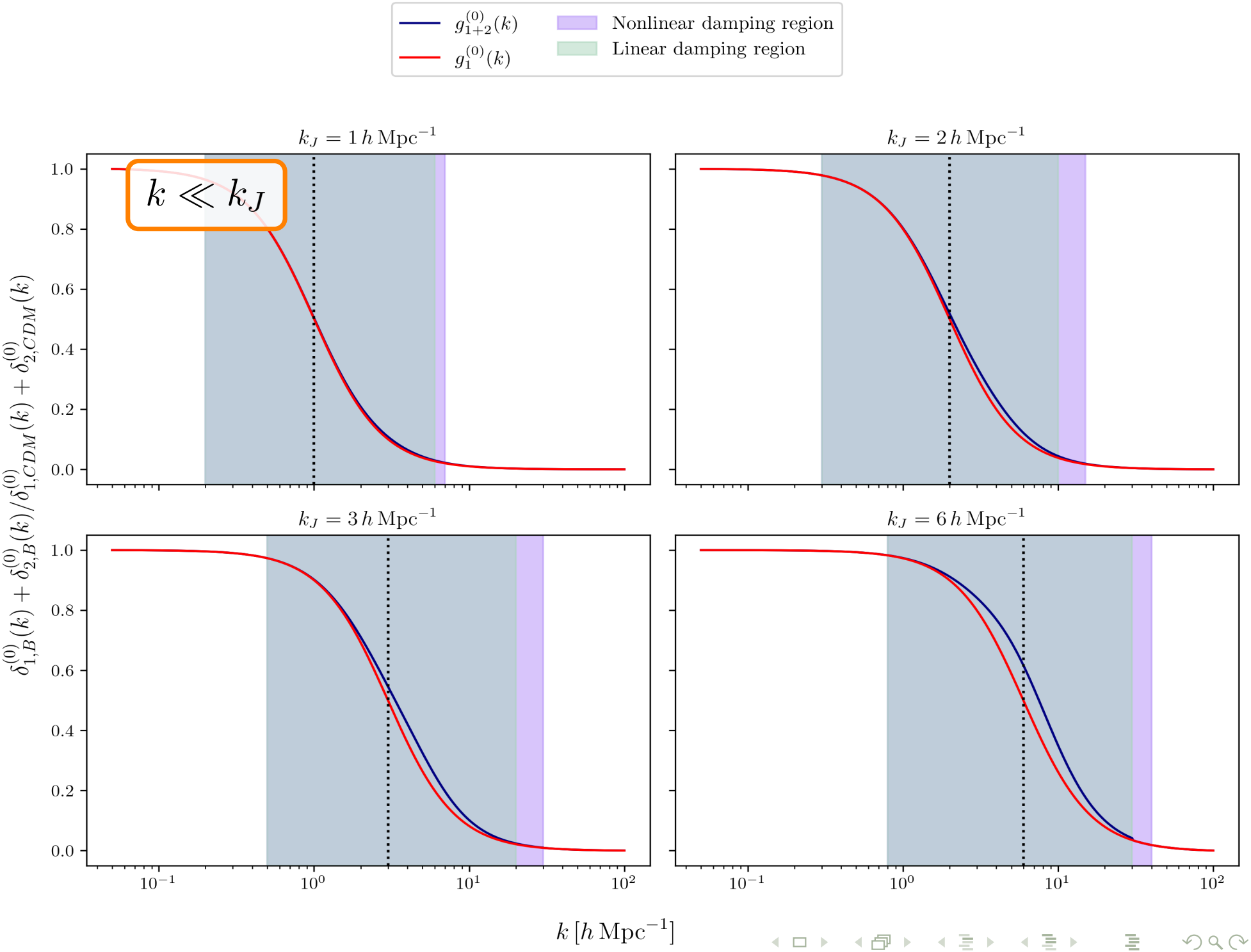
$$\boxed{g^{(0)}_{1+2}(k) \propto \frac{g^{(0)}_1(k)\sqrt{P_{\mathrm{lin}}(k)} + g^{(0)}_2(k)C(k)\mathcal{I}(k)}{\sqrt{P_{\mathrm{lin}}(k)} + C(k)\mathcal{I}(k)}} \tag{26}$$

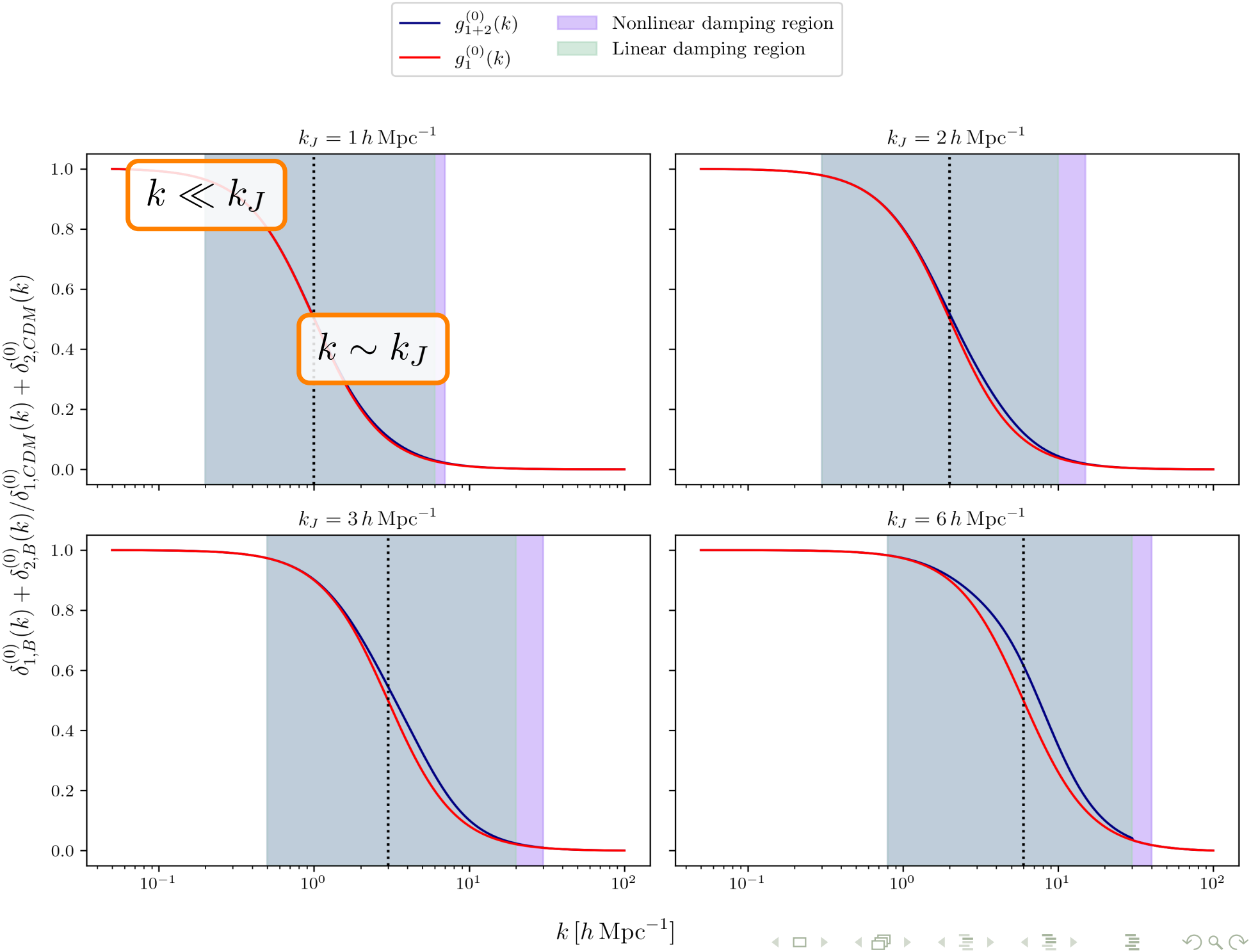
Where

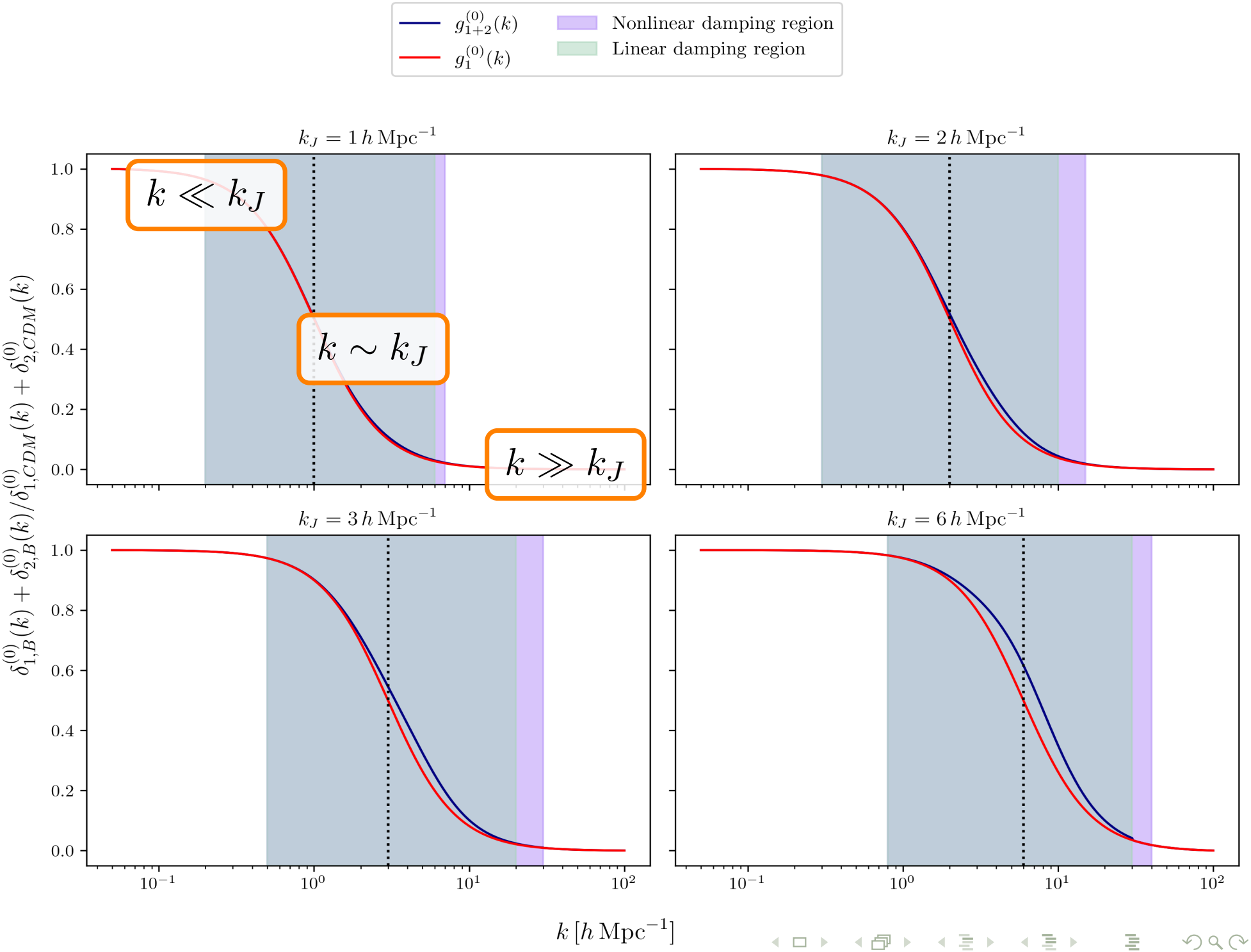
$$\mathcal{I}(k) \equiv \int_0^\infty \int_{-1}^1 dx\, dr\, r^{3/2} \frac{3r + 7x - 10rx^2}{(1 + r^2 - 2rx)^{3/4}} \overline{T}(k, r, x).$$

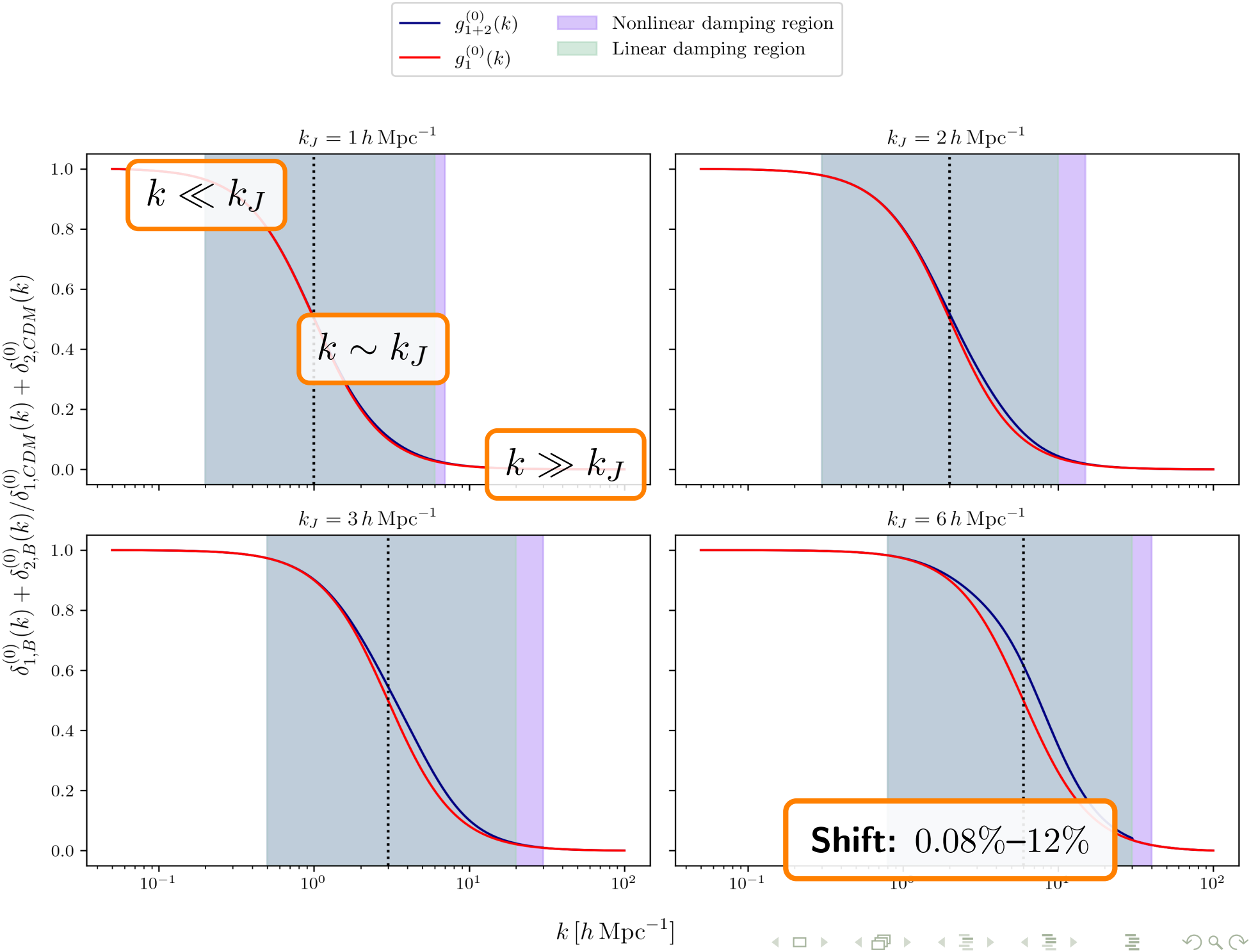
$$C(k) \equiv \frac{k^4 A}{14(2\pi)^2}.$$


$g^{(0)}_{1+2}(k)$
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Nonlinear damping region
Linear damping region
$k_J = 1\,h\,\mathrm{Mpc}^{-1}$
$k_J = 2\,h\,\mathrm{Mpc}^{-1}$
$k_J = 3\,h\,\mathrm{Mpc}^{-1}$
$k_J = 6\,h\,\mathrm{Mpc}^{-1}$
$\delta^{(0)}_{1,B}(k) + \delta^{(0)}_{2,B}(k)/\delta^{(0)}_{1,CDM}(k) + \delta^{(0)}_{2,CDM}(k)$
$k\,[h\,\mathrm{Mpc}^{-1}]$


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Shift: 0.08%–12%
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$k\,[h\,\mathrm{Mpc}^{-1}]$

# Nonlinear Shift of the Filtering Scale (Conclusion 1)

## Physical interpretation

The nonlinear solution undergoes damping at smaller spatial scales (larger $k$) than the linear solution, indicating a delayed suppression of second-order fluctuations. Consequently, the effective filtering scale $k_F$ differs from the characteristic linear Jeans scale $k_J$.

| $k_J\ [h\,\mathrm{Mpc}^{-1}]$ | 1 | 2 | 3 | 6 |
|---|---|---|---|---|
| $k_F/k_J$ | 1.21 | 1.26 | 1.69 | 1.48 |

## Main result

$$\boxed{k_F/k_J \sim 1.2 - 1.7}$$

The effective filtering scale is obtained from an exponential fit to the full scale-dependent suppression function, characterizing the overall damping profile rather than the precise onset of suppression **Gnedin et al. (1998)**.

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## Main result

$k_F \approx 1.4\,k_J$, inferred indirectly from the nonlinear matter power spectrum using third-order perturbation theory.

**Shoji et al. (2009)**

# Physical Implications of the Filtering Scale Shift

**Baryonic Temperature**

$$T_B \propto k_J^{-2}$$

$$\uparrow (1.46 - 2.85)$$

Relative to linear theory

**Filtering Mass**

$$M_F \propto k_F^{-3}$$

$$\downarrow (1.77 - 3.24)$$

Relative to linear estimate

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Temperature shift: $\sim 2$ Filtering mass shift: $\sim 3$

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# Semi-Analytical Modeling of Baryonic Fluctuations

At this stage, we focus on the divergence of the velocity field.

$$h_2^{(0)}(k) \propto -7\frac{\mathcal{I}_1(k)}{\mathcal{I}_2(k)} + \frac{2}{\frac{10}{3}+\frac{k^2}{k_J^2}}\frac{\mathcal{I}_3(k)}{\mathcal{I}_2(k)}\left[1+\frac{7}{3}\frac{\mathcal{I}_4(k)}{\mathcal{I}_3(k)}\right]$$

Fonseca, Castañeda & García (2026)

$$\mathcal{I}_1(k) = \int_0^\infty\int_{-1}^{1} dx\,dr \frac{r^{3/2}(r+x-2rx^2)}{D^{3/4}Q_1Q_2}\overline{T}(k,r,x),$$

$$\mathcal{I}_2(k) = \int_0^\infty\int_{-1}^{1} dx\,dr \frac{r^{3/2}(7x-r-6rx^2)}{D^{3/4}}\overline{T}(k,r,x),$$

$$\mathcal{I}_3(k) = \int_0^\infty\int_{-1}^{1} dx\,dr \frac{r^{3/2}(3r+7x-10rx^2)}{D^{3/4}}\overline{T}(k,r,x),$$

$$\mathcal{I}_4(k) = \int_0^\infty\int_{-1}^{1} dx\,dr \frac{\left[r^{3/2}\frac{3r+7x-10rx^2}{D^{3/4}} + 3\frac{k^2}{k_J^2}D^{1/4}r^{3/2}\right]}{Q_1Q_2}\overline{T}(k,r,x)$$

With $D = 1+r^2-2rx$, $Q_1 = 1+r^2\frac{k^2}{k_J^2}$ and $Q_2 = 1+D\frac{k^2}{k_J^2}$

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With $D = 1+r^2-2rx$, $Q_1 = 1+r^2\frac{k^2}{k_J^2}$ and $Q_2 = 1+D\frac{k^2}{k_J^2}$

# Semi-Analytical Modeling of Baryonic Fluctuations

To estimate the impact on the velocity power spectrum, we must consider the total contribution.

$$\tilde{\theta}(\boldsymbol{k},\tau) = \tilde{\theta}_{1}(\boldsymbol{k},\tau) + \tilde{\theta}_{2}(\boldsymbol{k},\tau) \sim \frac{\tilde{\theta}_{1,\mathrm{B}}(\boldsymbol{k}) + \tilde{\theta}_{2,\mathrm{B}}(\boldsymbol{k})}{\tilde{\theta}_{1,\mathrm{C}}(\boldsymbol{k}) + \tilde{\theta}_{2,\mathrm{C}}(\boldsymbol{k})} \propto h_{1+2}\boldsymbol{k}). \tag{27}$$

We can write

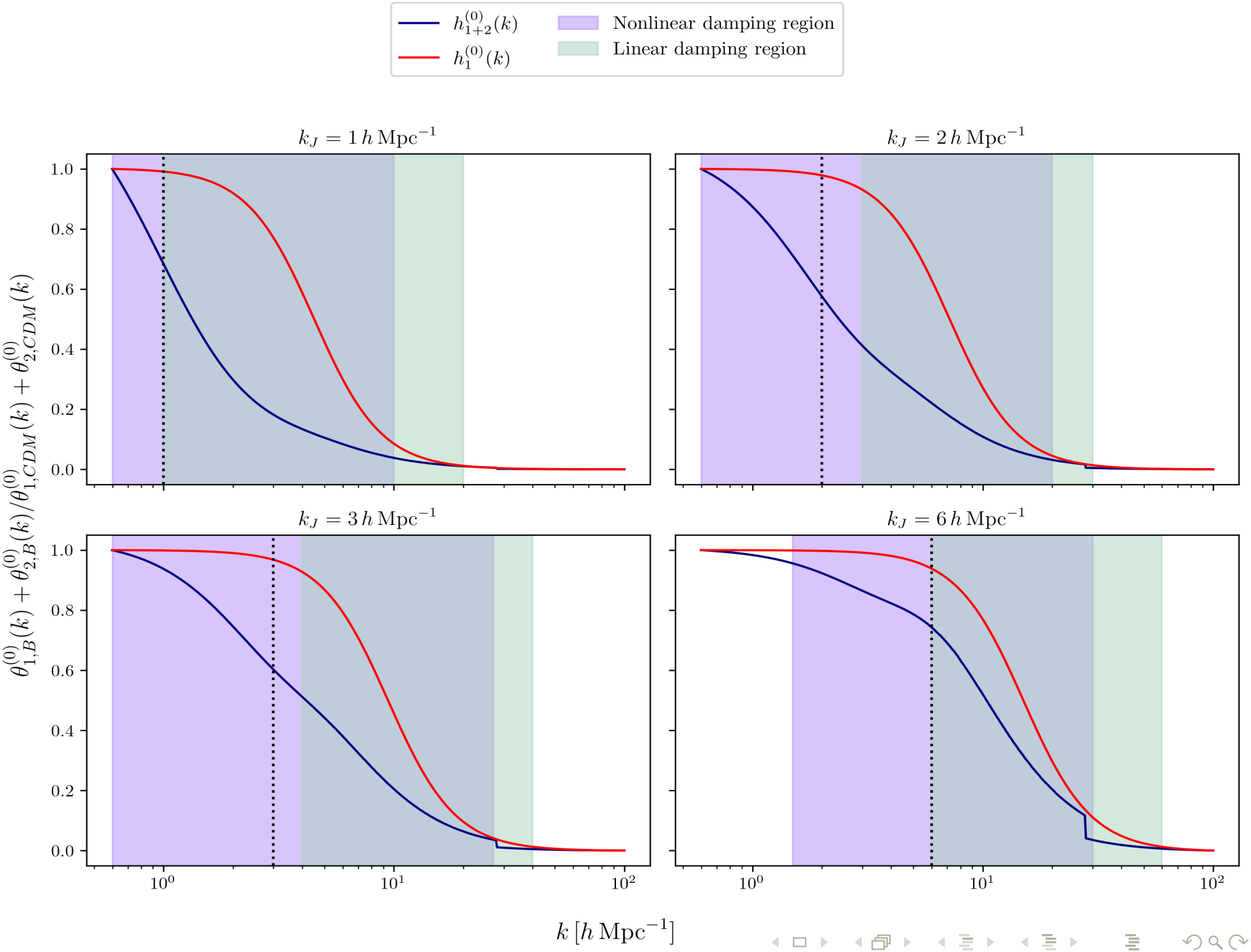
$$\boxed{h^{(0)}_{1+2}(k) \propto \frac{h^{(0)}_{1}(k)\tilde{\theta}^{(0)}_{1,\mathrm{C}}(k) + h^{(0)}_{2}(k)\tilde{\theta}^{(0)}_{2,\mathrm{C}}(k)}{\tilde{\theta}^{(0)}_{1,\mathrm{C}}(k) + \tilde{\theta}^{(0)}_{2,\mathrm{C}}(k)}} \tag{28}$$

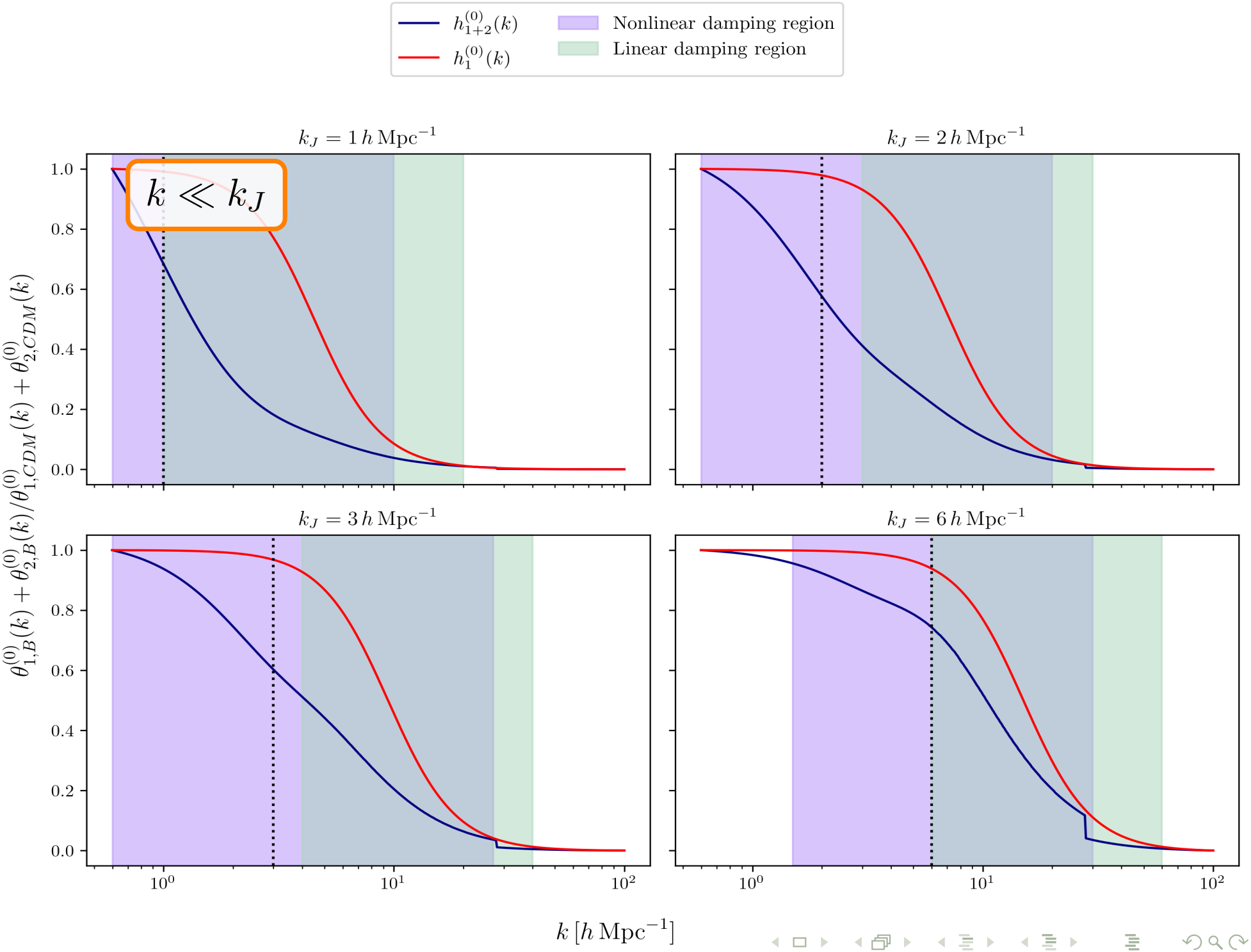
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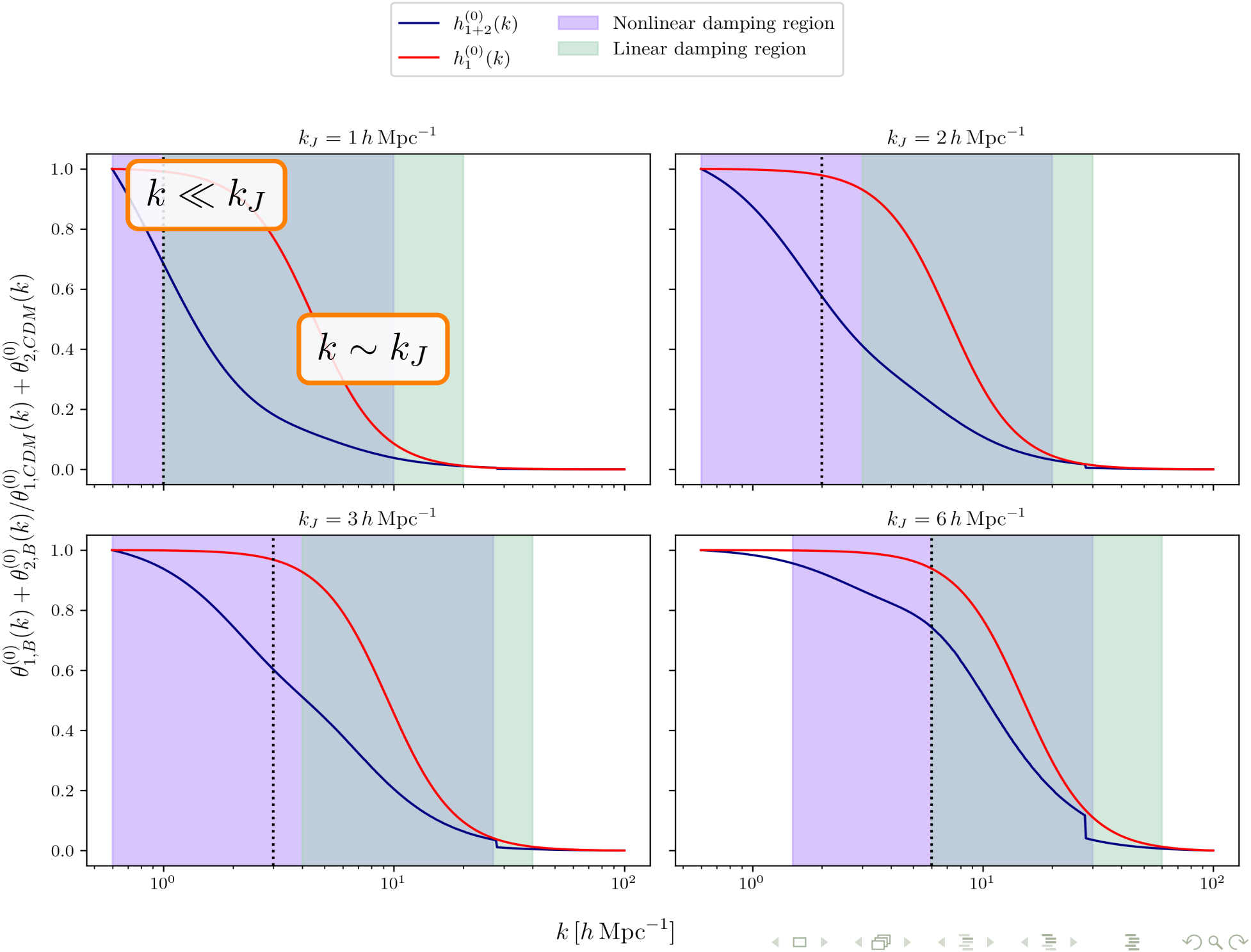
$$\boxed{h^{(0)}_{1+2}(k) \propto \frac{g^{(0)}_{1}(k)\sqrt{P_{\mathrm{lin}}(k)} + h^{(0)}_{2}(k)C(k)\mathcal{J}(k)}{\sqrt{P_{\mathrm{lin}}(k)} + C(k)\mathcal{J}(k)}} \tag{29}$$

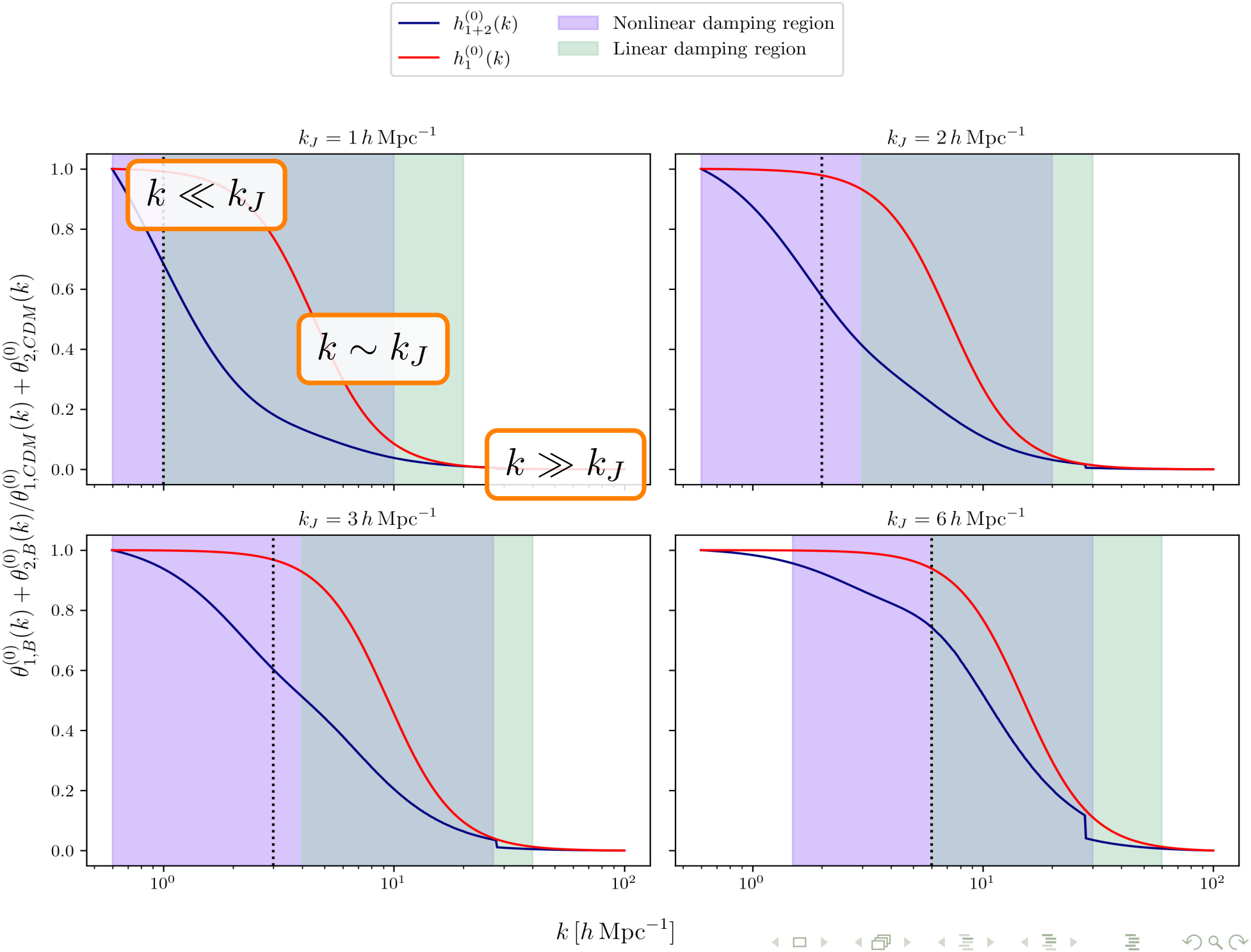
where

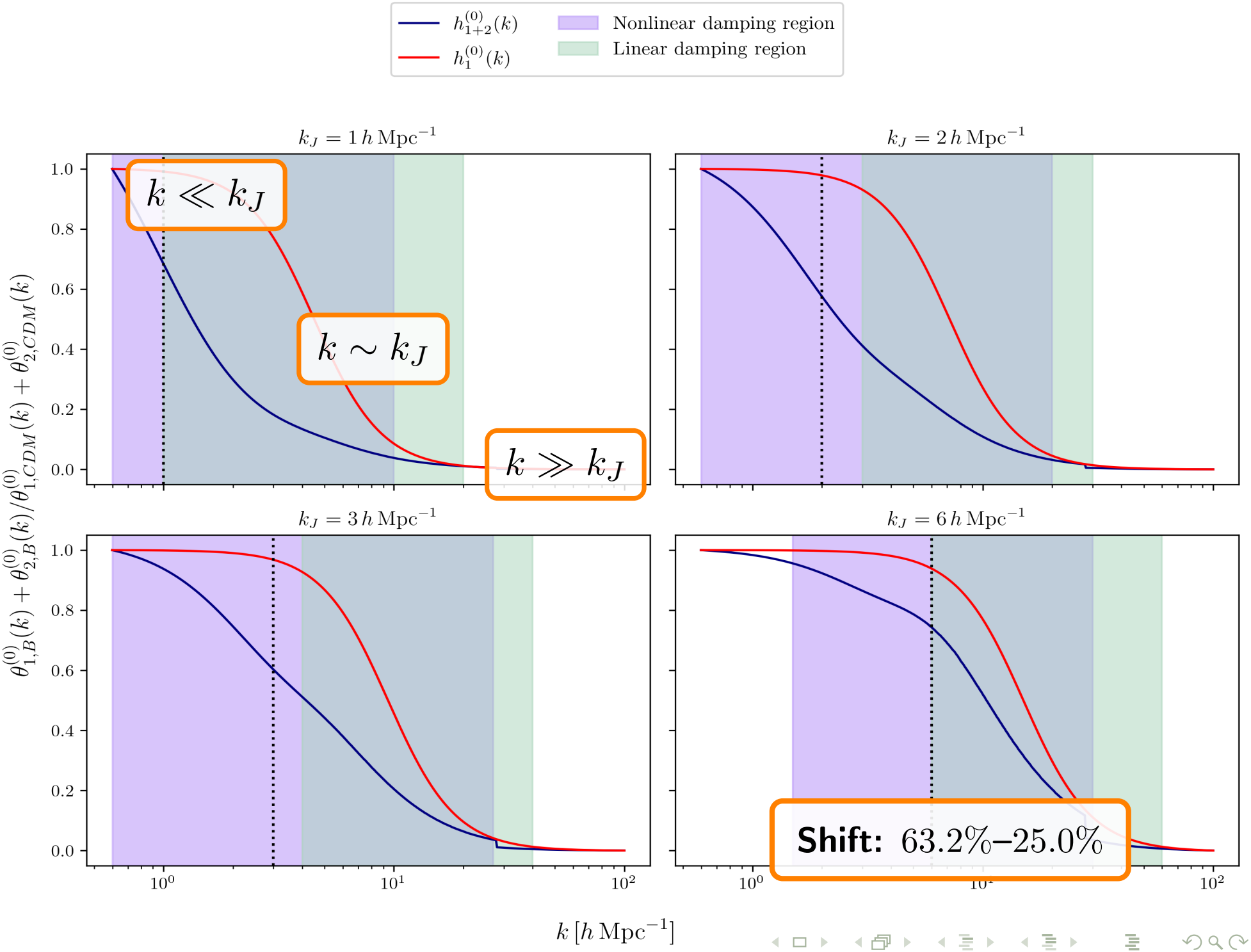
$$\mathcal{J}(k) \equiv \int_{0}^{\infty}\int_{-1}^{1} dx\, dr\, r^{3/2} \frac{7x - r - 6rx^2}{(1 + r^2 - 2rx)^{3/4}} \overline{T}(k, r, x).$$


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$k\,[h\,\mathrm{Mpc}^{-1}]$
0.0
0.2
0.4
0.6
0.8
1.0
$10^0$
$10^1$
$10^2$


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# Nonlinear Filtering of the Velocity Field (Conclusion 2)

## Second-order velocity-divergence filtering

$$\boxed{h_2^{(0)}(\boldsymbol{k}), \qquad h_{1+2}^{(0)}(\boldsymbol{k})}$$

Direct semi-analytical computation of baryonic pressure effects beyond the linear regime.

| $k_J\ [h\,\mathrm{Mpc}^{-1}]$ | 1 | 2 | 3 | 6 |
|---|---|---|---|---|
| $k_F/k_J$ | 1.67 | 1.53 | 1.70 | 2.03 |

**Main Physical Result**

Nonlinear effects modify the scale dependence of the velocity field more strongly than that of the density field. **Earlier and stronger suppression** relative to the linear velocity solution.

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# Future Work

## 1. General Second-Order Solutions

Incorporate decaying modes and appropriate initial conditions to investigate small-scale oscillations and obtain more realistic baryonic filtering functions.

## 2. Velocity-Based Cosmological Observables

Compute the nonlinear velocity power spectrum and assess its impact on the kinetic Sunyaev–Zel'dovich (kSZ) effect.

## 3. Time-Dependent Jeans Filtering

Extend the semi-analytical framework to a time-dependent Jeans scale, $k_J(z)$, to model the thermal evolution of baryons and its impact on cosmic structure formation.

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# Selected References


1. **Bernardeau et al. (2002).** *Large-scale structure of the Universe and cosmological perturbation theory.* Phys. Rept. **367**, 1–248.
2. **Makino, Sasaki & Suto (1992).** *Analytic approach to the perturbative expansion of nonlinear gravitational fluctuations in cosmological density and velocity fields.* Phys. Rev. D **46**, 585–602.
3. **Gnedin & Hui (1998).** *Probing the Universe with the Ly$\alpha$ forest: Hydrodynamics of the low-density intergalactic medium.* MNRAS **296**, 44–55.
4. **Shoji & Komatsu (2009).** *Third-order perturbation theory with nonlinear pressure.* Astrophys. J. **700**, 705–719.
5. **Somogyi & Smith (2010).** *Cosmological perturbation theory for baryons and dark matter: One-loop corrections in the renormalized perturbation theory framework.* Phys. Rev. D **81**, 023524.
6. **Fonseca & Castañeda (2024).** *A brief review of first-order cosmological perturbations including baryonic matter from an Eulerian perspective.* Momento **69**, 17–46.