Cosmological Viability of Non-Extensive Entropy Formalisms

within the Framework of Thermodynamic Gravity

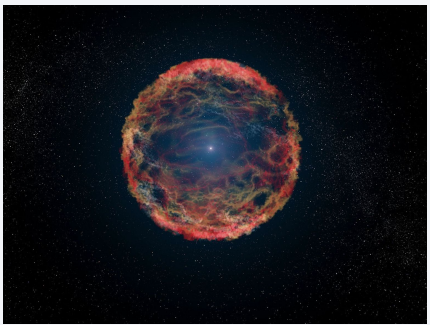
Cai-Kim thermodynamic gravity · BBN & CMB viability tests

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Overview



NASA: Artist's Impression of Supernova 1993J

The accelerated expansion of the Universe

- **Type Ia Supernovae (SNe Ia):** Late-time cosmic expansion.
- **Baryon Acoustic Oscillations (BAO):** The cosmic standard ruler.
- **Cosmic Microwave Background (CMB):** Early-Universe physics and cosmic geometry.

↓

The standard ΛCDM model

Overview

The Divide in Einstein's Field Equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

PATH 1. Modify the Right-Hand Side ($T_{\mu\nu}$):

Postulate an unknown component with negative pressure, known as **Dark Energy**.

PATH 2. Modify the Left-Hand Side ($G_{\mu\nu}$)

Alter the geometric laws of gravity at cosmological scales: **Modified Gravity**.

THE UNIFIED PERSPECTIVE. Thermodynamic Gravity:

Interpret Einstein's field equations as an equation of state emerging from thermodynamic principles:

$$\delta Q = T\, d\, S$$

Non-Extensive entropy on the apparent horizon

Apparent horizon: $R_A = 1/H$ (flat FLRW)

↓

Hawking temperature: $T_h = 1/2\pi R_A$

(Cai, Cao and Hu 2009)

↓

Tsallis entropy: $S_h = S_h(S_{BH})$

↓

First law: $\delta Q = T_h \, dS_h$

↓ *integrate*

Modified Friedmann equations

General Equation:

$$-4\,\pi\,G\,(\rho + P) = S_h'(S_{BH})\,\dot{H}$$

Two equivalent approaches:

- Effective gravitational coupling

$$G \longrightarrow G_{\text{eff}} = \frac{G}{S_h'(S_{BH})}$$

- Effective fluid description

$$T_{\mu\nu} \longrightarrow T^{\text{eff}}_{\mu\nu} \rightarrow \rho + P + \rho^{\text{DE}}_{\text{eff}} + P^{\text{DE}}_{\text{eff}}$$

Tsallis Entropy:

$$S_T = \frac{\alpha}{4G} A^{\delta}$$

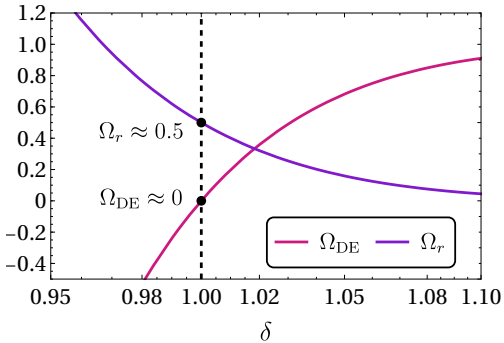
Closed-form dark energy parameter:

$$1 - \Omega_{\rm DE} = H^2_{(mr)} \left\{ \frac{2-\delta}{\alpha\delta} \left[\frac{\Lambda}{3} + H^2_{(mr)} \right] \right\}^{\frac{1}{\delta-2}}$$

$H^2_{(mr)} \equiv H_0^2 \{\Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4\}$

Acceleration condition ($q < 0$ today), but late-time acceleration $\neq$ early-Universe viability!

$\Omega_{\rm DE}$ **and** Ω_r **at** $z \approx 3200$ **as functions of** δ



BBN bound ($\Omega_{\rm DE}|_{z\sim3200} < 0.045$):

$1.00 \leq \delta < 1.00038$

Essentially indistinguishable from $\delta = 1$!

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Tsallis cosmology as a perturbative deformation of ΛCDM

Modified expansion law

$$\frac{\alpha\delta}{2-\delta}H^{2(2-\delta)} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \equiv H_\Lambda^2$$

Expanding around $\delta = 1$ ($\hat{\alpha} = 1$):

$$H^2 \approx H_\Lambda^2 + \delta H_{\Lambda,1}^2(\delta-1) + \delta H_{\Lambda,2}^2(\delta-1)^2$$

where the correction grows as:

$$\delta H_{\Lambda,1}^2 \sim H_\Lambda^2 \ln\frac{H_\Lambda^2}{H_0^2}$$

At high z: $\ln(H_\Lambda^2/H_0^2) \propto \ln(1+z)^4 \to \infty$

Impact on radiation era

$$\Omega_r \propto \frac{\rho_r}{H_\Lambda^2}\left[1 + \frac{\delta H_{\Lambda,1}^2}{H_\Lambda^2}(\delta-1) + \cdots\right]^{-1}$$

Correction factor $\propto \ln(1+z)^4$:

- $\delta < 1$: $\Omega_r > 1$ at high $z \Rightarrow \Omega_{\rm DE} < 0$
- $\delta > 1$: excess early dark energy, violates BBN

Key insight: Tsallis corrections are *not* small perturbations around ΛCDM. They grow **non-perturbatively** toward the past, invalidating the model for any $\delta \neq 1$.

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Rényi Entropy:

$$S_R = \frac{1}{\lambda} \ln\left(1 + \lambda S_{\rm BH}\right)$$

Dark energy parameter:

$$\Omega_{\rm DE} = \frac{\tilde{\lambda}}{H^2} \ln\left(1 + \frac{H^2}{\tilde{\lambda}}\right) + \frac{\tilde{\Lambda}}{3H^2}$$

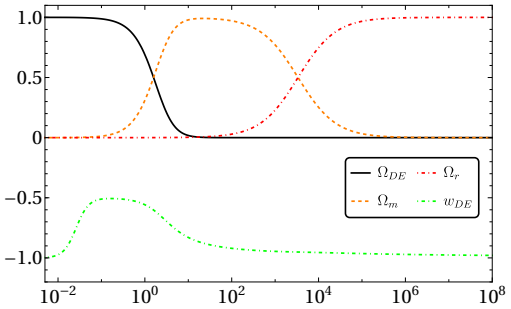
$H^2 \equiv H_{(mr)}/(1 - \Omega_{\rm DE})$

Acceleration condition ($q < 0$ today), but late-time acceleration $\neq$ early-Universe viability!

Viability Range: $0 \leq \tilde{\lambda} \leq 0.98341$

Equation of State (EoS):

$$w_{\rm DE} = -1 + \frac{2}{3\Omega_{\rm DE}} \frac{\dot{H}}{H^2} \left(\frac{1}{1 + \tilde{\lambda} H^{-2}} - 1\right)$$



Sharma–Mittal Entropy:

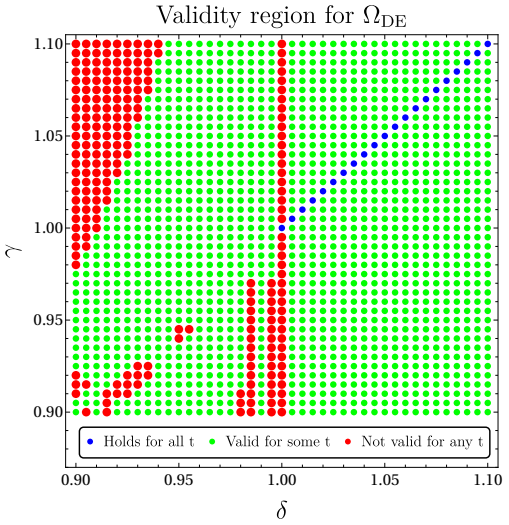
$$S_{\gamma,\delta} = \frac{1}{\gamma}\left((1+\delta S_T)^{\gamma/\delta} - 1\right)$$

Density:

$$\frac{8\pi G}{3}\rho = {}_2F_1(\tilde{a}, b, c, \tilde{z})\frac{(4\pi)^{\delta-1}\delta\tilde{\alpha}}{2-\delta}H^{2(2-\delta)} - \frac{\tilde{\Lambda}}{3}$$

$$\tilde{a} = 1 - \tfrac{\gamma}{\delta},\ b = \tfrac{\delta-2}{\delta},\ c = \tfrac{2(\delta-1)}{\delta},\ \tilde{z} \sim H^{-2\delta}$$

$$\tilde{z} \xrightarrow[a\to 0]{} 0 \implies \textbf{Tsallis limit}$$



Kaniadakis Entropy:

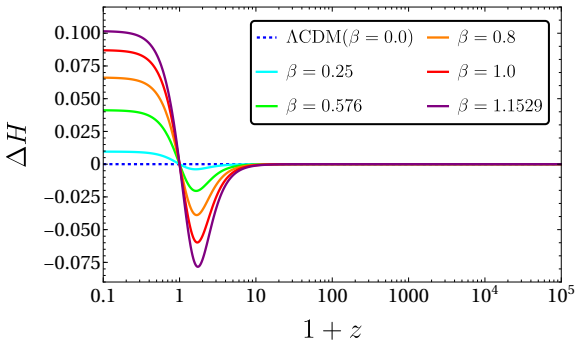
$$S_\kappa = \frac{1}{\kappa}\sinh\left(\kappa\, S_{\rm BH}\right)$$

Dark energy parameter:

$$\Omega_{\rm DE} = \frac{\tilde{\Lambda}}{3H^2}1 - \cosh\left(\frac{\beta}{H^2}\right) + \frac{\beta}{H^2}Shi\left(\frac{\beta}{H^2}\right)$$

Acceleration condition ($q < 0$ today), but late-time acceleration $\neq$ early-Universe viability!

Viability Range: $0 \leq \beta \leq 1.377$



Summary and implications

What Have We Shown?

- **Generalized thermodynamic framework**: We derive a general equation within the Cai–Kim approach to horizon thermodynamics.
- **Modified Friedmann equations**: We obtain the corresponding Friedmann equations within this framework.
- **Theoretical viability**: We assess the viability of the models and establish the allowed parameter ranges for the entropic parameters.

Future Perspectives

- **Observational constraints**: Apply Markov Chain Monte Carlo (MCMC) methods to constrain the model parameters using cosmological observational data.
- **Structure formation**: Investigate cosmic structure formation within this framework using the spherical collapse model.

Theoretical viability must be assessed at each stage of the Universe's evolution.



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Thank you!

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