Cosmology in Colombia: CoCo 2026

PAULI BLOCKING MECHANISM IN THE STABILITY OF SCALAR DARK MATTER IN THE EXPANDING UNIVERSE

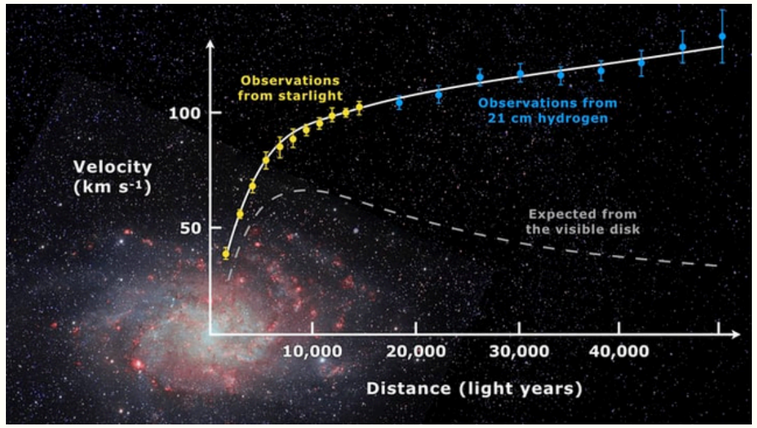
Cristian Eduardo Martínez Torres

INTRODUCTION

Evidence for dark matter

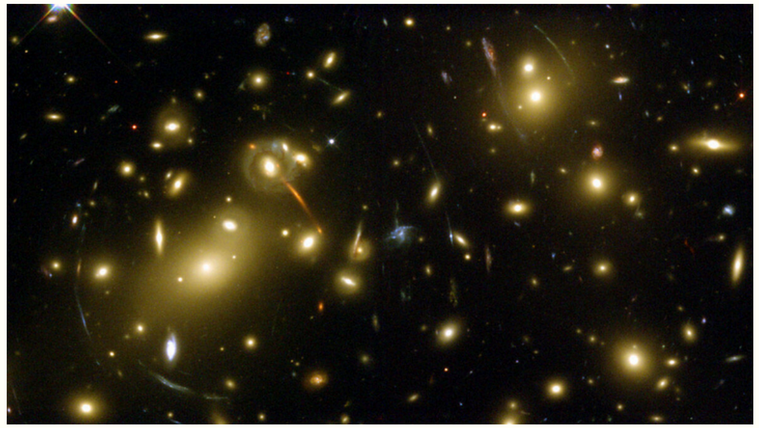
Due to many astrophysical and cosmological observations, we know about gravitational interactions that cannot be explained by visible matter alone.

- **Flat rotation curves of galaxies**



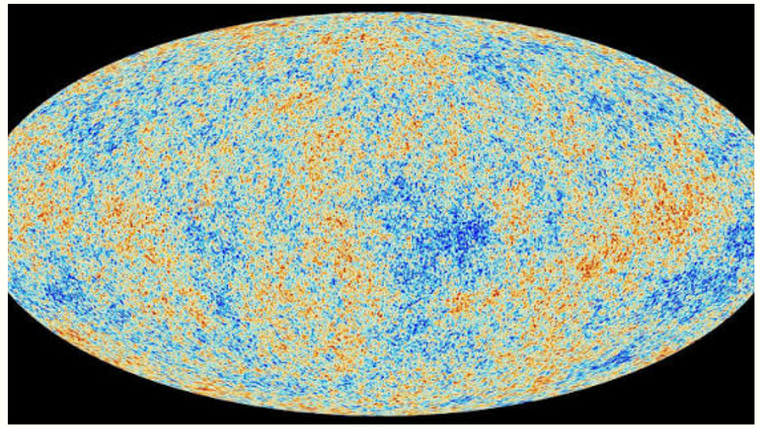
[Source: Corbelli & Salucci (2000).]

- **Gravitational lensing observations**



[Source: NASA, ESA, Andrew Fruchter and the ERO Team (2000).]

- **Measurements of CMB anisotropies**

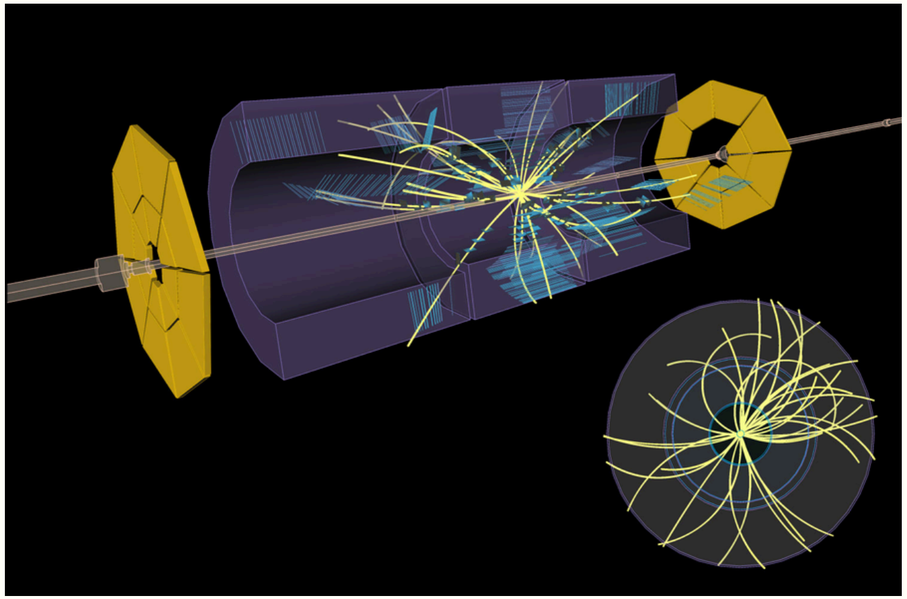


[Source: ESA and Planck Collaboration (2013).]

Requirements for a DM candidate

For a dark matter model to adequately account for our observations, it must satisfy three important requirements:

- It must interact feebly with the Standard Model particles, which explains why its detection remains highly challenging.

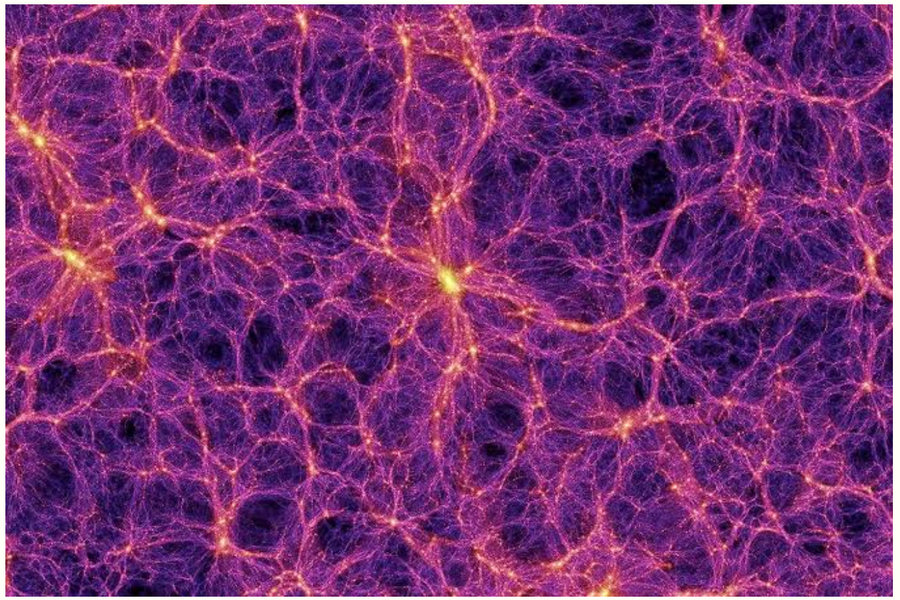


[Source: CERN, CMS Collaboration.]

Requirements for a DM candidate

For a dark matter model to adequately account for our observations, it must satisfy three important requirements:

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- It must be "cold," i.e., non-relativistic, to account for the large-scale structure that we observe today.

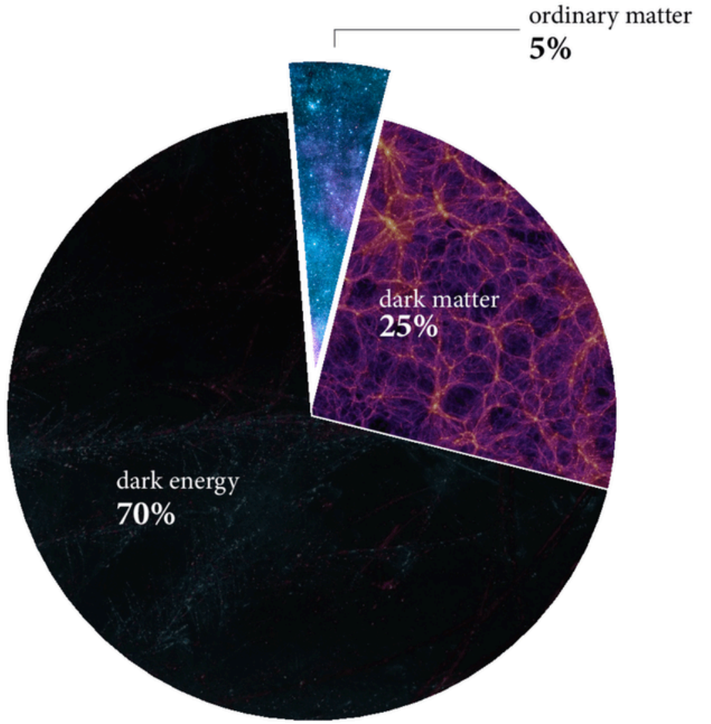


[Source: Volker Springel / Max Planck Institute For Astrophysics]

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For a dark matter model to adequately account for our observations, it must satisfy three important requirements:

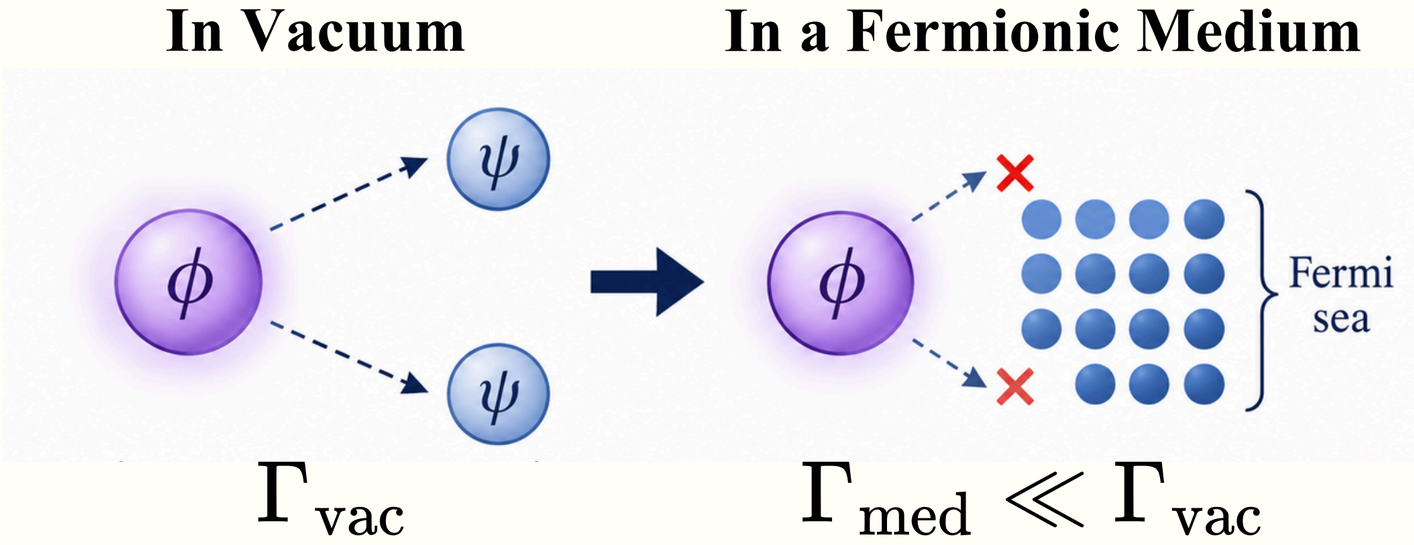
- It must interact feebly with the Standard Model particles, which explains why its detection remains highly challenging.
- It must be "cold," i.e., non-relativistic, to account for the large-scale structure that we observe today.
- It must be stable, meaning its lifetime must exceed the current age of the universe



[Source: Max-Planck-Institute for Astrophysics, Garching.]

Approach adopted in this work

In this work, we follow the approach proposed by Brian Batell and Wen Yin in [9]. We show that accounting for the quantum statistical nature of a dark matter particle and its possible interactions with fermions can lead to a highly stable candidate, even when its vacuum decay rate is large.



[Illustration generated with AI]

DARK MATTER MODEL

Dark matter coupling

We consider a minimal model in which a scalar dark matter particle ϕ is coupled to a pair of Majorana fermions ψ through the Lagrangian term:

$$\mathcal{L} \supset -\frac{y}{2}\phi\psi\bar{\psi}$$

where y is the coupling constant.

If the mass of the scalar particle is denoted by m_ϕ and the mass of the Majorana fermions by m_ψ, the following relations must hold:

$$m_\phi \neq 0, \quad m_\psi < m_\phi/2.$$

Vacuum decay rate

From the interaction term introduced above, we can calculate the vacuum decay rate of a boson decaying into two fermions, obtaining:

$$\Gamma_{\phi\to\psi\psi} = \frac{y^2 m_\phi}{16\pi}\beta_\psi^3, \qquad \beta_\psi = \sqrt{1-\frac{4m_\psi^2}{m_\phi^2}}$$

Note: In our simulations, we consider massless fermions, for which $\beta_\psi = 1$. The vacuum decay rate then becomes:

$$\Gamma_{\phi\to\psi\psi} = \frac{y^2 m_\phi}{16\pi}$$

Mass constraint

We estimate an upper bound on the mass of the scalar particle by requiring its energy density to dominate over that of the fermionic Fermi sea, i.e.,

$$\rho_\phi \gg \frac{g_\psi}{2(2\pi)^3} \int_{p<p_F} \varepsilon(p) d^3p \simeq \frac{m_\phi^4}{128\pi^2}$$

Using the measured dark matter energy density in the present $\rho_\phi = \rho_{\rm DM} \simeq (2\,{\rm meV})^4$, we obtain:

$$m_\phi \ll 0.01\,{\rm eV}$$

This places this model in a sub-eV mass range.

GENERAL BOLTZMANN EQUATION

The Boltzmann equation

The Boltzmann equation is a powerful tool for describing the evolution of a distribution function f in out-of-equilibrium systems. It can be written as

$$\hat{L}[f] = \hat{C}[f]$$

where the left-hand side is the Liouville operator and the right-hand side is known as the collision term.

$$\hat{L} \longrightarrow \text{Macroscopic effects.}$$

$$\hat{C} \longrightarrow \text{Microscopic effects.}$$

The cosmological Liouville operator

Using General Relativity and the FLRW metric, we can obtain the macroscopic term in an expanding universe.

$$\hat{L}[f] = E\left(\frac{\partial f}{\partial t} - H|\vec{p}|\frac{\partial f}{\partial |\vec{p}|}\right)$$

where H is the Hubble parameter, E is the energy and $\vec{p}$ the momentum of the particle.

Note: Although f depends in general on the four-position and the four-momentum, in this scenario it only depends on the energy and time due to the cosmology cal principle.

General collision term

The most general form of the collision term for a particle of species α is given by

$$\hat{C}[f_\alpha] = \frac{1}{2S} \sum_{\text{processes}} \prod_{i \neq \alpha} \left(\int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{1}{2E_i} \sum_{g_i} \right) (2\pi)^4 \delta^{(4)}(p_\alpha + p_A + p_B + \cdots - p_X - p_Y - \cdots)$$
$$\times \Delta N_\alpha \Big[|\mathcal{M}(p_X, p_Y, \cdots \to p_\alpha, p_A, p_B, \ldots)|^2 \, f_X f_Y \cdots (1 \pm f_\alpha)(1 \pm f_A)(1 \pm f_B) \cdots$$
$$- |\mathcal{M}(p_\alpha, p_A, p_B, \cdots \to p_X, p_Y, \ldots)|^2 \, f_\alpha f_A f_B \cdots (1 \pm f_X)(1 \pm f_Y) \cdots \Big],$$

which considers processes of the form:

$$\alpha, A, B, \cdots \longleftrightarrow X, Y, \cdots$$

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where:

S : symmetry factor

ΔN_α : number of changing per process

g_i : degrees of freedom of i

$\mathcal{M}$: matrix element

p_i : 4-momentum of i

f_i : distribution function of i

Cosmological Boltzmann equation

Using the following definitions:

$$C^{\alpha} \equiv \hat{C}[f_{\alpha}]/E_{\alpha}, \quad p_{\alpha} \equiv |\vec{p}_{\alpha}|$$

The cosmological Boltzmann equation that describes the time evolution of the distribution function of a particle of a species α is:

$$\frac{\partial f_{\alpha}(p_{\alpha}, t)}{\partial t} - p_{\alpha} H \frac{\partial f_{\alpha}(p_{\alpha}, t)}{\partial p_{\alpha}} = C_{\alpha}(p_{\alpha}, t)$$

Note: The most known form of the Boltzmann equation, which is in terms of number densities, can be obtained through integration over the momentum space.

STATISTICAL FORMULATION

Phase space distribution functions

Since bosons can form condensates, the distribution function of our dark matter candidate can be decomposed into a condensate and a thermal component:

$$f_\phi(p_\phi, t) = \hat{f}_\phi(p_\phi, t) + f_\phi^{\mathrm{th}}(p_\phi, t)$$

while the distribution function of the Majorana fermions is completely thermal, then:

$$f_\psi(p_\psi, t) = f_\psi^{\mathrm{th}}(p_\psi, t)$$

Note: In our notation, every quantity with a “hat” denotes the condensate, and the superscript “th” denotes thermal components.

Phase space distribution functions

For this treatment, we use the following ansatz for the distribution functions:

$$\hat{f}_\phi(p_\phi, t) = (2\pi)^3 \hat{n}_\phi(t) \delta^3(\vec{p}_\phi)$$

$$f_\phi^{\text{th}}(p_\phi, t) = \left[\exp\left(\frac{E_\phi - \mu_\phi(t)}{T(t)} \right) - 1 \right]^{-1}$$

$$f_\psi^{\text{th}}(p_\psi, t) = \left[\exp\left(\frac{E_\psi - \mu_\psi(t)}{T(t)} \right) + 1 \right]^{-1}$$

where $\hat{n}_\phi$ is the number density of the bosons in the condensate state, μ_i are the chemical potentials of $i = \phi, \psi$ and T is the temperature.

Statistical quantities

From the distribution functions, quantities such as the number density, energy density, and pressure can be calculated from their statistical definitions as:

$$n_i(t) = g_i \int \frac{d^3p_i}{(2\pi)^3} f_i(p_i, t)$$

$$\rho_i(t) = g_i \int \frac{d^3p_i}{(2\pi)^3} E_i f_i(p_i, t)$$

$$P_i(t) = g_i \int \frac{d^3p_i}{(2\pi)^3} \frac{p_i^2}{3E_i} f_i(p_i, t)$$

for $i = \phi, \psi$.

BOLTZMANN EQUATIONS FOR THIS MODEL

Boltzmann equations for this model

A direct application of the general form of the Boltzmann equation for our two particle species results in:

$$\frac{\partial f_\phi(p_\phi, t)}{\partial t} - p_\phi H \frac{\partial f_\phi(p_\phi, t)}{\partial p_\phi} = C^\phi(p_\phi, t)$$

$$\frac{\partial f_\psi(p_\psi, t)}{\partial t} - p_\psi H \frac{\partial f_\psi(p_\psi, t)}{\partial p_\psi} = C^\psi(p_\psi, t)$$

However, the integrated versions are usually more useful.

Integrated Boltzmann equations

Integrating on both sides of the form of the previous equations:

$$g_i \int \frac{d^3 p_i}{(2\pi)^3} \left[\frac{\partial f_i}{\partial t} - H p_i \frac{\partial f_i}{\partial p_i} \right] = g_i \int \frac{d^3 p_i}{(2\pi)^3} C^i(p_i, t)$$

We obtain the Boltzmann equations in terms of the particle number densities:

$$\dot{n}_i + 3 H n_i = g_i \int \frac{d^3 p_i}{(2\pi)^3} C^i(p_i, t)$$

Integrated Boltzmann equations

And defining the integrated collision term as:

$$\mathcal{C}^i \equiv g_i \int \frac{d^3 p_i}{(2\pi)^3} C^i(p_i, t)$$

We can express the equations for the particles ϕ and ψ as:

$$\dot{\hat{n}}_\phi + 3H\hat{n}_\phi = \mathcal{C}^{\phi_c}$$

$$\dot{n}_\phi^{\mathrm{th}} + 3Hn_\phi^{\mathrm{th}} = \mathcal{C}^{\phi_{\mathrm{th}}}$$

$$\dot{n}_\psi + 3Hn_\psi = \mathcal{C}^{\psi}$$

Independent variables

Recalling the dependence of the distribution functions:

$$\hat{f}_\phi(p_\phi, t) = (2\pi)^3 \hat{n}_\phi(t) \delta^3(\vec{p}_\phi)$$

$$f_\phi^{\mathrm{th}}(p_\phi, t) = \left[\exp\left(\frac{E_\phi - \mu_\phi(t)}{T(t)} \right) - 1 \right]^{-1}$$

$$f_\psi^{\mathrm{th}}(p_\psi, t) = \left[\exp\left(\frac{E_\psi - \mu_\psi(t)}{T(t)} \right) + 1 \right]^{-1}$$

This means that we have four independent variables and three equations. Therefore, we need one more equation to solve this system.

Energy conservation equation

Doing a similar integration process, but first multiplying by the energy:

$$g_i \int \frac{d^3p_i}{(2\pi)^3} E_i \left[\frac{\partial f_i}{\partial t} - Hp_i \frac{\partial f_i}{\partial p_i} \right] = g_i \int \frac{d^3p_i}{(2\pi)^3} E_i C^i(p_i, t)$$

we obtain:

$$\dot{\rho}_i + 3H(\rho_i + P_i) = \mathcal{E}^i$$

where we have defined:

$$\mathcal{E}^i = g_i \int \frac{d^3p_i}{(2\pi)^3} E_i C^i(p_i, t)$$

Energy conservation equation

After applying the projection operators to the boson density equation, we obtain:

$$\dot{\hat{\rho}}_\phi + 3H(\hat{\rho}_\phi + \hat{P}_\phi) = \mathcal{E}^{\phi_c}$$
$$\dot{\rho}_\phi^{\mathrm{th}} + 3H(\rho_\phi^{\mathrm{th}} + P_\phi^{\mathrm{th}}) = \mathcal{E}^{\phi_{\mathrm{th}}}$$
$$\dot{\rho}_\psi + 3H(\rho_\psi + P_\psi) = \mathcal{E}^{\psi}$$

Then, if we define the total energy density and the total pressure as:

$$\rho_{\mathrm{tot}} \equiv \hat{\rho}_\phi + \rho_\phi^{\mathrm{th}} + \rho_\psi$$

$$P_{\mathrm{tot}} \equiv \hat{P}_\phi + P_\phi^{\mathrm{th}} + P_\psi$$

Energy conservation equation

After applying the projection operators to the boson density equation, we obtain:

$$\dot{\hat{\rho}}_\phi + 3H(\hat{\rho}_\phi + \hat{P}_\phi) = \mathcal{E}^{\phi_c}$$
$$\dot{\rho}^{\mathrm{th}}_\phi + 3H(\rho^{\mathrm{th}}_\phi + P^{\mathrm{th}}_\phi) = \mathcal{E}^{\phi_{\mathrm{th}}}$$
$$\dot{\rho}_\psi + 3H(\rho_\psi + P_\psi) = \mathcal{E}^{\psi}$$

and add up the previous three equations:

$$\dot{\rho}_{\mathrm{tot}} + 3H(\rho_{\mathrm{tot}} + P_{\mathrm{tot}}) = \mathcal{E}^{\phi_c} + \mathcal{E}^{\phi_{\mathrm{th}}} + \mathcal{E}^{\psi} = 0$$

where the last equality comes from conservation of energy.

Extra considerations

However, we can further simplify this equation by assuming the following ansatz for the condensate and thermal components:

$$\hat{\rho}_\phi = m_\phi \hat{n}_\phi, \quad \hat{P}_\phi = 0,$$

$$P_\phi^{\mathrm{th}} = \frac{\rho_\phi^{\mathrm{th}}}{3}, \quad P_\psi = \frac{\rho_\psi}{3}$$

and denoting the sum of thermal energy densities as ρ_{DR} (for "Dark Radiation"), the final equation is:

$$\dot{\rho}_{\mathrm{DR}} + 4H\rho_{\mathrm{DR}} = -m_\phi \mathcal{C}^{\phi_c}$$

Final system of equations

As a summary, our final system of differential equations is:

$$\dot{\hat{n}}_\phi + 3H\hat{n}_\phi = \mathcal{C}^{\phi_c}$$

$$\dot{n}_\phi^{\mathrm{th}} + 3Hn_\phi^{\mathrm{th}} = \mathcal{C}^{\phi_{\mathrm{th}}}$$

$$\dot{n}_\psi + 3Hn_\psi = \mathcal{C}^{\psi}$$

$$\dot{\rho}_{\mathrm{DR}} + 4H\rho_{\mathrm{DR}} = -m_\phi \mathcal{C}^{\phi_c}$$

which allows us to determine the solution of our four independent variables.

HUBBLE PARAMETER

Hubble parameter

The first Friedmann equation relates the Hubble parameter to the energy density of the universe as follows:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

Here, we work in a flat universe, i.e., $k = 0$.

In particular, in $\Lambda\mathrm{CDM}$, the Hubble parameter considers the contributions from radiation, matter, and the cosmological constant using the expression:

$$H^2(a) = H_0^2 \left[\Omega_{r,0}\, a^{-4} + \left(\Omega_{b,0} + \Omega_{\mathrm{DM},0}\right) a^{-3} + \Omega_{\Lambda,0}\right]$$

Hubble parameter

In our model, we solve for the evolution of the energy density of dark matter; therefore, we must replace the standard contribution with our step-by-step solution, then:

$$H_0^2 \Omega_{\mathrm{DM},0} a^{-3} \longrightarrow \frac{8\pi G}{3} \hat{\rho}_\phi = \frac{8\pi G}{3} m_\phi \hat{n}_\phi$$

and we must also add the contributions of the dark radiation, since they are considered in the standard model. As a result, the Hubble parameter in this scenario is

$$H^2(a) = H_0^2 \left[\Omega_{r,0}\, a^{-4} + \Omega_{b,0}\, a^{-3} + \Omega_{\Lambda,0} \right] + \frac{8\pi G}{3} \left(m_\phi \hat{n}_\phi + \rho_{\mathrm{DR}} \right)$$

COLLISION TERMS

General collision term (Reminder)

The most general form of the collision term for a particle of species α is given by

$$\hat{C}[f_\alpha] = \frac{1}{2S} \sum_{\text{processes}} \prod_{i \neq \alpha} \left(\int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{1}{2E_i} \sum_{g_i} \right) (2\pi)^4 \delta^{(4)}(p_\alpha + p_A + p_B + \cdots - p_X - p_Y - \cdots)$$
$$\times \Delta N_\alpha \Big[|\mathcal{M}(p_X, p_Y, \cdots \to p_\alpha, p_A, p_B, \ldots)|^2 \, f_X f_Y \cdots (1 \pm f_\alpha)(1 \pm f_A)(1 \pm f_B) \cdots$$
$$- |\mathcal{M}(p_\alpha, p_A, p_B, \cdots \to p_X, p_Y, \ldots)|^2 \, f_\alpha f_A f_B \cdots (1 \pm f_X)(1 \pm f_Y) \cdots \Big],$$

where:

S : symmetry factor

ΔN_α : number of changing per process

g_i : degrees of freedom of i

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p_i : 4-momentum of i

f_i : distribution function of i

Processes

The collision terms depend on the interaction processes that are considered. In particular, for this model:

- **Decay/inverse decay:**

$$\phi(p_\phi) \leftrightarrow \psi(p_1) + \psi(p_2)$$

- **Annihilation/inverse annihilation:**

$$\phi(p_\phi) + \phi(p_2) \leftrightarrow \psi(p_3) + \psi(p_4)$$

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- **Decay/inverse decay:**

$$\phi(p_\phi) \leftrightarrow \psi(p_1) + \psi(p_2)$$

$$\phi_c \leftrightarrow \psi + \psi$$

$$\phi_{\mathrm{th}} \leftrightarrow \psi + \psi$$

- **Annihilation/inverse annihilation:**

$$\phi(p_\phi) + \phi(p_2) \leftrightarrow \psi(p_3) + \psi(p_4)$$

$$\phi_c + \phi_c \leftrightarrow \psi + \psi$$

$$\phi_c + \phi_{\mathrm{th}} \leftrightarrow \psi + \psi$$

$$\phi_{\mathrm{th}} + \phi_{\mathrm{th}} \leftrightarrow \psi + \psi$$

Collision terms

From the general form of the collision terms, we notice that they can be decomposed into individual process contributions. As we have five individual processes, we have five different collision terms, denoted by

Decay/inverse decay:

$$\mathcal{C}_{\phi_c \leftrightarrow \psi\psi}$$

$$\mathcal{C}_{\phi_{\mathrm{th}} \leftrightarrow \psi\psi}$$

Annihilation/inverse annihilation:

$$\mathcal{C}_{\phi_c \phi_c \leftrightarrow \psi\psi}$$

$$\mathcal{C}_{\phi_c \phi_{\mathrm{th}} \leftrightarrow \psi\psi}$$

$$\mathcal{C}_{\phi_{\mathrm{th}} \phi_{\mathrm{th}} \leftrightarrow \psi\psi}$$

Collision terms summary

From the individual collision terms, we can construct the total collision terms as follows:

$$\mathcal{C}^{\phi_c} = \mathcal{C}_{\phi_c \leftrightarrow \psi\psi} + \mathcal{C}_{\phi_c\phi_c \leftrightarrow \psi\psi} + \mathcal{C}_{\phi_c\phi_{\rm th} \leftrightarrow \psi\psi},$$
$$\mathcal{C}^{\phi_{\rm th}} = \mathcal{C}_{\phi_{\rm th} \leftrightarrow \psi\psi} + \mathcal{C}_{\phi_c\phi_{\rm th} \leftrightarrow \psi\psi},$$
$$\mathcal{C}^{\psi} = -2\left(\mathcal{C}_{\phi_c \leftrightarrow \psi\psi} + \mathcal{C}_{\phi_{\rm th} \leftrightarrow \psi\psi} + \mathcal{C}_{\phi_c\phi_{\rm th} \leftrightarrow \psi\psi}\right) - \mathcal{C}_{\phi_c\phi_c \leftrightarrow \psi\psi}$$

Final system of equations (Reminder)

As a summary, our final system of differential equations is:

$$\dot{\hat{n}}_\phi + 3H\hat{n}_\phi = \mathcal{C}^{\phi_c}$$

$$\dot{n}_\phi^{\mathrm{th}} + 3Hn_\phi^{\mathrm{th}} = \mathcal{C}^{\phi_{\mathrm{th}}}$$

$$\dot{n}_\psi + 3Hn_\psi = \mathcal{C}^{\psi}$$

$$\dot{\rho}_{\mathrm{DR}} + 4H\rho_{\mathrm{DR}} = -m_\phi \mathcal{C}^{\phi_c}$$

which allows us to determine the solution of our four independent variables.

RESULTS: NON-EXPANDING SCENARIO

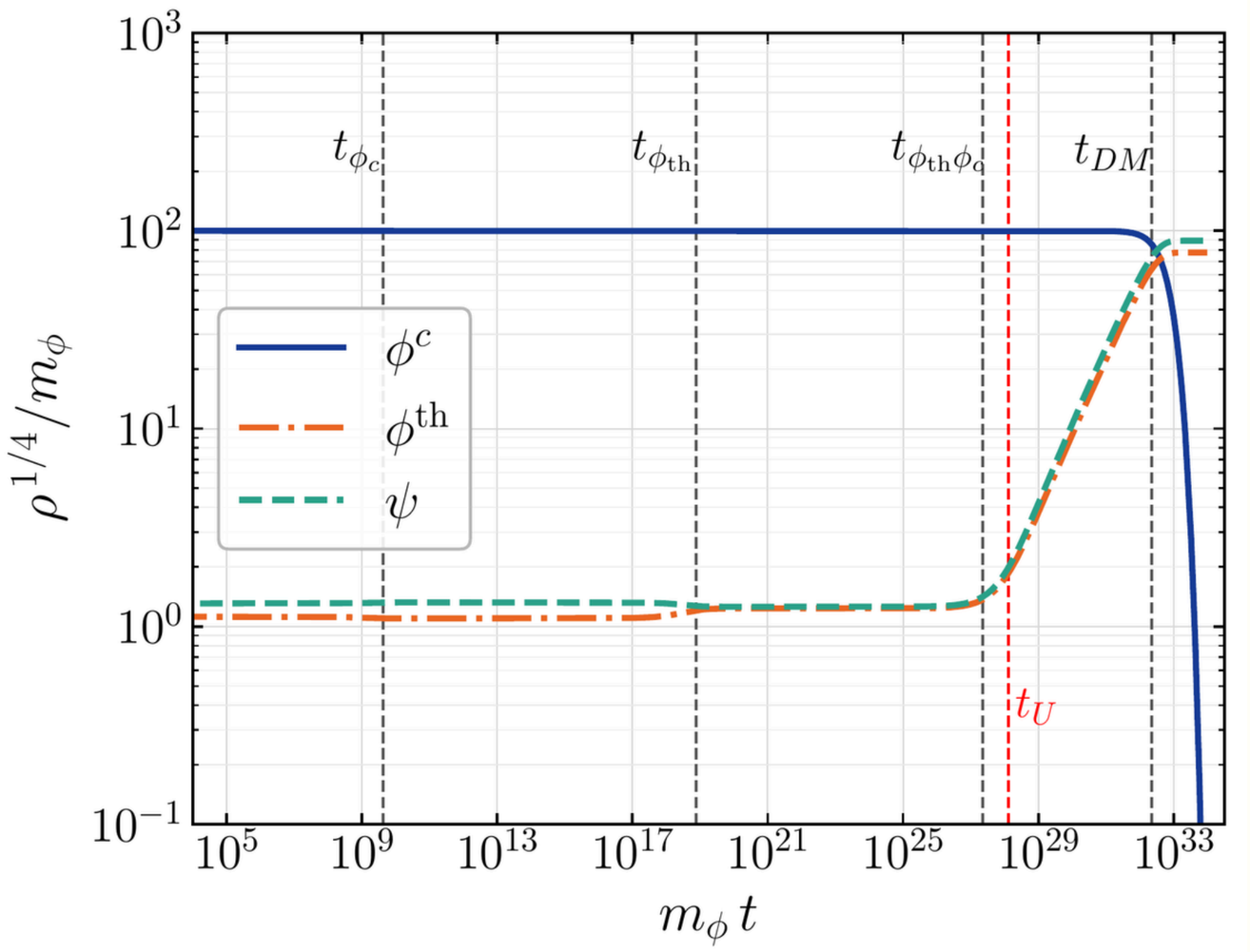
Energy density evolution

Time evolution of the energy density for condensate bosons, thermal bosons, and massless Majorana fermions as a function of dimensionless time.

The simulation was carried out with the following initial conditions:

$$\hat{n}_\phi(0) = 10^8 m_\phi^3, \quad T(0) = 1.5\, m_\phi$$

$$\mu_\phi(0) = \mu_\psi(0) = 0$$

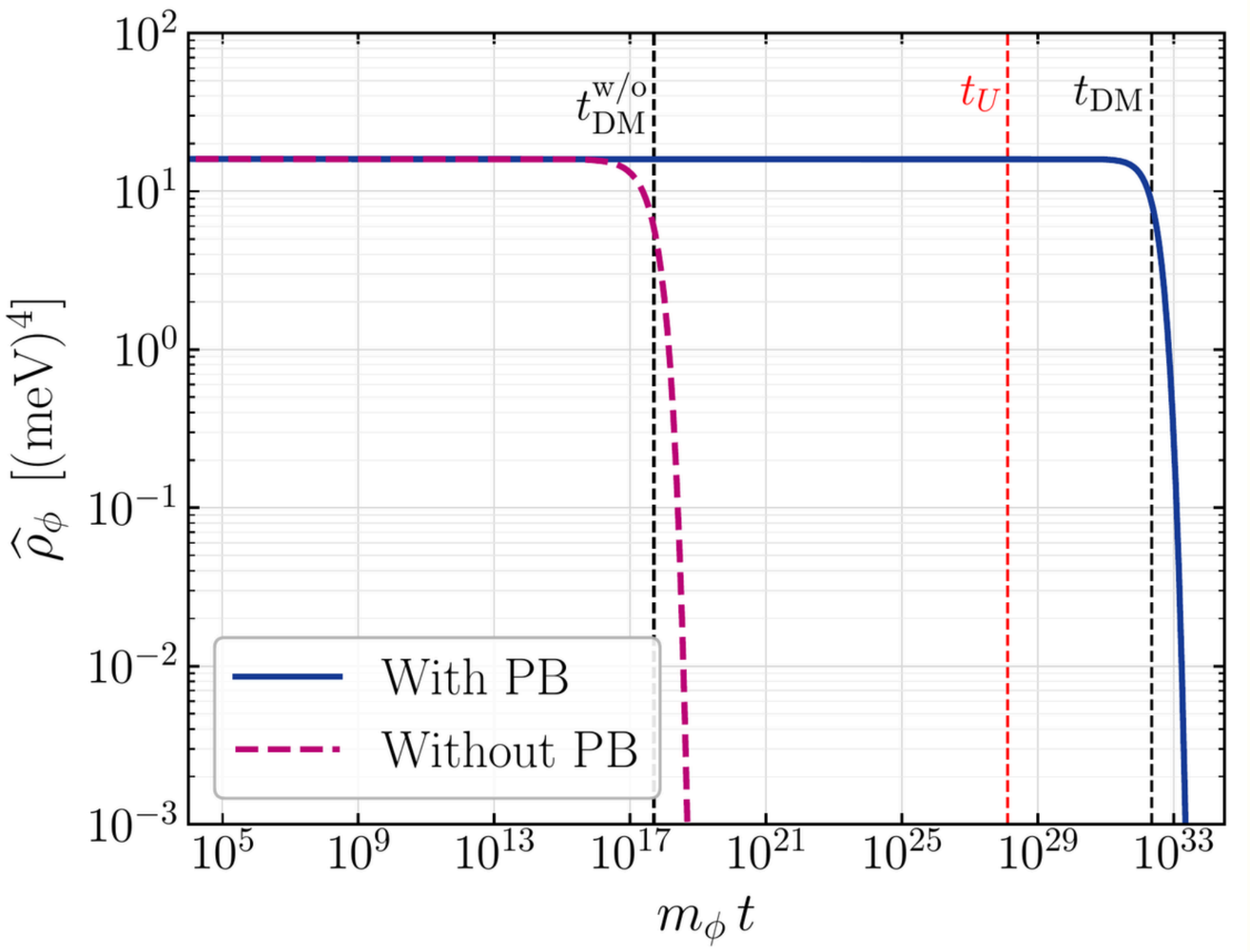


Comparison with and without PB

Condensate energy density evolution with and without the Pauli blocking mechanism. Incorporating fermion suppression significantly delays decay compared to the unblocked baseline.

Natural units were calculated using a dark matter mass value:

$$m_\phi = 2 \times 10^{-5}\,\mathrm{eV}$$



RESULTS: EXPANDING SCENARIO

Energy density evolution

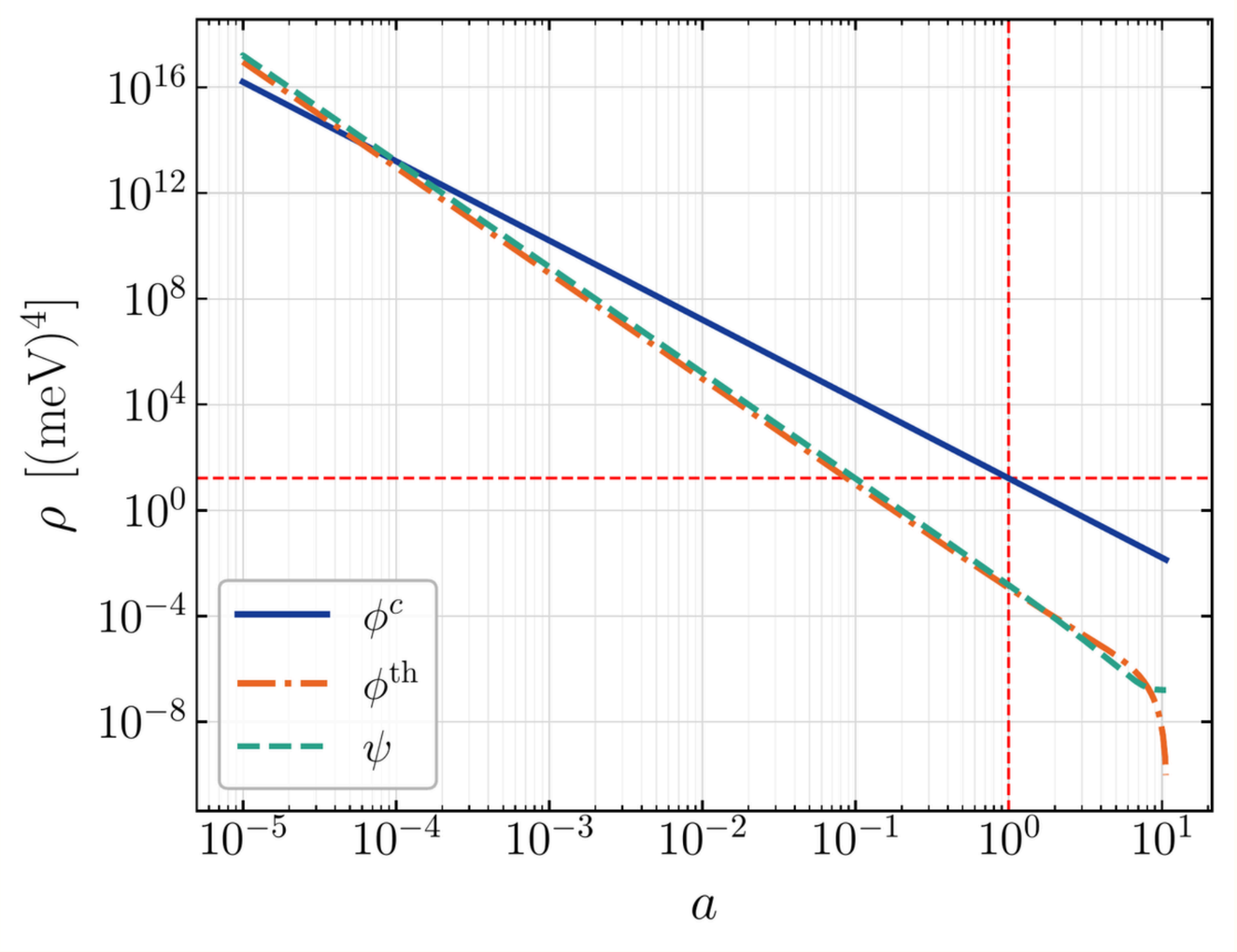
Evolution with the scale factor of the energy densities for the condensate, thermal bosons, and Majorana fermions. Here, $m_\phi = 10^{-4}\,\mathrm{eV}$. The simulation was carried out with the following initial conditions:

$$a_i = 10^{-5}$$

$$\hat{n}_\phi(a_i) = (2\,\mathrm{meV})^4 a_i^{-3}/m_\phi$$

$$T(a_i) = (0.23\,\mathrm{meV}) a_i^{-1}$$

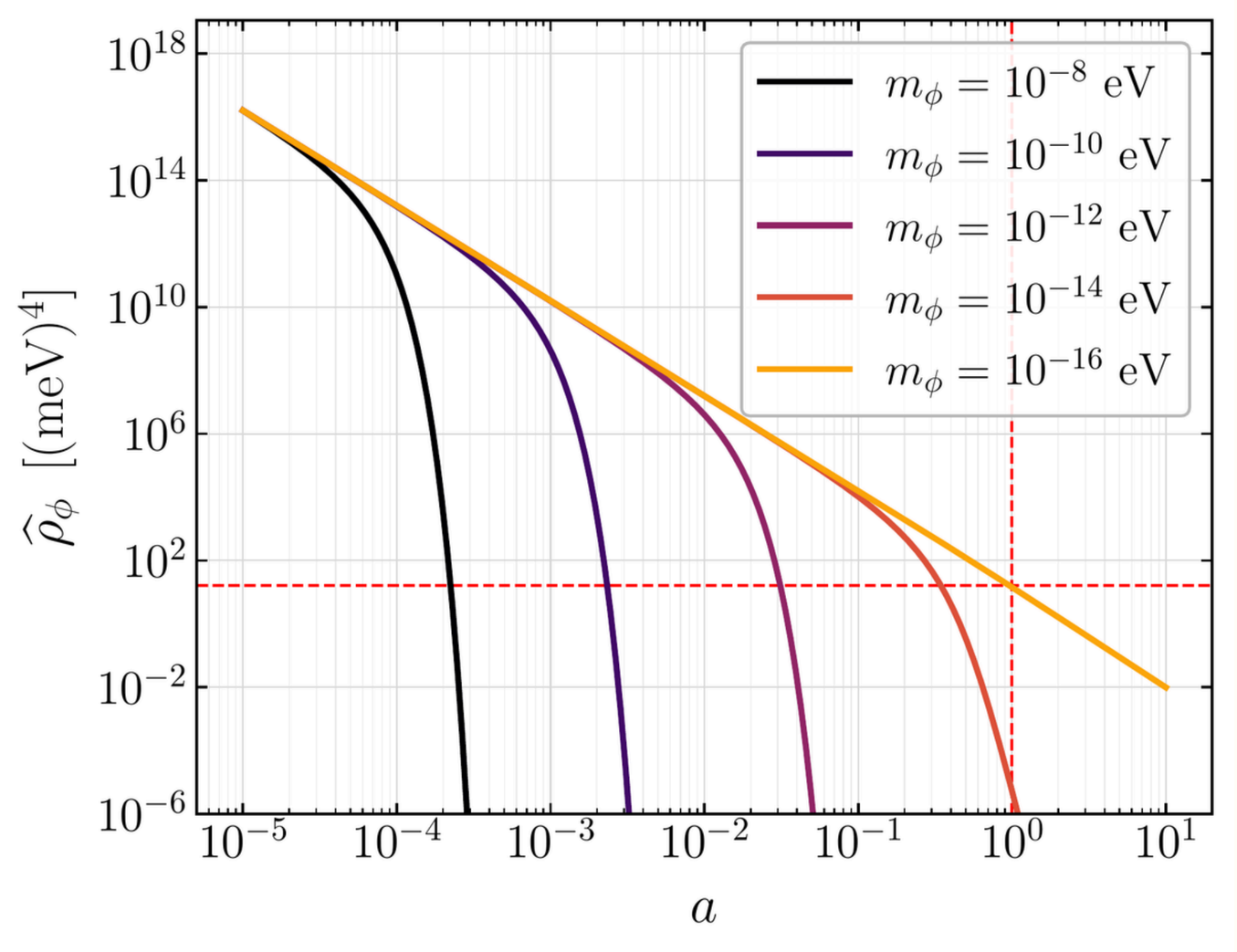
$$\mu_\phi(a_i) = \mu_\psi(a_i) = -10\,\mathrm{meV}$$



Evolution without PB

Physical energy densities of the condensate bosons across different scalar mass values m_ϕ, calculated without Pauli blocking.

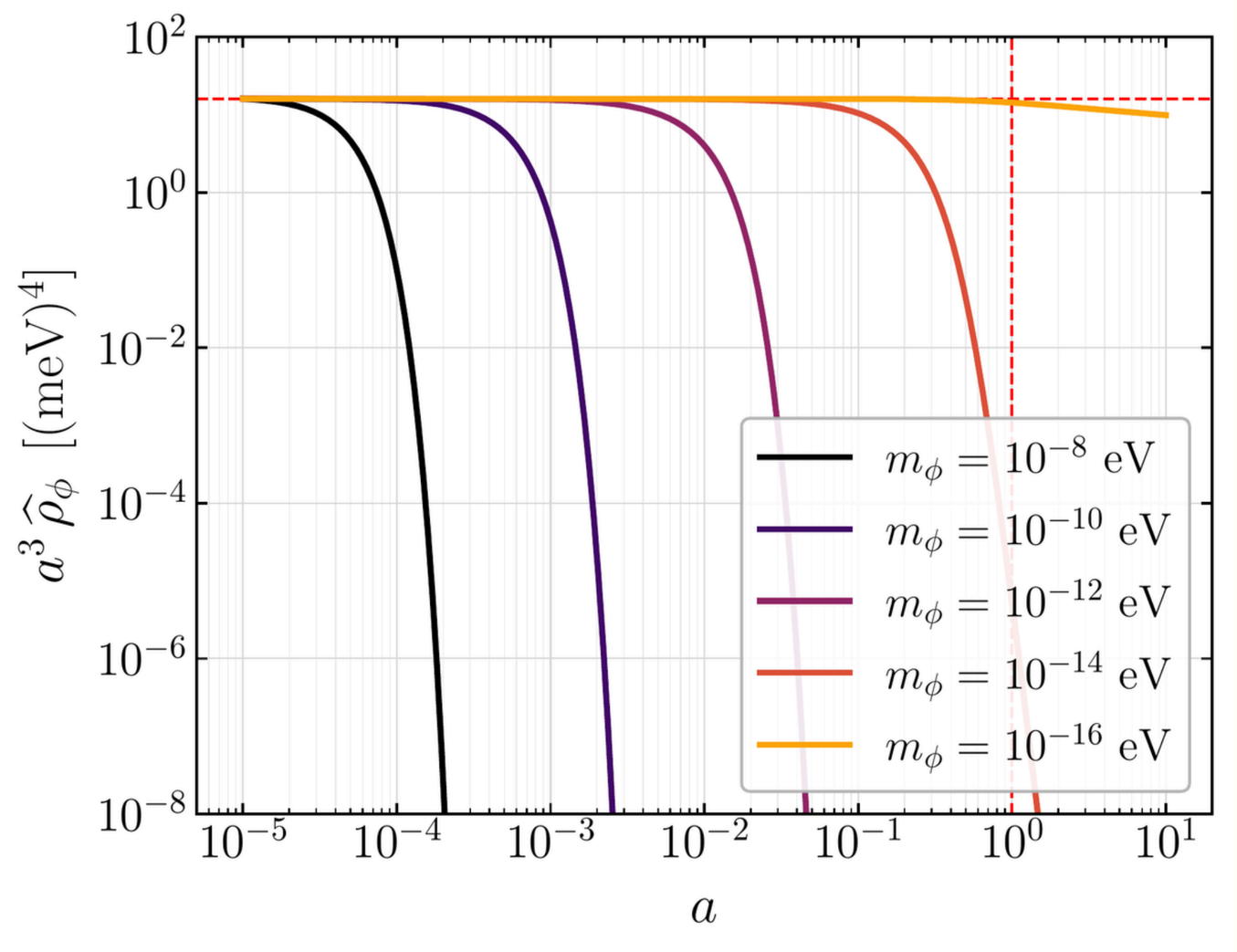
Extremely small mass values are required to achieve stability until the current age of the universe.



Evolution without PB

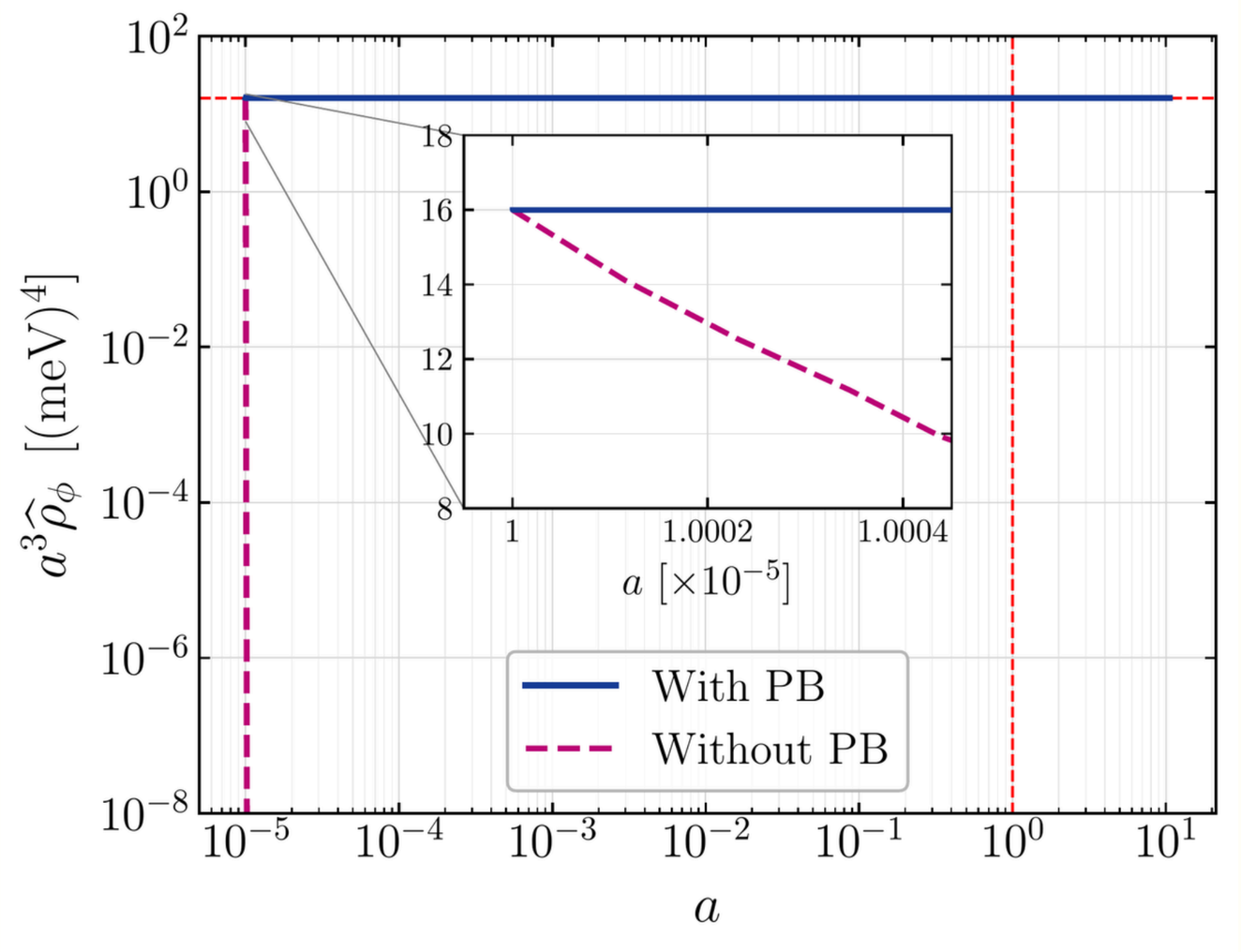
Comoving energy densities of the condensate bosons across different scalar mass values m_ϕ, calculated without Pauli blocking.

Extremely small mass values are required to achieve stability until the current age of the universe.



Comparison with and without PB

Comparison of comoving dark matter energy densities with and without Pauli blocking for a scalar mass $m_\phi = 10^{-4}$ eV. The inset highlights the epoch where the two curves diverge.



CONCLUSIONS

Conclusions

- **Dark matter stability in a static universe:** We showed that Pauli blocking can suppress the evaporation of a dark matter condensate in a non-expanding universe.
- **Cosmological evolution:** We extended this analysis to an expanding universe, incorporating the contributions of dark matter, dark radiation, and the standard components.
- **Consistency with cosmological observations:** We reproduced the standard evolution of the energy densities, while recovering the present-day dark matter abundance and obtaining a negligible dark radiation contribution.
- **Impact of quantum statistics:** We showed that, without Pauli blocking, dark matter stability requires ultralight scalar masses. Including quantum statistical effects allows stable dark matter with larger masses by many orders of magnitude.

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APPENDICES

Projection operators

As we are working with the condensate and thermal component of ϕ independently, it is useful to define an operator that separates them from integrated quantities. Such an operator can be defined as:

$$\hat{O} \int \frac{d^3 p_\phi}{(2\pi)^3} \equiv \frac{1}{(2\pi)^3} \int d\Omega_\phi \int_\epsilon^\infty p_\phi^2 \, dp_\phi$$

$$\left[1 - \hat{O}\right] \int \frac{d^3 p_\phi}{(2\pi)^3} \equiv \frac{1}{(2\pi)^3} \int d\Omega_\phi \int_0^\epsilon p_\phi^2 \, dp_\phi$$

where ϵ is an infinitesimal parameter.

Condensate and thermal separation

The statistical quantities can be written as:

$$n_\phi = \hat{n}_\phi + n_\phi^{\rm th}, \quad \rho_\phi = \hat{\rho}_\phi + \rho_\phi^{\rm th}, \quad P_\phi = \hat{P}_\phi + P_\phi^{\rm th}$$

using the projection operators as follows:

$$\hat{n}_\phi = [1 - \hat{O}]\, n_\phi, \quad n_\phi^{\rm th} = \hat{O}\, n_\phi,$$
$$\hat{\rho}_\phi = [1 - \hat{O}]\, \rho_\phi, \quad \rho_\phi^{\rm th} = \hat{O}\, \rho_\phi,$$
$$\hat{P}_\phi = [1 - \hat{O}]\, P_\phi, \quad P_\phi^{\rm th} = \hat{O}\, P_\phi$$

Integrated Boltzmann equations

And defining the integrated collision term as:

$$\mathcal{C}^i \equiv g_i \int \frac{d^3 p_i}{(2\pi)^3} C^i(p_i, t)$$

We can express the equations for the particles ϕ and ψ as:

$$\dot{n}_\phi + 3Hn_\phi = \mathcal{C}^\phi$$

$$\dot{n}_\psi + 3Hn_\psi = \mathcal{C}^\psi$$

Integrated Boltzmann equations

And defining the integrated collision term as:

$$\mathcal{C}^i \equiv g_i \int \frac{d^3 p_i}{(2\pi)^3} C^i(p_i, t)$$

We can express the equations for the particles ϕ and ψ as:

$$\dot{n}_\phi + 3Hn_\phi = \mathcal{C}^\phi \xrightarrow{[1-\hat{O}]} \dot{\hat{n}}_\phi + 3H\hat{n}_\phi = \mathcal{C}^{\phi_c}$$

$$\dot{n}_\psi + 3Hn_\psi = \mathcal{C}^\psi$$

Integrated Boltzmann equations

And defining the integrated collision term as:

$$\mathcal{C}^i \equiv g_i \int \frac{d^3 p_i}{(2\pi)^3} C^i(p_i, t)$$

We can express the equations for the particles ϕ and ψ as:

$$\dot{n}_\phi + 3Hn_\phi = \mathcal{C}^\phi \xrightarrow{[1-\hat{O}]} \dot{\hat{n}}_\phi + 3H\hat{n}_\phi = \mathcal{C}^{\phi_c}$$

$$\dot{n}_\psi + 3Hn_\psi = \mathcal{C}^\psi \qquad \hat{O} \qquad \dot{n}_\phi^{\mathrm{th}} + 3Hn_\phi^{\mathrm{th}} = \mathcal{C}^{\phi_{\mathrm{th}}}$$

Hubble parameter

In particular, in $\Lambda\mathrm{CDM}$, the Hubble parameter considers the contributions from radiation, matter and the cosmological constant using the expression:

$$H^2(a) = H_0^2 \left[\Omega_{r,0}\, a^{-4} + \left(\Omega_{b,0} + \Omega_{\mathrm{DM},0} \right) a^{-3} + \Omega_{\Lambda,0} \right]$$

Where the density parameters have values

$$\Omega_{r,0} \approx 9.4 \times 10^{-5}, \quad \Omega_{b,0} \approx 0.05, \quad \Omega_{\mathrm{DM},0} \approx 0.27, \quad \Omega_{\Lambda} \approx 0.68$$

and the Hubble constant:

$$H_0 = 2.13 \times 10^{-33} h \,\mathrm{eV}, \quad h \approx 0.67 \pm 0.01$$

1. $\phi(p_\phi) \leftrightarrow \psi(p_1) + \psi(p_2)$

$$\boxed{\mathcal{C}^{\phi_c}_{\phi_c \leftrightarrow \psi\psi} = -\Gamma_{\phi\to\psi\psi}(1 - 2f_\psi(E_\psi = m_\phi/2))\,\hat{n}_\phi}$$

$$\boxed{\mathcal{C}^{\phi}_{\phi \leftrightarrow \psi\psi} = -\frac{m_\phi \Gamma_{\phi\to\psi\psi}}{2\pi^2 \beta_\psi} \int_{m_\phi}^{\infty} dE_\phi \int_{E^-}^{E^+} dE_1 \left[f_\phi^{\rm th}(1 - f_\psi(E_1) - f_\psi(E_2)) - f_\psi(E_1) f_\psi(E_2)\right]}$$

2. $\phi(p_\phi) + \phi(p_2) \leftrightarrow \psi(p_3) + \psi(p_4)$

$$\mathcal{C}^{\phi_c}_{\phi_c\phi_c\leftrightarrow\psi\psi} = -\frac{N_\phi}{S_\phi S_\psi}\frac{\hat{n}_\phi^2}{32\pi m_\phi^2}\sum_{\text{spins}}\left|\mathcal{M}_{\phi\phi\to\psi\psi}\right|^2\Big|_{\vec{p}_\phi=\vec{p}_2=0}\left(1-\frac{m_\psi^2}{m_\phi^2}\right)^{1/2}(1-2f_\psi(E_\psi=m_\phi))$$

$$\mathcal{C}^{\phi_c}_{\phi_c\phi\leftrightarrow\psi\psi} = -\frac{N_\phi}{S_\phi S_\psi}\frac{\hat{n}_\phi}{64\pi^3 m_\phi}\sum_{\text{spins}}\int_{m_\phi}^{\infty}dE_2\int_{E_3^-}^{E_3^+}dE_3\left|\mathcal{M}_{\phi\phi\to\psi\psi}\right|^2 \times\left[f_\phi^{\text{th}}(p_2)(1-f_\psi(p_3)-f_\psi(p_4))-f_\psi(p_3)f_\psi(p_4)\right]$$

$$\mathcal{C}^{\phi}_{\phi_c\phi\leftrightarrow\psi\psi} = \mathcal{C}^{\phi_c}_{\phi_c\phi\leftrightarrow\psi\psi}$$

$$\mathcal{C}^{\phi}_{\phi\phi\leftrightarrow\psi\psi} = 0$$

Explicit ODE system

$$\dot{\hat{n}}_\phi = \mathcal{C}^{\phi_c} - 3H\hat{n}_\phi$$

$$\begin{pmatrix} \dot{T} \\ \dot{\mu}_\phi \\ \dot{\mu}_\psi \end{pmatrix} = \frac{1}{a_1 b_2 c_2 + a_2(b_1 c_3 - b_2 c_1)} \begin{pmatrix} b_2 c_2 & a_2 c_3 & a_2 b_2 \\ b_1 c_3 - b_2 c_1 & -a_1 c_3 & -a_1 b_2 \\ -b_1 c_2 & a_1 c_2 - a_2 c_1 & -a_2 b_1 \end{pmatrix} \begin{pmatrix} \mathcal{C}^{\phi} - 3H n_\phi^{\mathrm{th}} \\ \mathcal{C}^{\psi} - 3H n_\psi \\ \mathcal{C}^{\phi_c} + 4H\rho_{DR} \end{pmatrix}$$

System coefficients

$$a_1 = \frac{1}{T}(I_2 - \mu_\phi I_1)$$

$$a_2 = I_1$$

$$b_1 = \frac{2T}{\pi^2}\left[\mu_\psi \mathrm{Li}_2(-e^{\mu_\psi/T}) - 3T\mathrm{Li}_3(-e^{\mu_\psi/T})\right]$$

$$b_2 = -\frac{2T^2}{\pi^2}\mathrm{Li}_2(-e^{\mu_\psi/T})$$

$$c_1^\psi = \frac{6T^2}{\pi^2}\left[\mu_\psi \mathrm{Li}_3(-e^{\mu_\psi/T}) - 4T\mathrm{Li}_4(-e^{\mu_\psi/T})\right]$$

$$c_1^\phi = \frac{1}{T}(I_3 - \mu_\phi I_2)$$

$$c_1 = c_1^\psi + c_1^\phi$$

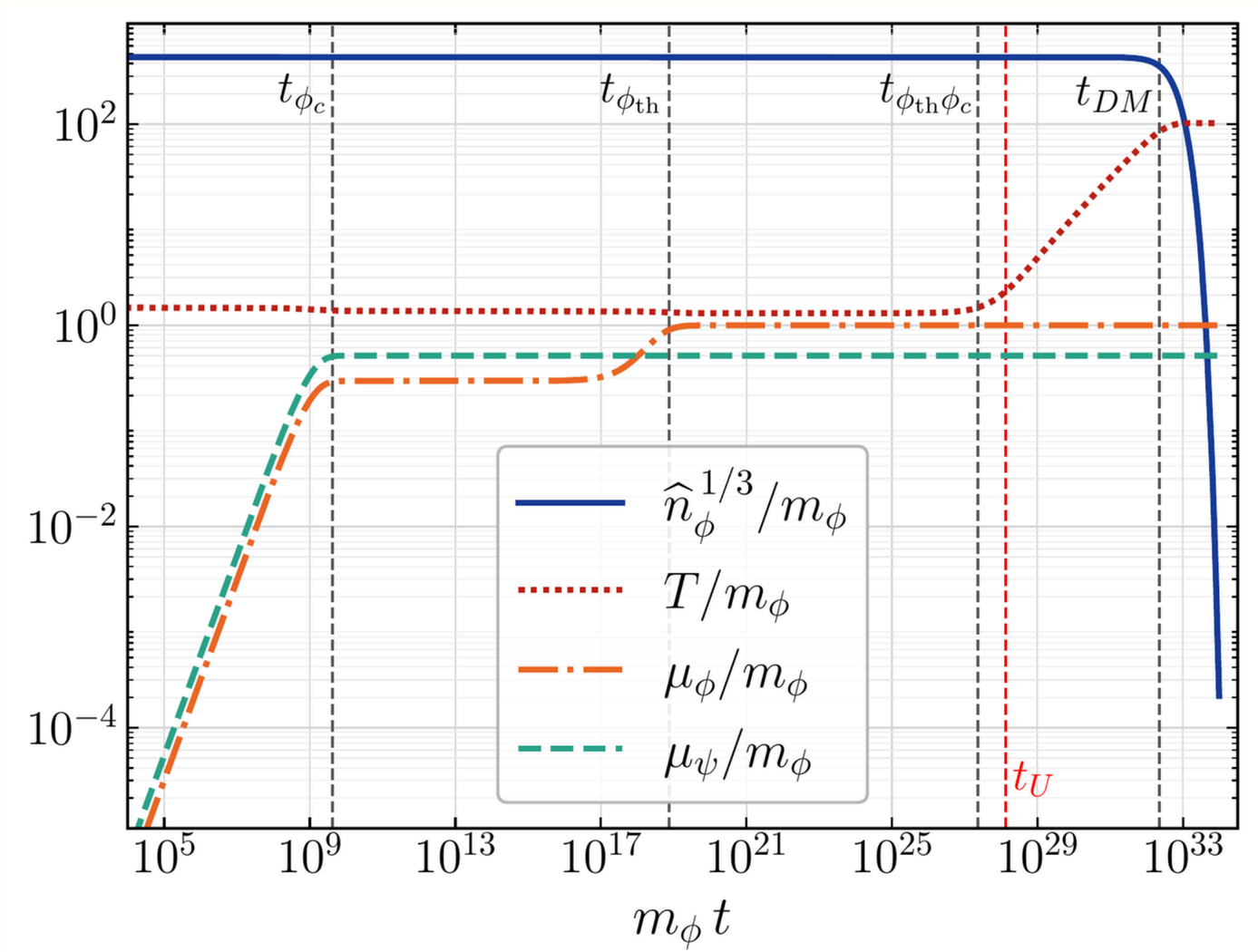
$$c_2 = I_2$$

$$c_3 = -\frac{6T^3}{\pi^2}\mathrm{Li}_3(-e^{\mu_\psi/T})$$

$$D = \frac{1}{a_1 b_2 c_2 + a_2(b_1 c_3 - b_2 c_1)}$$

$$I_n = \frac{1}{2\pi^2 T}\int_1^\infty E_\phi^n \sqrt{E_\phi^2 - 1} f_\phi^{\mathrm{th}}(1 + f_\phi^{\mathrm{th}})\, dE_\phi$$

Variables graphs H0



Variables graphs HH

