Systematic tracking of linear cosmological perturbations beyond the standard quasi-static approximation in Horndeski gravity

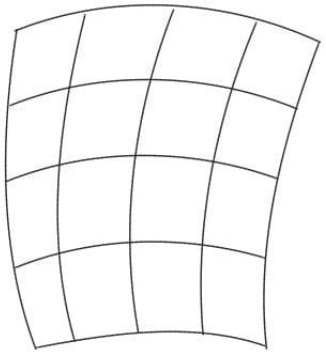
PhD(c) Santiago García-Serna

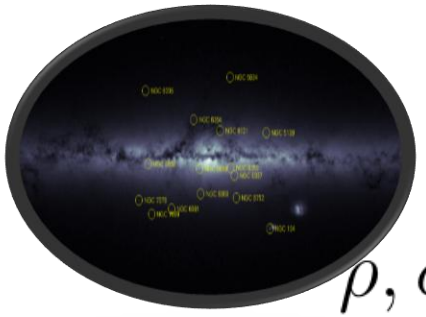
Prof. César A. Valenzuela-Toledo and Prof. J. Bayron Orjuela-Quintana

Greco

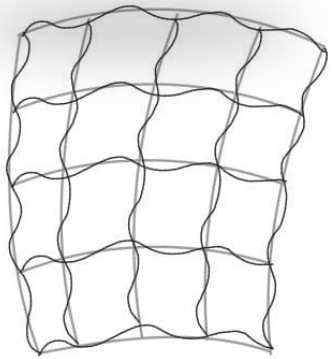
Facultad de Ciencias Naturales y Exactas
Universidad del Valle - Colombia

Input





ρ, ϕ



SOLVERS

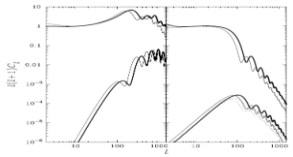
Horndeski in the Cosmic Linear Anisotropy Solving System

hi_class

CMBFAST

CLASS

the Cosmic Linear Anisotropy Solving System



MGCAMB

Code for Anisotropies in the Microwave Background

Observables and projects

DARK ENERGY SPECTROSCOPIC INSTRUMENT

U.S. Department of Energy Office of Science

SCIENCE & EXPLORATION

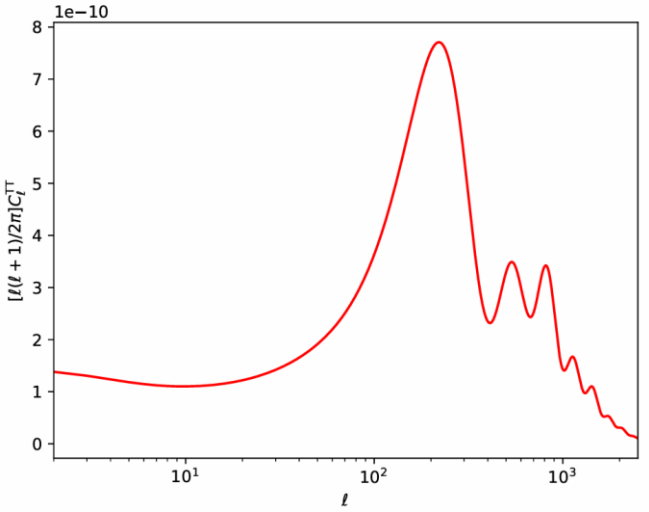
euclid

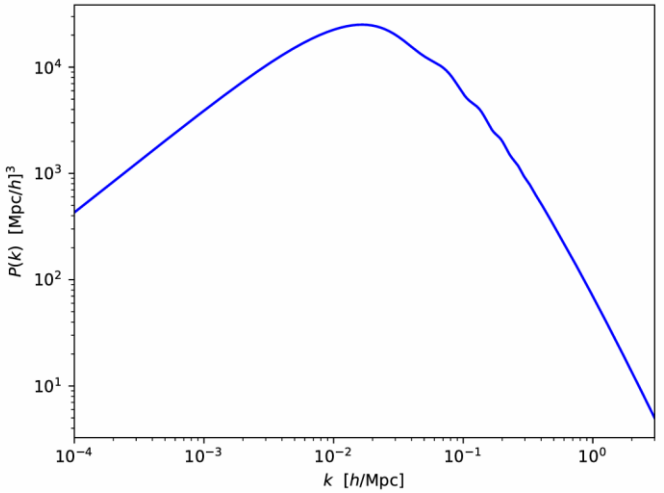
Exploring the dark Universe

- CMB, LSST, BAOS, RSD,…

VERA C. RUBIN OBSERVATORY

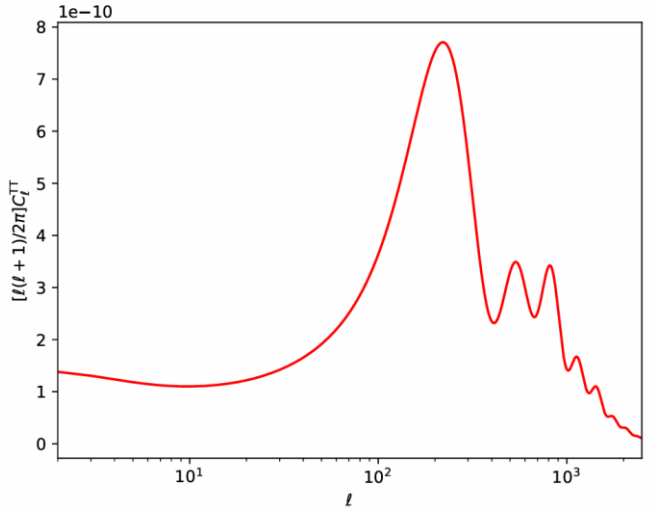
Output and comparison

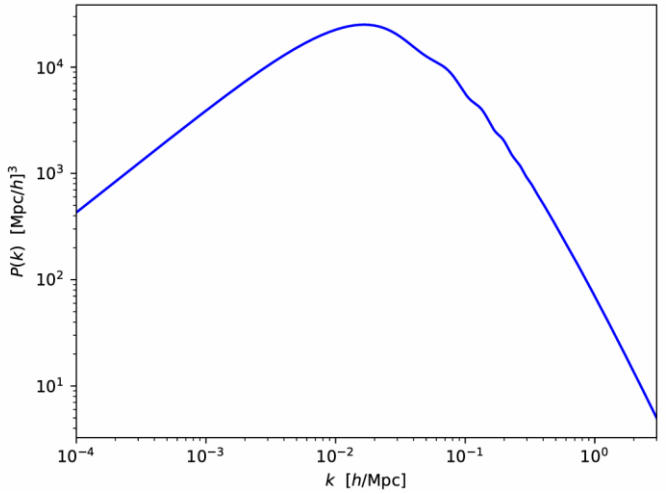




Cosmological model

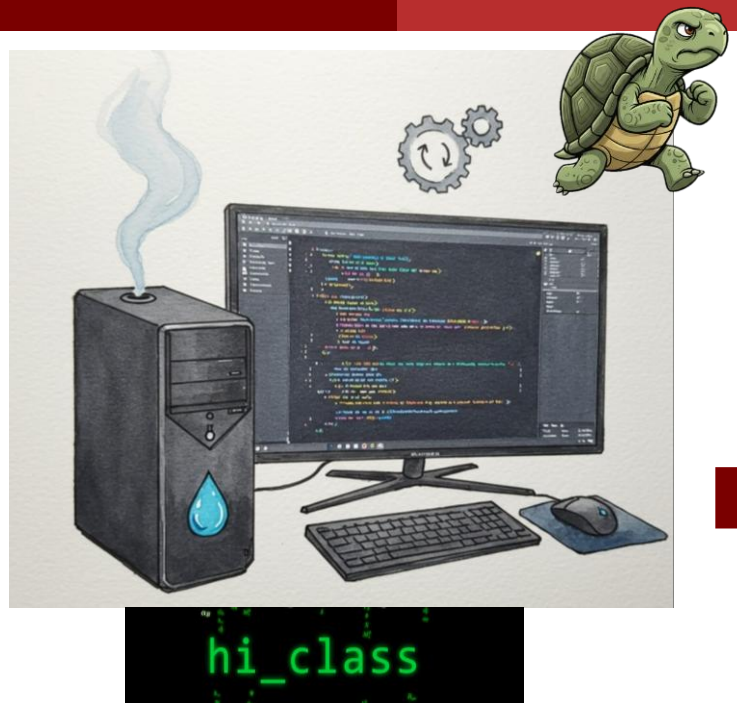
Output



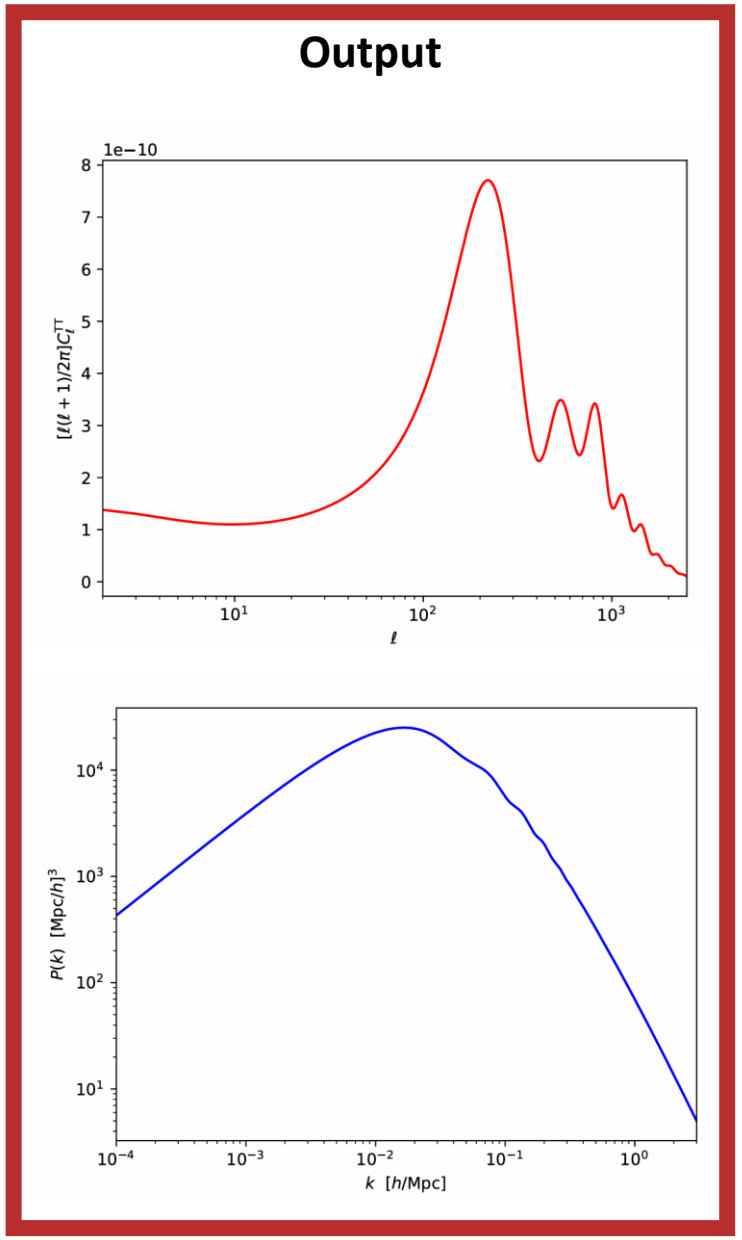


Cosmological model

FULL



=



Cosmological model

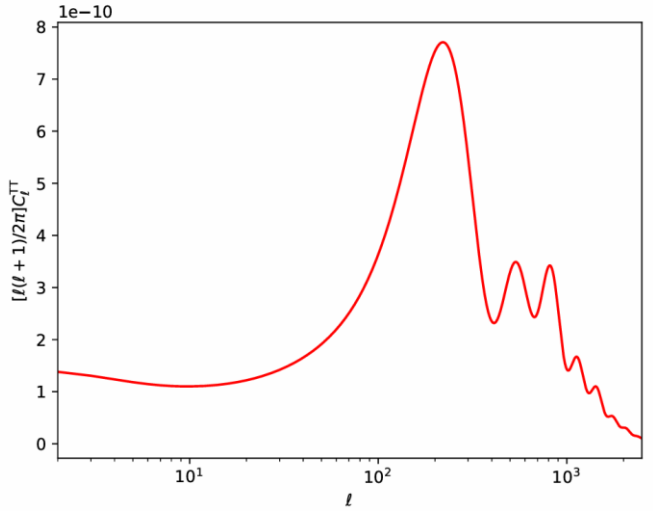
$QSA - SHA$

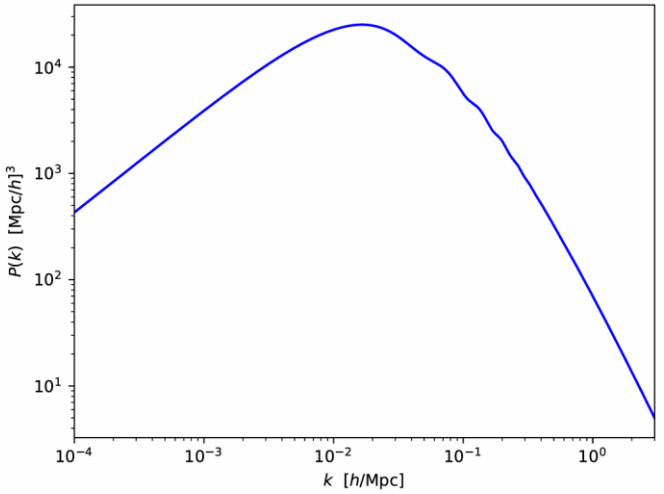


Einstein Boltzmann solver

$\approx$

Output





Cosmological model

FULL



$QSA - SHA$

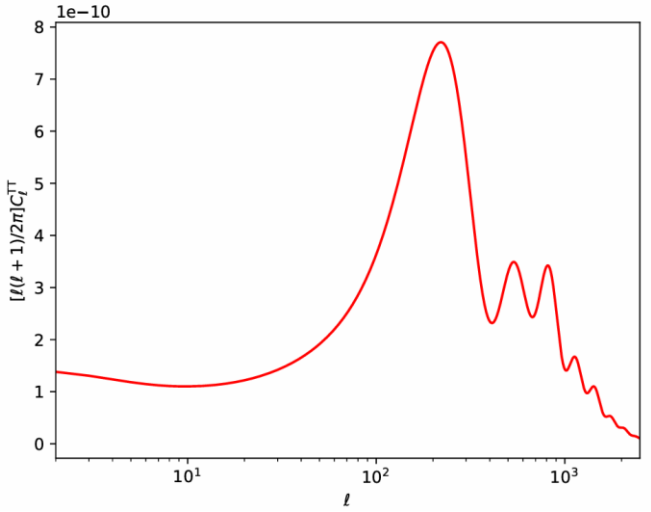


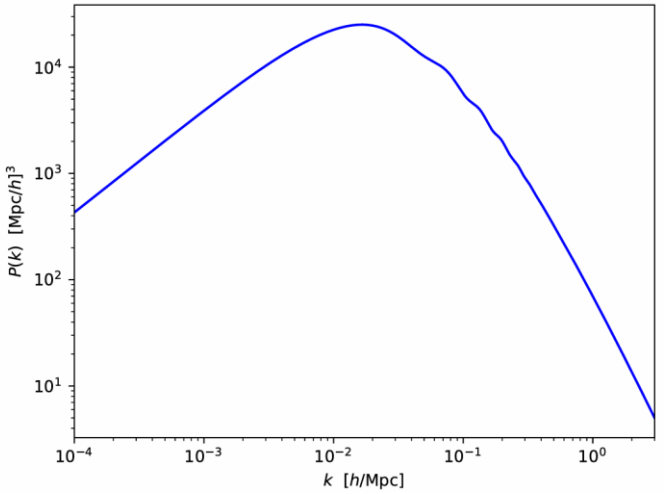
Einstein Boltzmann solver

=

≈

Output

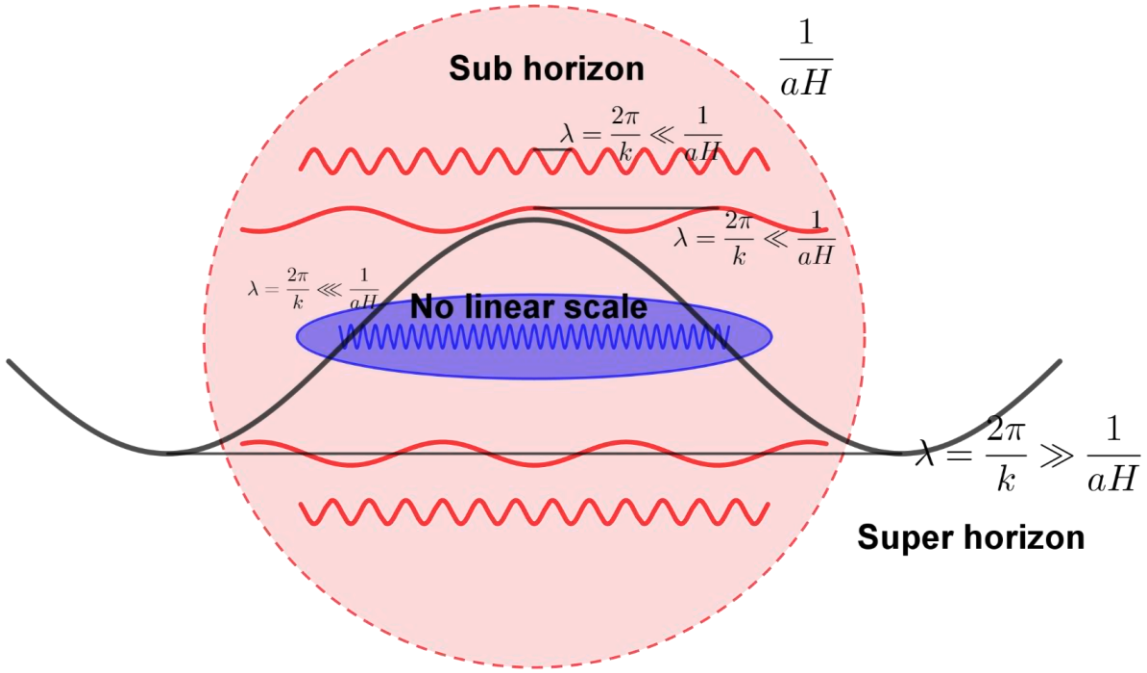




What is the QSA and SHA?

$$ds^2 = -\{1 + 2\Psi(\boldsymbol{x}, t)\}dt^2 + a(t)^2\{1 + 2\Phi(\boldsymbol{x}, t)\}\delta_{ij}\, dx^i\, dx^j$$

$$k \gg aH, \quad \Phi \gg \frac{\dot{\Phi}}{H}, \quad \Psi \gg \frac{\dot{\Psi}}{H}$$



What is the QSA and SHA?

$$\mathrm{d}s^2 = -\{1 + 2\Psi(\boldsymbol{x}, t)\}\mathrm{d}t^2 + a(t)^2\{1 + 2\Phi(\boldsymbol{x}, t)\}\delta_{ij}\ \mathrm{d}x^i\ \mathrm{d}x^j$$

$$k \gg aH, \quad \Phi \gg \frac{\dot{\Phi}}{H}, \quad \Psi \gg \frac{\dot{\Psi}}{H}$$

$$\delta R = 6\ddot{\Phi} + 24H\dot{\Phi} - 6H\dot{\Psi} + 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\dot{H} + 24H^2\right)\Psi$$

What is the QSA and SHA?

$$\mathrm{d}s^2 = -\{1 + 2\Psi(\boldsymbol{x}, t)\}\mathrm{d}t^2 + a(t)^2\{1 + 2\Phi(\boldsymbol{x}, t)\}\delta_{ij}\ \mathrm{d}x^i\ \mathrm{d}x^j$$

$$k \gg aH, \quad \Phi \gg \frac{\dot{\Phi}}{H}, \quad \Psi \gg \frac{\dot{\Psi}}{H}$$

$$\delta R = 6\ddot{\Phi} + 24H\dot{\Phi} - 6H\dot{\Psi} + 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\dot{H} + 24H^2\right)\Psi$$

$$\overset{\text{QSA}}{\approx} 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\dot{H} + 24H^2\right)\Psi$$

What is the QSA and SHA?

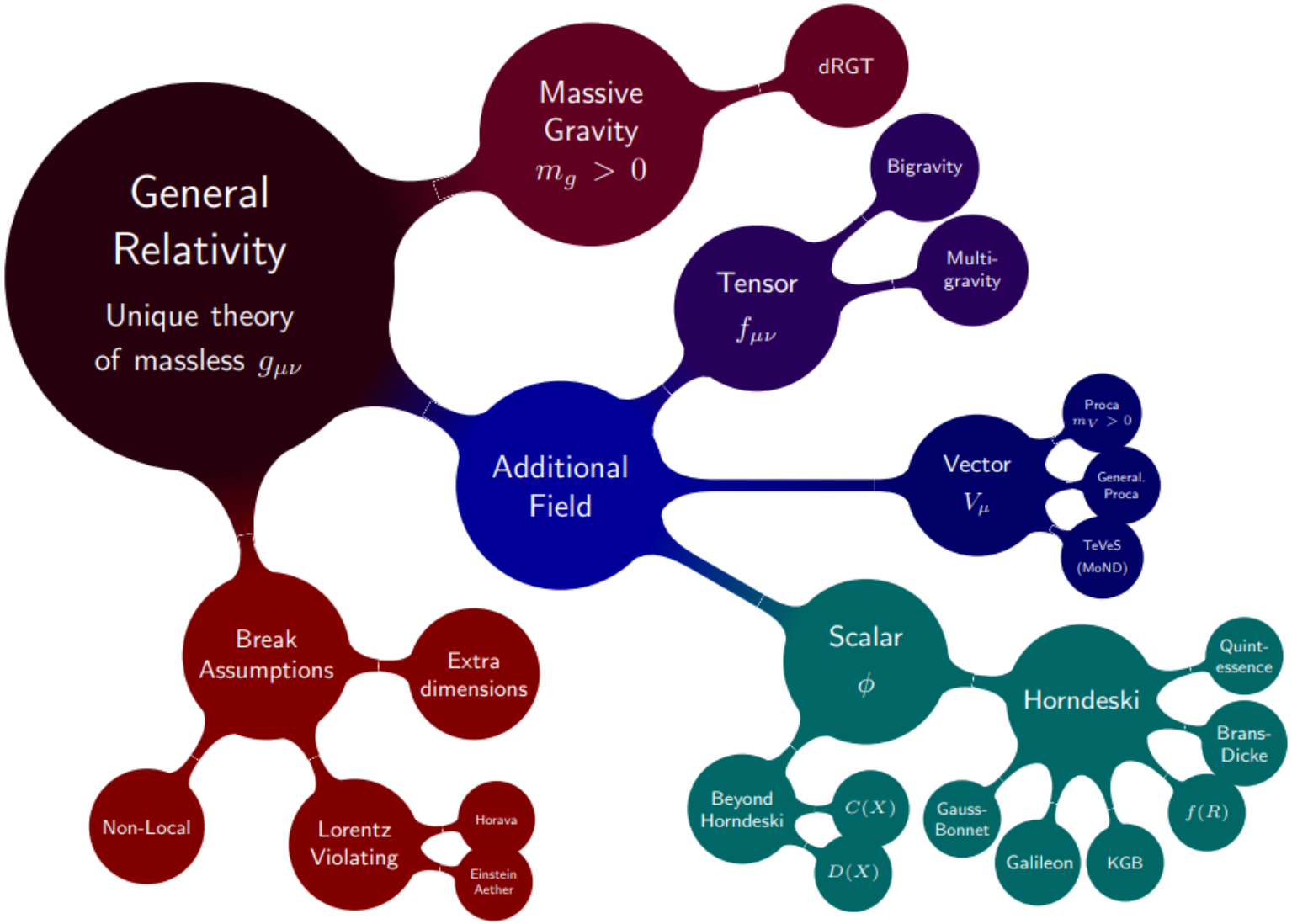
$$\mathrm{d}s^2 = -\{1 + 2\Psi(\boldsymbol{x}, t)\}\mathrm{d}t^2 + a(t)^2\{1 + 2\Phi(\boldsymbol{x}, t)\}\delta_{ij}\ \mathrm{d}x^i\ \mathrm{d}x^j$$

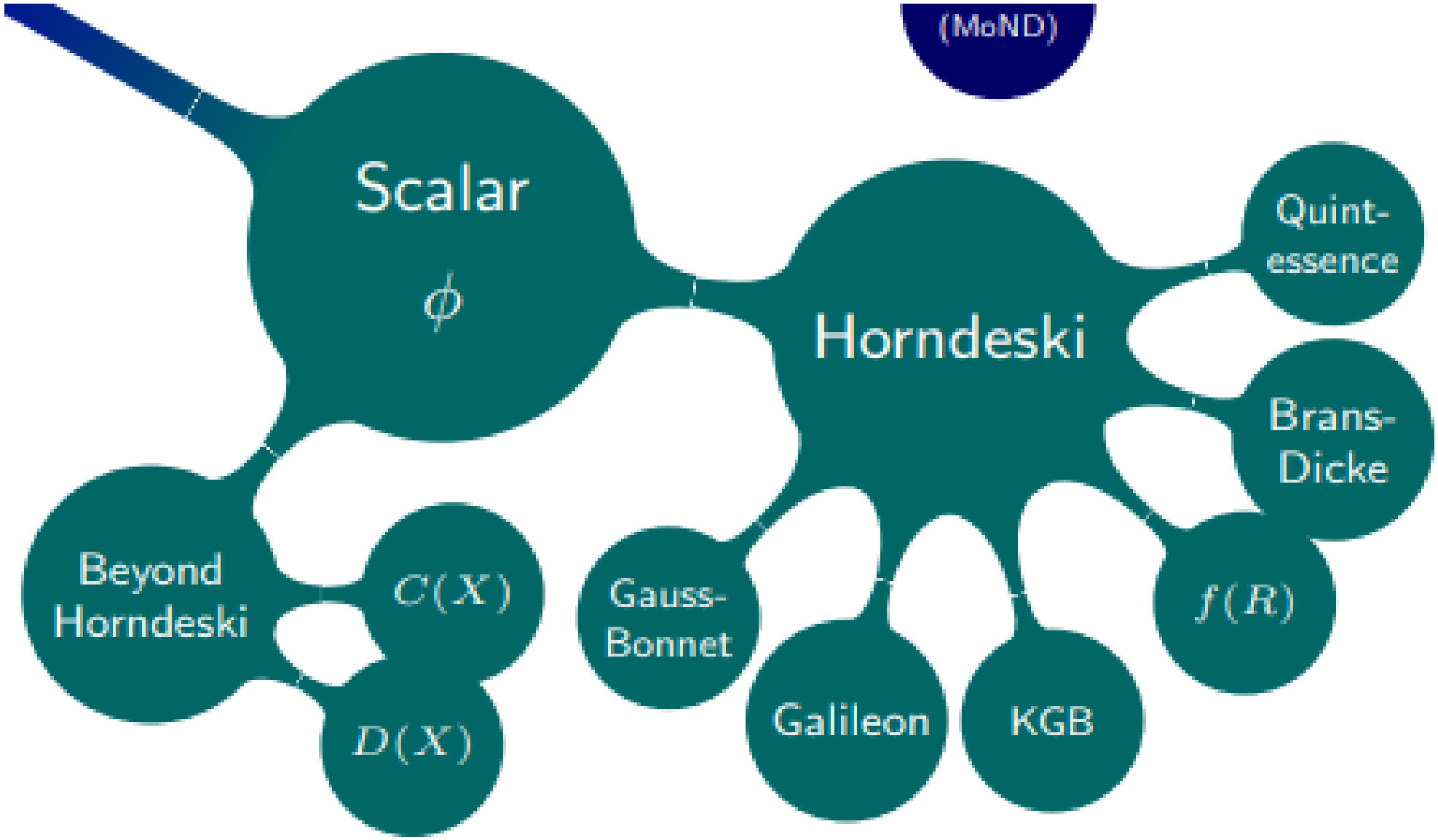
$$k \gg aH, \quad \Phi \gg \frac{\dot{\Phi}}{H}, \quad \Psi \gg \frac{\dot{\Psi}}{H}$$

$$\delta R = 6\ddot{\Phi} + 24H\dot{\Phi} - 6H\dot{\Psi} + 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\dot{H} + 24H^2\right)\Psi$$

$$\overset{\text{QSA}}{\approx} 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\dot{H} + 24H^2\right)\Psi$$

$$\overset{\text{SHA}}{\approx} 4\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\Psi.$$


General Relativity
Unique theory of massless $g_{\mu\nu}$
Massive Gravity $m_g > 0$
dRGT
Bigravity
Multi-gravity
Tensor $f_{\mu\nu}$
Additional Field
Vector V_μ
Proca $m_V > 0$
General. Proca
TeVeS (MoND)
Scalar ϕ
Horndeski
Quint-essence
Brans-Dicke
$f(R)$
KGB
Galileon
Gauss-Bonnet
Beyond Horndeski
$C(X)$
$D(X)$
Break Assumptions
Extra dimensions
Non-Local
Lorentz Violating
Horava
Einstein Aether


(MoND)
Scalar
ϕ
Horndeski
Quint-
essence
Brans-
Dicke
$f(R)$
KGB
Galileon
Gauss-
Bonnet
Beyond
Horndeski
$C(X)$
$D(X)$

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad G_{\mu\nu} \to \bar{G}_{\mu\nu} + \delta G_{\mu\nu} \qquad T_{\mu\nu} \to \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

$$0 = -\kappa\bar{\rho}_m\delta_m + A_1\dot{\Phi} + A_2\dot{\Psi} + \left(A_3 + A_4\frac{k^2}{a^2}\right)\Phi + \left(A_5 + A_6\frac{k^2}{a^2}\right)\Psi,$$

$$0 = -\frac{a\kappa\bar{\rho}_m V_m}{k^2} + C_1\dot{\Phi} + C_2\dot{\Psi} + C_3\Phi + C_4\Psi,$$

$$0 = B_1\ddot{\Phi} + B_2\ddot{\Psi} + B_3\dot{\Phi} + B_4\dot{\Psi} + B_5\Phi + B_6\Psi,$$

$$0 = D_1\ddot{\Phi} + D_2\dot{\Phi} + D_3\dot{\Psi} + \left(D_4 + D_5\frac{k^2}{a^2}\right)\Phi + \left(D_6 + D_7\frac{k^2}{a^2}\right)\Psi.$$

Designing Horndeski and the effective fluid approach

Rubén Arjona,* Wilmar Cardona,† and Savvas Nesseris‡

arXiv:1904.06294v2

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad G_{\mu\nu} \to \bar{G}_{\mu\nu} + \delta G_{\mu\nu} \qquad T_{\mu\nu} \to \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

$$A_3 \frac{k^2}{a^2}\Psi + A_6 \frac{k^2}{a^2}\delta\phi - \kappa\rho_m\delta_m \simeq 0,$$

$$B_6 \frac{k^2}{a^2}\Psi + B_8 \frac{k^2}{a^2}\Phi + B_7 \frac{k^2}{a^2}\delta\phi \simeq 0,$$

$$D_7 \frac{k^2}{a^2}\Psi + \left(D_9 \frac{k^2}{a^2} - M^2\right)\delta\phi + D_{10}\frac{k^2}{a^2}\Phi \simeq 0,$$

$$G_4(\Psi + \Phi) + G_{4\phi}\delta\phi = 0$$

Designing Horndeski and the effective fluid approach

Rubén Arjona,* Wilmar Cardona,† and Savvas Nesseris‡

arXiv:1904.06294v2

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad G_{\mu\nu} \to \bar{G}_{\mu\nu} + \delta G_{\mu\nu} \qquad T_{\mu\nu} \to \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

$$A_3\frac{k^2}{a^2}\Psi + A_6\frac{k^2}{a^2}\delta\phi - \kappa\rho_m\delta_m \simeq 0,$$

$$B_6\frac{k^2}{a^2}\Psi + B_8\frac{k^2}{a^2}\Phi + B_7\frac{k^2}{a^2}\delta\phi \simeq 0,$$

$$D_7\frac{k^2}{a^2}\Psi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + D_{10}\frac{k^2}{a^2}\Phi \simeq 0,$$

$$G_4(\Psi + \Phi) + G_{4\phi}\delta\phi = 0$$

$$D_9 = -K_X + 2G_{3\phi} - 4H\dot{\phi}G_{3X} - \ddot{\phi}\left(2G_{3X} + \dot{\phi}^2 G_{3XX}\right) - \dot{\phi}^2 G_{3\phi X}$$

Designing Horndeski and the effective fluid approach

Rubén Arjona,* Wilmar Cardona,† and Savvas Nesseris‡

arXiv:1904.06294v2

QSA-SHA coeffiecients

$$\varepsilon \equiv \frac{aH}{k}, \qquad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \qquad \xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \quad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$
$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \quad \varepsilon_\Psi \equiv \frac{\dot{\Psi}}{\Psi H}, \quad \chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \quad \chi_\Psi \equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}.$$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

J. Bayron Orjuela-Quintana, Savvas Nesseris arXiv:2303.14251 **[gr-qc]**

QSA-SHA coeffiecients

$$\varepsilon \equiv \frac{aH}{k}, \qquad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \qquad \xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \qquad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$
$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \quad \varepsilon_\Psi \equiv \frac{\dot{\Psi}}{\Psi H}, \quad \chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \quad \chi_\Psi \equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}.$$

$$\kappa\delta\rho_{\rm DE} = W_1\dot{\Phi} + W_2\dot{\Psi} + \left(W_3 + W_4\frac{k^2}{a^2}\right)\Phi + \left(W_5 + W_6\frac{k^2}{a^2}\right)\Psi,$$

$$\kappa\delta P_{\rm DE} = Y_1\ddot{\Phi} + Y_2\ddot{\Psi} + Y_3\dot{\Phi} + Y_4\dot{\Psi} + \left(Y_5 + Y_6\frac{k^2}{a^2}\right)\Phi + \left(Y_7 + Y_8\frac{k^2}{a^2}\right)\Psi,$$

$$\frac{a\kappa\bar{\rho}_{\rm DE}}{k^2}V_{\rm DE} = Z_1\dot{\Phi} + Z_2\dot{\Psi} + Z_3\Phi + Z_4\Psi,$$

$$\kappa\bar{\rho}_{\rm DE}\pi_{\rm DE} = -\frac{k^2}{a^2}(\Phi + \Psi),$$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

J. Bayron Orjuela-Quintana, Savvas Nesseris arXiv:2303.14251 [gr-qc]

QSA-SHA coeffiecients

$$\varepsilon \equiv \frac{aH}{k}, \qquad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \qquad \xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \qquad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$
$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \qquad \varepsilon_\Psi \equiv \frac{\dot{\Psi}}{\Psi H}, \qquad \chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \qquad \chi_\Psi \equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}.$$

$$V_{\rm DE} = \frac{a\dot{F}}{2F}\frac{F + 6\frac{k^2}{a^2}F_R}{3\frac{k^2}{a^2}F_R + F}\frac{\bar{\rho}_m}{\bar{\rho}_{\rm DE}}\delta_m \approx 0$$

$$\dot{F}\delta_m \approx 0$$

$$\dot{F} \propto H \times \text{Perturbation} \sim 0$$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

J. Bayron Orjuela-Quintana, Savvas Nesseris arXiv:2303.14251 **[gr-qc]**

QSA-SHA coeffiecients

$$\varepsilon \equiv \frac{aH}{k}, \qquad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \qquad \xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \qquad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$
$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \quad \varepsilon_\Psi \equiv \frac{\dot{\Psi}}{\Psi H}, \quad \chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \quad \chi_\Psi \equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}.$$

$$V_{\rm DE} = \frac{a}{F\left(F + 3\frac{k^2}{a^2}F_R\right)}\left\{F(F-1) + \frac{k^2}{a^2}(3F-4)F_R\right\}\frac{\bar{\rho}_m}{\bar{\rho}_{\rm DE}}\delta_m\frac{k}{a}\varepsilon$$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

J. Bayron Orjuela-Quintana, Savvas Nesseris arXiv:2303.14251 **[gr-qc]**

QSA-SHA coeffiecients

$$\varepsilon \equiv \frac{aH}{k}, \quad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \quad \xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \quad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$
$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \quad \varepsilon_\Psi \equiv \frac{\dot{\Psi}}{\Psi H}, \quad \chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \quad \chi_\Psi \equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}.$$

QSA-SHA "0-order"

$$V_{\rm DE} = \frac{a\dot{F}}{2\bar{F}} \frac{F + 6\frac{k^2}{a^2}F_R}{3\frac{k^2}{a^2}F_R + F} \frac{\bar{\rho}_m}{\bar{\rho}_{\rm DE}}\delta_m \qquad \dot{F} = 6\frac{k^3}{a^3}(\delta + \xi - 2)F_R\varepsilon^3$$

$$V_{\rm DE} = \frac{a}{F\left(F + 3\frac{k^2}{a^2}F_R\right)}\left\{F(F-1) + \frac{k^2}{a^2}(3F-4)F_R\right\}\frac{\bar{\rho}_m}{\bar{\rho}_{\rm DE}}\delta_m\frac{k}{a}\varepsilon$$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

J. Bayron Orjuela-Quintana, Savvas Nesseris arXiv:2303.14251 [gr-qc]

QSA-SHA coeffiecients

$$\varepsilon \equiv \frac{aH}{k}, \qquad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \qquad \xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \quad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$
$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \quad \varepsilon_\Psi \equiv \frac{\dot{\Psi}}{\Psi H}, \quad \chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \quad \chi_\Psi \equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}.$$

- Standar approach $V_{\rm DE} \sim \mathcal{O}(\epsilon^3)$
- QSA-SHA coeffiecients $V_{\rm DE} \sim \mathcal{O}(\epsilon)$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

J. Bayron Orjuela-Quintana, Savvas Nesseris arXiv:2303.14251 **[gr-qc]**

Horndeski

$QSA - SHA$



$$\varepsilon \equiv \frac{aH}{k}, \qquad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H},$$
$$\xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \qquad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$
$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \qquad \varepsilon_\varphi \equiv \frac{\dot{\varphi}}{\varphi H},$$
$$\chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \qquad \chi_\varphi \equiv \frac{\dot{\varepsilon}_\varphi}{\varepsilon_\varphi H}.$$

- $\pi_{DE} = 0$

Coefficients 0 - order

$$\varepsilon \equiv \frac{aH}{k}, \qquad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H},$$
$$\xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \qquad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$
$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \qquad \varepsilon_\Psi \equiv \frac{\dot{\Psi}}{\Psi H},$$
$$\chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \qquad \chi_\Psi \equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}.$$

- $\pi_{DE} \neq 0$

Coefficients for Horndesky

- $\pi_{DE} \neq 0$

$$\varepsilon \equiv \frac{aH}{k}, \quad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \quad \xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \quad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$
$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \quad \varepsilon_\Psi \equiv \frac{\dot{\Psi}}{\Psi H}, \quad \chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \quad \chi_\Psi \equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}.$$

$$A = A_0 + \frac{k^2}{a^2}A_1 + \left(A_1 + \frac{k^2}{a^2}A_2 + \frac{k^4}{a^4}A_3\right)\varepsilon + \mathcal{O}\left(\varepsilon^2, \varepsilon_\Psi, \varepsilon_\Phi\right)$$

- $\pi_{DE} = 0$

$$\varepsilon \equiv \frac{aH}{k}, \quad \delta \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \quad \xi \equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, \quad \chi \equiv \frac{\dddot{\varepsilon}}{\varepsilon H^3},$$
$$\varepsilon_\Phi \equiv \frac{\dot{\Phi}}{\Phi H}, \quad \varepsilon_\varphi \equiv \frac{\dot{\varphi}}{\varphi H}, \quad \chi_\Phi \equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, \quad \chi_\varphi \equiv \frac{\dot{\varepsilon}_\varphi}{\varepsilon_\varphi H}.$$

$$A = A_0 + \frac{k^2}{a^2}A_1 + \left(A_1 + \frac{k^2}{a^2}A_2 + \frac{k^4}{a^4}A_3\right)\varepsilon + \mathcal{O}\left(\varepsilon^2, \varepsilon_\varphi, \varepsilon_\Phi\right)$$

$$V_{\mathrm{DE}} = a + b\varepsilon + c\varepsilon^2 + \cdots$$

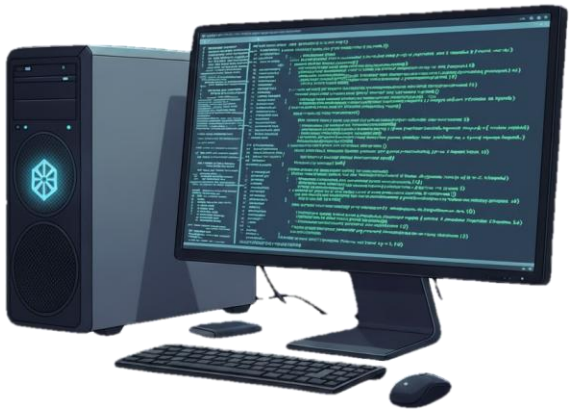
FULL



hi_class

Horndeski

$QSA - SHA$



Einstein Boltzmann solver

=

≈

Φ, Ψ, P(k), C(l), …

$$V_{\mathrm{DE}} = a + b\varepsilon + c\varepsilon^2 + \cdots$$

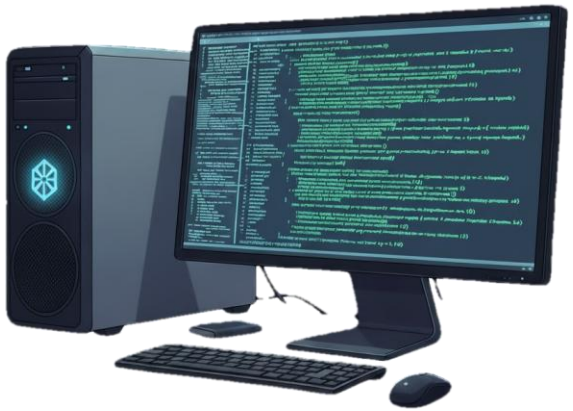
FULL



hi_class

Horndeski

$QSA - SHA$



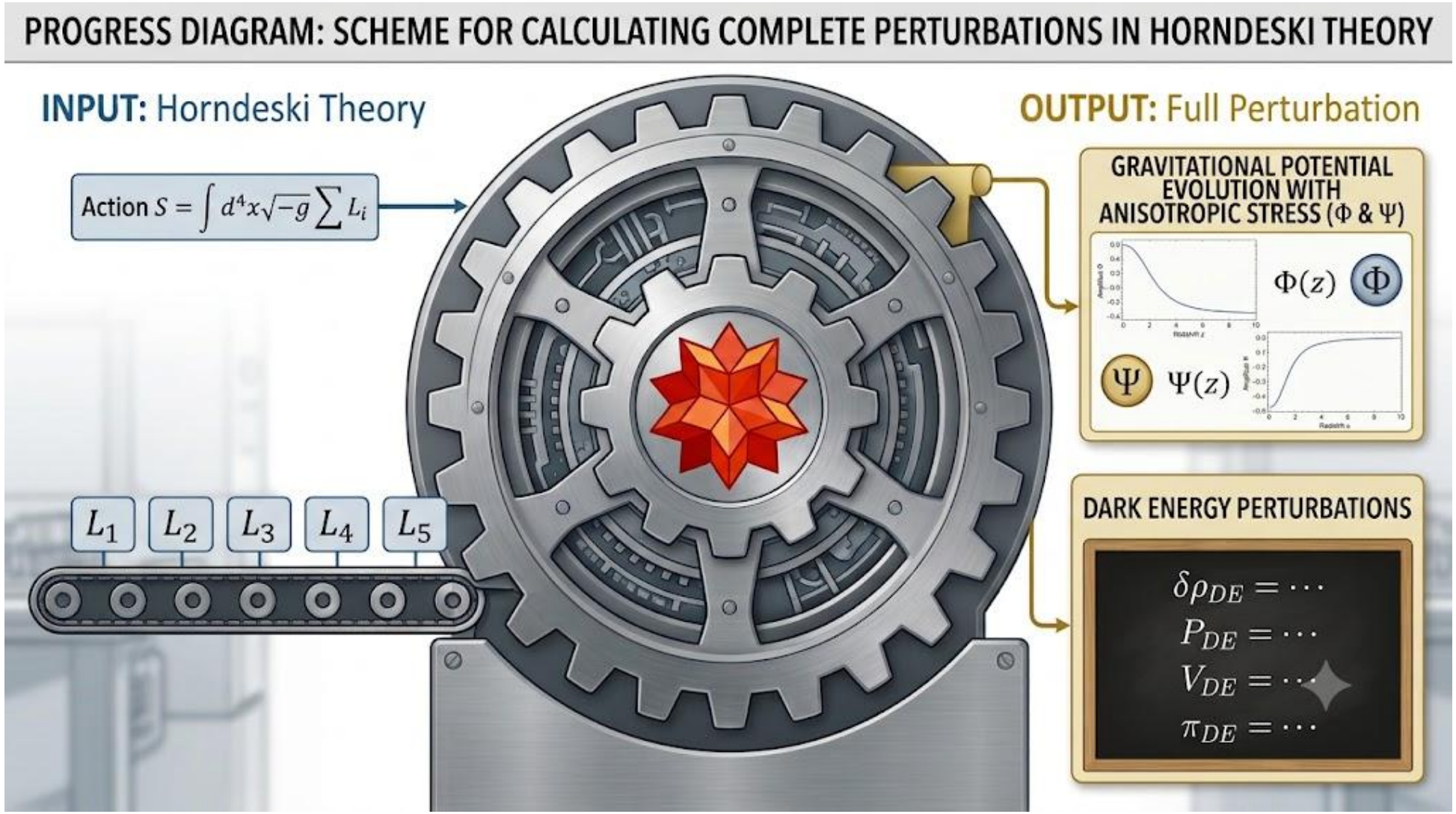
Einstein Boltzmann solver

=

≈

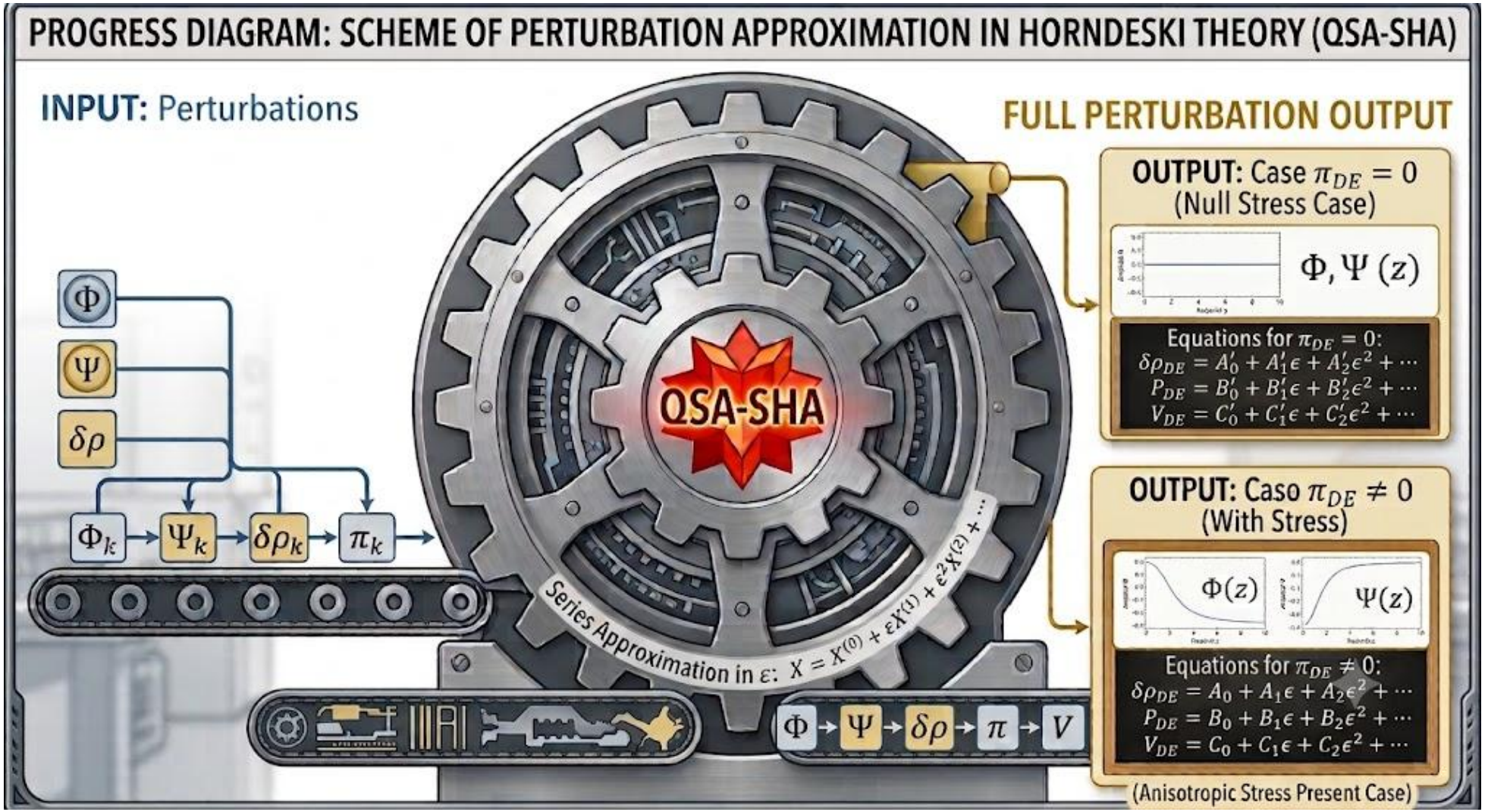
Φ, Ψ, P(k), C(l), …

What is done?



Google. (2026). *Gemini* (Versión julio 2026)

What is done?



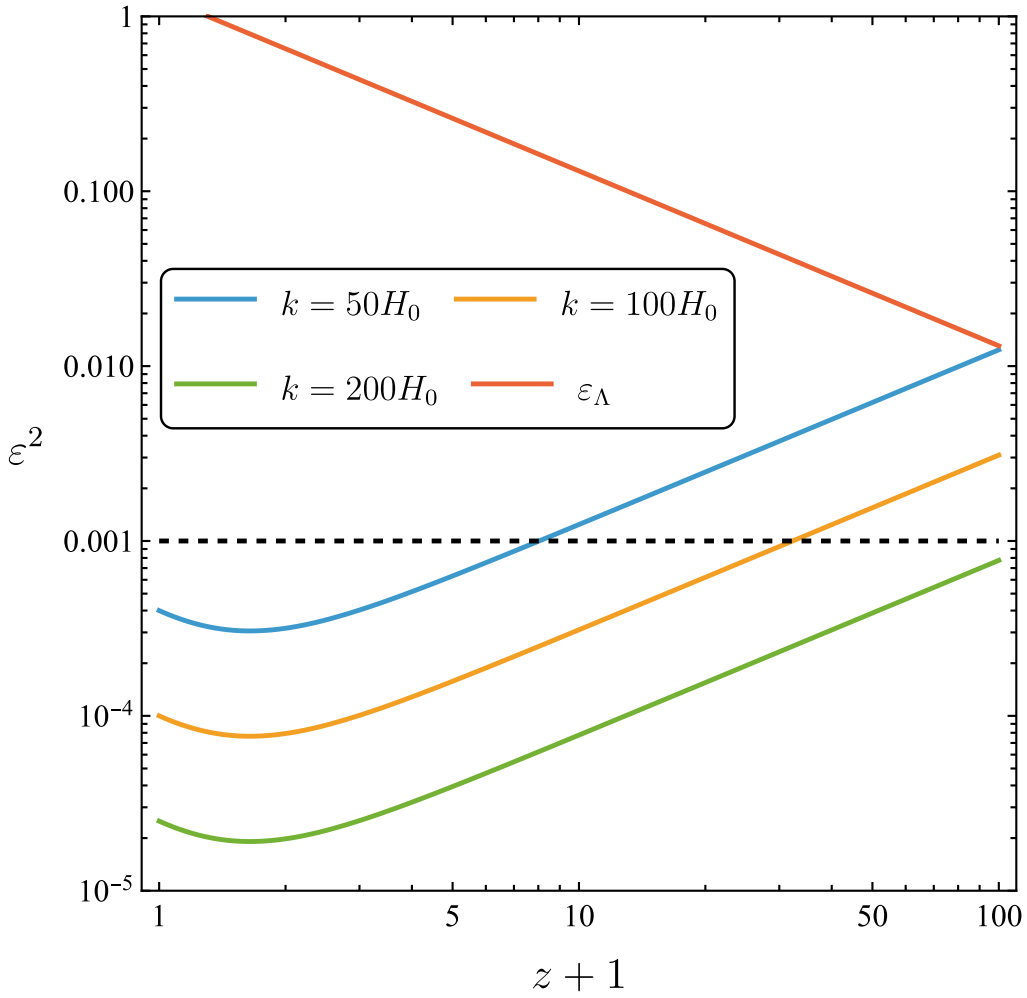
Google. (2026). *Gemini* (Versión julio 2026)

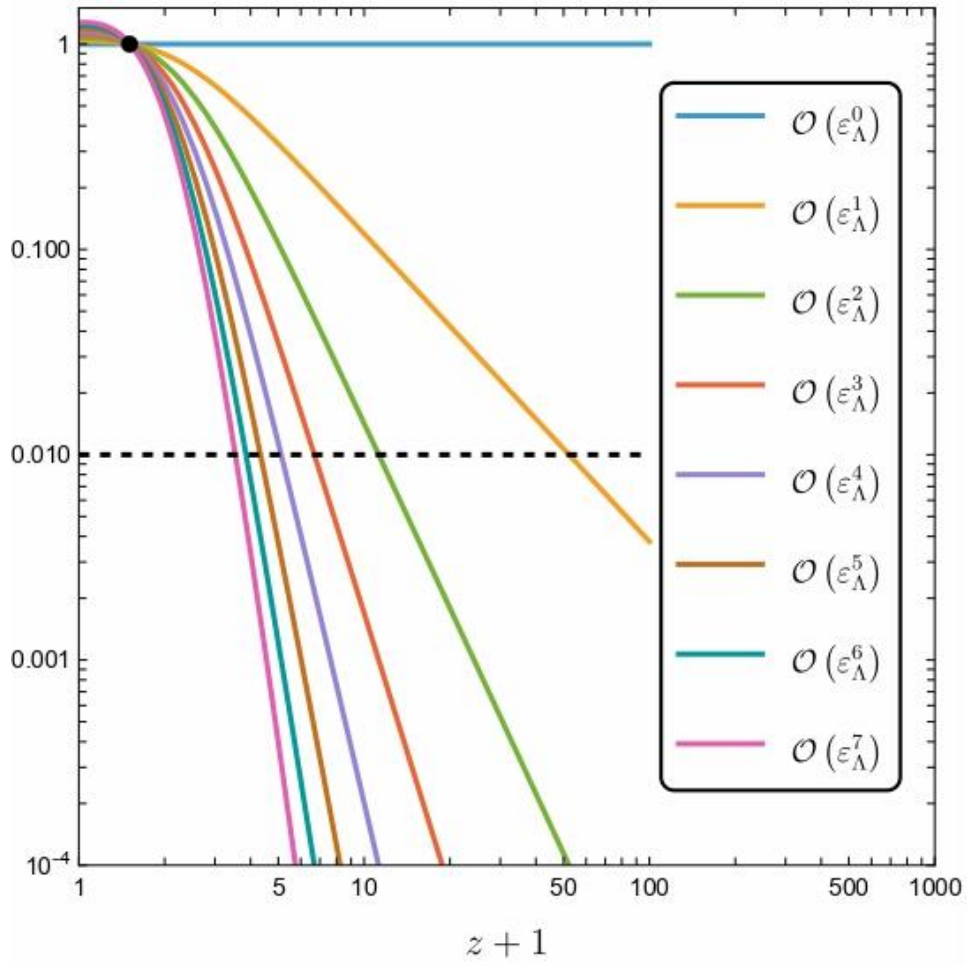
Deep-Matter Background Parameter

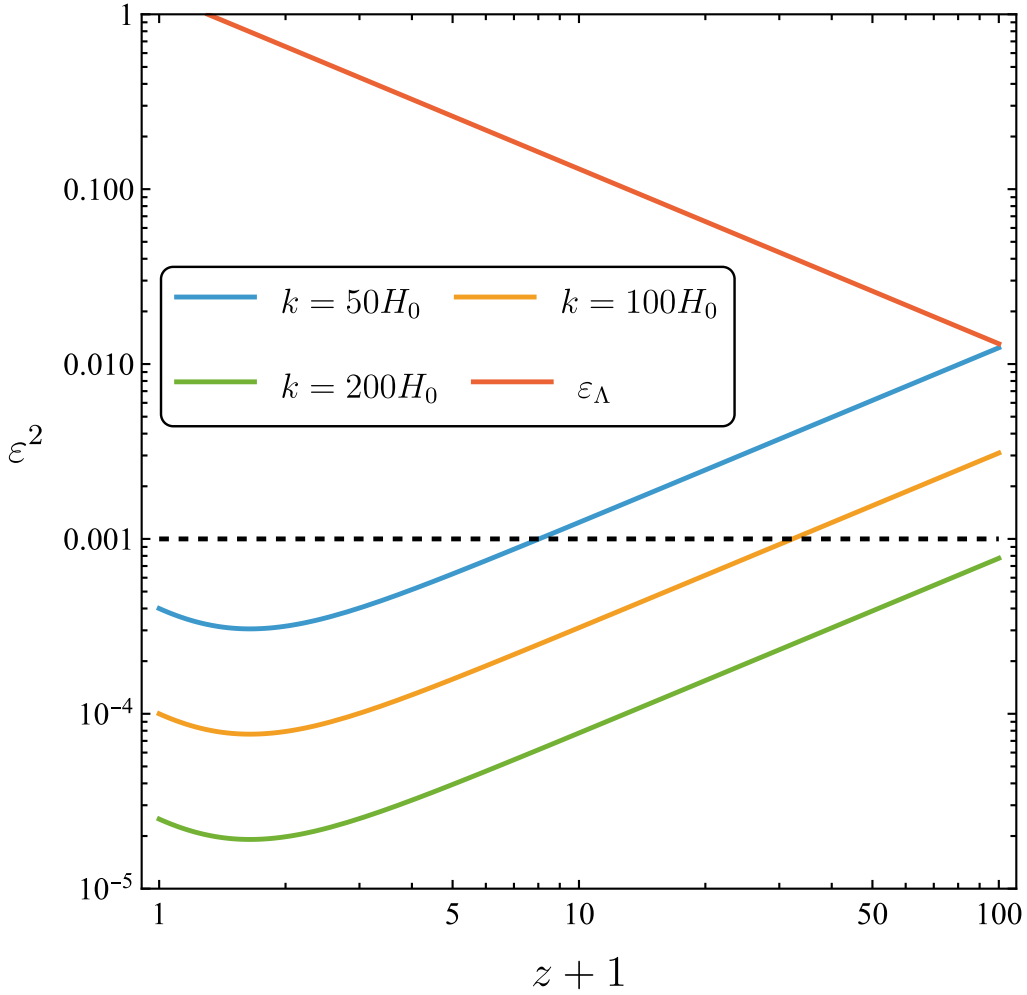
$$\underbrace{\left(\frac{H_0}{k}\right)^2 \Omega_m}_{\text{SHA Bound: } \epsilon \ll 1} \ll a \ll \underbrace{\left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3}}_{\text{Deep-Matter Bound: } \epsilon_\Lambda \ll 1}$$

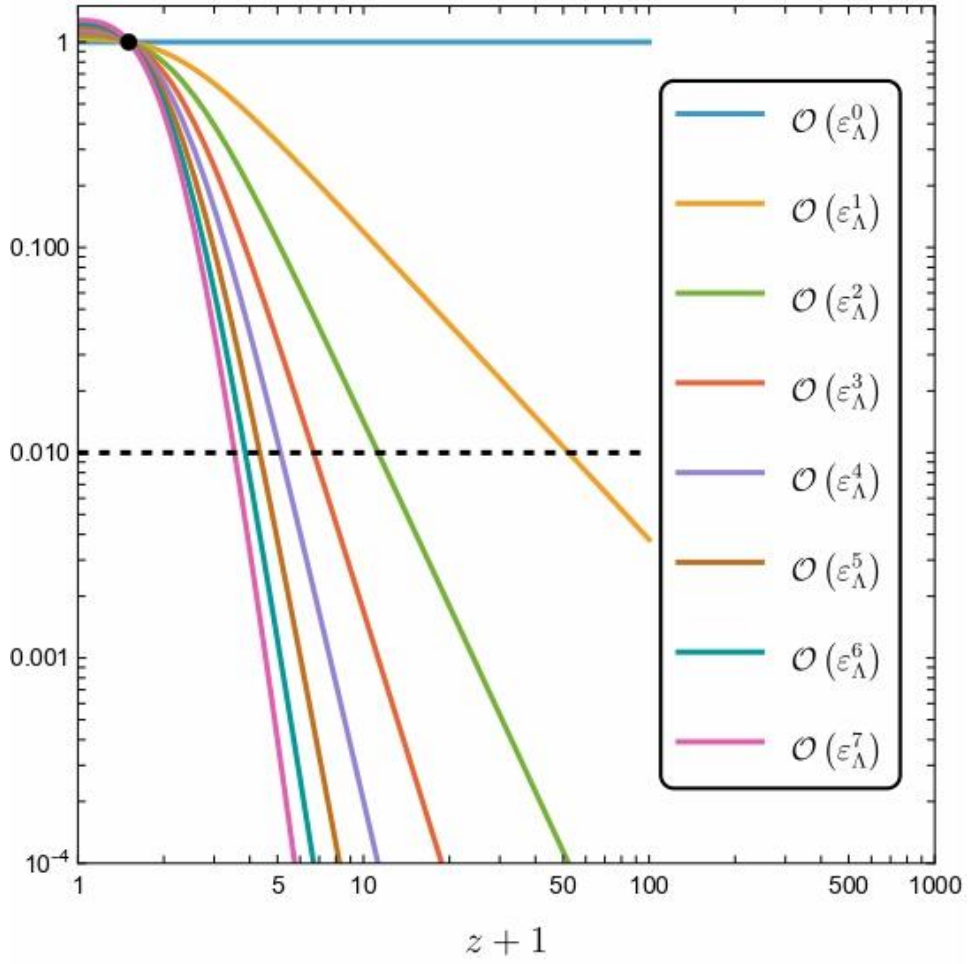
$$H(a) = H_0 \sqrt{\Omega_m a^{-3}} \left(1 + \epsilon_\Lambda^3\right)^{1/2}$$

$$\epsilon \equiv \frac{aH}{k} \qquad \epsilon_\Lambda \equiv \left(\frac{\Omega_\Lambda}{\Omega_m}\right)^{1/3} a$$


1
0.100
0.010
0.001
10^{-4}
10^{-5}
ε^2
$k = 50H_0$
$k = 100H_0$
$k = 200H_0$
ε_Λ
1
5
10
50
100
$z + 1$

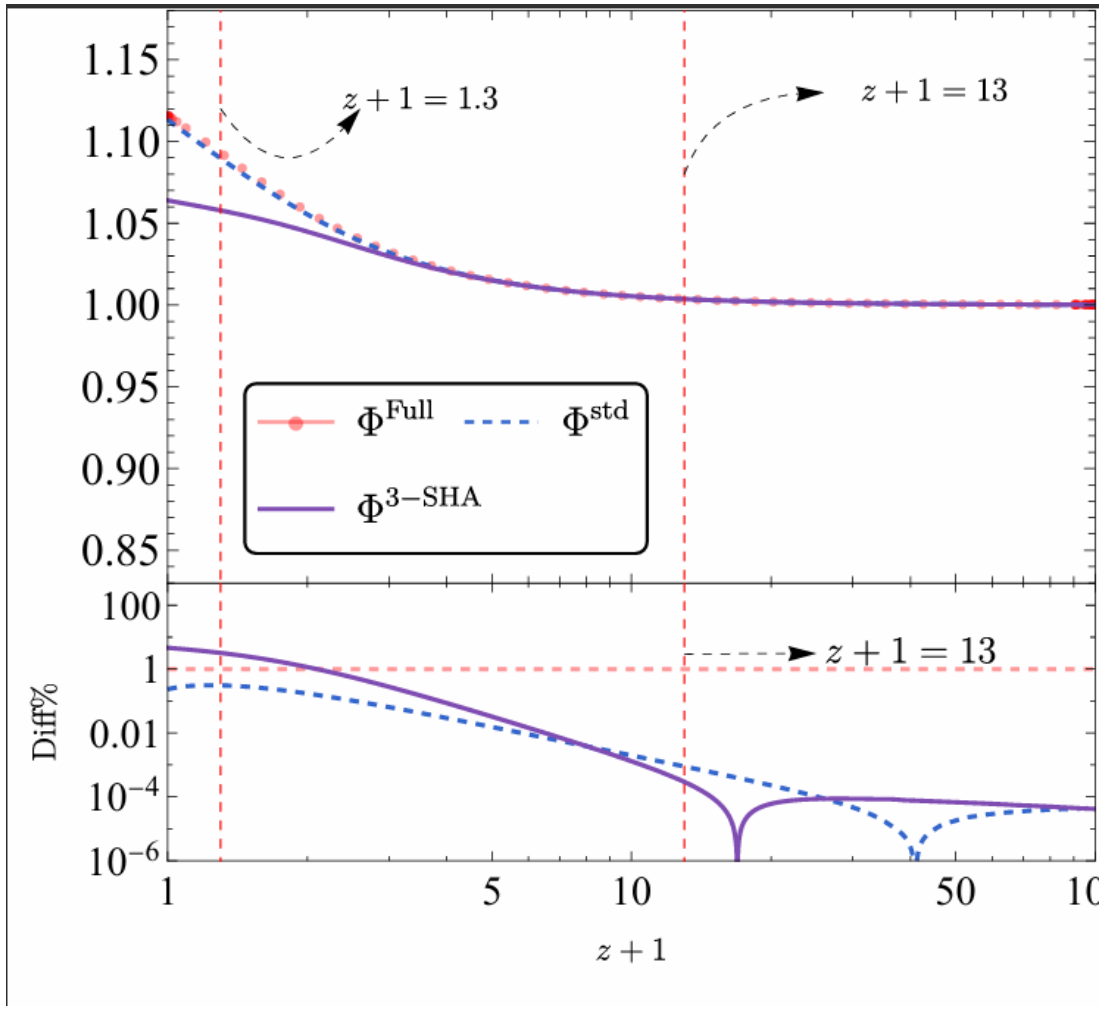

1
0.100
0.010
0.001
10^{-4}
$\mathcal{O}\left(\varepsilon_\Lambda^0\right)$
$\mathcal{O}\left(\varepsilon_\Lambda^1\right)$
$\mathcal{O}\left(\varepsilon_\Lambda^2\right)$
$\mathcal{O}\left(\varepsilon_\Lambda^3\right)$
$\mathcal{O}\left(\varepsilon_\Lambda^4\right)$
$\mathcal{O}\left(\varepsilon_\Lambda^5\right)$
$\mathcal{O}\left(\varepsilon_\Lambda^6\right)$
$\mathcal{O}\left(\varepsilon_\Lambda^7\right)$
1
5
10
50
100
500
1000
$z + 1$

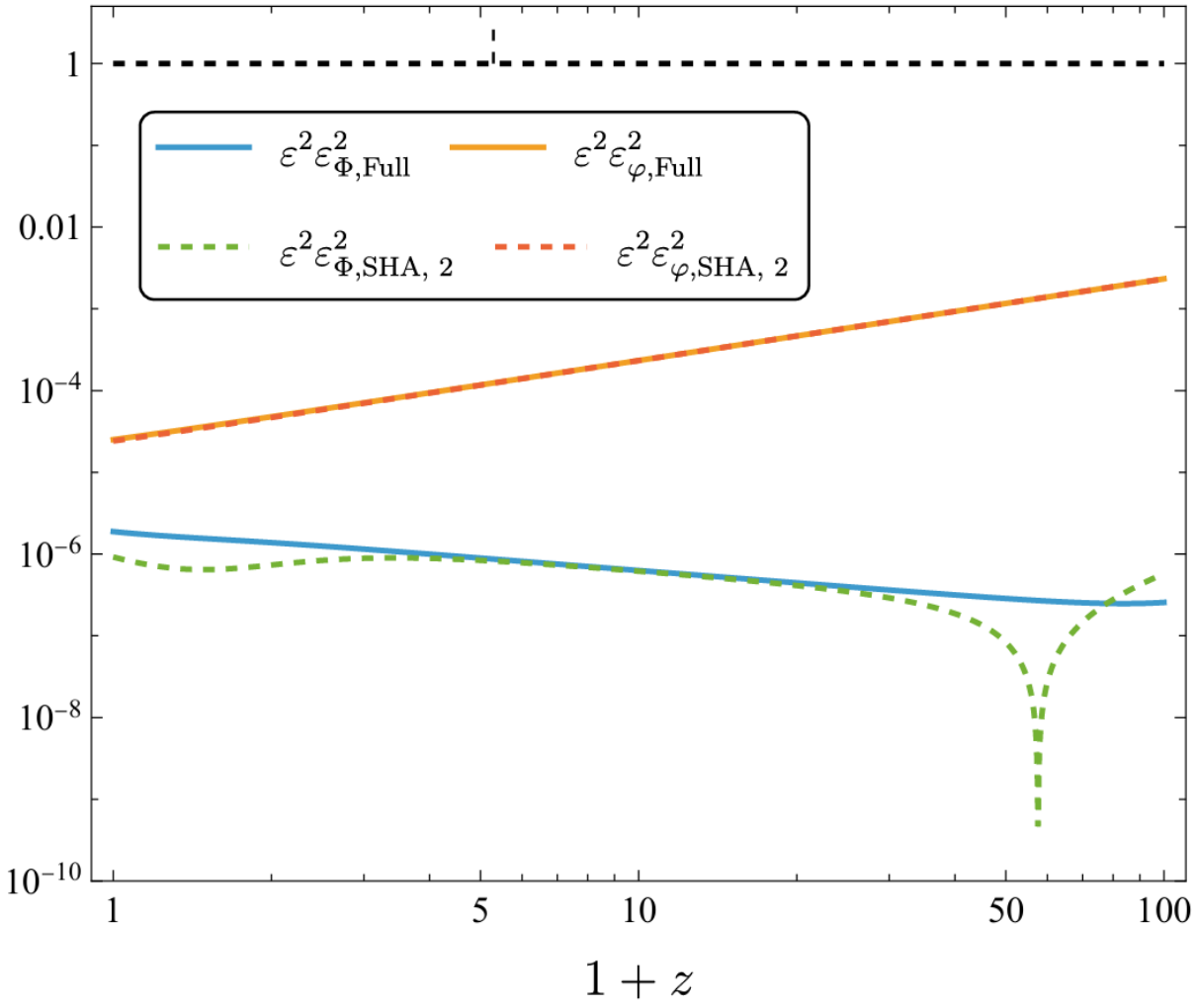




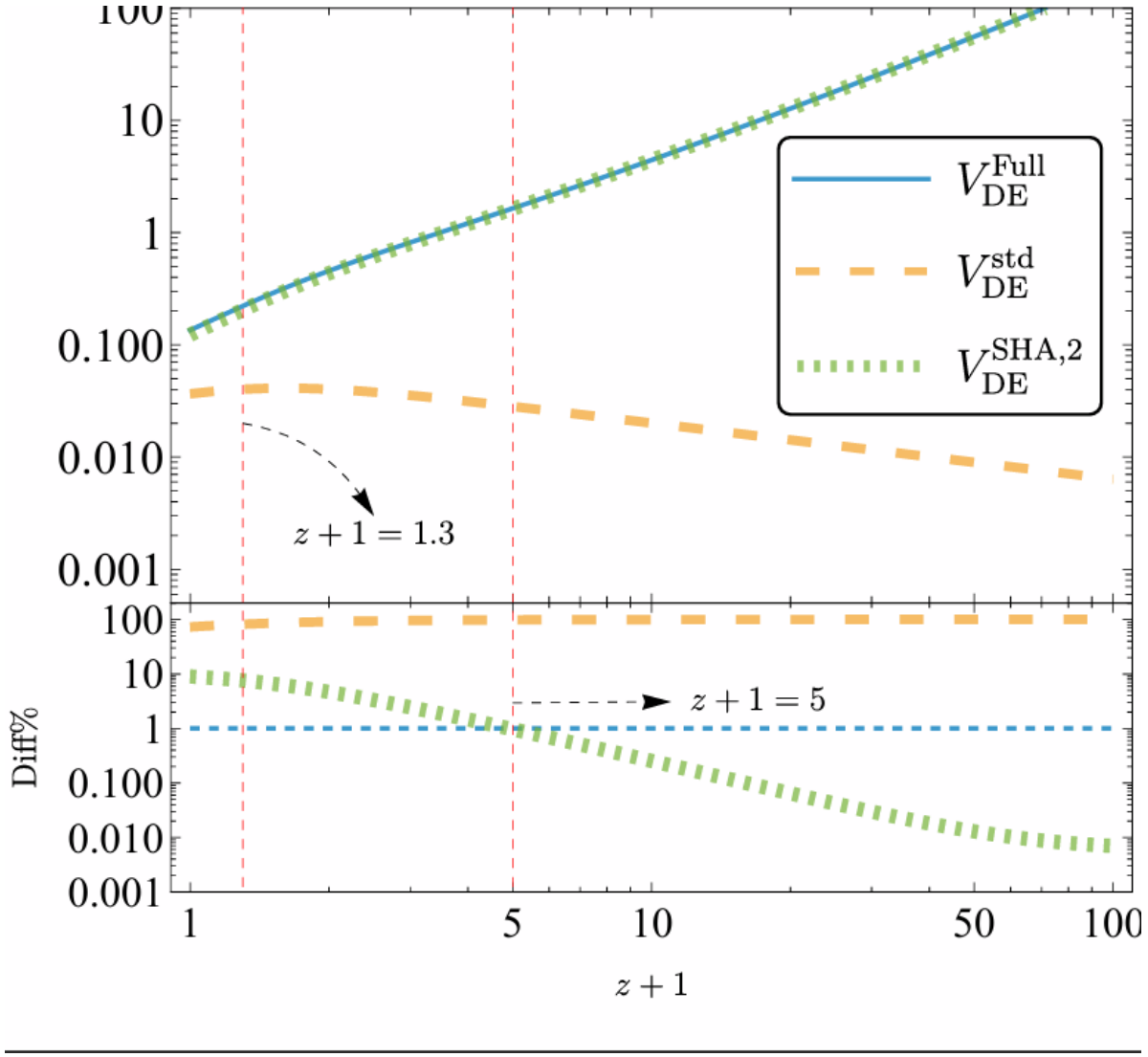
$$5 \lesssim z \lesssim 100$$

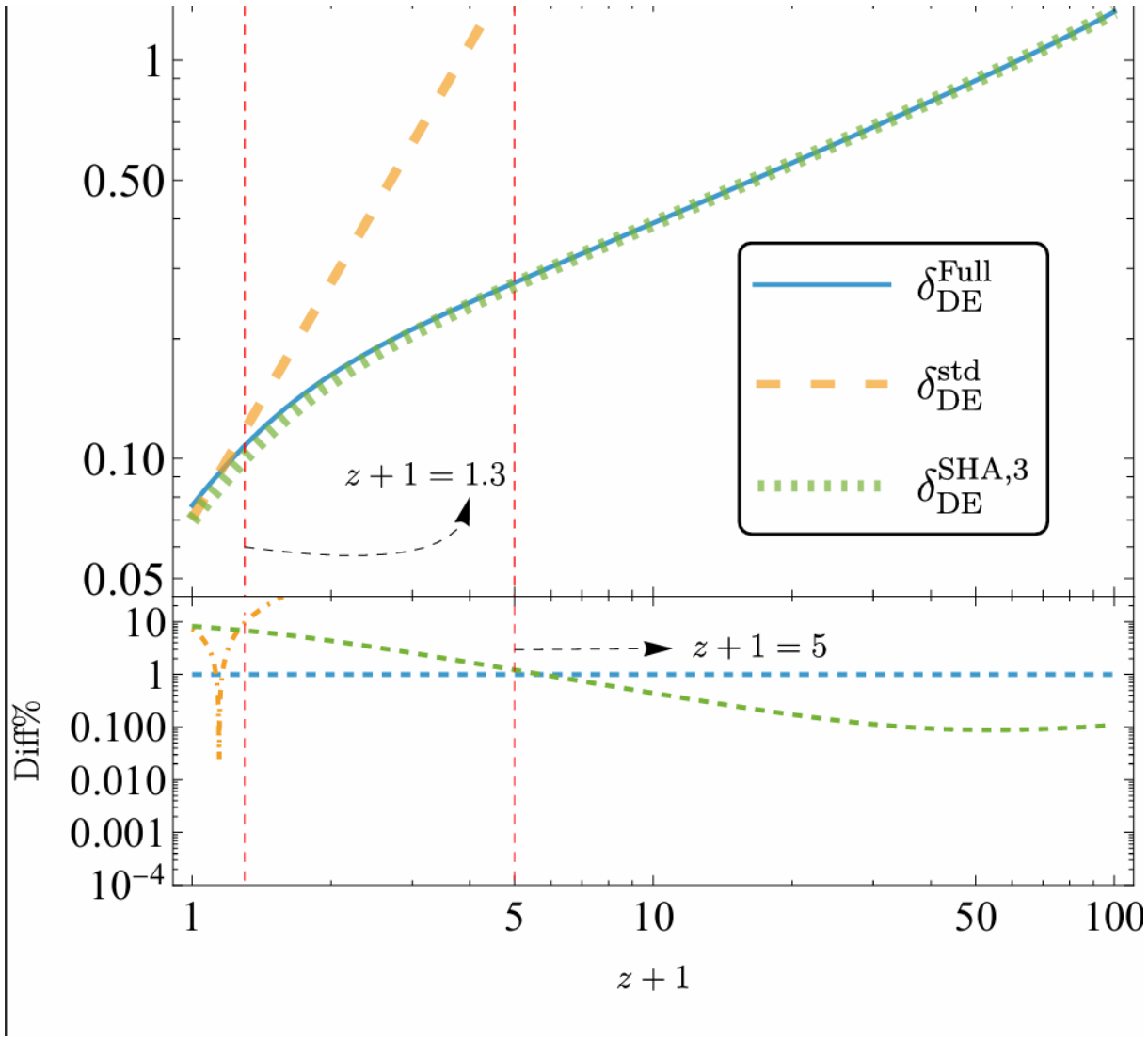
Consequences in the perturbations for HDES



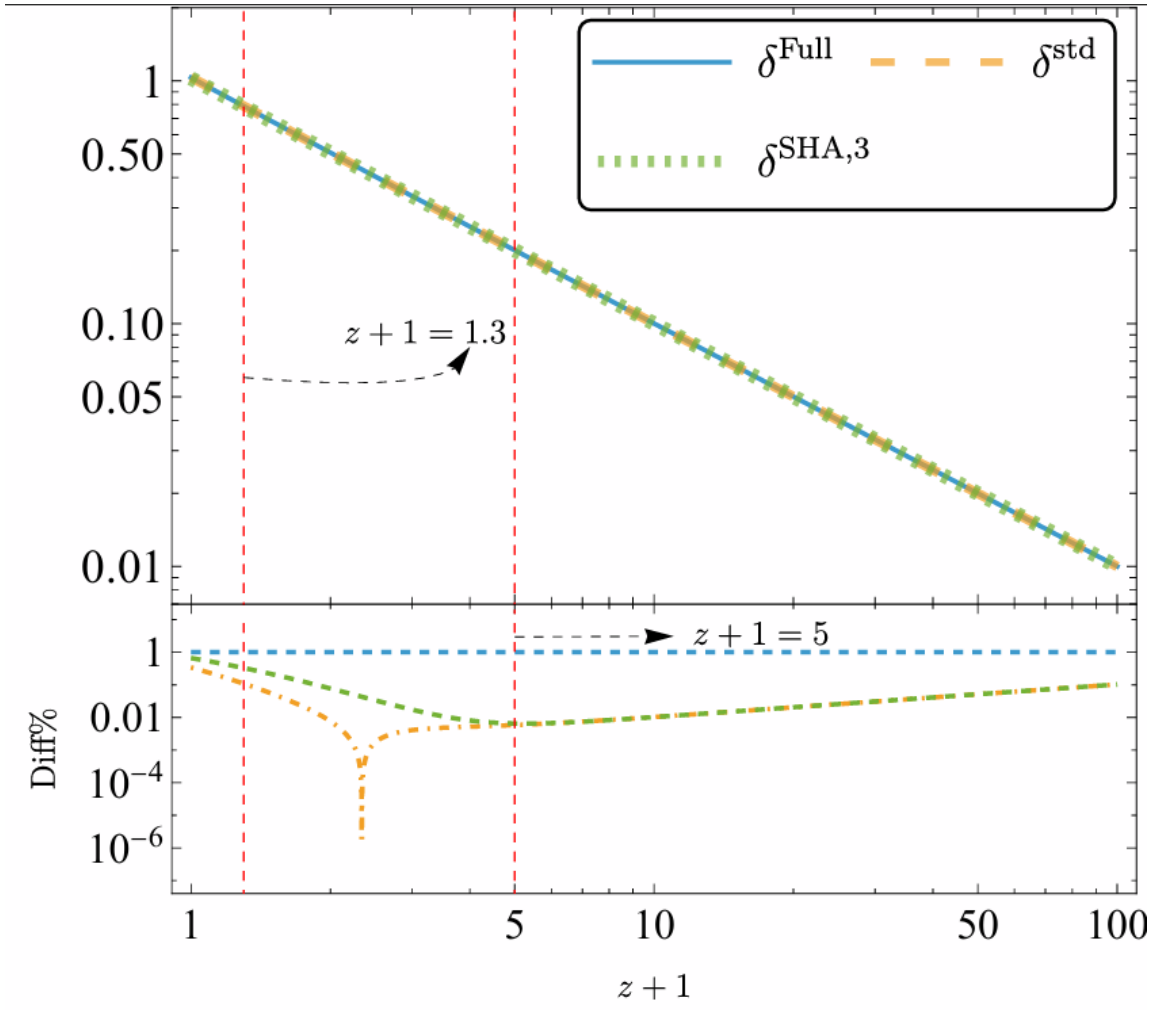


Consequences in the perturbations for HDES





Consequences in the perturbations for HDES




DESCUBRE EL CONOCIMIENTO
HECHO EN
UNIVALLE
QUE TRANSFORMA VIDAS