A Geometric* Approach to the Quantization of Gauged Nonlinear Sigma Models

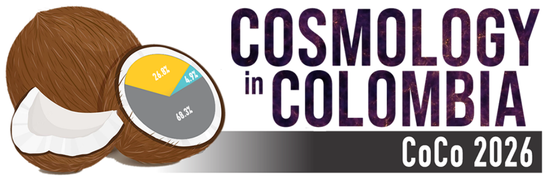
(*via Killing fields)

arXiv:2609.28992 [hep-th]

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Contents

- Non Linear σ Model (NLσM) Foundations
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- BRST Procedure (Geometry connection)
- Scenarios for potential applications

Pragmatic question

¿Is there a geometrical connection among fields representations susceptible to be crucial conceptually in highly symmetric QFTs with potential applications in particle physics (strong and EW sectors), cosmology and astrophysics for the dark sectors?

Non Linear σ Model (NLM) Foundations

Foundations NLσM

$$\mathscr{L} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \frac{\lambda}{4}\left(\Phi^T\Phi - \frac{\mu^2}{\lambda}\right)^2 .$$

N Fields

$\Phi^T = (\pi_1, \pi_2, \ldots, \pi_N, \sigma)$

π_{N+1}

@Gell-Mann y Lévy (60)

$$G = O(N+1) = \{R\left(\theta^\alpha\right) \in \mathbb{R}^{(N+1)} : R^T\left(\theta^\alpha\right) R\left(\theta^\alpha\right) = 1; \quad \theta^\alpha \in \mathbb{R}\}$$

@Coleman, Wess, Zumino (69), Weinberg (68)

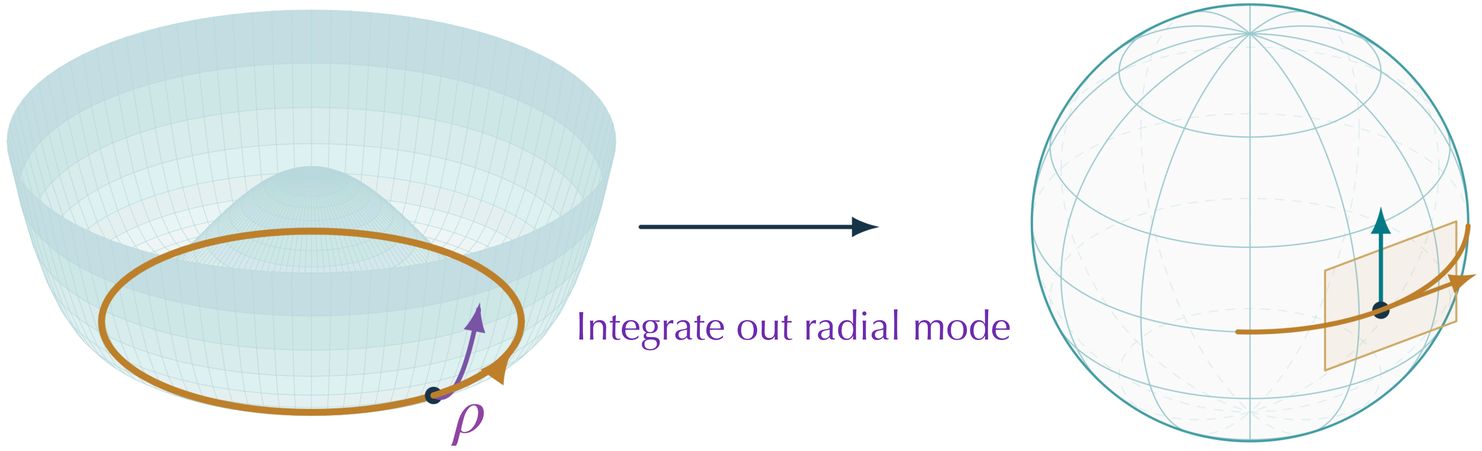
Foundations NLσM

Spontaneous symmetry breaking

$$O(N+1) \to O(N) \qquad G \to H \qquad G/H \simeq S^N$$

No Goldstone bosons = *dim*(*G*)-*dim*(*H*)

$O(4)/O(3) \simeq S^3$

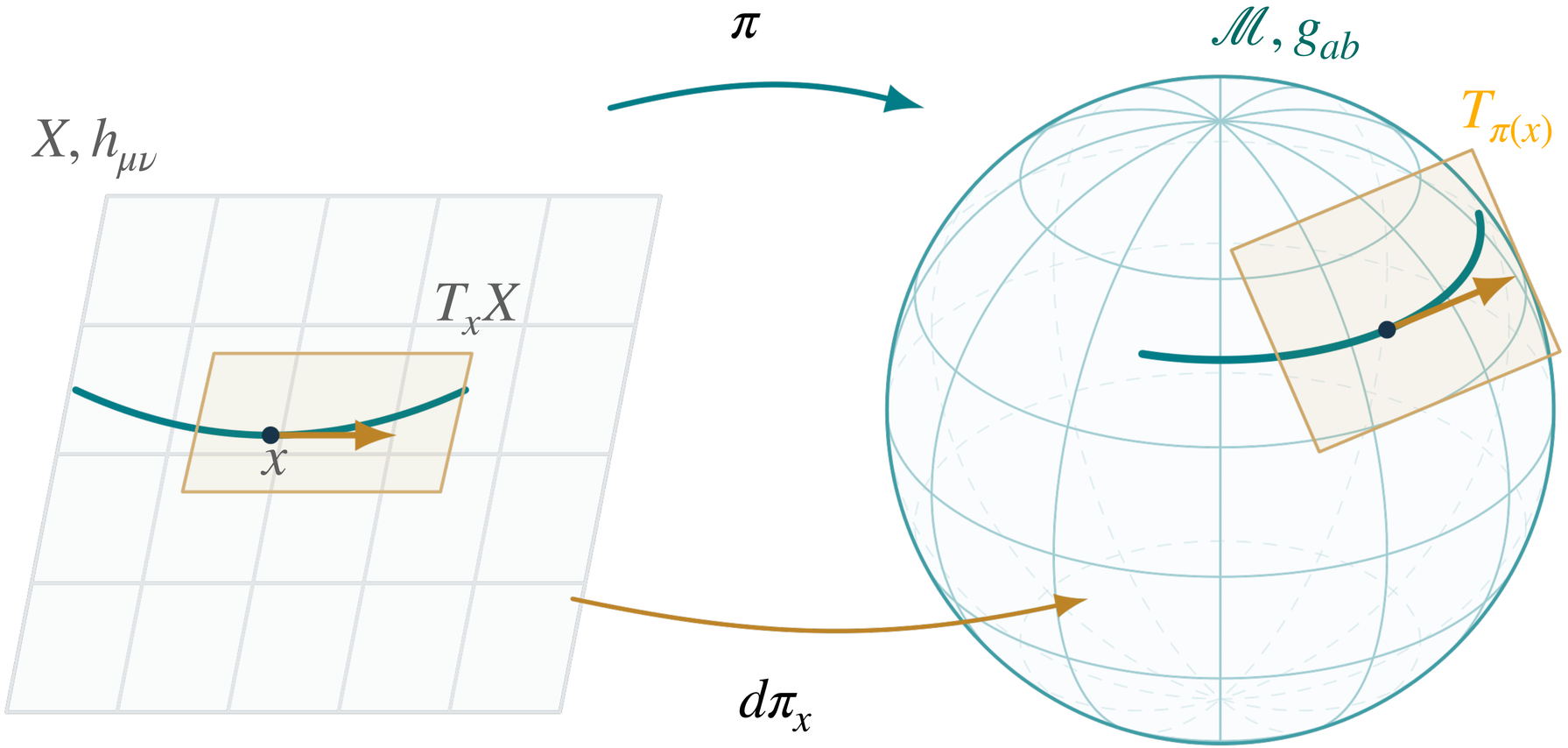


Radial mode (Higgs like)

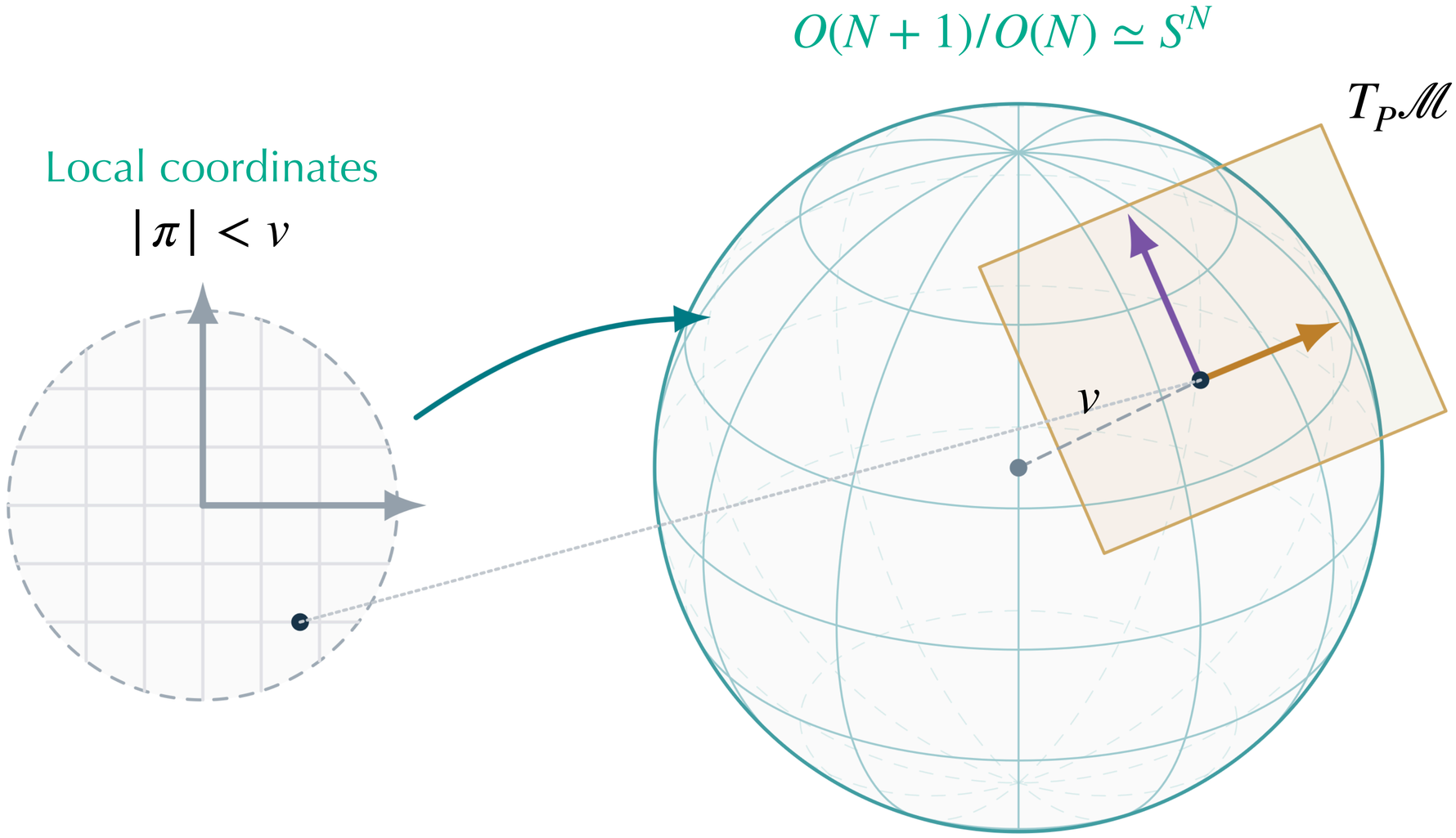
$SU(2)_L \times SU(2)_R/SU(2)_V \simeq S^3$

Foundations NLσM

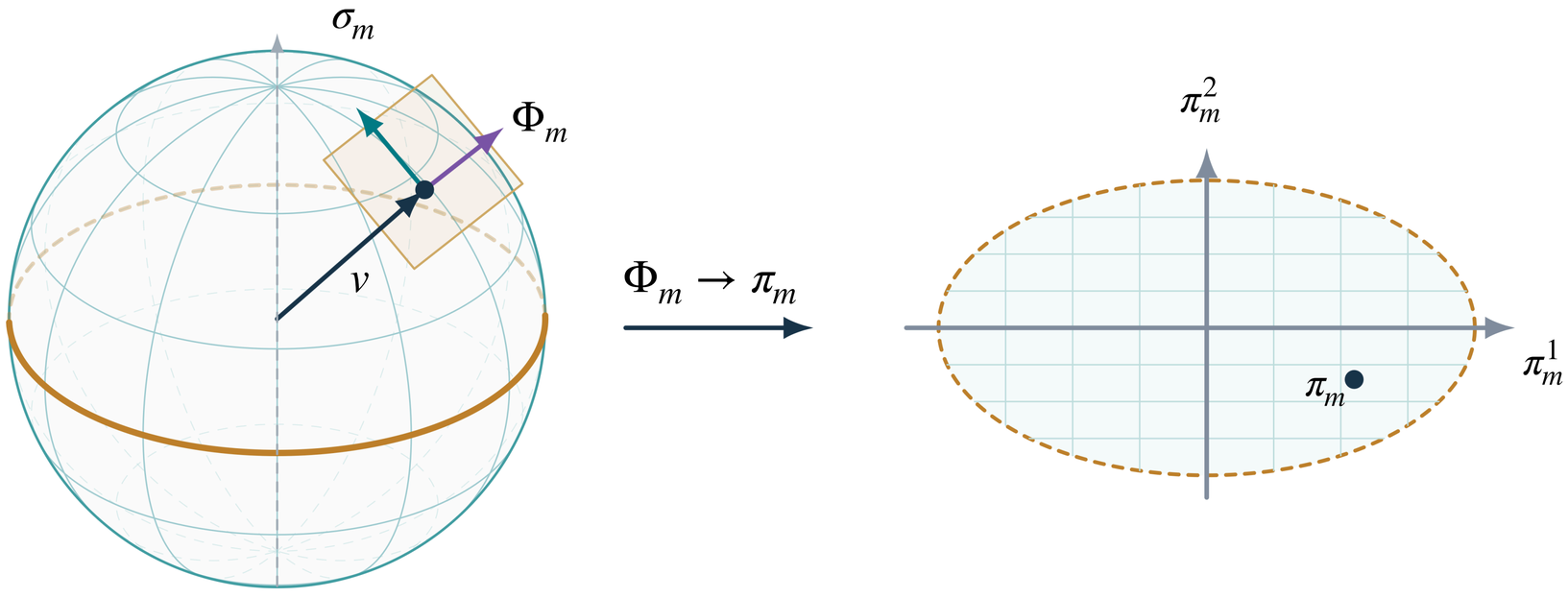
Target space: $\mathscr{M} \to G/H$



Killing fields closure relation



Foundations NLσM



$$\pi_m^T \pi_m + \sigma_m^2 = v^2$$

Constraint across minima

NLσM pions and metric appearance

$$\Phi^T = (\pi_1, \pi_2, \ldots, \pi_N, \sigma)$$

$$\mathscr{L} = \frac{1}{2} g_{ab} \partial_\mu \pi^a \partial^\mu \pi^b$$

$$g_{ab} \equiv \delta_{ab} + \frac{\pi_a \pi_b}{v^2 - |\pi|^2},$$

@Weinberg (68)

- The metric quantifies the kinetic cost of field variation in the target space

Motivations and historical developments

- Effective Field Theory for pions-Large N interactions for $\Lambda < \Lambda_{QCD}$

@Weinberg (68), t' Hooft (72), Witten (79)

- Effective Field Theory for $O(N)$ (for GUTS and Chiral Theories)

@Coleman, Wess, Zumino, Callan (69), Weinberg (68)

- Composite Higgs models $O(N+1)/O(N)$

@Georgi, Kaplan, Dimopoulos (84)

- Non linear realizations for topological induced models for baryons

@Skyrme (61-62), Witten (83)

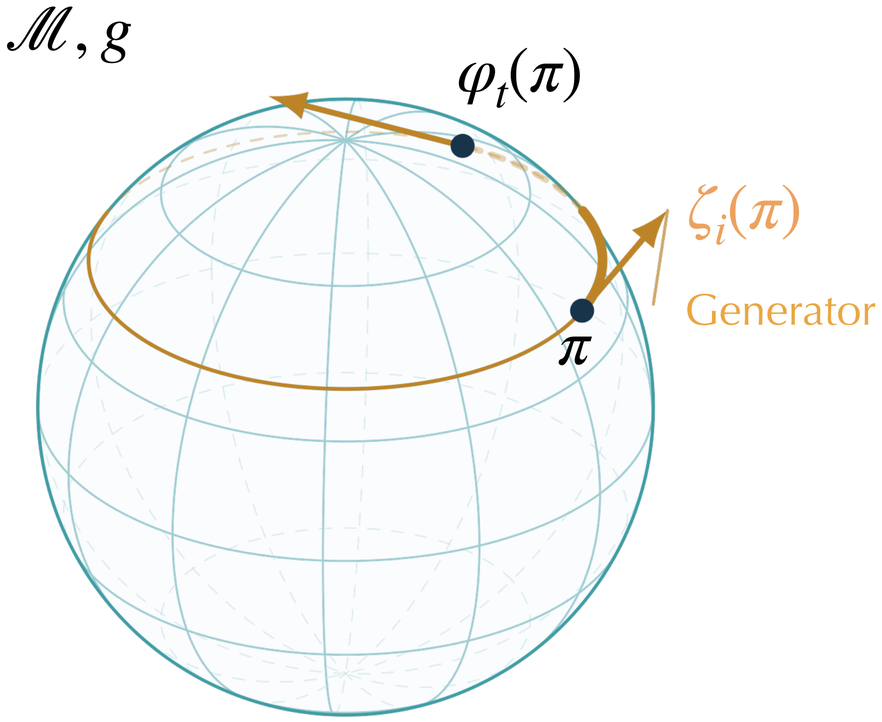
- Non linear realizations for topological dark matter models

@Murayama (09), Kitano (16) , Zurek (21)

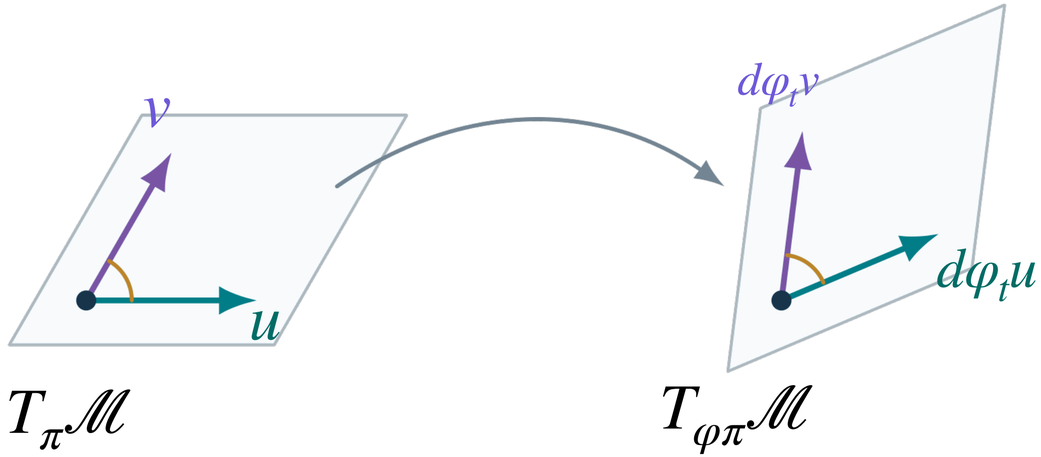
Killing fields description

Killing fields appearance

$$\mathscr{L} = \frac{1}{2} g_{ab} \partial_\mu \pi^a \partial^\mu \pi^b \qquad \mathscr{L}_{\zeta_i} g_{ab} = 0 \qquad \delta_\epsilon \pi^a = \epsilon^i \zeta_i^a(\pi)$$

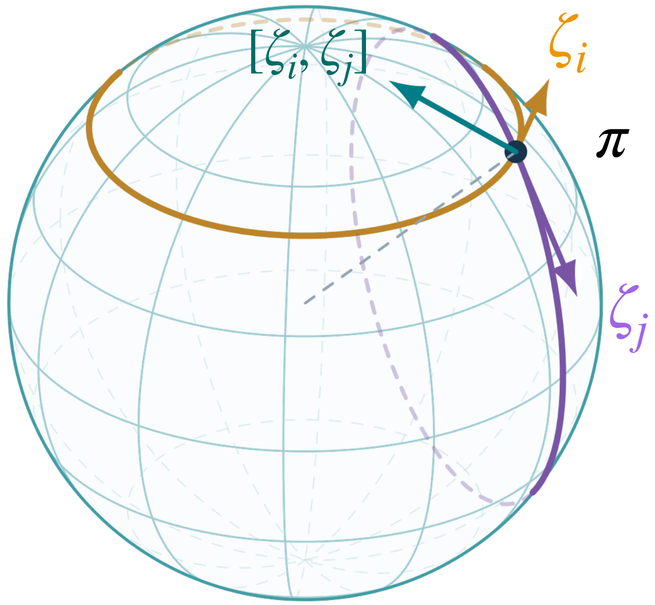


Killing vectors

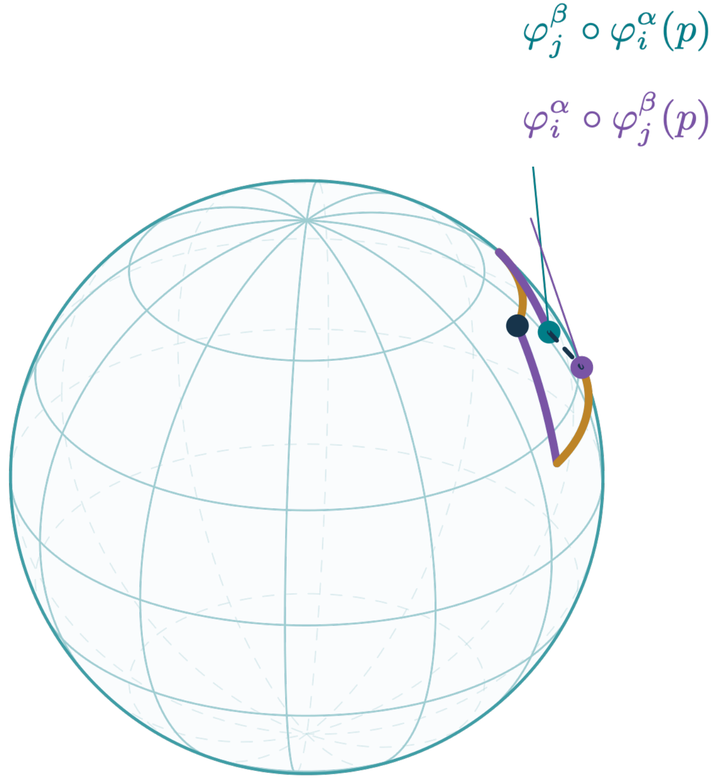


Killing vector fields generate the target-space field transformations under which the metric is invariant

Killing fields closure relation



$\zeta_I \in T_\pi \mathscr{M}$



$$[\zeta_i, \zeta_j]^a = \zeta_i^b \partial_b \zeta_j^a - \zeta_j^b \partial_b \zeta_i^a = f_{ij}{}^k \zeta_k^a \,.$$

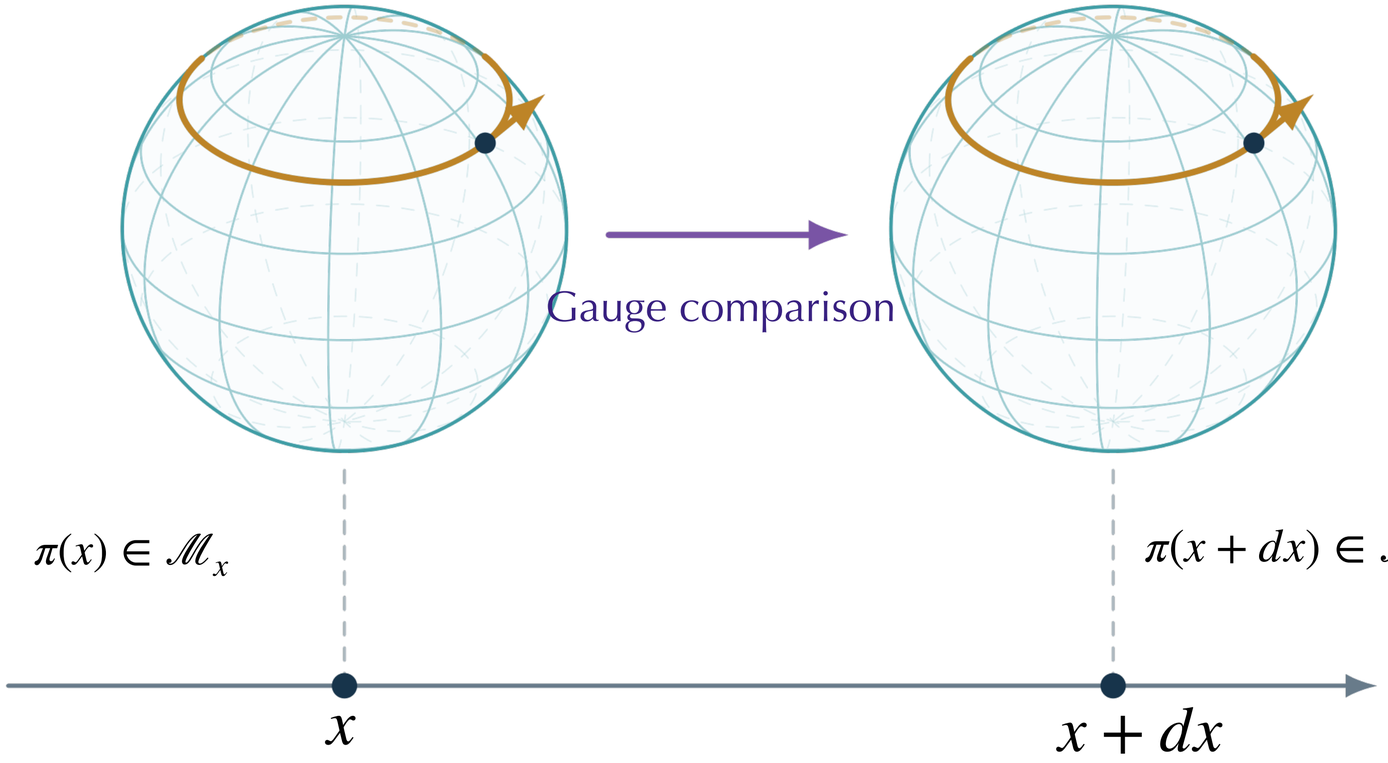
Closure relation

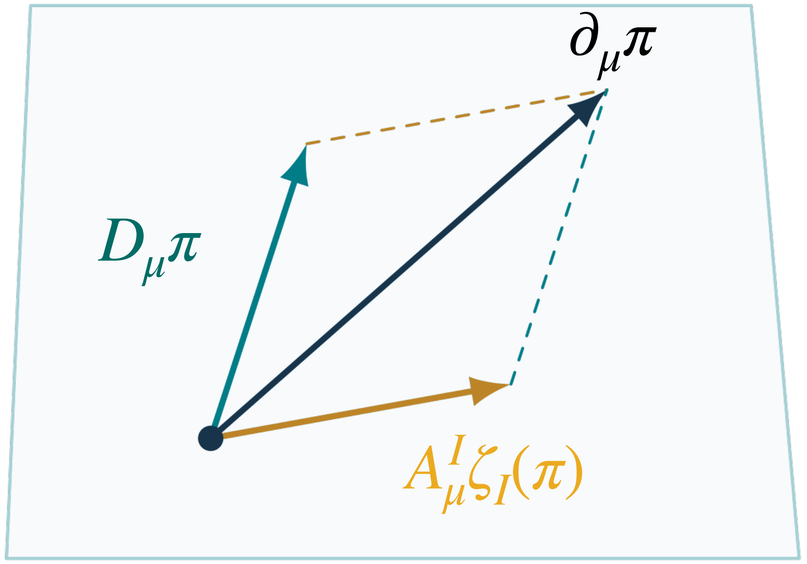
Gauging NLσM

Gauging NLσM

$$D_\mu \pi^a = \partial_\mu \pi^a - g A^I_\mu \zeta^a_I(\pi).$$

$$\mathscr{L}_{GB} = \frac{1}{2} g_{ab}(\pi) D_\mu \pi^a D^\mu \pi^b - \frac{1}{4} F^I_{\mu\nu} F_I^{\mu\nu}.$$





@Torres, Morales, Castillo (BJP 25)

Gauge fixing and ghosts

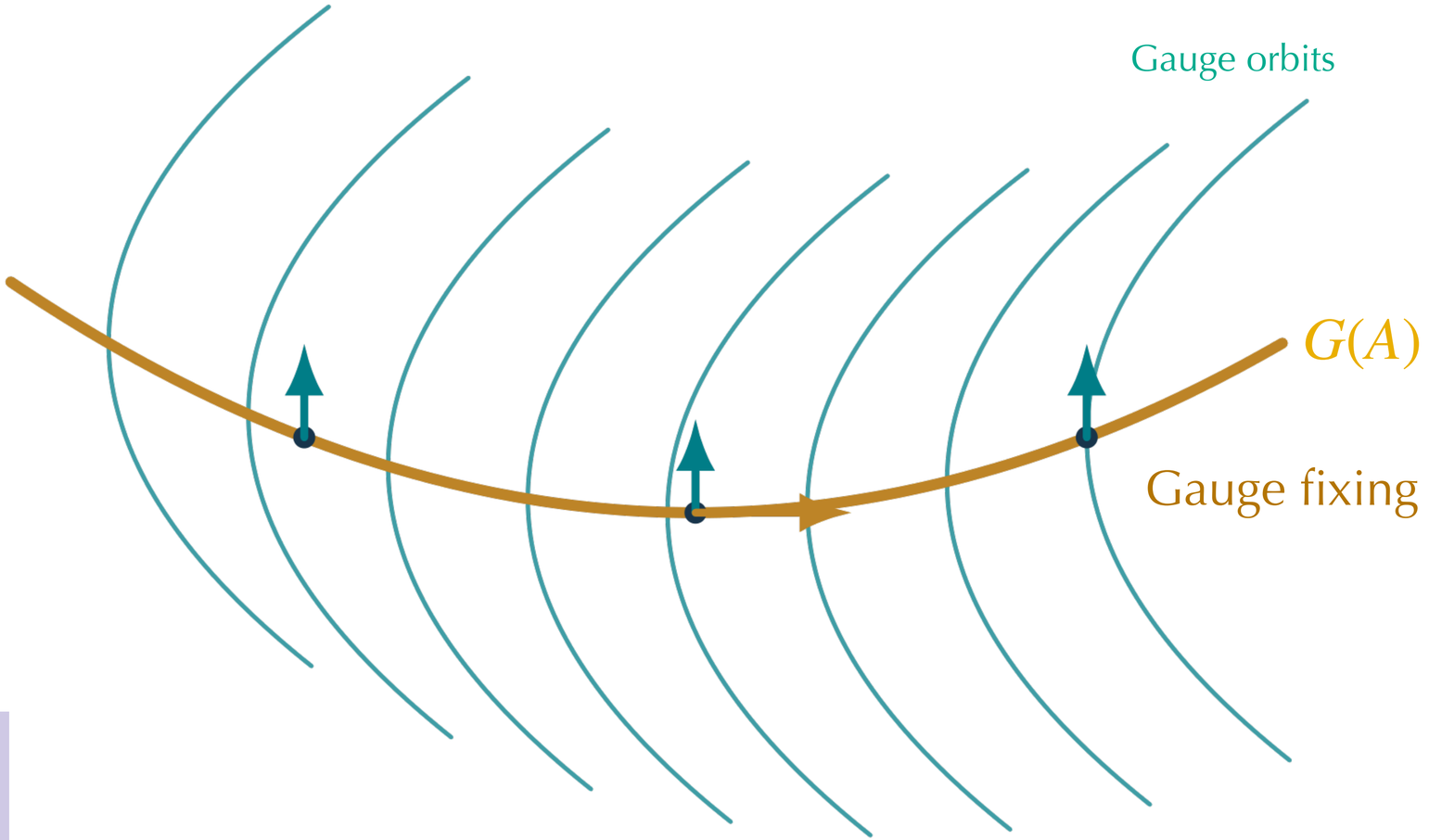
Gauge fixing and ghosts

$$Z = \int [\mathscr{D}A]\, e^{i\int d^4x\, \mathscr{L}_{YM}(A)}$$

Redundances on Z

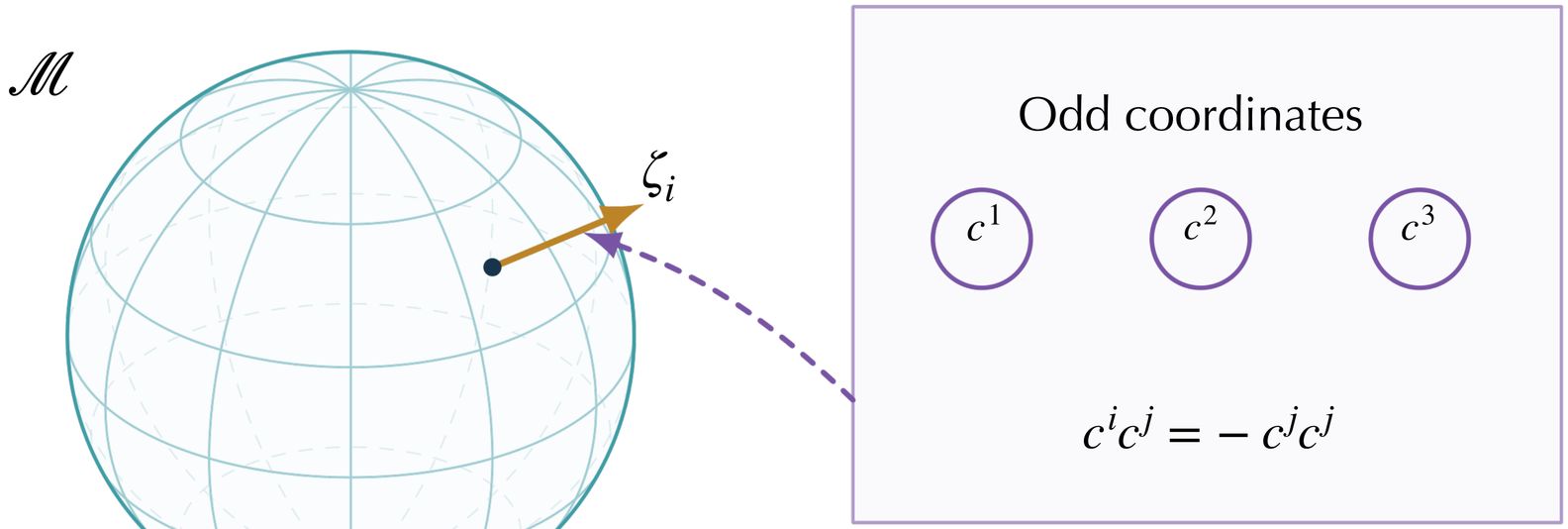
$$1 = \int [\mathscr{D}\theta]\, \delta\big(G(A^\theta)\big)\, \det\left(\frac{\delta G(A^\theta)}{\delta\theta}\right)$$

$c(\bar{c})$ Grasmman variables



Gauge fixing and ghosts

$$\mathscr{L}_{ext} = \frac{1}{2} g_{ab} D^{\mu}\pi^{a} D_{\mu}\pi^{b} - \frac{1}{4} F^{I}_{\mu\nu} F_{I}^{\mu\nu} - \frac{(\partial^{\mu} A^{I}_{\mu})^{2}}{2\xi} - \bar{c}_{I}(\partial^{\mu} D^{I}_{\mu J}) c^{J} \,.$$



$$\bar{c}^{i} \xrightarrow{s} b \xrightarrow{s} 0 \qquad s^2$$

(Cohomology)

BRST procedure

@Becchi-Rouet-Stora-Tyutin (70´s)

BRST procedure

$$-\frac{1}{2\xi}(\partial^\mu A^I_\mu)^2 \quad \longrightarrow \quad \frac{\xi}{2}B_I B^I + B_I \partial^\mu A^I_\mu,$$

B: Nakanishi-Lautrup Auxiliary Field

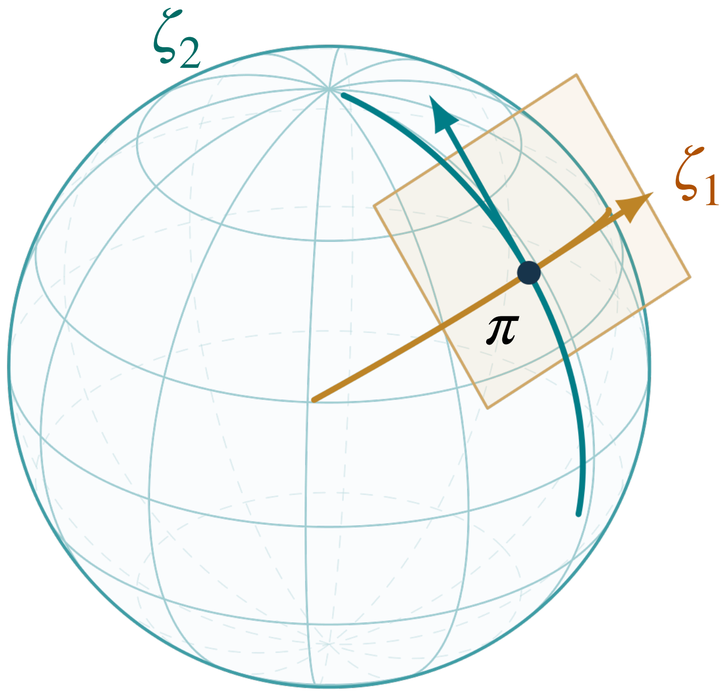
$$\mathscr{L}_{ext} = \frac{1}{2}g_{ab}D^\mu \pi^a D_\mu \pi^b - \frac{1}{4}F^I_{\mu\nu}F_I^{\mu\nu} + \frac{\xi}{2}B_I B^I + B_I \partial^\mu A^I_\mu - \bar{c}_I(\partial^\mu D_{\mu J}{}^I)c^J \,.$$

The extended Lagrangian linearizes the gauge-fixing term via the Nakanishi-Lautrup auxiliary field, making the exact BRST invariance of the coupled non-linear sigma model explicit.

BRST transformation

$$\delta_{\mathrm{BRST}}\pi^a = g\epsilon\, c^I(x)\zeta_I^a(\pi)$$

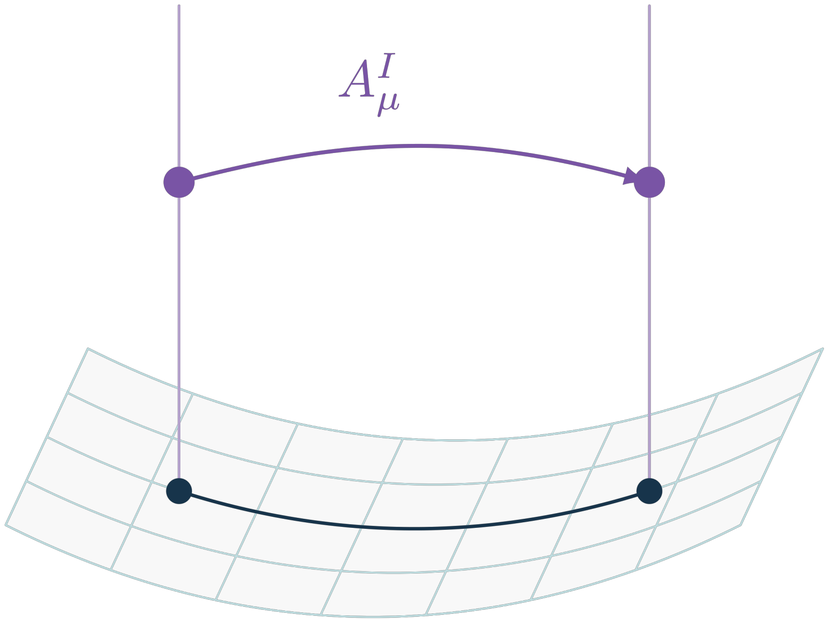
$$\delta_{\mathrm{BRST}}() = \epsilon s$$



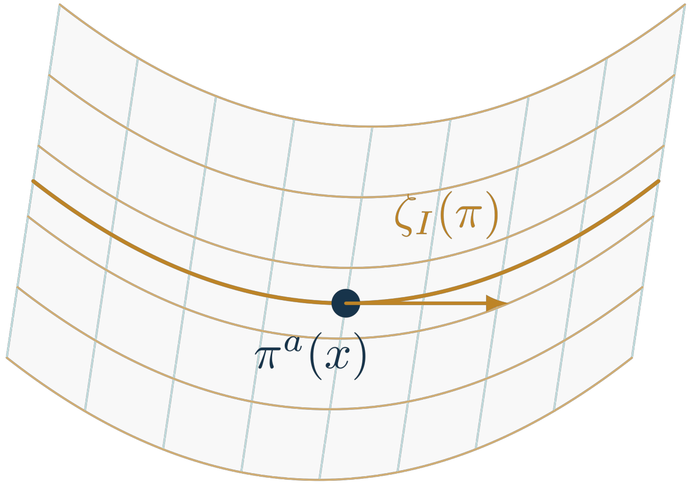
$$\zeta_I \in T_\pi \mathscr{M}$$

- Mapping local gauge parameters to the product of FP ghost field along the Killing vector.

BRST transformations (all fields)



$$sA^I_\mu = (D_\mu c)^I$$



$$s\pi^a = g c^I \zeta^a_I$$

$$sc^I = -\frac{g}{2} f_{JK}{}^I c^J c^K$$

$$s\bar{c}_I = B_I$$

$$sB^I = 0$$

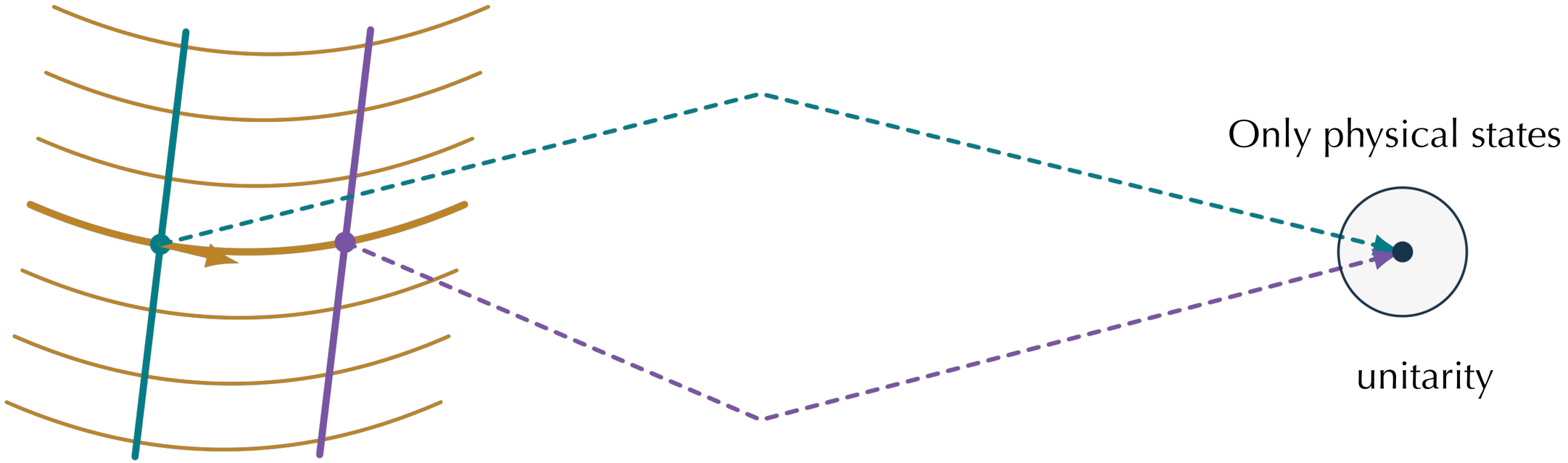
- Global symmetry across fields multiplets: j^μ, Q

BRST transformations

Nilpotency

$$s^2 A_\mu^I = s^2 c^I = s^2 \bar{c}_I = s^2 B^I = s^2 \pi^a = 0$$

Gauge orbits



Configuration space

@BRST (74, 75), Kugo y Ojima (78), Henneaux y Teitelboim (92), Barnich, Brandt y Henneaux (00

Geometric BRST

Geometric BRST

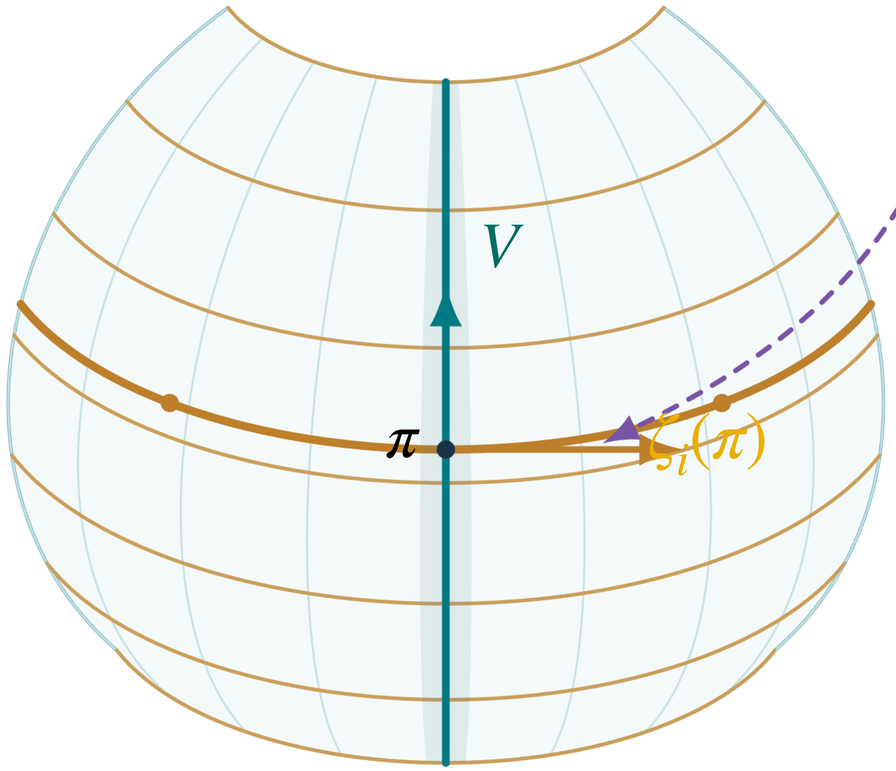
$$s^2\pi^a = -\frac{g^2}{2} c^I c^J \left(f_{IJ}{}^K \zeta_K^a - [\zeta_I, \zeta_J]^a \right) = 0.$$

@Torres, Morales, Castillo (26)

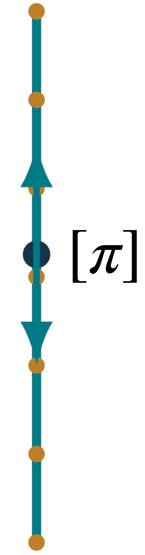
Ghost c^I

Grasmman odd

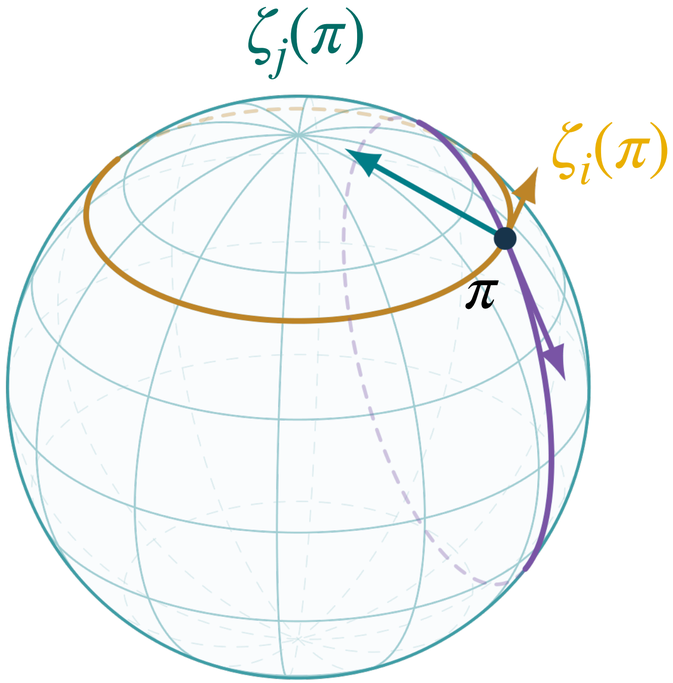
Local orbit space

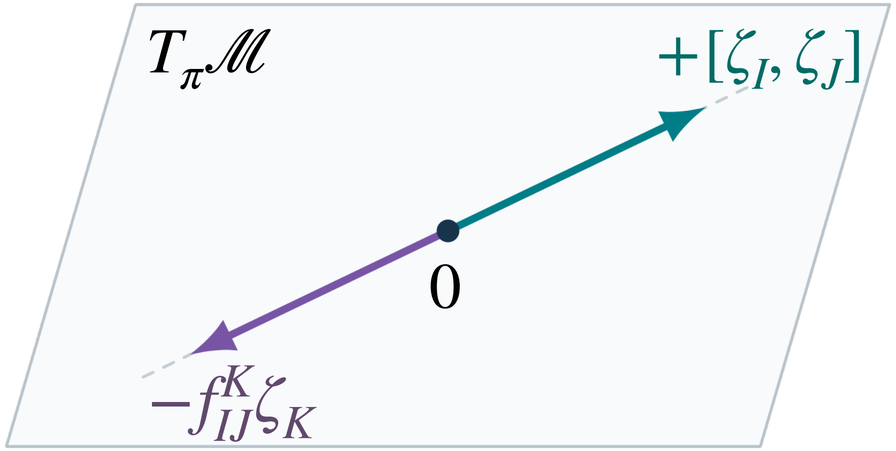


Orbit $K \cdot \pi$



Geometric BRST





$$s^2\pi^a = -\frac{g^2}{2}c^I c^J \left(f_{IJ}{}^K\zeta_K^a - [\zeta_I, \zeta_J]^a\right) = 0.$$

Main remark so far

Exploiting Killing vectors and their algebraic properties on target space leads to the geometric interpretation of gauge orbits and dictaminate the BRST transformations and its nilpotency (cohomology).

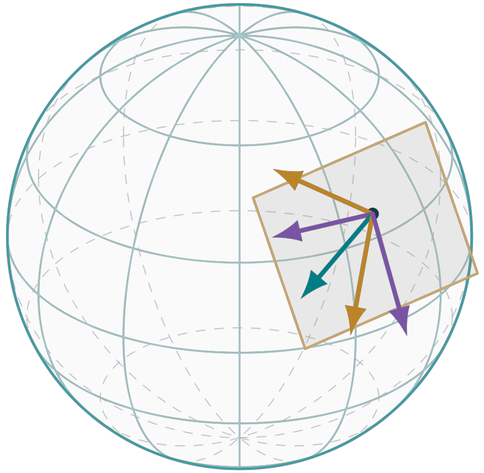
Scenarios for applications

Scenarios

- Large N models

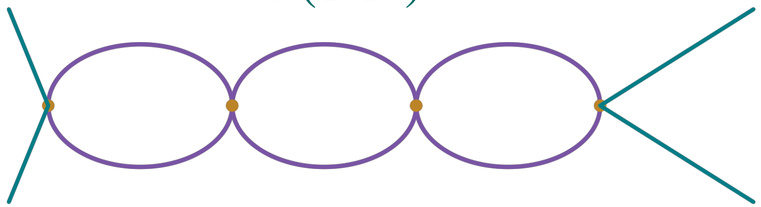
stay tuned!

N tangent models



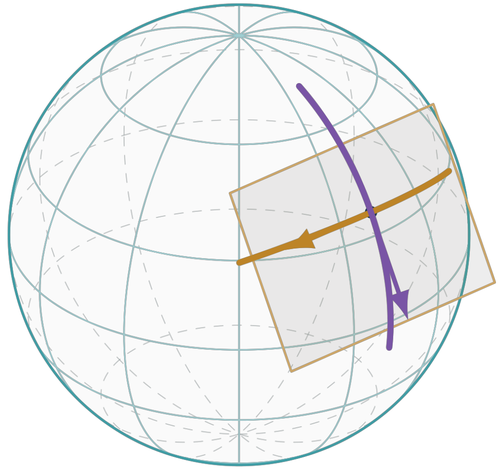
Loop expansion

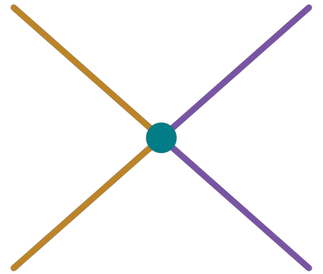
$O(1/N)$



+external fields

- Self (geometric) interactions





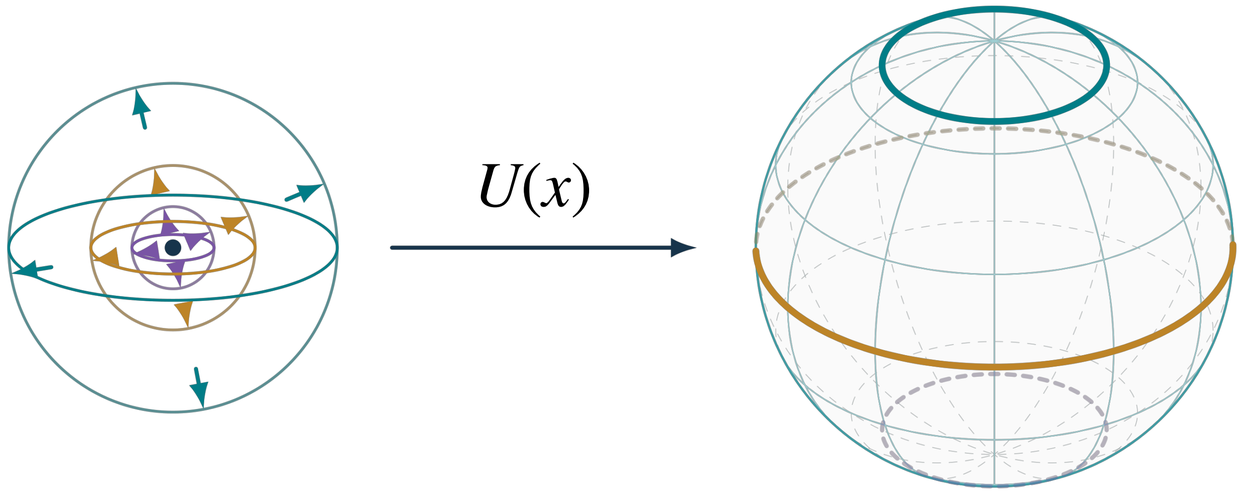
$O(p^6)$

$O(p^4)$

$O(p^2)$

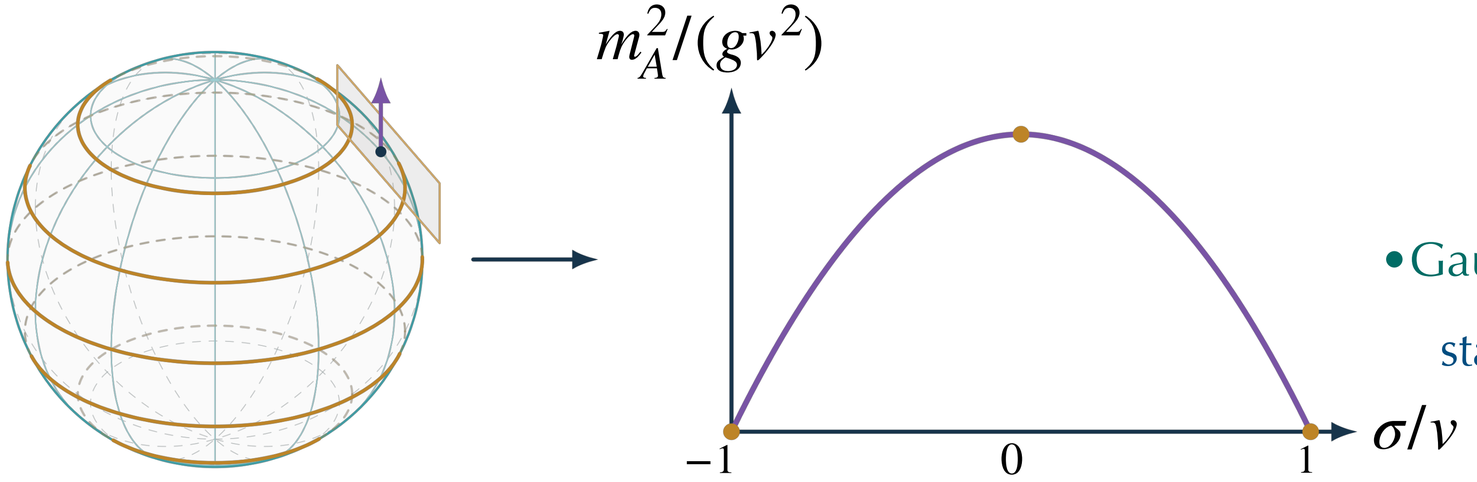
+external fields

Scenarios for applications (some prospects)



- Skyrme model for baryons and DM terms with NLσM and mediators

  stay tuned!



- Gauge portals determined by Killing fields

  stay tuned!

Concluding remarks

Concluding remarks

- Consistent interpretation of NLSM with Killing vectors and target space
- Gauge construction of the NLSM with isometries on the target space
- BRST procedure to quantize in consistent way the gauged NLSM
- Geometric BRST transformation for the goldstone bosons and gauge bosons
- Killing vectors on target space and their closure relations defines couplings and physical cohomology
- Potential applications to HEP (composite models), astrophysics & cosmology (DM)

Collaborators



John Morales & Teyrungunmu Torres

Gracias!