

A melonic quantum mechanical model without disorder

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Leinweber Network Workshop on Quantum Black Holes

arXiv:2601.08908



Motivation

- SYK: solvable at large N , nearly conformal in the IR, Schwarzian, maximal chaos. But the couplings are random.
- Tensor models get melons without disorder [Gurau–Witten; Klebanov–Tarnopolsky], but are hard to diagonalize numerically.
- Here: fermions in spin j rep of $SU(2)$, coupled by the $3j$ symbol. Similar models were studied as bosonic field theories in higher d [Amit–Roginsky '79; Benedetti–Delporte '20].
 - no disorder
 - melonic at large j
 - symmetry blocks are small, so exact diagonalization up to $j = 11$ ($N = 23$ fermions)
- Exponentially many BPS states, and for large $SU(2)$ charge they have a simple description.

The model

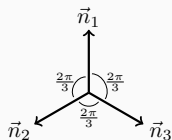
$N = 2j + 1$ complex fermions ψ_m , $m = -j, \dots, j$, in the spin j rep (j odd).

$$Q = \frac{\sqrt{2jN}}{3!} \sum_{m_i} \begin{pmatrix} j & j & j \\ m_1 & m_2 & m_3 \end{pmatrix} \psi_{m_1} \psi_{m_2} \psi_{m_3}, \quad H = \{Q, Q^\dagger\}$$

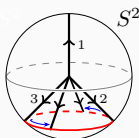
- $\mathcal{N} = 2$ SUSY, symmetry $SO(3) \times U(1)_R$, with $R = N_\psi/3$.
- Picture: lowest Landau level on S^2 with N flux quanta (fuzzy sphere). H is a strange non-local interaction on this sphere.
- Witten index (plus numerics): 3^j BPS states at each of $R = \pm \frac{1}{6}$, spread over a wide range of spins.

Melons

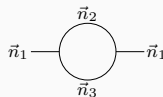
At large j , ψ_m behaves like a classical spin pointing along $\vec{n} \in S^2$. The $3j$ symbol forces $\vec{n}_1 + \vec{n}_2 + \vec{n}_3 = 0$ at every vertex.



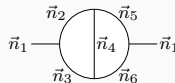
(a)



(b)



(c)



(d)

- In a melon the inner arrows can still rotate (b,c); in other diagrams they are stuck (d). So melons dominate.
- The Schwinger–Dyson equations are exactly those of $\mathcal{N} = 2$ SYK [Fu, Gaiotto, Maldacena, Sachdev '16].

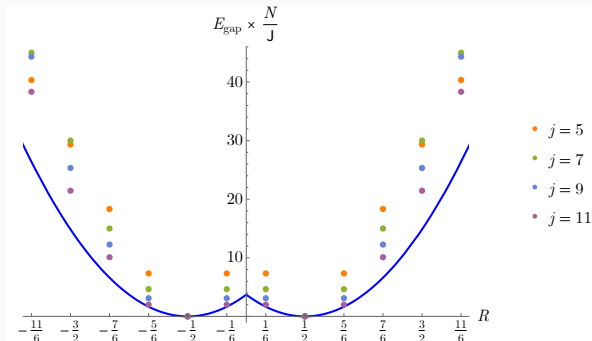
Low energies, small charges

Same IR as $\mathcal{N} = 2$ SYK [Fu, Gaiotto, Maldacena, Sachdev '16]:

$$\Delta_\psi = \frac{1}{6}, \quad \Delta_b = \frac{2}{3}$$

$$E_{\text{gap}}(R) = \frac{J}{8N\alpha_S} \left(|R| - \frac{1}{2} \right)^2$$

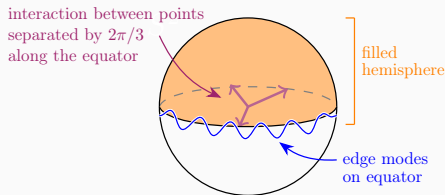
$$\alpha_S \approx 0.0084 \text{ [Heydemann, Turiaci, Zhao '22]}$$



2d CFT description near maximal spin

- Filled northern hemisphere, chiral edge modes on the equator. Bosonize, $\psi = e^{i\phi}$.
- For $|m| \ll j$ the $3j$ symbol couples points $2\pi/3$ apart:

$$Q \propto \int d\varphi e^{i[\phi(\varphi) + \phi(\varphi + \frac{2\pi}{3}) + \phi(\varphi - \frac{2\pi}{3})]}$$



- Write $\phi = \phi_z + \phi_{\pm}$ with $\phi_z = \sum_k \alpha_{3k} e^{i3k\varphi}$, $\phi_{\pm} = \sum_k \alpha_{3k\pm 1} e^{i(3k\pm 1)\varphi}$. Then $H \propto \int d\varphi (\partial\phi_z)^2$. The $\phi_{\pm}$ cost no energy, but they change J_3 .
- BPS states = ground state of $\phi_z \otimes$ anything in $\phi_{\pm}$:

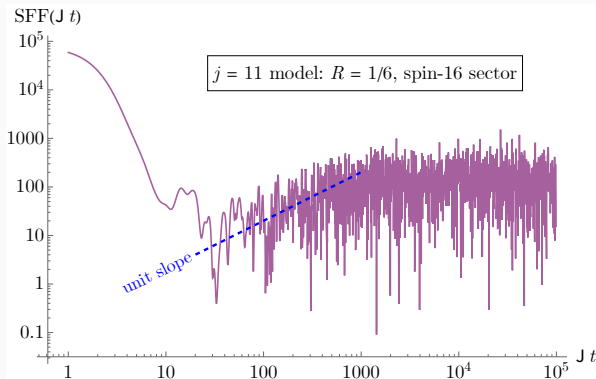
$$\text{Tr}_{\text{BPS}} q^{J_3^{\text{max}} - J_3} = 2 \prod_{k \geq 0} \frac{1}{(1 - q^{1+3k})(1 - q^{2+3k})}$$

Spectral form factor of the
non-BPS levels in one (R, ℓ) block:

$$\text{SFF}(t) = \left| \sum_i e^{-\lambda_i^2/w^2} e^{-i\lambda_i t} \right|^2$$

Ramp and plateau, as for a
chaotic system.

However, too low j to determine
Thouless time $\mathcal{O}(1)$ vs $\mathcal{O}(N)$.



Summary and future directions

- Fermions in a spin j rep with a 3j-symbol interaction.
- Same large N equations as $\mathcal{N} = 2$ SYK.
- Near maximal spin it becomes a 2d CFT.

Future directions:

- Most BPS states are at intermediate-sized $SU(2)$ charge, where we have no explicit description. Are they chaotic?
- Low energy action for $SU(2)$ mode? Can't be the usual $\int dt \text{Tr} (g^{-1} \dot{g})^2$ one.
- Generalizations: other groups, non-SUSY versions, $1 + 1$ dimensions...

Backup

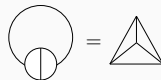
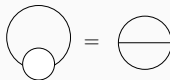
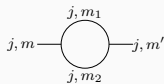
Why melons dominate

- The $3j$ bubble is $\propto \delta_{mm'}/N$, so each melon gives a factor J , independent of N . All melons agree with $\mathcal{N} = 2$ SYK.
- Count arrow configurations. A vacuum diagram has L lines, V vertices, $2L = 3V$.

$$N_{\text{free}} = 2L - 3(V - 1) = 3$$

which is just the overall rotation. A generic diagram has no extra freedom.

- A melon bubble can rotate about the external arrow: a circle on S^2 , worth $\sim \sqrt{j}$ states. Each bubble gets this enhancement.
- Explicitly: tetrahedron $\sim 6j$ symbol, down by $1/\sqrt{j}$ (not in our model). The $9j$ diagram vanishes. First correction is the cube ($12j$ symbol), down by $\log j/j$.



Operators with $SU(2)$ spin

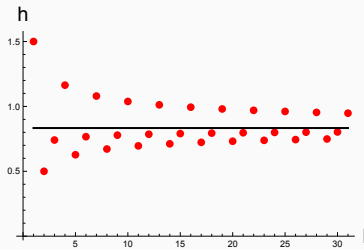
Bilinears in a spin ℓ rep,

$$O_m^\ell \sim \begin{pmatrix} j & j & \ell \\ m_1 & -m_2 & m \end{pmatrix} \psi_{m_2}^\dagger \partial^k b_{m_1}.$$

Ladder kernel = $\mathcal{N} = 2$ SYK kernel $\times$ a $6j$ ratio,
which goes to $P_\ell(-\frac{1}{2})$ at large j :

$$1 = P_\ell(-\frac{1}{2}) \kappa(h)$$

- $\ell = 0$: $h = \frac{3}{2}$, the Schwarzian mode.
- $\ell = 1$: $h = \frac{1}{2}$, multiplet of the $SU(2)$ current.
- Large ℓ : $h \rightarrow \Delta_\psi + \Delta_b$, two separate particles.
Not like KK modes on an S^2 .



Lowest h vs. ℓ

Exact diagonalization near maximal spin for $j = 11$

