

Chasing M-Theory Saddles with the 3d Superconformal Index

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- ▶ Also at **small** finite N (unrefined index)
- ▶ **Captures new phases:** grey galaxies

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Goal: Dressed black holes (DBH) at finite- N ?

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Specialization:

$$\mathcal{I}_N(x, y) = \text{Tr}_{\text{BPS}} \left[(-1)^F x^Q y^h \right] = \sum_{Q, h} d(Q, h) x^Q y^h$$

$$Q \equiv 4J + \sum_{l=1}^4 Q_l, \quad h \equiv Q_1 - Q_2 + Q_3 - Q_4$$

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- ▶ Compare: $\log |d(Q, h)|$ vs $S_{\text{BH}}(Q, h)$

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- ▶ Matrix Integral:

$$\mathcal{I}(q, \xi_\ell, \xi_F, \xi_T) = \frac{1}{N!} \sum_{\mathbf{m} \in \mathbb{Z}^N} \oint \prod_{i=1}^N \frac{dz_i}{2\pi i z_i} \xi_T^{m_i} \mathcal{I}_{\text{ved}}(q, \xi_F; \mathbf{z}, \mathbf{m}) \mathcal{I}_{\text{adj hyp}}(q, \xi_\ell, \xi_F; \mathbf{z}, \mathbf{m}) [\mathcal{I}_{\text{fund hyp}}(q, \xi_F; \mathbf{z}, \mathbf{m})]^{N_f}$$

- ▶ Factorized:

$$\mathcal{I}(x, \xi_\ell, \xi_F, \xi_T) = \sum_{|\mathcal{Y}|=N} Z_{\text{pert}}^{\mathcal{Y}}(x, \xi_\ell, \xi_F) Z_{\text{vort}}^{\mathcal{Y}}(x, \xi_\ell, \xi_F, \xi_T) Z_{\text{vort}}^{\mathcal{Y}}(x^{-1}, \xi_\ell^{-1}, \xi_F^{-1}, \xi_T^{-1})$$

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- ▶ Factorized: **Nonperturbative**

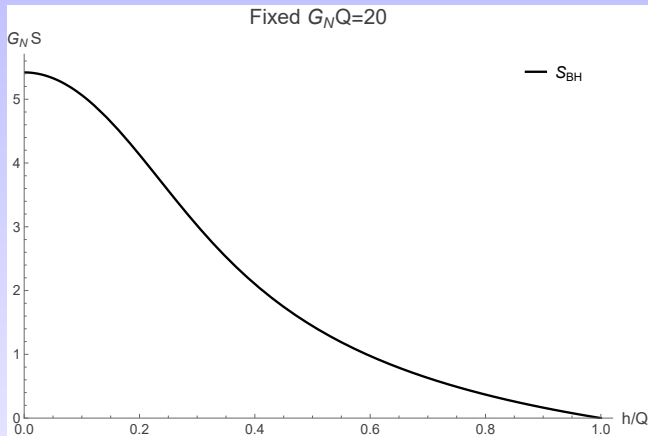
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AdS₄ Black Holes

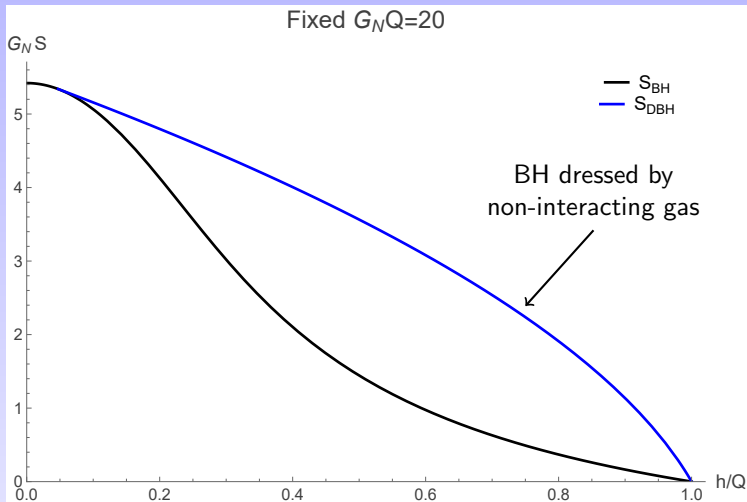
► Restrict to $Q_1 = Q_3, Q_2 = Q_4 \rightarrow \underline{Q = 4J + 2(Q_1 + Q_2)}, \quad \underline{h = 2(Q_1 - Q_2)}$

AdS₄ Black Holes

- ▶ Restrict to $Q_1 = Q_3$, $Q_2 = Q_4 \rightarrow \underline{Q = 4J + 2(Q_1 + Q_2)}, \quad \underline{h = 2(Q_1 - Q_2)}$
- ▶ $S_{\text{BH}}(Q, h)$ at fixed Q :

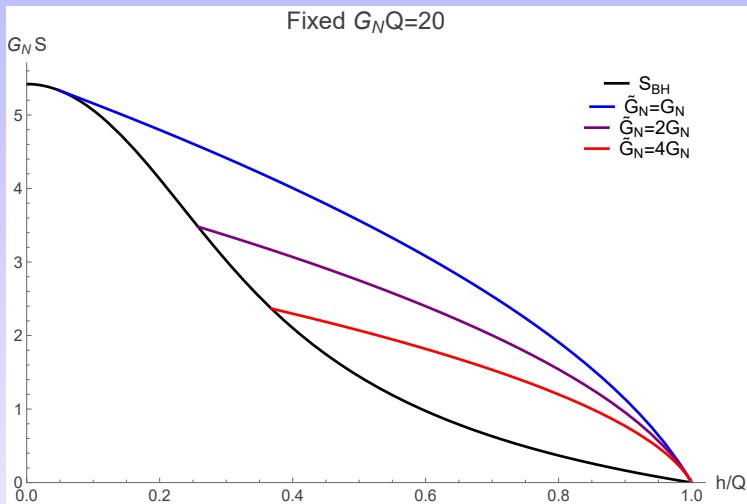


Dressed AdS_4 Black Hole Entropy

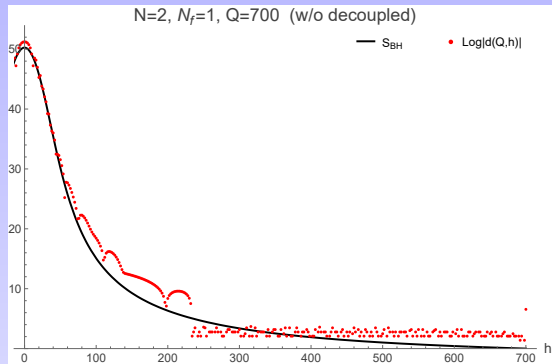
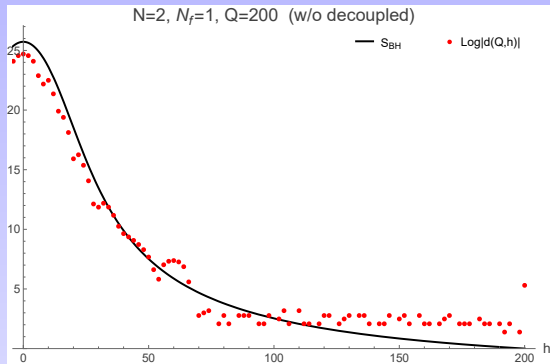


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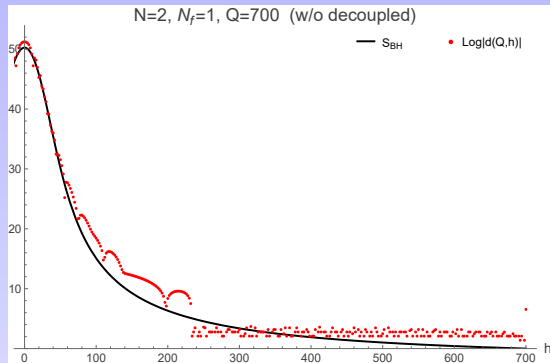
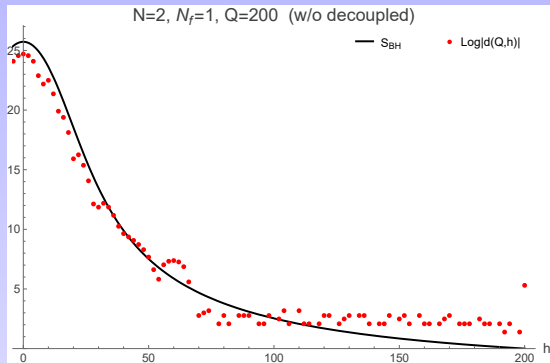
- If the dressing carries flux away from the central BH (M2 branes):



$N = 2$: no dressed BH



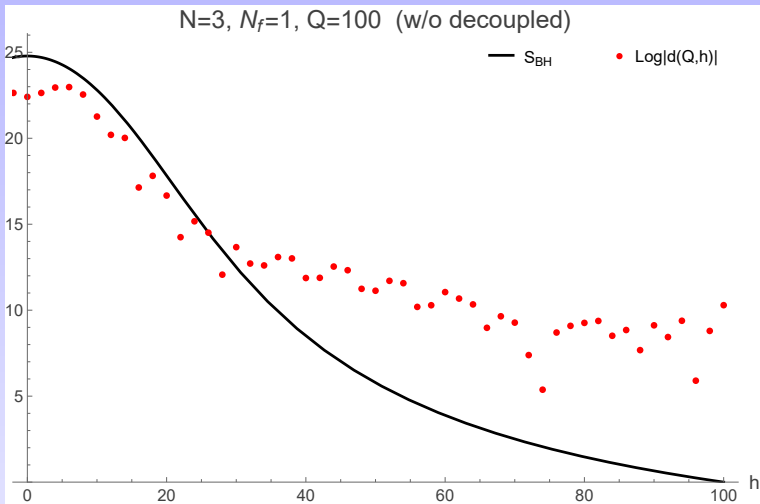
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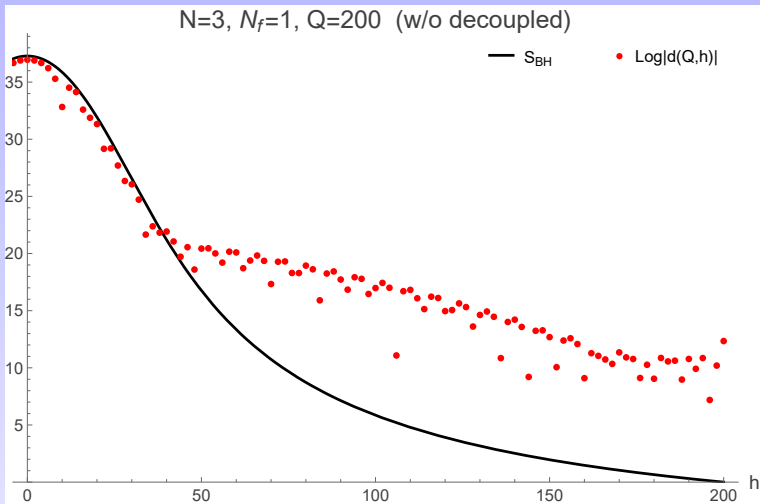
$\geq 1/8$ BPS

Higgs/Coulomb Index: $\log |d_Q| \sim 2N \log Q$

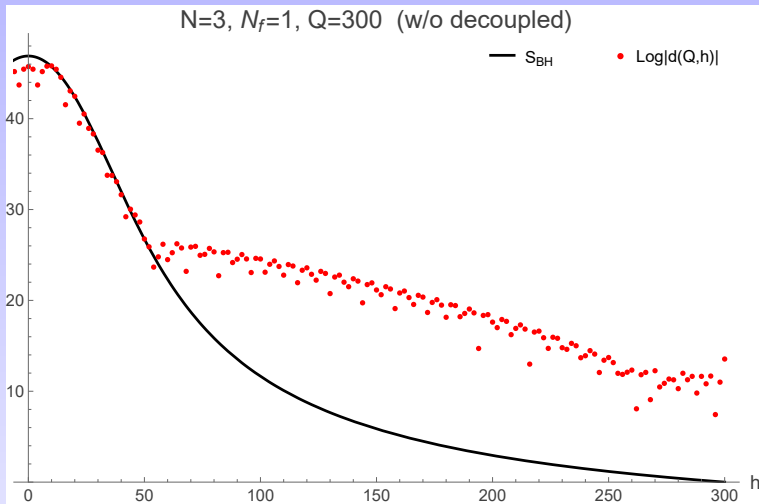
$N = 3$: dressed BH



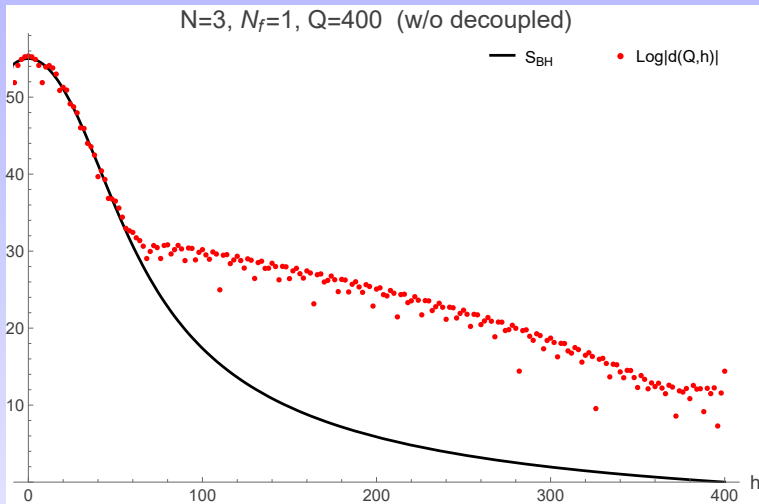
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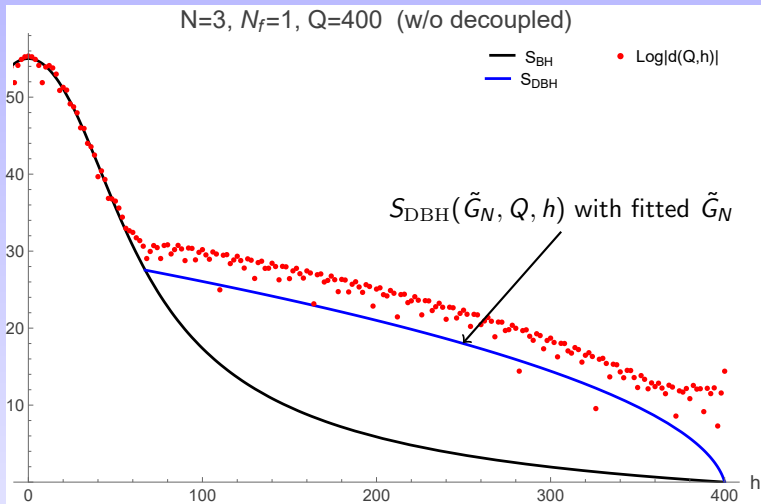
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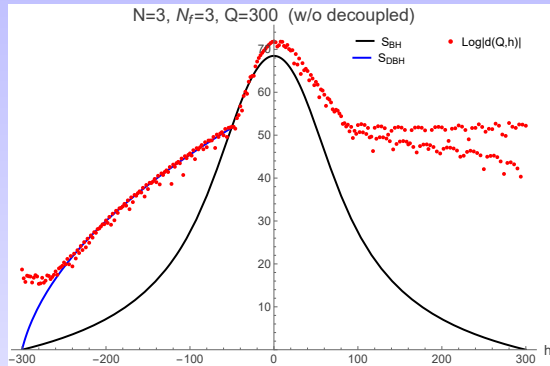
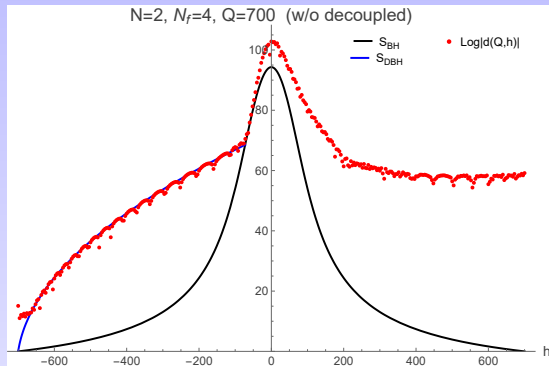
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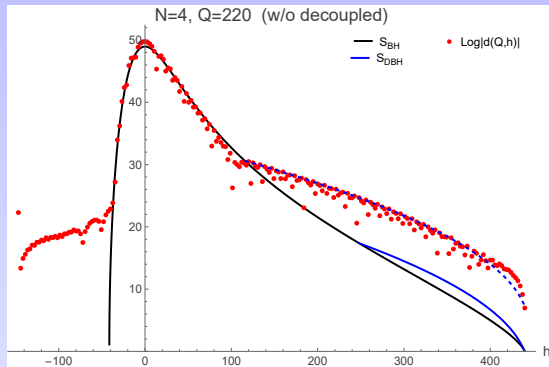
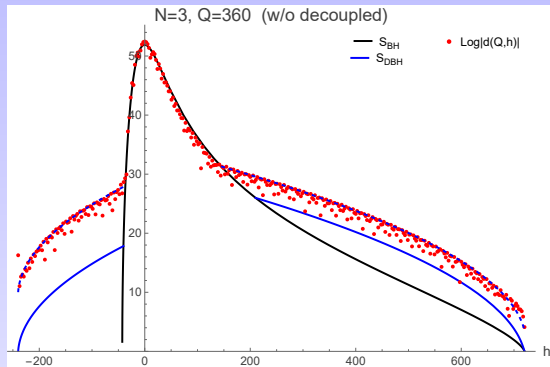
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Multiple flavors ($N_f > 1$)



Another charge limit: $Q_1 \neq Q_2 = Q_3 = Q_4$



Summary and future directions

- ▶ Precise evidence for dressed black holes with $N = 2, 3, 4$
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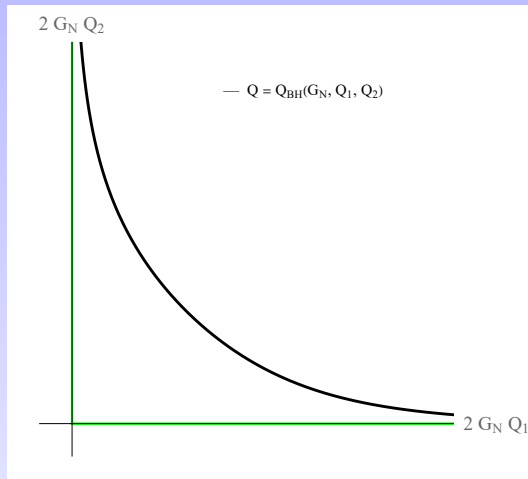
Open questions

- ▶ Direct identification of index saddles
- ▶ How does $\tilde{G}_N$ behave at larger N ?
- ▶ Construction of gravitational saddles beyond the probe brane limit

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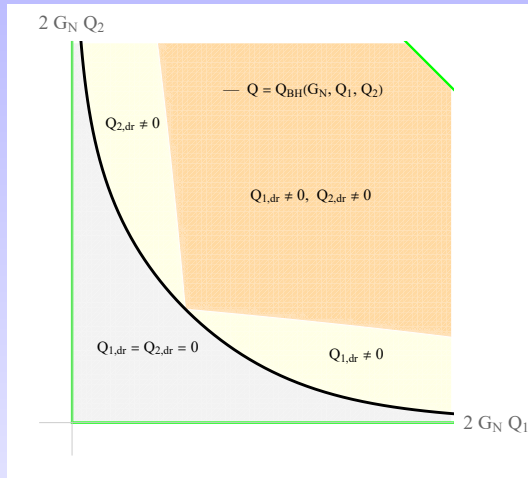


Dressed AdS_4 Black Holes

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- dressed black holes:
central BH w/ non-interacting “gas”

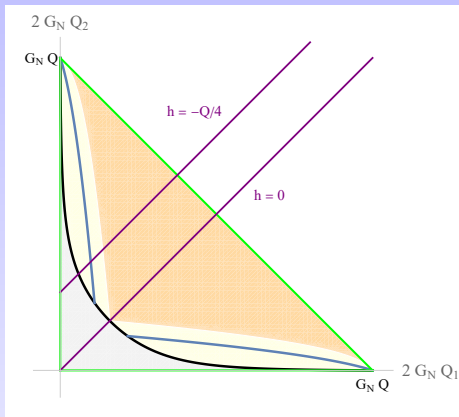
$$(J, Q_1, Q_2) = (J_{\text{bh}}, Q_{1,\text{bh}}, Q_{2,\text{bh}}) \\ + (J_{\text{dr}}, Q_{1,\text{dr}}, Q_{2,\text{dr}})$$

- Assume gas entropy is negligible



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