

Measuring the Acceleration-Dependent Temperature of the Minkowski Vacuum

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Check arXiv soon (v2) for pedagogical Supplemental Material!

Quantum Field Theory

- Quantum field theory is the basic framework that underlies the Standard Model of Particle Physics.
- It is the most precisely tested framework in all of physics, being used to produce predictions at the 13th decimal point in, for instance, measurements of the anomalous magnetic moment of the electron.
- Direct measurements of QFT phenomena in novel regimes are therefore worthwhile. This helps inform our confidence of extrapolation of QFT into more exotic regimes, such as in the study of black holes and Hawking radiation.
- Indeed, Hawking radiation has a close cousin phenomenon in flat spacetime, which has never been directly measured...

The Fulling-Davies-Unruh Effect

The Fulling-Davies-Unruh Effect predicts that an observer undergoing uniform acceleration in the Minkowski vacuum will not measure empty space. Instead, they will measure the Minkowski vacuum to be a *thermal bath of particles* at a temperature

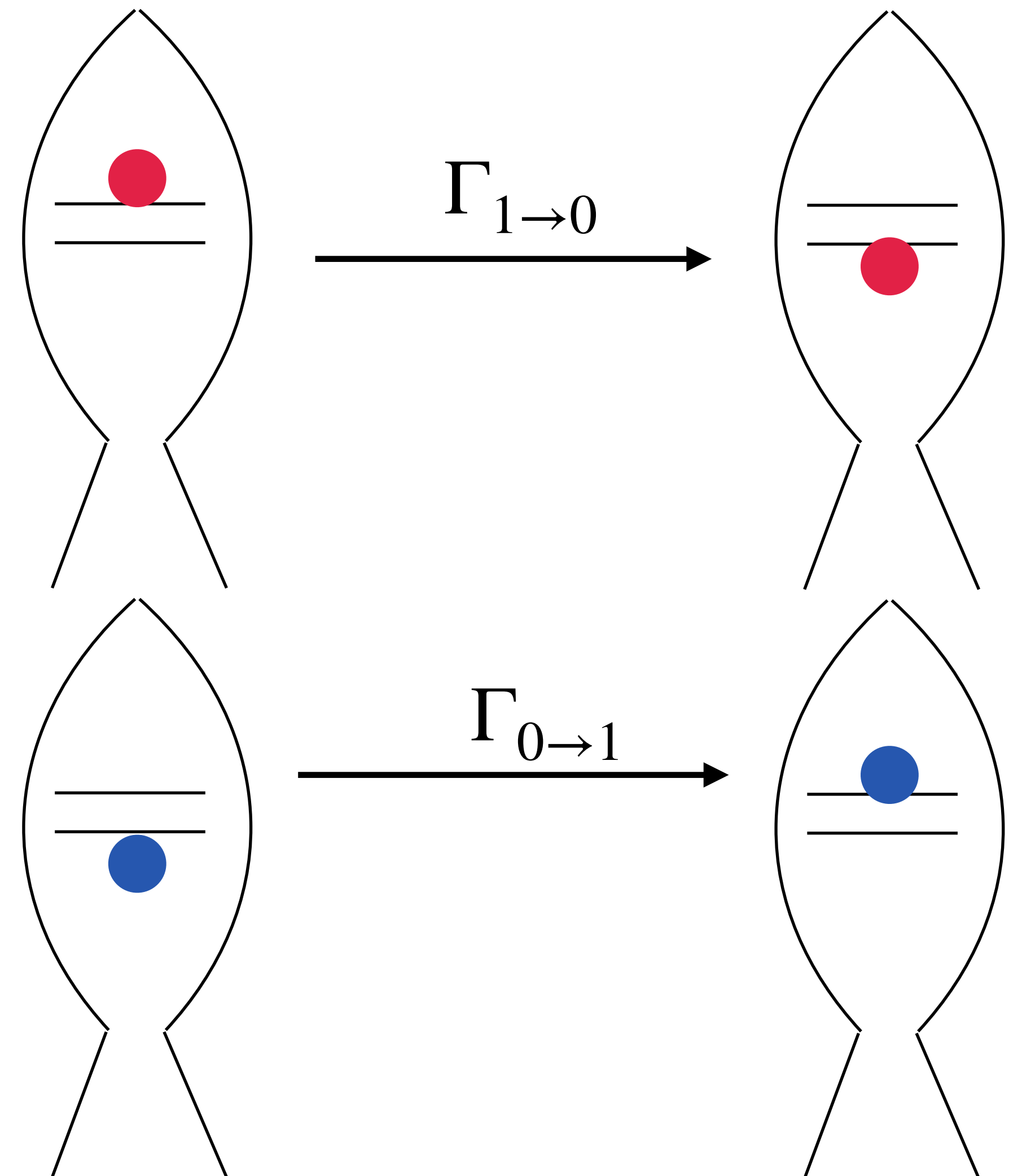
$$T_{\text{Unruh}} = \frac{\hbar a}{2\pi c k_{\text{B}}}.$$

What counts as measuring the Unruh Effect?

- The Unruh Effect is a foundational prediction of the quantum field theory.
- It is intrinsically tied to the local structure of quantum field theories. [Bisognano-Wichmann, 1975]
- It is possible to view even “classical” coherent radiation as being related to the Unruh effect, via stimulated emission. [Higuchi, Matsas, Sudarsky, 1992]
- What would count as “measuring” the Unruh effect?

How would an accelerating observer measure the effect?

- One protocol would be:
 - Construct a sequence n of different two-level systems with different energy-level spacings $\hbar f_{\text{acc.},n}$.
 - In a uniformly accelerating lab, we expect the relative rate of excitations and de-excitations to be given by
$$\Gamma_{0 \rightarrow 1} = \Gamma_{1 \rightarrow 0} \exp \frac{\hbar f_{\text{acc.},n}}{k_B T_U}$$



Quantum Detailed Balance

To see this, consider the rate of excitation and de-excitation.

$$\Gamma_{1 \rightarrow 0} \propto S^+ = \int d\Delta\tau e^{i2\pi f_{\text{acc.}}\Delta\tau} \langle \hat{E}(0) \hat{E}(\Delta\tau) \rangle$$

$$\Gamma_{0 \rightarrow 1} = S^- = \int d\Delta\tau e^{i2\pi f_{\text{acc.}}\Delta\tau} \langle \hat{E}(\Delta\tau) \hat{E}(0) \rangle$$

These are equivalent in any classical theory (including a stochastic theory),
but *not* in the quantum theory.

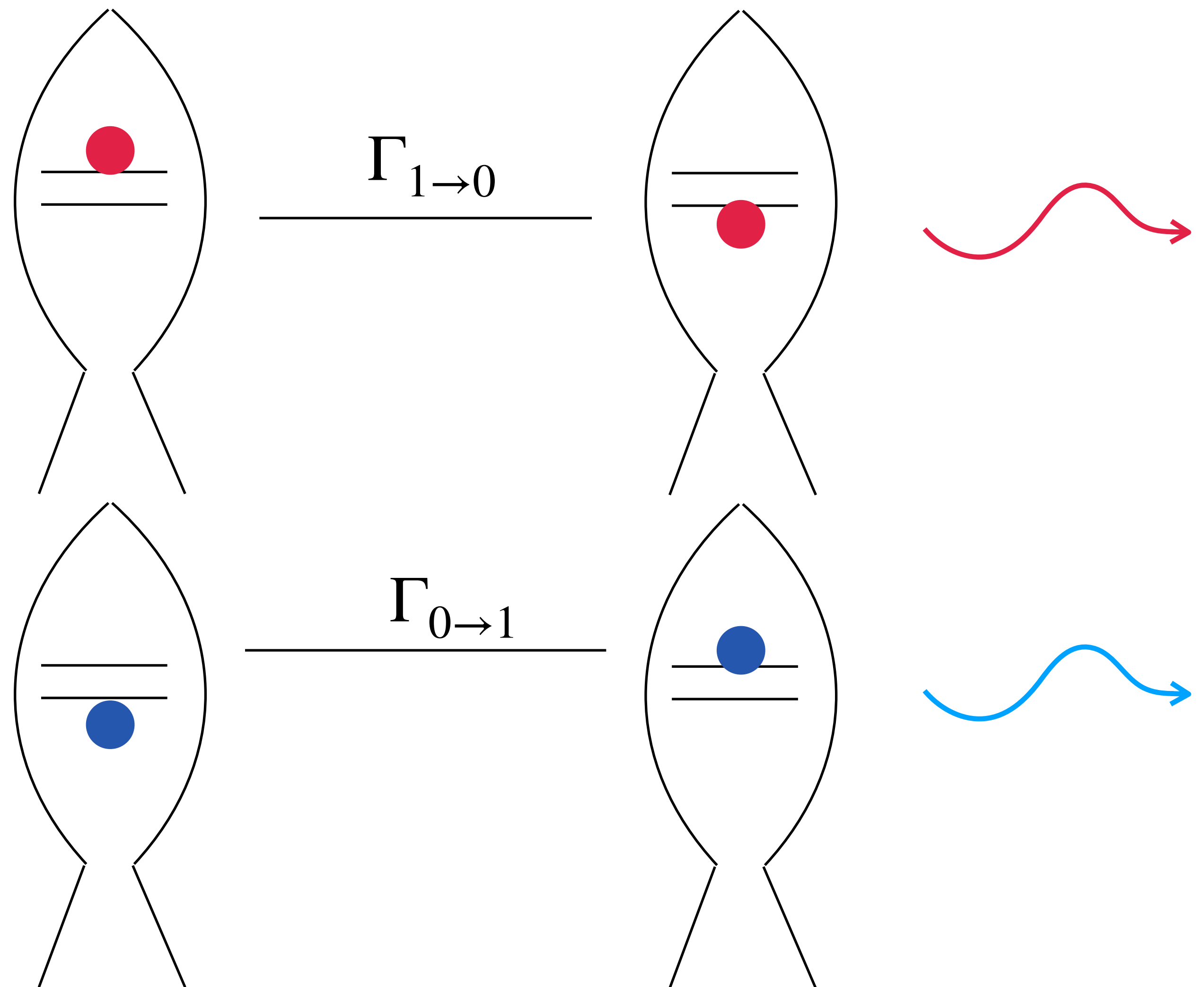
Quantum Detailed Balance:

$$S^- = \exp\left(\frac{hf_{\text{acc.}}}{k_B T_{\text{DB.}}}\right) S^+$$

Can an Inertial Observer Monitor the Detector?

- Unruh and Wald (1984): “What happens when an accelerating observer detects a Rindler particle?”
- The accelerating detector emits and absorbs Rindler modes: positive frequency excitations with respect to the uniformly accelerating time.
- These modes, viewed from the inertial point of view, can be measured as excitations above any coherent radiation.

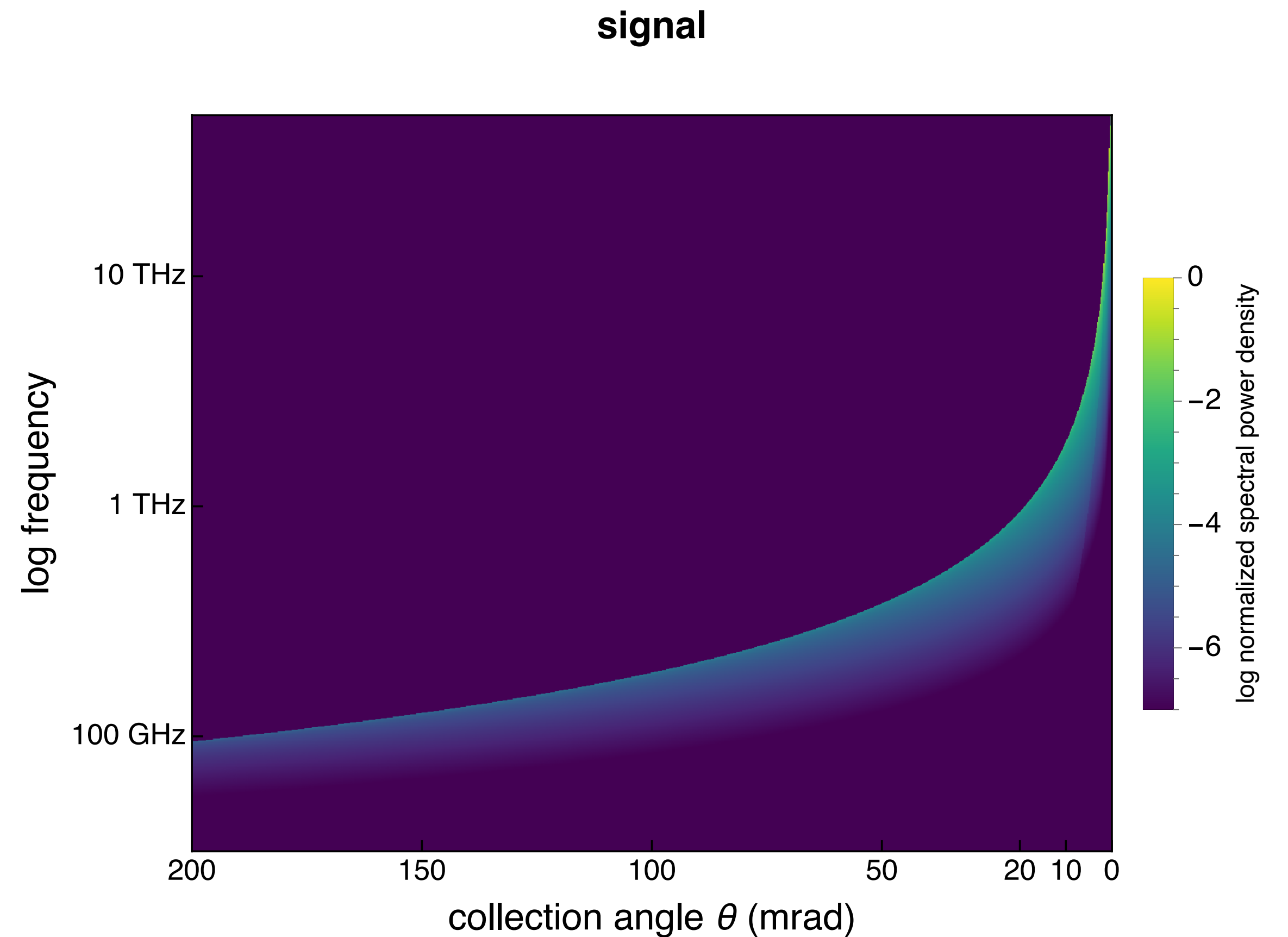
- Instantaneously (for a given $\beta(t)$),
$$\nu_{\text{lab}}(\beta, \theta) = \frac{f_{\text{acc.}}}{\gamma(1 - \beta \cos \theta)}$$



Chirped Radiation in the Inertial Lab

- Whenever the accelerating apparatus absorbs or emits an (approximately) definite-frequency Rindler mode, it radiates a characteristic *chirped* Minkowski mode.

- $$\nu_{\text{lab}}(\beta, \theta) = \frac{f_{\text{acc.}}}{\gamma(1 - \beta \cos \theta)}$$



x-axis ~ angle / rapidity / inertial time

Challenges to Measuring the Unruh Effect

- Many proposals have been put forth to measure the Unruh effect.
- **A modern-day wakefield accelerator, using the parameters we will use in our proposal, would require a median 10^{31000} lab-frame years to measure just one Rindler photon!**
- All methods rely on attempting to find some clever inertial signature.
- **To our knowledge, no existing proposal signature measures the Unruh effect in our sense, in that they do not allow one to read out the quantum, thermal response of a co-moving detector.**
- Case in point: a population of two level system coupled to a classical or stochastic radiation field will not respond in the same way, even at the same temperature.

The Connection to Classical Radiation

- Classical bremsstrahlung corresponds to coherent radiation in the quantum field Theory. This is equivalent to *zero energy* Rindler particles in the accelerating frame.
[Higuchi, Matsas, Sudarsky].
- Thus, any signature that can be derived from the *coherent radiation alone* would not signal the Unruh effect.

Can we use the Chirped Radiation?

- The system is radiating / absorbing Rindler modes. Written in terms of Minkowski particle modes, $\hat{b}_R^\dagger = \alpha \hat{a}_{M,I}^\dagger + \beta \hat{a}_{M,II}$
- A flux of positive frequency Rindler modes entails a flux of positive Minkowski modes.
- A *negative* frequency flux of Rindler modes also entails a flux of positive-frequency Minkowski modes, but with a **different mode function**: $\hat{b}_R = \alpha \hat{a}_{M,I} + \beta \hat{a}_{M,II}^\dagger$
- The relative emission of these distinct Minkowski modes measures the ratio of positive-to-negative *Rindler* modes, which gives the temperature via quantum detailed balance.

Counting Rindler Modes with an Inertial Detector

- A flux of positive frequency Rindler modes entails a flux of positive Minkowski modes.
- A *negative* frequency flux of Rindler modes also entails a flux of positive-frequency Minkowski modes, but with a **different mode function**.

Let $n_{\pm}$ denote the number of Rindler photons, N_I and N_{II} denote the number of each respective Minkowski modes. Let $M_{I,II}$ denote the measured covariance between the two Minkowski mode functions. Then

$$n_{\pm} = |u_f|^2 \left(N_I + \frac{1}{2} \right) + |v_f|^2 \left(N_{II} + \frac{1}{2} \right) + 2 \operatorname{Re}[u_f^* v_f e^{i\phi_f} M_{I,II}^*] \mp \frac{1}{2}$$

$$|u_f|^2 - |v_f|^2 = 1$$

Quantum Detailed Balance

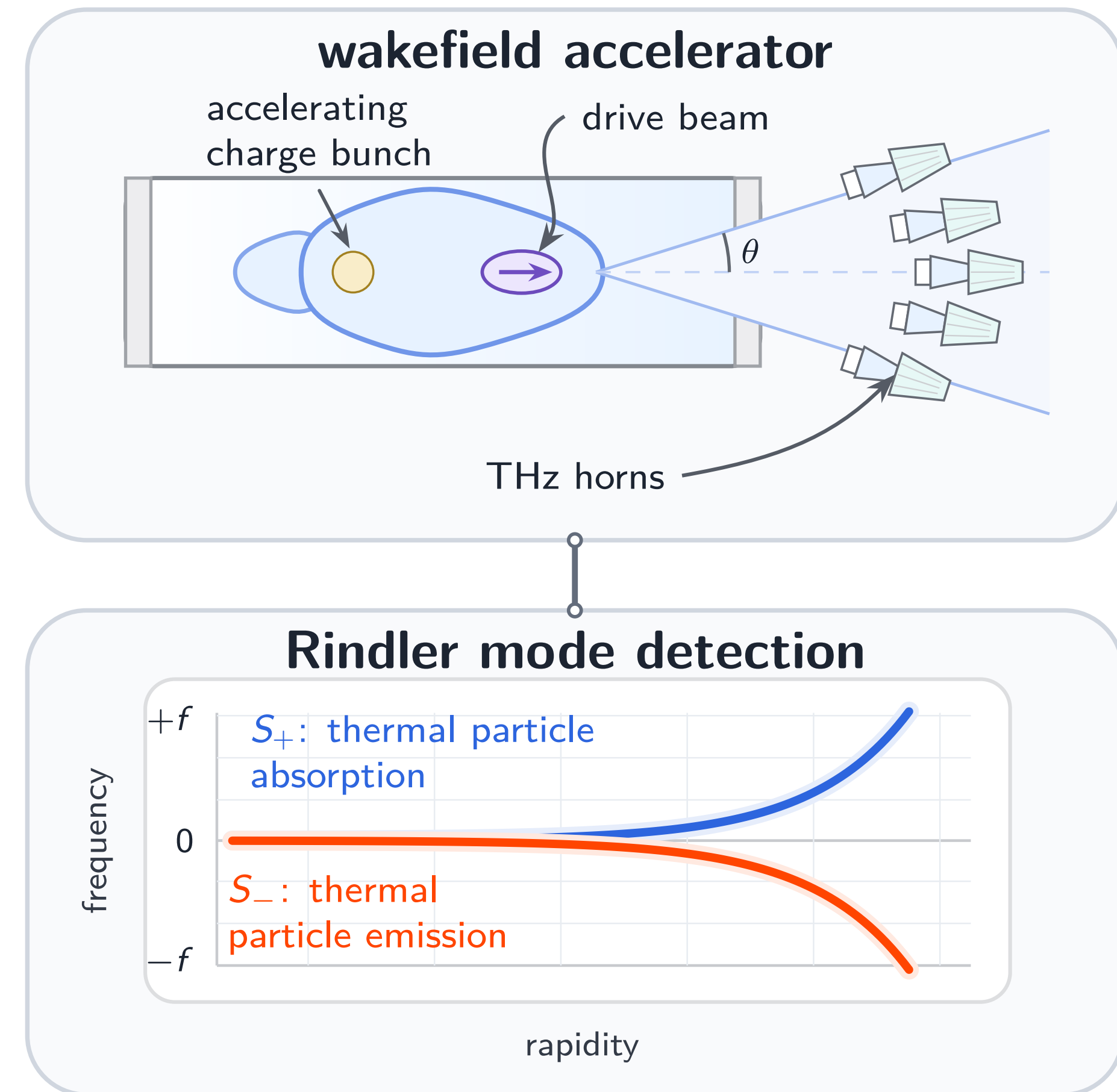
- By measuring separately the absorption and emission of Rindler modes of co-accelerating frequency $f_{acc.}$ by an accelerating charge, one obtains $S^+(f_{acc.})$ and $S^-(f_{acc.})$: the emission and absorption rates *of the charge in its own frame*.
- From this, one can read off the temperature $T_{DB}(f_{acc.}) = \frac{hf_{acc.}}{k_B \ln[S^-(f_{acc.})/S^+(f_{acc.})]}$.

The prediction of Fulling-Davies-Unruh is that $T_{DB} = T_{Unruh}$ for all $f_{acc.}$.

$$\text{Recall } T_{Unruh} = \frac{\hbar a}{2\pi c k_B}.$$

Measurement Proposal

- Our proposal is to carry out this measurement at a wakefield accelerator facility.
- Charge-bunch collective modes absorb and emit Rindler modes.
- Count the Rindler mode excess/deficit relative to the vacuum, after subtracting the coherent Lienard-Wiechert component.

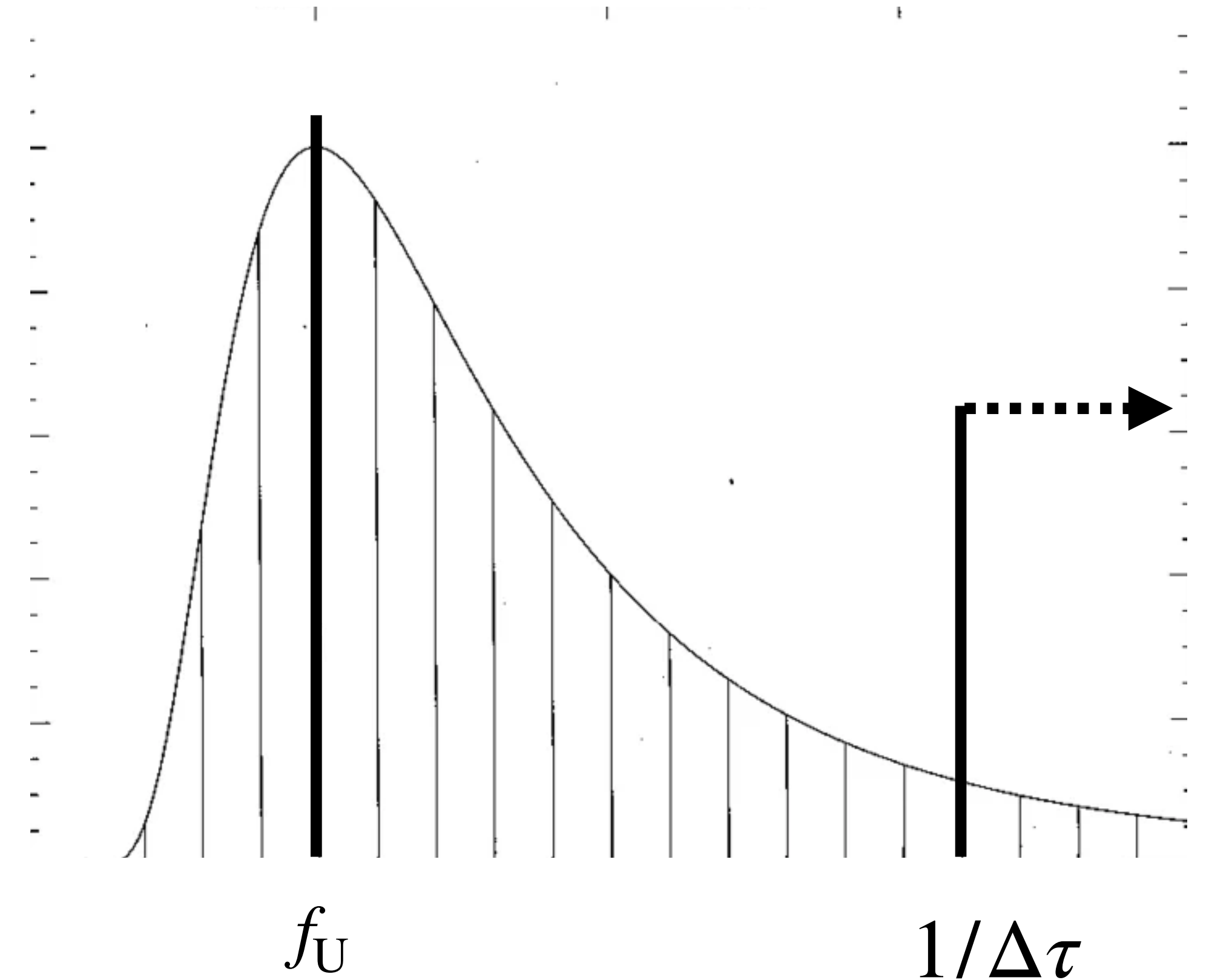


Isn't this Insane?

- Recall: A modern-day wakefield accelerator, using the parameters we will use in our proposal, would require a median 10^{31000} lab-frame years to measure just one Rindler photon!!!
- How is it that we claim to beat this?
- Answer: Time dilation kills the naive estimate. Proper Time is incredibly time dilated after just a short inertial time. This time dilation is a function of *velocity*, not acceleration. Thus our proposal uses a pulsed source, in which each pulse starts from inertial rest. Wakefield accelerators cycle through new inertial bunches at about 30-100 Hz.

Sampling the Tails

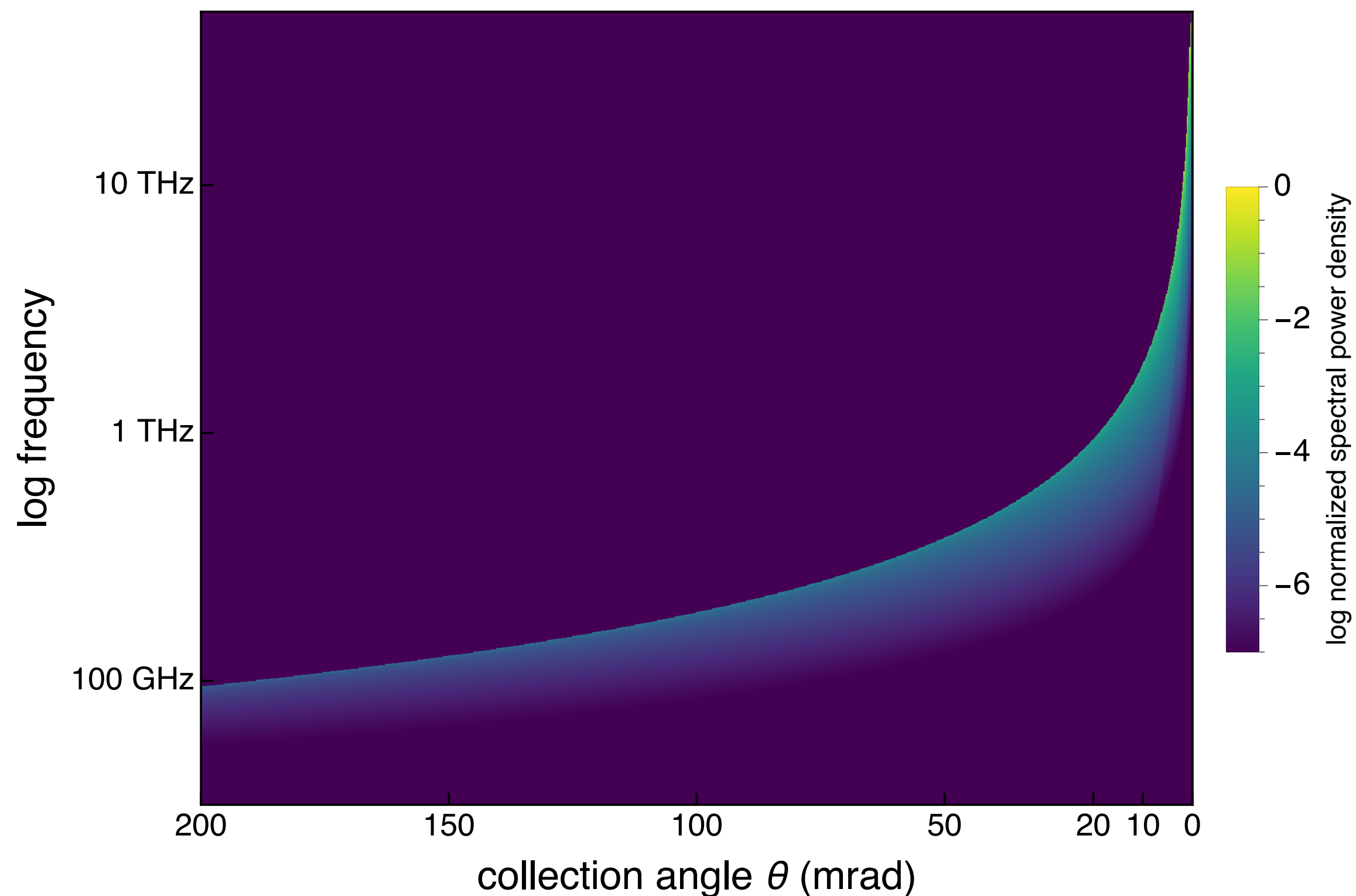
- Modern wakefield accelerators probe proper accelerations well over $10^{19}g$. This is an Unruh temperature of the order 0.7K. The radiation from interactions at 0.7K is extremely weak...
- Indeed, there is an effective IR cutoff due to the small acceleration duration ~ 2.5 ns, or about 12 ps comoving (4 K)
- But the high-energy thermal tails of the distribution still obey quantum detailed balance.



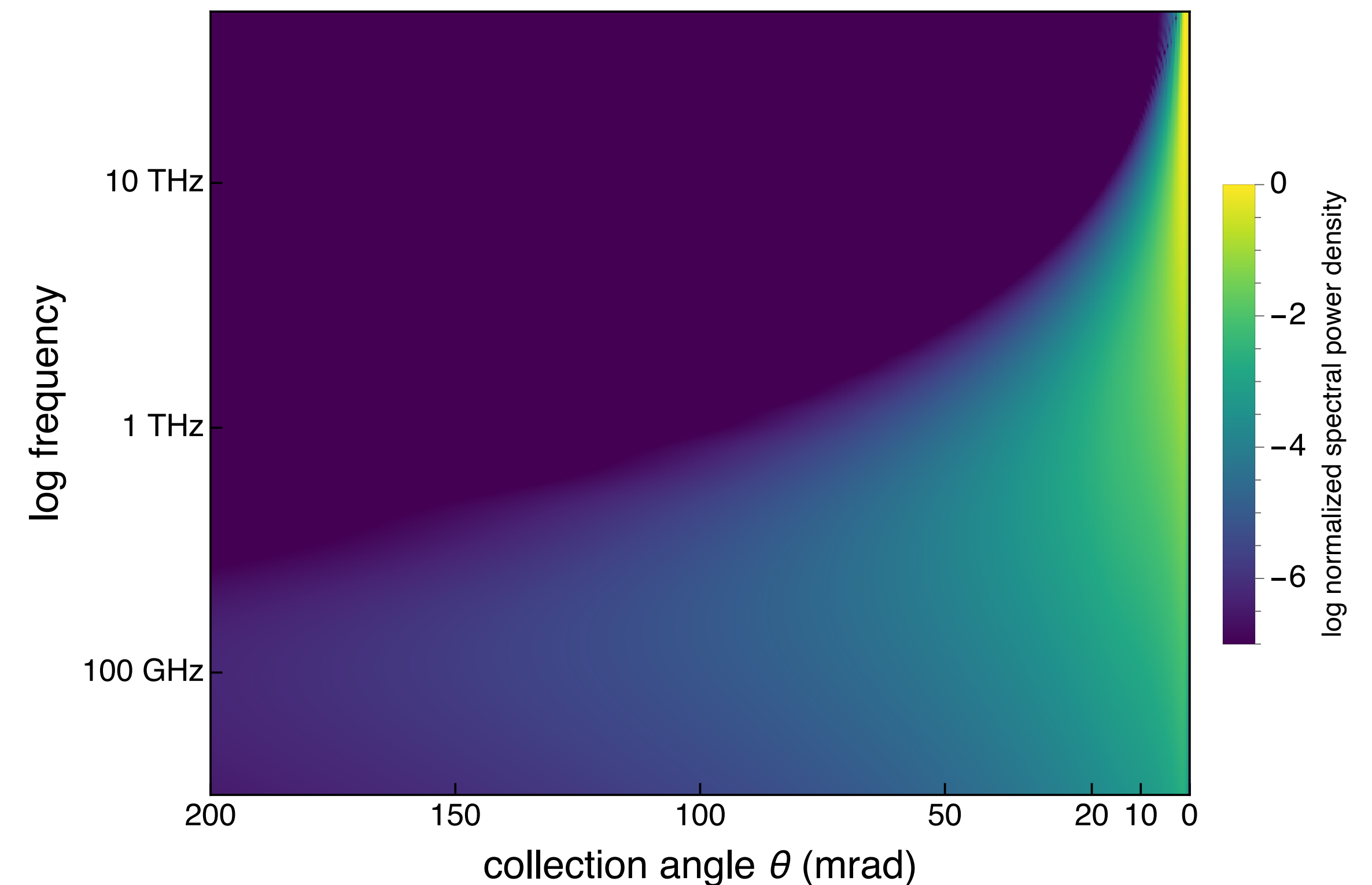
Measurement Proposal

- Our proposal is to carry out this measurement at a wakefield accelerator facility.
- A dominant source of *background* for us is the coherent classical radiation, which corresponds to the zero-frequency Rindler photons [Higushi, Matsas, Sudarsky].

signal



background



Might this be Feasible?

- This would be an extremely challenging experiment.
- In particular, the coherent classical field vastly dominates the strength of the chirped Minkowski modes we need to measure. Background rejection must be very good.
- Nevertheless, we do not as yet see any *component* of the proposal that is beyond present day technology, on its own.
- Integrating all of these things into one experiment is very much an open problem, which we are actively pursuing with experimental groups.
- See Supplemental Material for detailed estimates.

Significant Technical Challenges

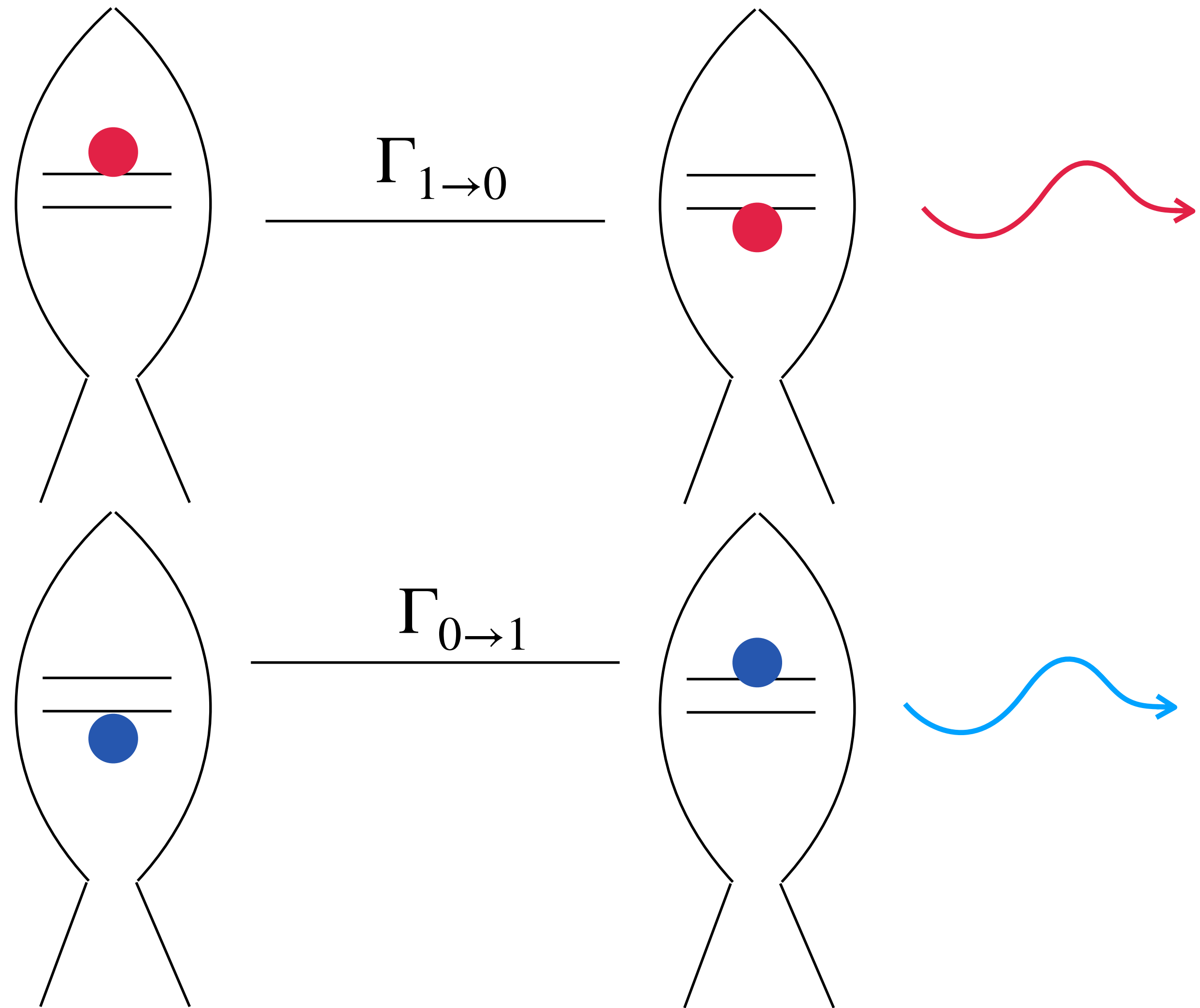
- Great challenges remain to overcome.
 - Quantum-coherent mode selection must be done with extremely good precision across GHz - THz bands in the lab frame.
 - Detailed bunch modeling must be carried out to better estimate its Rindler mode emission and absorption rates. Likely will introduce other correlated backgrounds.
 - Need to model / control the **noise environment** in the wakefield facility.

Controls

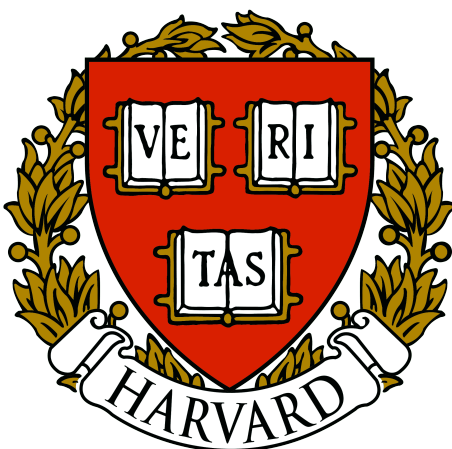
- The detailed balance *ratio* involves an overall cancellation of multiplicative uncertainties.
- The experiment should be carried out at many frequencies, and many accelerations. The relation $T_{\text{Unruh}}(f_{acc.}) = \frac{hf_{acc.}}{k_B \ln[S^-(f_{acc.})/S^+(f_{acc.})]}$ must hold for all.

Summary

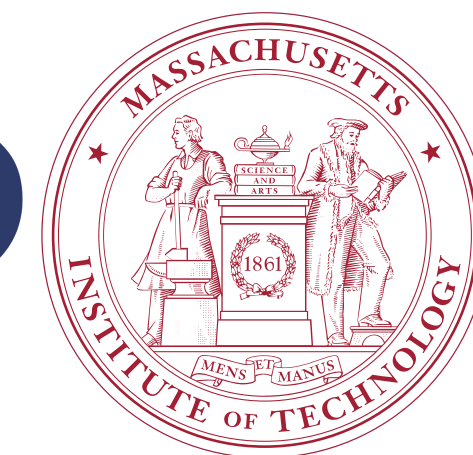
- What happens when an accelerating observer detects a Rindler particle? An inertial observer sees them radiate!
[Unruh Wald '87]
- This radiation can be detected.
Indeed, the Rindler mode absorption and emission ratio can be measured, giving a temperature.
- This is nothing more than the thermal excitation and de-excitation of the particle detector.
- This might be within reach of present-day wakefield facilities.



Thank you!



**BLACK HOLE
INITIATIVE**



$$\begin{aligned} \overline{P}_{\text{II,det}}^{\text{arrival}}(\kappa, E) &\simeq 4.7 \times 10^{-15} \text{ W} \left(\frac{Q_b}{0.65 \text{ nC}} \right) \left(\frac{\mathcal{R}_{\text{src}}}{0.16} \right) \\ &\times \left(\frac{\eta_{\text{count}}^{(N)}}{0.018} \right) \left(\frac{E}{1 \text{ GV/m}} \right)^2 \left(\frac{\kappa}{1.2} \right)^2 \frac{\nu_{\text{II,out}}}{300 \text{ THz}} \\ &\times \exp \left[-\frac{4\pi^2}{\Delta\eta} (\kappa - \kappa_*) \right]. \end{aligned}$$

$$\begin{aligned} \sigma_{\Xi}^{\text{stat}} &\simeq 2.3 \times 10^{-5} \left(\frac{Q_b}{0.65 \text{ nC}} \right) \left(\frac{0.16}{\mathcal{R}_{\text{src}}} \right) \left(\frac{1 \text{ GV/m}}{E} \right) \\ &\times \left(\frac{1.2}{\kappa} \right)^3 \frac{e^{x(\kappa)} - 1}{e^{x(1.2)} - 1} \left(\frac{\sigma_A}{10^{-3}} \right)^2 \left(\frac{L_{\text{tom}}}{2 \times 10^{-16}} \right) \left(\frac{3 \times 10^9}{N_{\text{eff}}} \right)^{1/2}. \end{aligned}$$

Quantity	Reference value
<i>Trajectory and source</i>	
Effective accelerating field E	1 GV/m
Bunch charge Q_b	0.65 nC
Rapidity endpoints (η_i, η_f)	(1, 8)
Rapidity span $\Delta\eta \equiv \eta_f - \eta_i$	7
Collection region $\mathcal{C}$	$0 \leq \theta \leq 0.2 \text{ rad}$
Selected bunch response $\mathcal{R}_{\text{src}}$	0.16
<i>Readout, fit, and rejection</i>	
Reference fit point $\kappa \equiv f_{\text{acc.}}/f_{\text{min,acc.}}$	1.2
Partner-counting efficiency $\eta_{\text{count}}^{(N)}$	0.018
Two-mode quadrature efficiency η_{tom}	0.09
Residual field-amplitude error σ_A	10^{-3}
Liénard field leakage L_{tom}	2×10^{-16}
Homodyne effective samples N_{eff}	3×10^9
Counting effective samples N_{count}	1.5×10^9
<i>Derived benchmark figures</i>	
Unruh frequency $f_{\text{U,acc.}}$	15 GHz
Minimum frequency $f_{\text{min,acc.}}$	84 GHz
Reference Rindler frequency $f_{\text{acc.}}$	100 GHz
Weak mode-II power $\overline{P}_{\text{II,det}}^{\text{arrival}}$	$4.7 \times 10^{-15} \text{ W}$
Absorption probability P_{abs}^R	6.7×10^{-5}
Raw energy dynamic range D_{cl}	6.3×10^{21}
Statistical contamination $\sigma_{\Xi}^{\text{stat}}$	2.3×10^{-5}
Classical contamination target Ξ_z^{tot}	$\lesssim 10^{-2}$