

Black Holes and Decoherence

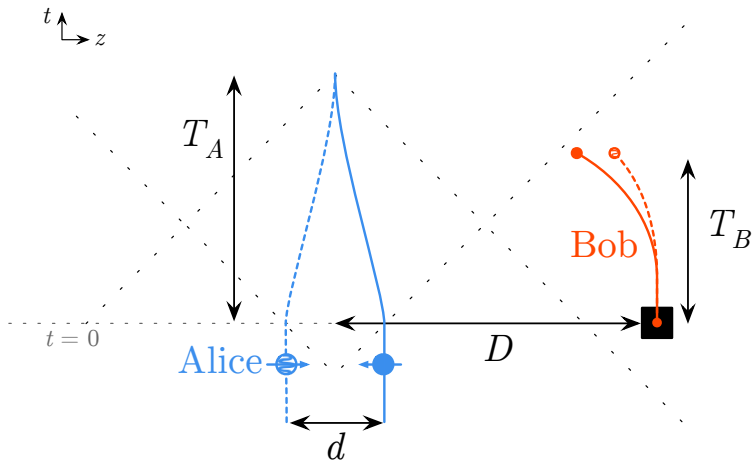
[“Black Holes are Watching You”]

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Key References:

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- 2) D. Danielson, G. Satishchandran, and R.M. Wald, Phys. Rev. D 105, 086001 (2022) [arXiv:2112.10798].
- 3) D. Danielson, G. Satishchandran, and R.M. Wald, Int. J. Mod. Phys. D 2241003 (2022) [arXiv:2205.06279].
- 4) D. Danielson, G. Satishchandran, and R.M. Wald, Phys. Rev. D 108, 025007 (2023) [arXiv:2301.00026].
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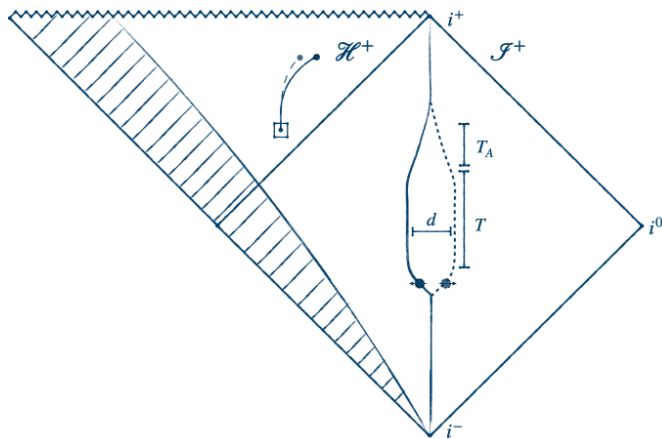
A Gedankenexperiment



Will Alice's particle maintain coherence?

A New Twist to the Gedankenexperiment: Black Holes

Now suppose a black hole is present and Bob performs the measurements from inside the black hole:



Decoherence of Alice's Particle Due to Emission of Radiation

Suppose Alice is in Minkowski spacetime and after recombination, she keeps her recombined particle stationary forever. At asymptotically late times, the state of the total system is of the form

$$\frac{1}{\sqrt{2}} (|\uparrow; A\rangle_{i+} \otimes |\Psi_1\rangle_{\mathcal{I}^+} + |\downarrow; A\rangle_{i+} \otimes |\Psi_2\rangle_{\mathcal{I}^+})$$

where $|\Psi_1\rangle_{\mathcal{I}^+}$ and $|\Psi_2\rangle_{\mathcal{I}^+}$ represent the quantum states of the electromagnetic radiation at future null infinity corresponding to whether Alice's particle followed “path 1” (spin $|\uparrow\rangle$) or “path 2” (spin $|\downarrow\rangle$) . The decoherence of Alice's superposition is

$$\mathcal{D} = 1 - |\langle \Psi_1 | \Psi_2 \rangle_{\mathcal{I}^+}| \ .$$

Decoherence in the Presence of a Black Hole

Now suppose Alice does her experiment in the presence of a black hole. Then the final state is now

$$\frac{1}{\sqrt{2}} \left(|\uparrow; A_1\rangle_{i+} \otimes |\Psi_1\rangle_{\mathcal{J}+} |\Phi_1\rangle_{\mathcal{H}+} + |\downarrow; A_2\rangle_{i+} \otimes |\Psi_2\rangle_{\mathcal{J}+} |\Phi_2\rangle_{\mathcal{H}+} \right)$$

where $|\Phi_1\rangle_{\mathcal{H}+}$ and $|\Phi_2\rangle_{\mathcal{H}+}$ represent the quantum states of the electromagnetic radiation that passes through the event horizon of the black hole. If Alice does the recombination sufficiently slowly, the decohering radiation that reaches infinity will be negligible, i.e., $|\langle\Psi_1|\Psi_2\rangle_{\mathcal{J}+}| \approx 1$. The decoherence of Alice's superposition is

$$\mathcal{D}_{\text{BH}} = 1 - |\langle\Phi_1|\Phi_2\rangle_{\mathcal{H}+}|.$$

We will see that the decoherence cannot be made small by doing the recombination slowly.

Electromagnetic Radiation Through the Horizon

Let n^a denote the (affinely parametrized) normal to the horizon. Electromagnetic radiation through the horizon is described by the pullback, E_A , of the “electric field” $E_a = F_{ab}n^b$ to the horizon, $\mathcal{H}^+$, (where capital latin indices are used for dual vector fields on the horizon that are orthogonal to n^a , i.e., having “purely angular” components). For a static point charge outside the horizon, we have $E_A = 0$, i.e., there is no radiation through the horizon. However, $E_r \neq 0$, on $\mathcal{H}^+$, where $E_r = F_{ab}l^an^b$ where l^a is a past directed null vector with $l^an_a = 1$.

Now, suppose that the point charge is moved to a new location. By Maxwell’s equations,

$$D^A E_A = -\partial_V E_r$$

so there will necessarily be some radiation through the horizon. Furthermore, $\int D^A E_A dV$ is constrained by the initial and final values of E_r , and is independent of how slowly the charge is moved.

Radiation of Through the Horizon

If the point charge is moved very slowly, the total energy radiated into the black hole $\propto \int E^A E_A r^2 d\Omega V dV$ can be made arbitrarily small. However, as I shall now explain, if the charge remains in its new position forever, the number of photons radiated into the black hole is infinite!

For an unperturbed black hole formed by gravitational collapse, the state of the electromagnetic field on the horizon of the black hole is described by the Unruh vacuum. However, we will be concerned here only with low frequency phenomena ($\omega \ll c^3/GM$), in which case the Unruh and Hartle-Hawking vacua near the horizon are essentially indistinguishable. In the Fock space associated with the Hartle-Hawking vacuum, a “particle” corresponds to a solution that is purely positive frequency with respect to affine parameter on the horizon.

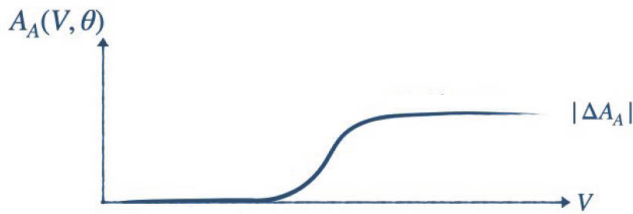
Soft Photons

A classical charge-current source will perturb the state of the quantum field from the Unruh/Hartle-Hawking vacuum to a coherent state associated with the classical retarded solution. In a gauge where $A_a n^a = 0$, the number of photons radiated through the horizon is given by

$$\langle N \rangle_{\mathcal{H}^+} = \frac{1}{\pi \hbar} \int r^2 d\Omega \int_0^\infty \omega d\omega |\hat{A}_A(\omega, x^A)|^2$$

where $\hat{A}_A$ is the Fourier transform of A_A with respect to affine parameter V . In this gauge, $E_A = -\partial_V A_A$, so since $\int E_A dV \neq 0$, it follows that A_A does not return to its initial value at late times. But this implies that $\hat{A}_A$ diverges as $1/\omega$ as $\omega \rightarrow 0$, which, in turn, implies that $\langle N \rangle_{\mathcal{H}^+} = \infty$. This is a precise analog of the infrared divergences that occur in scattering theory (for $d = 4$).

Infrared Divergence



$$\Delta A_A \neq 0 \implies \tilde{A}_A(\omega, \theta) \sim \frac{1}{\omega}$$

$$\implies \langle N \rangle = ||A||^2 \sim \int_0^\infty \frac{d\omega}{\omega}$$

Thus moving a point charge to a different location and leaving it there forever results in the radiation of an infinite number of “soft photons” through the horizon!

How Large is ΔA_A ?

Radial electric field of a point charge located a distance b from the black hole:

$$E_r \sim \frac{q}{b^2}$$

If the charge is moved a distance $d \ll b$, we have

$$\Delta E_r \sim \frac{qd}{b^3}$$

Since we have

$$\Delta E_r = - \int dV D^A E_A = \int dV D^A \partial_V A_A = D^A (\Delta A_A)$$

and the horizon is at $r \sim M$, we obtain

$$|\Delta A_A| \sim \frac{M^2 q d}{b^3}$$

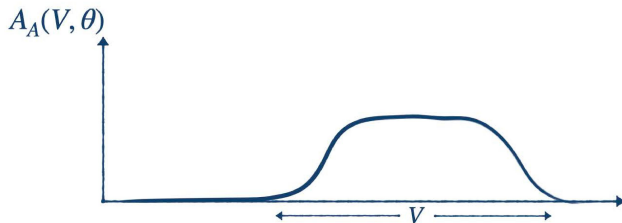
Radiation When Point Charge is Moved Back to Original Position after time T

The infrared divergence is avoided if the charge is moved back to its original position. Now we obtain

$$\langle N \rangle_{\mathcal{H}^+} \sim |\Delta A_A|^2 \ln V \sim \frac{M^4 q^2 d^2}{b^6} \ln V$$

But $V = e^{\kappa T}$ with $\kappa = 1/4M$, so

$$\langle N \rangle_{\mathcal{H}^+} \sim \frac{M^3 q^2 d^2}{b^6} T$$



Alice's experiment

In a stationary lab outside of the black hole, Alice puts her particle in a spatial superposition, keeps it in the superposition for a (proper) time T , and then recombines the particle. The above estimate for $\langle N \rangle_{\mathcal{H}+}$ holds for the number of “entangling soft photons” emitted into the black hole in this process. If $\langle N \rangle_{\mathcal{H}+} \gtrsim 1$, then her particle will have decohered. Thus, Alice's particle will decohere in a time

$$\begin{aligned} T_D &\sim \frac{b^6}{M^3 q^2 d^2} \\ &\sim 10^{43} \text{ years} \left(\frac{b}{\text{a.u.}} \right)^6 \cdot \left(\frac{M_\odot}{M} \right)^3 \cdot \left(\frac{e}{q} \right)^2 \cdot \left(\frac{\text{m}}{d} \right)^2 \end{aligned}$$

(But $T_D \sim 5$ minutes if Alice's lab is at $b = 6M$ rather than at 1 a.u.)

Gravitational Case

The analysis of the gravitational case follows in close analogy with the electromagnetic case, with the electric part of the Weyl tensor $E_{ab} = C_{acbd}n^cn^d$ playing the role analogous to E_a . For a static point mass, the only nonvanishing component of E_{ab} on the horizon is $E_{rr} = C_{acbd}l^an^cl^bn^d$. Radiation through the horizon is described by the pullback, E_{AB} , of E_{ab} to the horizon. In parallel with Maxwell's equations, the Bianchi identity yields

$$D^A E_{AB} = -\partial_V E_{rB}, \quad D^A E_{rA} = -\partial_V E_{rr}$$

which implies

$$D^A D^B E_{AB} = \partial_V^2 E_{rr}.$$

Similarly, we now have

$$E_{AB} = -\frac{1}{2}\partial_V^2 h_{AB}.$$

Gravitational Decoherence

An important difference that occurs in the gravitational case is that Alice does not create a “mass dipole” in the superposition—her lab moves oppositely to cancel the mass dipole of the superposition. Thus, it is now the effective mass quadrupole md^2 of the superposition that now enters, rather than the effective electrostatic dipole qd . Otherwise, the analysis proceeds in complete parallel, and decoherence time is now

$$\begin{aligned} T_D^{\text{GR}} &\sim \frac{b^{10}}{M^5 m^2 d^4} \\ &\sim 10 \mu\text{s} \left(\frac{b}{\text{a.u.}} \right)^{10} \cdot \left(\frac{M_\odot}{M} \right)^5 \cdot \left(\frac{M_{\text{Earth}}}{m} \right)^2 \cdot \left(\frac{R_{\text{Earth}}}{d} \right)^4 \end{aligned}$$

Generalizations

The only crucial property of black holes used in the above analysis is that the event horizon is a Killing horizon. A closely parallel analysis (in both the electromagnetic and gravitational cases) holds in other cases where a Killing horizon is present.

This includes the case where Alice's lab is accelerating in Minkowski spacetime (Rindler horizon) and the case of de Sitter spacetime (cosmological horizon). For an inertial lab in de Sitter spacetime, in the electromagnetic case, we have

$$T_D \sim \frac{R_H^3}{q^2 d^2}$$

whereas in the gravitational case, we have

$$T_D \sim \frac{R_H^5}{m^2 d^4}.$$

Conclusions

Black holes—and more generally, Killing horizons—harvest information about quantum superpositions of spatially separated components. They do so by absorbing “soft photons/gravitons” associated with the long range fields sourced by the matter comprising these components. Eventually, a black hole will decohere any quantum superposition. Although this is not likely to be of practical importance for experiments, it may be of significance for our understanding of the nature of black holes in a quantum theory of gravity.