

Analytic Continuation of Planar Diagram Theories as a Function of N

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The background has to do with the question of what sort of “averaging” is involved in wormhole contributions to the gravitational path integral. In the SYK model, for example, we average over a lot of coupling parameters. But holographic theories in higher dimensions generally depend on only a very small number of parameters, in some important examples (like $\text{AdS}_7 \times S^4$ or $\text{AdS}_4 \times S^7$) only a single integer N .

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(here H_N is the Hamiltonian for given N) is believed to vary erratically with t (or N) if t is large enough. The same is true of its absolute value squared $|a_N(t)|^2$, which is known as the spectral form factor. Can we claim that what gravity computes for us, with the help of wormholes, is a smooth approximation of some sort to these wildly N -dependent functions?

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Though this is tempting, there is a serious problem: N doesn't have enough values. To explain this, let us recall that based on the double cone wormhole it is argued (by Saad, Shenker, and Stanford 1806.06840) that $a_N(t)$ behave like Gaussian random variables with zero mean, while on the other hand the double cone wormhole gives us an answer with a non-trivial $1/N$ expansion for its modulus

$$|a_N(t)|^2 = b_0(t) + \frac{1}{N^2} b_1(t) + \frac{1}{N^4} b_2(t) + \cdots .$$

If the numbers $a_N(t)$ really are a sequence of Gaussian random variables (maybe with a variance that has an expansion in powers of $1/N^2$) then we could define the limiting average value

$$b_0(t) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{N'=1}^N |a_{N'}(t)|^2.$$

But we wouldn't be able to define the higher order terms $b_1, b_2, \dots$ in the $1/N$ expansion, since the partial sums $\sum_{N'=1}^N |a_{N'}(t)|^2$ would actually have fluctuations of order $1/\sqrt{N}$, much too big to allow us to define the desired $1/N$ expansion of $|a_N(t)|^2$.

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For related perspectives on averaging over N , see H. Liu, 2512.13807, 2608.02743.

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For this idea to make any sense, field theories that have holographic duals must have natural analytic continuations in N . So we (J. Kudler-Flam, A. Biggs, and I) wanted to investigate this. However, that is hard. What we were able to do is to find what appear to be natural analytic continuations of certain matrix field theories – or, as in the title of my talk, certain theories with planar diagram expansions.

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$$I = \int d^n x \left(\frac{1}{2} (d\vec{\phi})^2 + \frac{1}{2} m^2 \vec{\phi}^2 + \frac{\lambda}{2N} (\vec{\phi}^2)^2 \right)$$

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Integrating out σ brings us back to I ; on the other hand integrating out $\vec{\phi}$ gives an effective action that depends only on σ in which N appears only as a parameter. In that form analytic continuation in N is no problem.

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Such a lattice theory is expected to converge to the continuum theory if $\beta \sim N \log(1/a)$ as the lattice spacing $a \rightarrow 0$. However, we were not able to analytically continue this version of the theory.

A version that works better is to take the argument of the lattice path integral to be

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This is expected to have the same continuum limit if $h \sim N \log(1/a)$. Notice in particular that near the continuum limit, $h \gg N$. Since $|\det(1 + U_x U_y^{-1})|^2 = \det(U_x + U_y) \det(U_x^\dagger + U_y^\dagger)$, an equivalent form is

$$\prod_{x,y} \det^h(U_x + U_y) \det^h(U_x^\dagger + U_y^\dagger).$$

The model has $U(N) \times U(N)$ global symmetry acting by $U_x \rightarrow AU_x B^{-1}$ for $A, B \in U(N)$.

We can represent all the determinants by fermion integrals. For each edge $e = \langle xy \rangle$ and each “flavor” $a = 1, \dots, h$, introduce complex fermions $\psi_{e,a}, \bar{\psi}_{e,a}$ transforming as $(1, N)$ and $(\bar{N}, 1)$ under $U(N) \times U(N)$. (Despite the notation, $\bar{\psi}$ isn't the complex conjugate of ψ .) Then

$$(\det(U_x + U_y))^h = \int d\bar{\psi}_{e,a,k} d\psi_{e,a}^j \exp(\bar{\psi}_{e,a,k} (U_x^k_j + U_y^k_j) \psi_{e,a}^j).$$

Similarly we introduce another family of fermions $\chi, \bar{\chi}$ with

$$(\det(U_x^\dagger + U_y^\dagger))^h = \int d\bar{\chi}_{e,a,k} d\chi_{e,a}^j \exp(\bar{\chi}_{e,a,k} (U_x^{\dagger k}_j + U_y^{\dagger k}_j) \chi_{e,a}^j).$$

For any given lattice vertex x , the dependence of the integrand on U_x is

$$\exp \left(\sum_e \sum_{a=1}^h (\bar{\psi}_{e,a,i} U_x^i{}_j \psi_{e,a}^j + \bar{\chi}_{e,a,i} U_x^{\dagger i}{}_j \chi_{e,a}^j) \right).$$

To simplify the notation, we combine e, a to an index α that has $F_x = h \deg(x)$ values, where $\deg(x)$ is the number of edges that end at x .

For any given lattice vertex x , the dependence of the integrand on U_x is

$$\exp \left(\sum_e \sum_{a=1}^h (\bar{\psi}_{e,a,i} U_x^i j \psi_{e,a}^j + \bar{\chi}_{e,a,i} U_x^{\dagger i} j \chi_{e,a}^j) \right).$$

To simplify the notation, we combine e, a to an index α that has $F_x = h \deg(x)$ values, where $\deg(x)$ is the number of edges that end at x . So we have the integral over all fermions of

$$\exp \left(\sum_{\alpha} (\bar{\psi}_{\alpha,i} U_x^i j \psi_{\alpha}^j + \bar{\chi}_{\alpha,i} U_x^{\dagger i} j \chi_{\alpha}^j) \right).$$

A remarkable “color-flavor” transformation of Zirnbauer (for a review see 2109.10272) lets us replace the integral over U with an integral over an $F_x \times F_x$ complex matrix Z_x :

$$\begin{aligned} & \exp\left(\sum_{\alpha} (\bar{\psi}_{\alpha,i} U_{xj}^i \psi_{\alpha}^j + \bar{\chi}_{\alpha,i} U_{xj}^{\dagger i} \chi_{\alpha}^j)\right) \\ &= C_{N,F_x} \int \frac{d^{2F_x^2} Z_x}{(\det(1 + Z_x Z_x^{\dagger}))^{N+2F_x}} \exp\left(\sum_{\alpha\beta} (\bar{\psi}_i^{\alpha} Z_{\alpha\beta} \chi^{\beta i} - \bar{\chi}_{\alpha,i} Z^{\dagger\alpha\beta} \psi_{\beta}^i)\right). \end{aligned}$$

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Since the fermions appear only quadratically in the exponent, the fermion integral gives a determinant again. And since the index i in these formulas takes N values, we actually get a determinant raised to the N^{th} power.

Finally, and crucially, the determinant that we get here is *positive*. (Some superficially similar things that we've tried failed because of getting a complex determinant here.) So finally the lattice path integral is

$$\prod_x C_{N,F_x} \int \frac{d^{2F_x^2} Z_x}{(\det(1 + Z_x Z_x^\dagger))^{N+2F_x}} (\det M)^N,$$

where $\det M > 0$ is the determinant produced by the fermion integral. In this form, analytic continuation in N is manifest.

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On a lattice, all this works in any dimension, but the sigma-model only has a continuum limit in $d = 2$ (or $d = 1$). However, one can get a matrix ϕ^4 theory in 3 spacetime dimensions by starting with

$$\prod_s |\det(b_s + U_x + U_y)|^{2h_s}$$

with suitable (positive and negative) constants b_s , and suitable h_s , adjusted so that the ϕ^2 coefficient that is produced is small and does not dominate over the less relevant ϕ^4 coupling. The h_s are taken large enough so that the group manifold $U(N)$ decompactifies and reduces to $\mathbb{R}^{N^2}$, giving a theory of matrix-valued scalar fields.

As far as we know, all this is only good for analytic continuation. It does not shed light on the dynamics, because the dual theory in terms of Z is (as far as we understand) strongly coupled.

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It would be nice to believe that $\mathcal{N} = 4$ super Yang-Mills – at least in the strong coupling region $g^2 N \gg 1$ – can be analytically continued in N , but we certainly have not found a way to do that.

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